Optimal placement of wind turbines in a windfarm using L-SHADE algorithm

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Optimal Placement of Wind Turbines in a Windfarm using L-SHADE algorithm

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***Abstract*—Setting of turbines in a windfarm is a complex task as several factors need to be taken into consideration. During recent years, researchers have applied various evolutionary algorithms to windfarm layout problem by converting it to a single objective and at the most two objective optimization problem. The prime factor governing placement of turbines is the wake effect attributed to the loss of kinetic energy by wind after it passes over a turbine. Downstream turbine inside the wake region generates less output power. Optimizing the wake loss helps extract more power out of the wind. The cost of turbine is tactically entwined with generated output to form single objective of cost per unit of output power e.g. cost/kW. This paper proposes an application of L-SHADE algorithm, an advanced form of Differential Evolution (DE) algorithm, to minimize the objective cost/kW. SHADE is a success history based parameter adaptation technique of DE. L-SHADE improves the performance of SHADE with linearly reducing the population size in successive generations. DE has historically been used mainly for optimization of continuous variables. The present study suggests an approach of using algorithm L-SHADE in discrete location optimization problem. Case studies of varying wind directions with constant and variable wind speeds have been performed and results are compared with some of the previous studies.**

Keywords—windfarm power, wake loss, efficiency, wind turbine placement, LSHADE algorithm, location optimization.

I. Introduction

Wind energy is one of the most popular forms of renewable energy as it is abundant and present everywhere. Wind flows across the turbine blades and kinetic energy imparted in the process rotates the turbine blades, thereby producing electricity. To get an aggregated output, usually many turbines are grouped together in a windfarm, supposedly a windy site. However, the placement of turbines requires careful study as wake resulted from a turbine affects the generated output from a turbine located downstream in the wake region. Minimizing the wake loss to increase output power poses the challenge for various optimization algorithms applied for this layout optimization problem. The cost factor incorporated in the single objective of 'cost/kW', proposed by Mosetti *et al.* [3], has extensively been used by many researchers. Genetic algorithm [3-6], binary PSO [7], ant colony algorithm [8], greedy algorithm [9,10] are some of the evolutionary algorithms used to optimize windfarm layout with objective of minimizing cost per unit power output.

The present study applies LSHADE algorithm to find optimum locations of the turbines with objective of optimization being 'cost/kW'. Case studies with a) constant wind speed and variable wind direction and b) variable wind speed and variable wind direction are performed. The simulation results are compared with studies performed using GA by Mosetti *et al.* [3] and Grady *et al.* [4] and using binary PSO by Pookpunt *et al.* [7]. Though many literatures have used other optimization algorithms for the same problem, conscious efforts are made for prudent and valid comparisons with study results adopting same mathematical models and parameters that influence the objective function cost/kW.

II. Windfarm Modelling

Free stream of wind when encounters the turbine blades, its speed decreases and turbulence intensity increases forming a wake. The wake travels downstream and expands laterally. Jensen linear wake decay model [1,2] is considered here for calculation of wind velocity in the wake region.

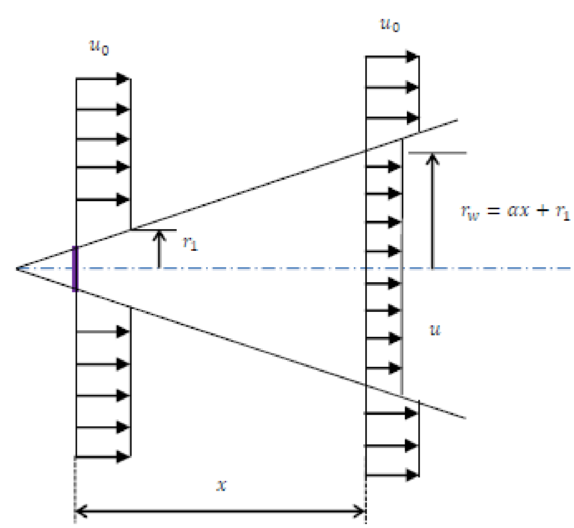


Fig. 1: Schematic of linear wake model

Fig. 1 shows the schematic of the linear wake model. Assuming momentum is conserved in the wake, wind speed in the wake region is given by –

$$u = u_0\left[1 - \frac{2a}{\left(1+\alpha\frac{x}{r_1}\right)}\right] \tag{1}$$

$$a = \frac{1-\sqrt{1-C_T}}{2} \tag{2}$$

$$r_1 = r\sqrt{\frac{1-a}{1-2a}} \tag{3}$$

$$\alpha = \frac{0.5}{\ln(\frac{h}{z_0})} \tag{4}$$

where, u_0 is local wind speed at the turbine without considering any wake effect, x is the distance downstream of the turbine, r is the radius of turbine rotor, r_1 is the downstream rotor radius, h is the hub height of turbine, α is the entrainment constant, a is the axial induction factor, C_T is the thrust coefficient of the wind turbine rotor and z_0 is the surface roughness of the windfarm.

When one turbine encounters multiple wakes, the kinetic energy of the mixed wake is assumed to be equal to the sum of kinetic energy deficits. The resultant velocity of *i*-th turbine downstream of N_T turbines is:

$$u_i = u_0\left[1 - \sqrt{\sum_{j=1}^{N_T}\left(1 - \frac{u_{ij}}{u_0}\right)^2}\right] \tag{5}$$

where u_{ij} is the wind velocity at *i*-th turbine under the influence of *j*-th turbine. The wake region is conical for the linear wake model and radius of the wake region is defined as wake influence radius calculated by:

$$r_w = \alpha x + r_1 \tag{6}$$

Mosetti *et al.* [3] proposed a simplified cost model where number of turbines in a windfarm is the only variable. Non-dimensional cost/year of a single turbine is assumed as 1 with a maximum 1/3 cost reduction for each additional turbine if a large number of turbines are installed in the windfarm. Mathematically the cost model with *N* turbines in a wind park is expressed as:

$$Cost = N\left(\frac{2}{3} + \frac{1}{3}e^{-0.00174N^2}\right) \tag{7}$$

III. Case Studies and Objective Formulation

Three case studies have been performed in [3] and [4] for a windfarm area of 2km x 2km equally divided into 100 cells with center of any cell able to accommodate a turbine. The first case, case-1 of the study considers a situation when the wind blows at constant speed from a fixed direction. Grady *et al.* [4] found the most optimal layout analytically with considering one column of 10 cells along wind direction in the optimization process and the optimal arrangement of one column was extended in same order to other nine columns. The results have further been validated by González *et al.* [6] and Pookpunt *et al.* [7]. This paper focuses on remaining two cases, the scenarios of which are described below:

a) Case 2: constant wind speed and variable wind direction - In this case constant wind speed (u_0) of 12m/s is considered blowing with equal probability from all directions. The direction is discretized in 36 segments of 10° each.

b) Case 3: variable wind speed and variable wind direction - This case considers more complicated scenario where both wind speed and direction are variable. Three possible wind speeds of 8m/s, 12m/s and 17m/s are considered. Like in earlier case, the direction is discretized into 36 segments of 10° each. Wind from north direction is represented with angle 0° and incremental angle 10° is considered clockwise. Therefore, angle of 90° implies wind from east direction and 270° from west direction. Wind distribution diagram for the case is indicated in Fig. 2. Probability of occurrence of a particular wind speed from a specific direction is given in the wind distribution diagram with the sum of all probabilities being equal to 1.

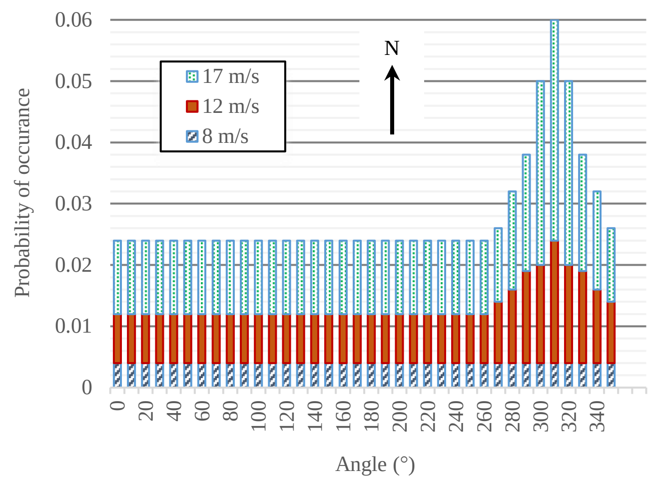


Fig. 2: Wind probability distribution diagram for case 3

Turbine and other relevant data are listed in Table I for the optimization problem and are in line with the data in [3] and [4]:

TABLE I. Numerical Data

Sl. No.	Parameter	Value
1	Rotor diameter, 2*r*	40 m
2	Thrust coefficient, C_T	0.88
3	Hub height, *h*	60 m
4	Rotor efficiency, C_P	0.4
5	Air density, ρ	1.2254 kg/m^3
6	Surface roughness of windfarm, z_0	0.3 m

Power output from *i*-th turbine in kW is given by:

$$P_i = 0.5 * \rho * \pi * r^2 * u_i^3 * C_P/1000 \tag{8}$$

Equation (8) is re-written and used in [3] and [4] with little numerical approximation as in equation (9). In order to compare with previous results equation (9) is applied for various cases of power calculations in this paper.

$$P_i = 0.3u_i^3 \tag{9}$$

Total power output of the windfarm having *N* turbines,

$$P_T = \sum_{k=0}^{360}\sum_{i=1}^{N} f_k P_i(u_i) \tag{10}$$

where f_k is the wind probability for a wind speed from a specific direction and $\sum_{k=0}^{360} f_k = 1$. P_i is the actual power output from *i*-th turbine as a function of wind speed u_i.

The objective function 'cost/kW' is calculated as:

$$\text{cost/kW} = \frac{Cost}{P_T} \tag{11}$$

Windfarm efficiency η is found using formula:

$$\eta = \frac{\sum_{k=0}^{360}\sum_{i=1}^{N} f_k P_i(u_i)}{\sum_{k=0}^{360}\sum_{i=1}^{N} f_k P_{i,max}(u_{i,max})} \tag{12}$$

where, $P_{i,max}$ is the maximum power output from *i*-th turbine as a function of maximum possible wind speed $u_{i,max}$ had there been no wake effect.

IV. APPLICATION OF L-SHADE ALGORITHM

Differential Evolution (DE) is a stochastic, population based optimization algorithm where the individuals in the population evolve and improve their fitness through probabilistic operators like recombination and mutation. The performance of DE is found to be highly dependent on the control settings which are: the scaling factor (*F*), the crossover rate (*CR*), the population size (*Np*), and the chosen mutation/crossover strategies [11,12]. Many researchers studied adaptive mechanisms for adjusting the control parameters on-line during the search process. Consequently JADE [13], which proposed a novel mutation strategy, and thereafter SHADE [14], a history based parameter adaptation scheme were introduced. Our research presented in this paper uses L-SHADE [15] optimization algorithm, an extension of SHADE with Linear Population Size Reduction (LPSR), which continually decreases the population size according to a linear function. A brief description of the algorithm and its application in windfarm layout optimization problem is provided in this section.

A. Initialization

The first step in the DE optimization process is to create an initial population of candidate solutions by assigning random values to each decision vector of the population. Such values must lie inside the feasible bounds between lower and upper limits of the decision vector. We may initialize *j*-th component of the *i*-th decision vector as:

$$x_{i,j}^{(0)} = x_{min,j} + rand_{ij}[0,1](x_{max,j} - x_{min,j}) \tag{13}$$

where $rand_{ij}[0,1]$ is a uniformly distributed random number between 0 and 1 and superscript '0' represents initialization. If '*Np*' is the population size and '*d*' is the dimension of decision vector, then *i* = 1,2,…,*Np* and *j* = 1,2,…,*d*.

The windfarm has 100 possible locations where the turbines can be placed. Each decision vector $x_i^{(0)}$ for *i* = 1,2,….,*Np* is formulated as a 100 dimensional vector (i.e. *d* = 100) with each element (i.e. decision variable) $x_{i,j}^{(0)}$ of the vector assuming a discrete integer value of either 0 or 1 representing status of that location. A value of '0' means the location is empty while '1' indicates the location has a turbine. During evolution through mutation and crossover the value of any element may become a fraction in which case it is rounded off to the nearest integer value of either 0 or 1.

B. Mutation

After initialization, DE creates a donor/mutant vector $v_i^{(t)}$ corresponding to each population member or target vector $x_i^{(t)}$ in the current generation through mutation *(the superscript 't' denotes the value at t-th generation)*. There are quite a few strategies for mutation. The one used here is 'current-to-*p*best':

$$v_i^{(t)} = x_i^{(t)} + F_i^{(t)}.\left(x_{pbest}^{(t)} - x_i^{(t)}\right) + F_i^{(t)}.\left(x_{R_1^i}^{(t)} - x_{R_2^i}^{(t)}\right) \tag{14}$$

The indices R_1^i & R_2^i are mutually exclusive integers randomly chosen from the population range 1 to *Np*; $x_{pbest}^{(t)}$ is chosen from the top $Np \times p$ ($p \in [0,1]$) best individuals of current generation. The scaling factor $F_i^{(t)}$ is a positive control parameter for scaling the difference vectors at *t*-th generation. During mutation if an element $v_{i,j}^{(t)}$ goes outside the search range boundaries $[x_{min,j}, x_{max,j}]$ i.e. [0,1] for all decision variables in this problem, it is corrected as:

$$v_{i,j}^{(t)} = \begin{cases} (x_{min,j} + x_{i,j}^{(t)})/2 \ if \ v_{i,j}^{(t)} < x_{min,j} \\ (x_{max,j} + x_{i,j}^{(t)})/2 \ if \ v_{i,j}^{(t)} > x_{max,j} \end{cases} \tag{15}$$

C. Parameter Adaptation

At each generation *t*, each individual has its own $F_i^{(t)}$ and $CR_i^{(t)}$ parameters that are used to generate new trial vectors. These two parameters are adapted as -

$$F_i^{(t)} = randc(\mu F_r^{(t)}, 0.1)$$
$$CR_i^{(t)} = randn(\mu CR_r^{(t)}, 0.1) \tag{16}$$

where $randc(\mu F_r^{(t)}, 0.1)$ & $randn(\mu CR_r^{(t)}, 0.1)$ are two values sampled from Cauchy and Normal distributions respectively with mean values $\mu F_r^{(t)}$ & $\mu CR_r^{(t)}$ and a variance of 0.1. $\mu F_r^{(t)}$ & $\mu CR_r^{(t)}$ are randomly chosen from means of scale factor and crossover rate of successful candidates of previous generations stored in a memory. The two mean values are initialized to 0.5 and subsequently modified by weighted Lehmer mean, the detail of which can be referred in [14,15].

D. Crossover

Through crossover the donor vector mixes its components with the target vector $x_i^{(t)}$ to form the trial/offspring vector

$u_i^{(t)} = (u_{i,1}^{(t)}, u_{i,2}^{(t)}, \ldots\ldots, u_{i,d}^{(t)})$. Binomial crossover, which is adopted here, operates on each variable whenever a randomly generated number between 0 and 1 is less than or equal to the adapted parameter $CR_i^{(t)}$, the crossover rate. The scheme is expressed as:

$$u_{i,j}^{(t)} = \begin{cases} v_{i,j}^{(t)} \; if \; j = K \; or \; rand_{i,j}[0,1] \leq CR_i^{(t)}, \\ \qquad\qquad x_{i,j}^{(t)} \; otherwise \end{cases} \tag{17}$$

where *K* is any randomly chosen natural number in {*1,2,….,d*}, *d* being the dimension of real-valued decision vectors.

E. Selection

Selection determines whether the target (parent) or the trial (offspring) vector survives to the next generation at *t+1*. The selection operator performs fitness checks as:

$$x_i^{(t+1)} = \begin{cases} \mathrm{u}_i^{(t)} \; if \; f(\mathrm{u}_i^{(t)}) \leq f(x_i^{(t)}), \\ \qquad x_i^{(t)} \; otherwise \end{cases} \tag{18}$$

where *f(.)* is the objective function to be minimized. The concerned problem here of windfarm layout optimization has objective function of 'cost/kW' as given in equation (11).

F. Linear population size reduction

It has been observed that success history based adapted values of scaling factor *F* and crossover rate *CR* improve the DE performance. In addition, population size *Np* which if dynamically reduced, accelerates performance of SHADE. L-SHADE introduced the concept of population size reduction employing a linear function. After each generation *t*, the population size in the next generation, *t+1*, is computed by –

$$Np(t+1) = round\left[\left(\frac{Np_{min} - Np_{ini}}{NFE_{max}}\right).NFE + Np_{ini}\right] \tag{19}$$

Np_{min} is set to 4 because mutation strategy adopted here requires minimum 4 individuals. Np_{ini} is the population size at first generation. NFE is the current number of fitness evaluations and NFE_{max} is the maximum number of fitness evaluations. If $Np(t+1) < Np(t)$, the $[Np(t) - Np(t+1)]$ worst ranking individuals are deleted from the population [15]. A plot of dynamic reduction of population size vs the number of fitness evaluations will be a non-smooth line with negative slope as the population size finally reduces to Np_{min}.

A population size of Np_{ini} = 300 has been considered for the optimization problem during initialization. Maximum 30,000 (NFE_{max}) number of fitness evaluations are performed for the objective function during each run of the algorithm.

TABLE II. L-SHADE ALGORITHM USER-DEFINED PARAMETER

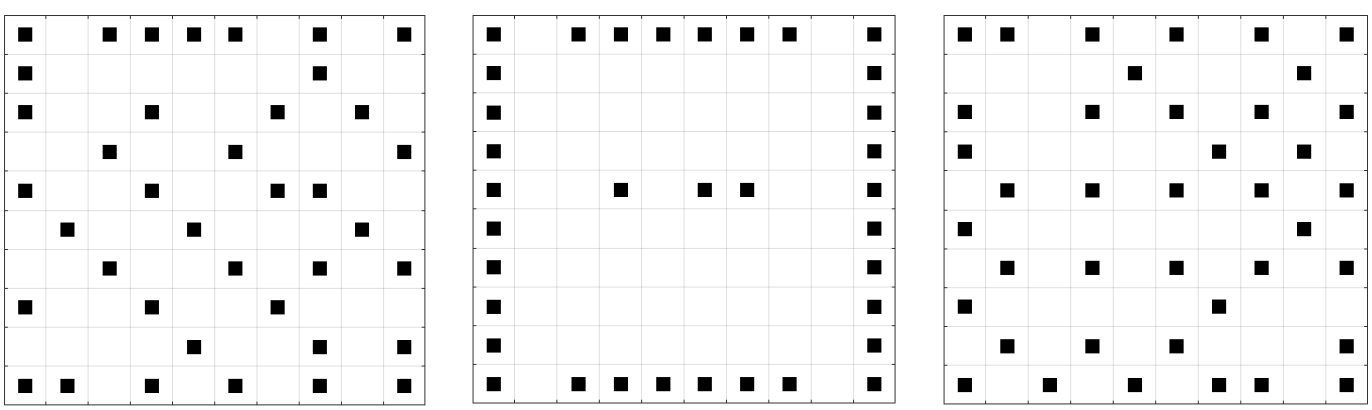
Parameter	**Value**
Dimension of optimization problem, d	100
Initial population size, Np_{ini}	300
Decision variable range $[x_{min}, x_{max}]$ for all	[0,1]
Initial mean scale factor, $\mu F_r^{(0)}$	0.5
Initial mean crossover rate, $\mu CR_r^{(0)}$	0.5
Maximum no. of fitness evaluations, NFE_{max}	30,000

The user-defined parameters of L-SHADE algorithm is summarized in Table II. Subsequently, simulation results are discussed in section V.

V. SIMULATION RESULTS AND COMPARISONS

The optimization algorithm is run for more than 30 times and the best values achieved for the objective function during multiple runs of various case studies are selected as final results. A computer with Intel Core i5 CPU@2.7GHz and 4GB RAM can perform one run of the algorithm on MATLAB platform in about 13.5 minutes for case-2 and in about 15.5 minutes for case-3. The reasonably moderate run time is attributed mainly to discretization of wind direction into 36 segments rather than to high dimension of the problem. The algorithm is to perform power computation for 36 different wind directions.

Table III compares the results of present study with previous studies [3,4,7] for case-2 when wind blows at constant speed



(i) Genetic Algorithm [4] (ii) Binary PSO-TVAC [7] (iii) L-SHADE

Fig. 3: Windfarm optimal configurations using various algorithms for case 2 – constant wind speed and variable wind direction

TABLE III. COMPARISON OF RESULTS OF VARIOUS OPTIMIZATION METHODS FOR CASE 2 – CONSTANT WIND SPEED AND VARIABLE WIND DIRECTION

Reference Method for Optimization	**No. of turbines**	**Power (kW)**	**Windfarm Efficiency**	**Cost/kW**
GA [3]	19	9245	93.86%	0.0017371
GA [4]	39	17220	85.17%	0.0015666
BPSO-TVAC [7]	35	15796	87.06%	0.0015648
L-SHADE	40	17920	86.42%	**0.0015341**

from all directions with equal probability. In this case, distance between two wind turbines is the main factor affecting wake loss and hence output from the farm. Grady *et al*. [4] could arrive at an optimal layout given in Fig. 3(i). Pookpunt *et al.* [7] applied binary particle swarm optimization (BPSO) algorithm with time varying acceleration coefficients (TVAC) and quoted to have obtained better fitness function. However, discrepancy is observed in the literature as tabulated result mentioned of 40 turbines while the layout in the paper shows 35 turbines only. The 35 turbines arranged as in Fig. 3(ii) can generate output of about 15796kW with fitness function as given in Table III.

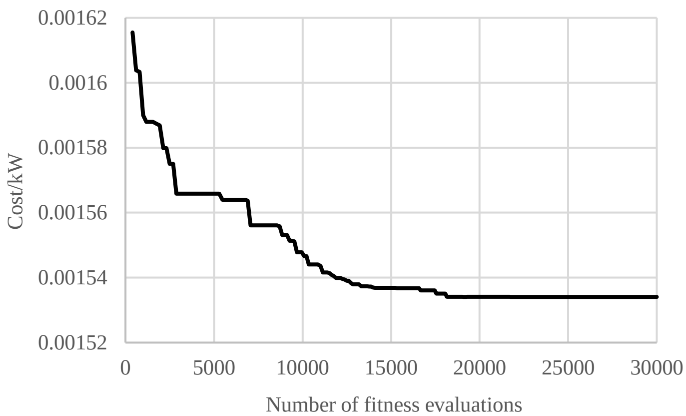


Fig. 4: Convergence of L-SHADE algorithm for case 2

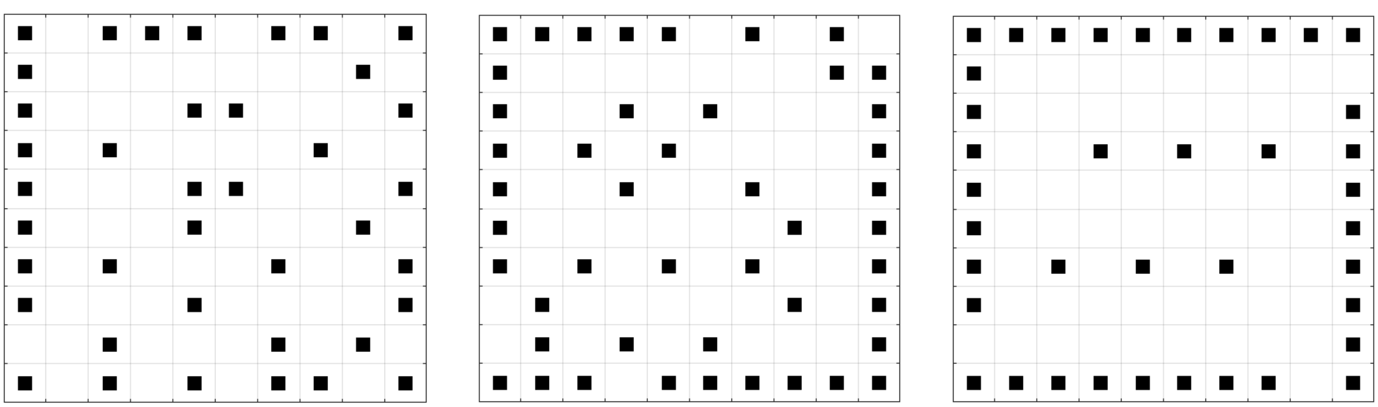
Optimum layout resulted from L-SHADE algorithm as shown in Fig. 3(iii) portrays similarity with layout proposed by Grady *et al.* [4]. The objective cost/kW is slightly improved by the L-SHADE optimization with one additional turbine totaling to 40 numbers. Total power output is also increased with more efficient turbine configuration. Layout proposed by Mosetti *et al.* [3] gives highest efficiency because of lowest number of turbines in the same windfarm area. Nevertheless, efficiency is not a criterion for the optimization problem. If 'cost/kW' is targeted to be the objective function, layout suggested by L-SHADE algorithm provides the lowest fitness value. The convergence of the algorithm to fitness value is represented in Fig. 4.

Table IV summarizes output results for evaluation of case-3 with variable wind speed and variable wind direction. Fitness function achieved by Grady *et al.* [4] with layout in Fig. 5(i) was much improved than what had been attained by Mosetti *et al.* [3]. Pookpunt *et al.* [7] in applying optimization algorithm BPSO-TVAC described a power model where maximum output from a turbine is capped at its rated capacity of 630kW beyond wind speed of 12.8m/s. Though the calculation does not seem to have factored in the saturation of power for case-3 where peak wind speed reaches 17m/s. The statement can be corroborated with a simple observation that if output power from a turbine is restricted to 630kW, 46 turbines cannot produce a total output power of about 39359kW as claimed in [7]. Further, wind frequency distribution table in the literature shows values for 37 discretized segments from 0° upto 360° instead of 350°. Due to aforementioned contentious observations, straight comparison of results of [7] with those of [3] or [4] seems questionable. Objective function, power and efficiency are recalculated with analogous power model and wind probabilities for the layout arrangement as suggested in [7] and replicated here in Fig. 5(ii). The comparable results thus obtained are included in Table IV.

TABLE IV. COMPARISON OF RESULTS OF VARIOUS OPTIMIZATION METHODS FOR CASE 3 – VARIABLE WIND SPEED AND VARIABLE WIND DIRECTION

Reference Method for Optimization	**No. of turbines**	**Power (kW)**	**Windfarm Efficiency**	**Cost/kW**
GA [3]	15	13460	94.62%	0.0009941
GA [4]	39	32038	86.62%	0.0008403
BPSO-TVAC [7]	46	36433	82.76%	0.0008523
L-SHADE	39	32351	86.68%	**0.0008322**

Optimization with L-SHADE algorithm resulted layout with same number of turbines as in [4] but with reshuffled configuration as indicated in Fig. 5(iii). Turbines are found congregated at outermost cells of the meshed area with all outer cells occupied between 280° and 350° where high wind probabilities prevail. Unlike case-2, wind turbines are positioned



(i) Genetic Algorithm [4] (ii) Binary PSO-TVAC [7] (iii) L-SHADE

Fig. 5: Windfarm optimal configurations using various algorithms for case 3 – variable wind speed and variable wind direction

at adjacent cells along north-south and east-west directions. A probable explanation is because of low wind frequencies from straight north (0°), east (90°), south (180°) and west (270°) directions in case-3. Wake loss due to such juxtaposition is far more compensated by sparse allocation of turbines along other directions. The resulting fitness function cost/kW for the layout arrangement is the lowest with higher efficiency and greater output power than those recorded in [4]. A plot of convergence to fitness value by the proposed L-SHADE algorithm for case-3 is given in Fig. 6.

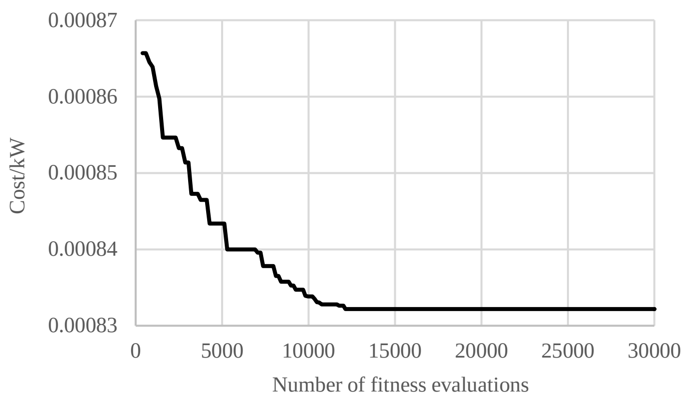


Fig. 6: Convergence of L-SHADE algorithm for case 3

VI. Conclusion

This paper proposes a novel approach and an effective application of L-SHADE algorithm in the problem of discrete location optimization of a windfarm for various scenarios of wind speed and direction. The optimal configurations of wind turbines obtained using the algorithm are more efficient, thereby producing high output power. The differences in final values of objective function using various evolutionary algorithms are subtle. Nonetheless, this study does improve the results and L-SHADE arguably performs better than any other optimization algorithm applied for the windfarm layout optimization problem.

Acknowledgment

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