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Wind turbine layout optimization with multiple hub height wind turbines using greedy algorithm

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ABSTRACT

Wind turbine layout optimization in wind farm is one of the most important technologies to increase the wind power utilization. This paper studies the wind turbine layout optimization with multiple hub heights wind turbines using greedy algorithm. The linear wake model and the particle wake model are used for wake flow calculation over flat terrain and complex terrain, respectively. Three-dimensional greedy algorithm is developed to optimize wind turbine layout with multiple hub heights for minimizing cost per unit power output. The numerical cases over flat terrain and complex terrain are used to validate the effectiveness of the proposed greedy algorithm for the optimization problem. The results reveal that it incurs lower computational costs to obtain better optimized results using the proposed greedy algorithm than the one using genetic algorithm. Compared to the layout with identical hub height wind turbines, the one with multiple hub height wind turbines can increase the total power output and decrease the cost per unit power output remarkably, especially for the wind farm over complex terrain. It is suggested that three-dimensional greedy algorithm is an effective method for more benefits of using wind turbines with multiple hub heights in wind farm design.

1. Introduction

In recent decades, traditional fossil fuel shortage and the serious environmental pollution raise much public concerns. As one of the effective ways to mitigate these problems, renewable energy has been taken worldwide attention. Among various kinds of renewable energies, wind energy is one of the most important ones as it is clean to environment, and rich in resources. In China, wind energy utilization develops rapidly in recent years. The total installed capacity of wind turbines is 318.1 GW all over the world up to 2013, while in China it has reached 91.4 GW, accounting for 29% of the total.

Wind energy is usually extracted and converted into electricity by wind turbine in wind farm. The power output of wind turbine is usually determined by the local wind speed. Large wind speed usually generates large power output, so local wind speed is desirable for maximum. However, due to the extraction of wind power and the disturbance of the wind rotor, there generates a speed decay region called wake region after wind turbine. The power output of one turbine will reduce when the turbine is inside the wake regions of other turbines. Therefore, the positions of the wind turbines should be arranged carefully to reduce the wake decay effect and increase the total wind power output of wind farm for more benefits. This procedure is called wind turbine layout optimization.

Wind turbine layout optimization is a critical topic in wind energy utilization research. Much research has been done on wind turbine layout optimization problem and some optimization methods have been developed. Genetic algorithm (GA) is one of the most popular optimization methods, which was first introduced in wind turbine layout optimization by Mosetti et al. [1] in 1994. GA is based on the mechanics of genetic and evolutionary. Wind turbine layouts are changed into chromosomes, and genetic operators including crossover, mutation and selection are implemented to

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search better solution when performing GA. In Mosetti's study, binary coding GA was employed and the wind speed deficit effect was calculated by linear wake model for wind farm design over flat terrain. After optimization with the objective of minimizing cost per unit power output, both the best number of wind turbines and the optimized layout were obtained. Later, some researchers improved the optimized results of GA through adopting better coding methods [2–4], introducing more realistic models [5–10], using various shapes of grid cells [11,12] and exchanging some individuals among multiple small sub-populations [13]. Greedy algorithm is another popular optimization method for wind turbine layout optimization. Greedy algorithm starts from an empty turbine layout and the turbines are placed into wind farm one by one. At each step, greedy algorithm chooses the position with the best objective value for this step to place the wind turbine. Compared to GA, greedy algorithm operates on a single layout and incurs lower computational costs. Ozturk and Norman [14] combined greedy algorithm with the adding, removing and moving operators to improve the optimized results. Zhang et al. [15] found that the wind turbine layout problem had the “submodular” property. Based on this property, the lazy algorithm was developed to reduce much computational costs when performing greedy algorithm. Chen et al. [16] developed the incremental calculation method, revealing the property that the wake flows of the original turbines did not need to be calculated again when adding a turbine into the wind farm. This method was combined with greedy algorithm, saving much computational time. Later Chen et al. [17] studied the tower height matching in wind turbine positioning problem. In the study, the wind turbines with identical tower height was considered. The height was optimized to maximize the power output per unit cost. A fitting method was developed to obtain the best height, which can save much computation compared to the enumeration method. The results showed that the objective value can be improved significantly through choosing appropriate tower height of wind turbines. In further study, an iteration method was developed to obtain the best tower height in shorter computational time for the problem [18]. Besides, more heuristic optimization approaches are introduced to solve wind turbine layout design problem, including Monte Carlo simulation [19], simulated annealing algorithm [20], particle swarm optimization [21], extended pattern search [22], ant colony algorithm [23], particle filtering approach [24], random search algorithm [25], sequential convex programming [26], and the hybrid algorithms [27,28].

In the above studies, the wind turbines considered were with identical hub height. Some scholars found that the total power output of the wind farm can be further increased through using wind turbines with multiple hub heights. Herbert-Acero et al. [29] investigated the situation that the turbines with two different hub heights were placed in a straight line. The results indicated that more power output can be extracted through using wind turbines with different hub heights. Mora et al. [30] treated the hub height of wind turbine as a variable in the coding of GA, which can be used to optimize the wind turbine layout problem with multiple hub heights. Chowdhury et al. [31] optimized the placement and the selection of wind turbines using an advanced mixed-discrete particle swarm optimization algorithm. The results showed that the normalized power output of the wind farm can be increased dramatically through using various types of wind turbines. Chen et al. [32] used the GA that contained two stages to optimize wind turbine layout with multiple hub heights. The positions of the wind turbines were determined through the first stage GA and the hub heights of the wind turbines were optimized using the second stage GA. However, in the existing studies, the linear wake model was employed and only flat terrain was considered. Furthermore, the optimization of hub height increased the number of optimization variables remarkably and high computational costs were needed.

In this paper, the wind turbine layout optimization with multiple hub height wind turbines is studied. Greedy algorithm is combined with three-dimensional grid system for the optimization problem. The problems over flat terrain and complex terrain are considered and optimized, respectively. The wake flow over flat terrain is calculated by linear wake model and the one over complex terrain is calculated by particle wake model. The effectiveness of greedy algorithm for wind turbine layout optimization with multiple hub height wind turbines is validated through comparing the results to the existing optimization results by GA. Two numerical cases both over flat terrain and complex terrain are implemented to further discuss the benefit of using multiple hub height wind turbines in wind farm.

The remaining parts of the paper are organized as follows. Section 2 presents the mathematical models used in the wind turbine layout optimization problem. Section 3 introduces the three-dimensional greedy algorithm. Section 4 discusses the numerical results of the test cases. Section 5 presents the conclusions.

2. Mathematical models

2.1. Wind speed profile in wind farm

Normally, wind speed will increase with the height from the ground in wind farm, which is commonly modeled by power law or logarithmic law [33]. In this paper, the logarithmic law is used, shown as

$$u = u_{\mathrm{ref}} \log(h/z_0) / \log\left(h_{\mathrm{ref}} \middle/ z_0\right) \tag{1}$$

where u_{ref} is the wind speed at the reference height h_{ref}, named the reference wind speed. z_0 is the ground roughness. As the wind speed increases with the height, the wind turbine with higher hub height can usually operate at larger power output.

2.2. Linear wake model

Wake effect is a critical factor that influences the wind resource distribution in wind farm, which should be considered for more accurate evaluation of the total power output. Computational Fluid Dynamics (CFD) is one of the most accurate methods for wind turbine wake flow calculation [34]. CFD method obtains the detail wake flow information of wind turbine through solving the governing equations of the air flow numerically. However, fine grids are needed to guarantee the calculation accuracy and the total calculation of CFD method is quite large. As wind turbine layout is altered during wind turbine layout optimization process, the wind turbine wake flow calculation needs to be implemented for hundreds of thousands of times, costing unacceptable computational time. Thus, CFD method is not efficient for wake flow calculation in wind turbine layout optimization. In order to reduce the computational time, some simplified wake models are developed. Among all the wake models, the linear wake model is the most popular one for wind turbine layout optimization over flat terrain. The linear wake model was developed by Jensen [35] in 1983. The wind speed inside the wake region was calculated by algebraic expression in this model. Later other researchers developed similar linear wake models and combined them with different optimization methods for wind turbine layout optimization [1,32,36]. In the present study, the linear wake model used by Chen et al. [32] is introduced for wake flow calculation over flat terrain. Ignoring the turbulent intensity of the near wake flow caused by the rotor blade, the shape of the wake region in the linear wake model can be simplified as a conical region, shown in Fig. 1(a). The velocity inside the wake region is calculated by [32].

$$u = u_0\left[1 - \frac{2a}{\left(1+\alpha\frac{x}{r_1}\right)^2}\right] \tag{2}$$

$$a = \frac{1-\sqrt{1-C_T}}{2} \tag{3}$$

$$r_1 = r\sqrt{\frac{1-a}{1-2a}} \tag{4}$$

$$\alpha = \frac{0.5}{\ln(h/z_0)} \tag{5}$$

where u_0 is the local wind speed without considering the wake flows of wind turbines. x is the distance downstream from the turbine rotor center. r_1 is the downstream rotor radius. a is the axial induction factor. α is the entrainment constant. C_T is the thrust coefficient of the wind turbine, which represents the thrust exerted on the wind rotor by the air. Wake effect is more obvious as C_T increases. r is the radius of the wind rotor and h is the hub height of the wind turbine. z_0 is the surface roughness of wind farm. The linear wake model considers the influence of the turbulence intensity caused by the ground through involving z_0.

As is shown in Fig. 1, the wake region is conical for the linear wake model, which is identified by the wake influenced radius R_w. R_w represents the radius of the wake region at a specified section along the crosswind, calculated by

$$R_w = \alpha x + r_1 \tag{6}$$

Usually one turbine is only covered partially by others' wake flow, shown in Fig. 1(b). A parameter A_{ij} is introduced to identify the actual influence area of the ith turbine by the jth turbine's wake flow, shown as the shaded area in Fig. 1(b). When one area is inside multiple wake flows, the velocity deficit will be enhanced. Considering the wake flow superimposed effect, the wind speed at the position of the ith turbine is calculated by [37].

$$u_i = u_{0i}\left(1 - \sqrt{\sum_{j=1}^{N_T}\left[\frac{A_{ij}}{\pi r_i^2}\left(1-\frac{u_{ij}}{u_{0j}}\right)\right]^2}\right) \tag{7}$$

where u_{0i} and u_{0j} are the local wind speeds at the ith and the jth turbines' positions without placing the turbines, respectively. They are equal to the inlet speed (u_0) of wind farm over flat terrain. u_{ij} is the wind speed at the wind rotor of ith turbine in the wake region of the jth turbine. N_T is the number of wind turbines in wind farm. r_i is the rotor radius of the ith turbine. A_{ij} is the rotor area of the ith turbine inside the jth turbine's wake.

2.3. Particle wake model

Sometimes wind farm is constructed over complex terrain. For complex terrain, the topology of the ground will alter the shape of the wind turbine's wake flow. Thus, the linear wake model cannot accurately reproduce the wake flow over complex terrain. Song et al. [38] proposed the particle wake model for wake flow calculation over complex terrain. In this model, particle simulation is introduced to simulate momentum diffusion process in the wake region and the velocity deficit is identified by the particle concentration. The steps of particle wake model are shown as follows.

1. The flow field of the wind farm without any wind turbines is calculated through Computational Fluid Dynamics (CFD) method. The obtained flow field is named the background flow filed.
2. The particle simulation time is discreted into time steps. At each time step, particles are generated uniformly within the wind rotor area and are carried by the background local wind speed.
3. At each time step, the convective displacement and the diffusive displacement satisfying Gaussian distribution are added to each particle. The displacement component is expressed as

$$\Delta x_i = u_i\Delta t + \sigma_d\sqrt{-2u_i r\Delta t \log R_{1i}}\cos(2\pi R_{2i}) \tag{8}$$

where u_i is the wind velocity component interpolated from the background flow field. r is the radius of the wind rotor. Δt is the span of a time step. σ_d is the diffusive intensity of the particle wake flow model. R_1 and R_2 are two independent uniform distributed random numbers within [0,1].

4. A cube with the side length as same as the wind rotor diameter is used to count the number of the particles at each point of the calculation domain. The number of the particles represents the particle concentration. After hundreds of time steps of simulation, the particle concentration of each time step is summed and averaged, obtaining the stable particle concentration distribution. The stable particles concentration is normalized by a characteristic particle concentration. The normalized particle

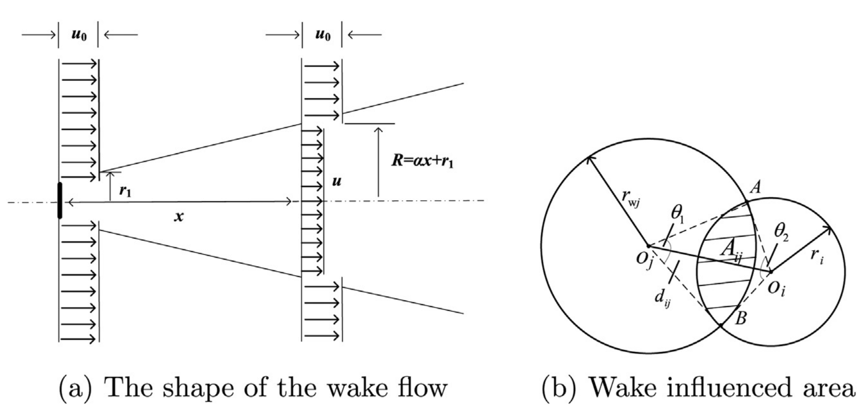


Fig. 1. Schematic of the linear wake model [17].

concentration is denoted by c, which represents the relative speed deficit intensity in the wake region.

5. The final velocity considering wake effect is calculated by the following transformation expression.

$$\mathbf{u}' = \mathbf{u}(1 - \beta c) \tag{9}$$

where $\mathbf{u}'$ is the wake influenced velocity and $\mathbf{u}$ is the velocity of the background flow field. β is the transformation constant and c is the normalized particle concentration.

In the particle wake model, σ_d and β are two parameters to be determined, which depend on the properties of wind turbine. As the particles are carried by the background flow field, particle wake flow model can reflect the turbulence effect on the velocity. Moreover, the shape of the wake flow can alter according to the terrain through involving the background flow field in transformation expression (9). Besides, particle simulation is a meshless simulation method. It doesn't need to refine the calculation meshes near the wind rotor, which can save much computational time. Therefore, the particle wake model can be used to calculate the wake flow over complex terrain in the wind turbine layout optimization. Previous study has validated the effectiveness of the particle wake flow model [38].

2.4. Power curve model of wind turbine

The power output of a wind turbine is calculated by power curve model, which identifies the power output for each wind speed. In this paper, the 3-order power law power curve with power control mechanisms is used to evaluate the power output of the wind turbine, expressed as [39].

$$P_e(u) = \begin{cases} 0, & u < u_c \quad \text{or} \quad u \geq u_f \\ u^3/u_r^3, & u_c \leq u < u_r \\ P_r, & u_r \leq u < u_f \end{cases} \tag{10}$$

where P_e is the actual power output with the wind turbine operating. P_r is the rated power output. u_c, u_r and u_f are the cut-in speed, the rated speed and the cut-out speed of wind turbine, respectively. From Equation (10), it can be seen that the power output does not always increase with wind speed. When the wind speed is in the range within $[u_r, u_f]$, the power output of the wind turbine will maintain the rated power output P_r.

2.5. Objective model

The total power output is desirable for maximum and the cost is desirable for minimum, both for more benefits of wind farm. It is difficult to solve such type of optimization problem due to the multiple objectives. In order to transform the multiple objectives into a single objective, the cost per unit power output is introduced as the objective for the wind turbine layout optimization with multiple hub heights in this paper, which is the same as Ref. [32]. The objective function is expressed as

$$\text{Objective} = \frac{\sum_i^{N_T} \text{Cost}_i}{P_{tot}} = \frac{\sum_i^{N_T} (\text{Cost}_h \cdot h_i + \text{Cost}_b)}{\sum_{i=1}^{N_d} \left[p_i \left(\sum_{j=1}^{N_T} P_e(u_{ij}) \right) \right]} \tag{11}$$

where the denominator is the total power output. N_d is the number of the wind cases and p_i is the probability of the ith wind case. N_T is the number of the wind turbines and P_e is the power output calculated by Equation (10). The total power output is the sum of the power outputs of all the turbines in wind farm. The cost for a wind farm is very complicated, including the turbine and support structure cost, the transmission wire cost, the taxes and insurance cost, the operation and maintenance cost [40]. Previous study has shown that the turbines cost dominates in the total investment for the onshore wind farm [41]. Therefore, the simplified cost model that only considers the cost of the wind turbine is introduced. The cost model is shown as the numerator in Equation (11), where h_i is the hub height of the ith wind turbine. Cost_h represents the cost per unit length of hub height and Cost_b represents the basic cost of the wind turbine, which are both constants. Note that the cost of the wind turbine increases with hub height linearly. In the present study, the minimum objective value is desirable. The power output and the cost both increase with the hub height of the wind turbine. There may have a best scheme of the positions and the hub heights of wind turbines to minimize cost per unit power output for wind turbine layout optimization.

2.6. Safe distance condition

The wind turbines should guarantee safe distance to avoid damaging others when falling down, which is called safe distance condition. In order to describe the distances among the wind turbines, a distance factor is defined as

$$d_F = \min_{1 \leq i,j \leq N_T} \left(\frac{D_{ij}}{R_{ij}} \right) \tag{12}$$

where N_T is the number of the wind turbines. R_{ij} is the sum of the heights of the ith and the jth wind turbines. D_{ij} is the horizontal distance of the ith and the jth wind turbines. d_F should be greater than 1 to make the safe distance condition satisfy.

3. Wind turbine layout optimization

3.1. Greedy algorithm

In this paper, greedy algorithm is introduced to optimize the wind turbine layout. The effectiveness of greedy algorithm for wind turbine layout optimization has been validated in previous study [16]. However, the study only considered the wind turbines with identical hub height and used the two-dimensional grid system to identify the positions of the wind turbines. In order to extent greedy algorithm to the wind turbine layout optimization with multiple hub heights, a three-dimensional grid system is applied. The steps of three-dimensional greedy algorithm are listed below.

1. The wind farm is divided by cubic grids and all the grid cells are numbered. The horizontal coordinates of the grids represent the positions to place the wind turbines and the vertical one identifies the hub heights of the wind turbines. The number of the vertical grid cells depends on the number of the optional hub heights of the wind turbines.
2. Add a turbine at the grid cell with the minimum evaluation value.
3. Try to place a wind turbine in each empty grid cell and calculate the evaluation value.
4. Add the turbine at the grid cell with the minimum evaluation value. When there are some grid cells having the same minimum evaluation value, it will choose the grid cells with small number to place the wind turbine.
5. If the domain contains the specified number of wind turbines, the process of greedy algorithm is completed. Otherwise, return to step 3 and continue the procedure.

When the process is finished, the obtained wind turbine layout is the final solution. The greedy algorithm developed in Ref. [16] contains two stages, including the locating stage and the adjusting stage. Previous results show that the adjusting stage improves the optimized result little when the grid of the domain is fine. Therefore, the grid is refined when optimizing and only the locating stage is considered in this paper. The incremental calculation method developed in Ref. [16] is used to accelerate the optimization process of greedy algorithm. Through this method, the influences of the adding turbine on other turbines are considered and much computational costs is saved.

3.2. Evaluation function in greedy algorithm

The evaluation function in greedy algorithm is usually chosen as the objective function. In order to adjust the distances among the wind turbines in the wind farm, an adjustment coefficient is introduced and is convert into a penalty term added into the objective function. The penalized objective function is treated as the evaluation function in greedy algorithm, expressed as

$$E = \begin{cases} 10^{10}, & d_F < \lambda \\ \text{Objective}, & d_F \geq \lambda \end{cases} \tag{13}$$

where λ is the adjustment coefficient. When the safe distance condition is satisfied ($d_F \geq \lambda$), the evaluation value is calculated by Equation (11). When the safe distance condition is failure ($d_F < \lambda$), the evaluation value is very large, such as 10^{10}. As the optimization desires the minimum objective value, the layout that does not satisfy the condition will be eliminated during the optimization. Note that λ should be greater than 1 to make the safe distance condition satisfy. Through introducing λ, the distances among the turbines can be adjusted automatically during the optimization process.

4. Numerical study

In this section, typical cases over flat terrain and complex terrain are introduced to test the three-dimensional greedy algorithm for wind farm layout optimization with multiple hub height wind turbines. The logarithmic law is introduced to model the vertical profile of the wind speed in the wind farm and the reference height is 78 m. The wind from west to east is defined as 0° and the one from south to north is defined as 90°. The linear wake model is used for the wake flow calculation over flat terrain and the particle wake model is used for the one over complex terrain. In the present study, the wind turbine with two optional hub heights 50 m and 78 m is introduced, the same as the one in Ref. [32]. The properties of the wind turbine are listed in Table 1.

Table 1
Wind turbine properties [32].

Property	Value
Rotor diameter D (m)	40
Trust coefficient C_T	0.8888
Hub height (m)	50 or 78
Cut-in speed (m/s)	2
Cut-out speed (m/s)	25
Rated speed (m/s)	13.0158
Rated power (kW)	680
Cost_b (k€)	593.87
Cost_h (k€/m)	1.5

4.1. Validation of the greedy algorithm

Before conducting the numerical cases, the effectiveness of three-dimensional greedy algorithm is validated through comparing the optimized result with the one of genetic algorithm (GA). GA has been used to optimize the wind turbine design problem with multiple hub height wind turbines in Chen's study [32]. In this section, the wind case with multi-directional wind and variable speeds in Ref. [32] is introduced as the test case. The wind rose of the test case is shown in Fig. 2. All the calculation parameters are chosen the same as the ones in Ref. [32]. The wind farm considered is a square area with the size of 500 m × 500 m. The ground roughness of the site is 0.3 m. The number of the wind turbines to be placed in the domain is 15. The wind farm is divided by 20 × 20 grid cells and the wind turbines are placed at the intersection points of the grid cells. The intersection points at the four edges of the wind farm are not included. Therefore, there are 361 potential positions for wind turbines in total. The distances among the turbines are at least 100 m. Wind turbine layout is optimized using greedy algorithm with the target of minimizing the cost per unit power output as expressed in Equation (11). Two situations are considered. For Situation 1, only the wind turbine with hub height at 78 m are placed in the wind farm. For Situation 2, the wind turbines with hub height at 50 m and 78 m are both considered. For each step, greedy algorithm will choose the wind turbine with appropriate hub height that makes the evaluation value best to add in the wind farm. The optimized results are compared to the one using GA by Chen et al. [32].

Table 2 shows the optimized results using greedy algorithm and GA. Fig. 3 shows the optimized wind turbine layouts of various situations. In the figures, the black triangles represent the wind turbines at 50 m and the black circles represent the wind turbines at 78 m. The optimized layout of Situation 2 is similar to the one of Situation 1. The difference is that some turbines at 78 m is replaced by the ones at 50 m. Nevertheless, the power outputs of Situation 2 is almost the same as Situation 1, while the total cost of Situation 2 is 2.4% less than the one of Situation 1. The total cost is effectively reduced through adopting some wind turbines with lower hub height. Meanwhile, the wake effects among the wind turbines are also reduced as the wind turbines are staggered in the height direction. The reduction of wake effect almost compensates the

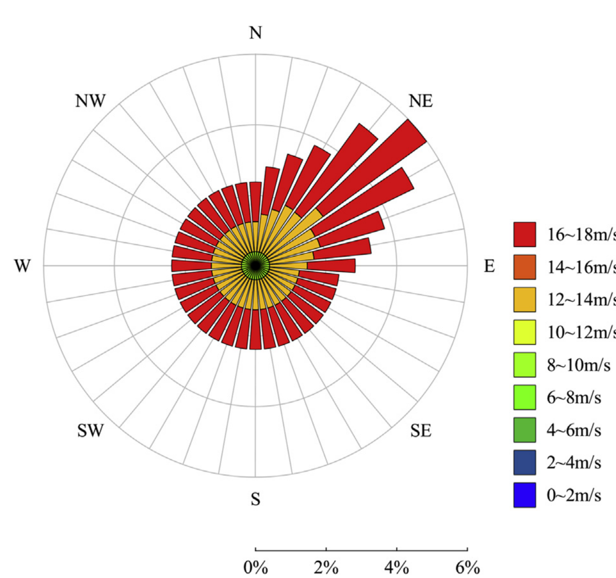


Fig. 2. The wind rose of the test case in Chen's study.

Table 2
Optimized results of the test case in Chen's study.

	Greedy algorithm		Genetic algorithm
	Situation 1	Situation 2	Chen [32]
Power output (MW)	7.022	7.050	6.805
Cost (M€)	10.663	10.411	10.453
Objective (€/W)	1.519	1.477	1.500
Number of turbines (50 m,78 m)	(0,15)	(6,9)	(5,10)

reduction of power output due to the lower wind speed. Therefore, the total power output of Situation 2 is not reduced compared to Situation 1. In summary, the objective value is reduced by 2.8% through placing the wind turbines with multiple hub heights. Compared to GA, more turbines at 50 m are selected by greedy algorithm. Therefore, the cost of the optimized layout by greedy algorithm is lower. Despite of more wind turbines at low hub height, the total power output of greedy algorithm increases by 3.6%. Finally, the optimized objective value of greedy algorithm decreases by 3.9% compared to GA. Furthermore, greedy algorithm operates one wind turbine layout and GA operates on the population of layouts. The computational time of the former is much less than the latter. For the wind turbine design problem, the calculation of wake flow costs most of the computational time. So the number of wind turbine wake flow calculation times (N_w) is used to evaluate the total computational costs of the two algorithms. For a wind turbines layout with N_T turbines, it should calculate the wake flow once for each two turbines, so the total N_w can be expressed as

$$N_{w,layout}(N_T) = C_{N_T}^2 = \frac{N_T(N_T-1)}{2} \tag{14}$$

For GA, N_w is proportional to the number of wind cases, the population size and the number of generations, so the total N_w can be calculated by

$$N_{w,GA} = N_d N_p N_g N_{w,layout}(N_T) = N_p N_g \frac{N_d N_T(N_T-1)}{2} \tag{15}$$

where N_d is the number of the wind cases, N_p is the population size and N_g is the number of generations. As an approximation, it is assumed that each individual in the population has the same number of wind turbines as N_T.

For greedy algorithm, the incremental calculation method [16] is introduced to save the total computational costs. Based on this method, the wake flows of the original turbines do not need to be calculated again when adding a turbine into the wind farm. When the kth wind turbines is added into the wind farm, only $(k-1)$ times of wake flow calculation is needed for each try of placing the kth turbine. $(k-1)(N_{grid}-k+1)$ times of wake flow calculation should be implemented when adding the kth turbines, where N_{grid} represents the number of the grid cells. Therefore, when optimizing wind turbine layout with N_T wind turbines, the total N_w can be calculated by

$$\begin{aligned} N_{w,Greedy} &= N_d \sum_{k=2}^{N_T} \left[(k-1)\left(N_{grid}-k+1\right)\right] \\ &= \frac{N_d N_T(N_T-1)}{2}\left[N_{grid} - \frac{(2N_T-1)}{3}\right] \end{aligned} \tag{16}$$

Note that some grid cells are not allowed to place the wind turbines due to the constraint of the safe distance condition during the optimization process. Thus, N_w calculated by Equation (16) always overestimates the total times of wake flow calculation. $N_p N_g$ in Equation (15) is usually greater than 10^5, while N_{grid} in Equation (16) is usually the same order as 10^4. Thus, the total computational costs of greedy algorithm is much lower than the one of GA. In this test case, GA used in Ref. [32] contained two stages, one for wind turbine positioning optimization and the other for hub height optimization. The population sizes for the two stages are 200 and 50, respectively. The numbers of generations are both 200. Thus, N_w for GA is 4.5×10^{12}. While for greedy algorithm, N_w is only 9.0×10^6. Compared to GA, the total calculation of greedy algorithm is reduced remarkably. In summary, it can be concluded that greedy algorithm is more effective method than GA for wind turbine layout optimization with multiple hub heights.

4.2. Numerical cases over flat terrain

In this section, two numerical cases are used to discuss the wind turbine layout optimization problem over flat terrain. The wind farm is chosen as a square area with the size of 1000 m × 1000 m. The ground roughness is 0.3 m. The wind farm is meshed by 50 × 50 grid cells. The same wind turbine as Ref. [32] is adopted and the properties are shown in Table 1. The wind turbines are placed in the center of the grid cells. The adjustment coefficient (λ) is chosen as 1.15, which satisfies the safe distance condition. Two types of wind cases are considered.

- Case 1: Single-directional wind with a single wind speed. The wind is from north to south and the magnitudes are chosen as 12 m/s, 13 m/s and 14 m/s, respectively.
- Case 2: 16-directional wind with variable speeds. The intervals of the wind directions are 22.5° and the wind rose is shown in Fig. 4.

For each case, three situations are considered for the wind turbine layout optimization using greedy algorithm, shown as follows.

- Situation 1: Only add the wind turbine with 50 m hub height into the wind farm.
- Situation 2: Only add the wind turbine with 78 m hub height into the wind farm.
- Situation 3: Consider both the two types of wind turbines.

Table 3 shows the optimized results with 22 wind turbines for various wind cases. For u_{ref} s at 12 m/s and 13 m/s, the optimized objective values of Situation 3 decrease by 10.9% and 0.3% compared to Situation 1 and Situation 2, respectively. While for u_{ref} at 14 m/s, the optimized objective values of Situation 3 decrease by 6.0% and 3.9% compared to Situation 1 and Situation 2. It is more likely to use the wind turbines with lower hub height when the wind speed is large. For Case 2, the domain cannot accomodate wind turbine layout with 22 wind turbines at 78 m when using greedy algorithm. In this situation, more turbines can be arranged in the wind farm through using the wind turbines with lower hub height. Compared to Situation 1, the power output of Situation 3 increases by 21.1% and the objective values decreases by 13.0%. Fig. 5 shows the optimized layouts with 22 wind turbines over flat terrain for Case 2. It can be seen in Fig. 5(b) that the 19 wind

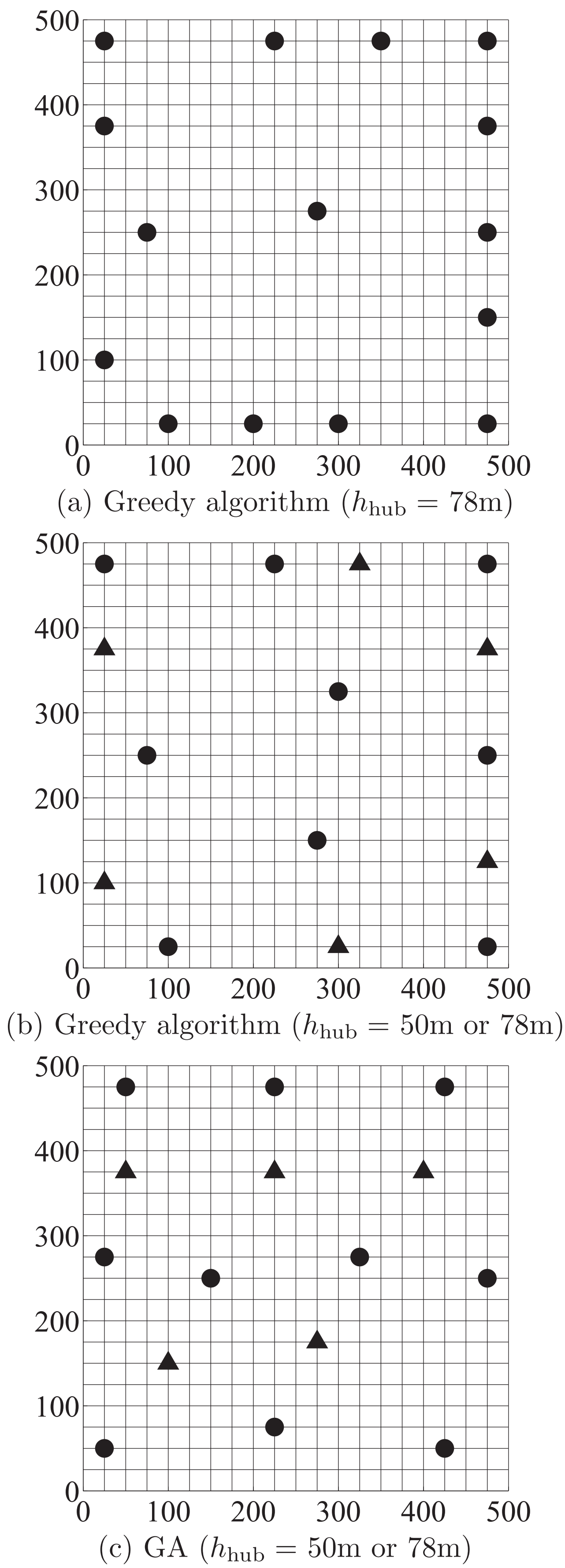


Fig. 3. Optimized layouts of the test case (Black O: turbines with 78 m, Black Δ: turbines with 50 m).

turbines at 78 m almost take up all the space of the site after considering the safe distance conditions. There is no space for other three turbines with the same height. In this situation, it can use three turbines at 50 m to insert in the spare place of the 78 m wind turbine layout. Therefore, Situation 3 can locate more wind turbines into the wind farm compared to Situation 2.

Fig. 6 shows the comparison of the three situations for various numbers of wind turbines. The horizontal ordinate represents the number of wind turbines, denoted as N_T. The vertical ordinate represents the optimized objective value for each N_T. When $u_{ref} = 12$ m/s, wind power at low height is small. Therefore, Situation 3 chooses the wind turbines at 78 m to extract more wind power when N_T is small. As N_T becomes large, the wind power at the height of 78 m is reduced due to the wake effect of the wind turbines. Thus, Situation 3 begins to choose the wind turbines at 50 m to avoid the wake flow of other turbines'. Through adopting wind turbines with multiple hub heights, the optimized objective value is smaller than the other two situations due to the reduction of the wake flow effect when the number of turbines is large. For the situation that $u_{ref} = 14$ m/s, the turbine can operates at large power output at low height. Situation 3 preferentially chooses the wind turbines at 50 m to reduce the installation cost when N_T is small. When N_T becomes large, the wake flow at 50 m height is very strong, so Situation 3 chooses the wind turbines at 78 m to avoid the wake flow of other turbines'. In summary, when N_T is large, the optimized objective value of Situation 3 is below the ones of other two situations for all the wind cases. According to the safe distance constraint, one turbine will take up a certain area in the domain. Turbine with higher height will take up more area. When N_T reaches some limited number, the space of the wind farm will be exhausted. As Situation 2 only adopts the higher turbines, the limited number reaches first. The limited number is 22 for Case 1 and it is 20 for Case 2. However, more wind turbines can be placed in the wind farm through introducing the turbine with lower height. For this perspective, the wind farm can arrange more wind turbines through adopting wind turbines with multiple hub heights and extract more wind power.

4.3. Numerical cases over complex terrain

In reality, the terrain is not completely flat, but has slope in the wind farm. So the wind speed is not uniform at a specified height. In this section, two numerical cases over complex terrain are conducted using greedy algorithm to discuss the influence of the terrain on the benefits of using wind turbines with multiple hub heights. The complex terrain is generated randomly, the contour of which is shown in Fig. 7(a). The size of the terrain is 1000 m × 1000 m and the ground roughness is 0.3 m. The same wind cases, the same wind turbines at 50 m and 78 m and three situations in Section 4.2 are considered. The flow field of the wind farm are calculated through CFD method. The inlet velocity is chosen as the logarithmic law. The particle wake flow model is introduced to calculate the turbine wake flow. For the wind turbines used, the parameters β and σ_d of the particle wake model are valued as 0.65 and 0.3, respectively [38]. The number of the grid cells for optimization is set as 50 × 50.

Table 4 shows the optimized results with 20 wind turbines for various wind cases. Similar conclusions can be obtained as flat terrain. For all the wind cases considered, the power outputs of Situation 3 are increased by about 15% compared to Situation 1 and increased by 1%–3% compared to Situation 2. The objective

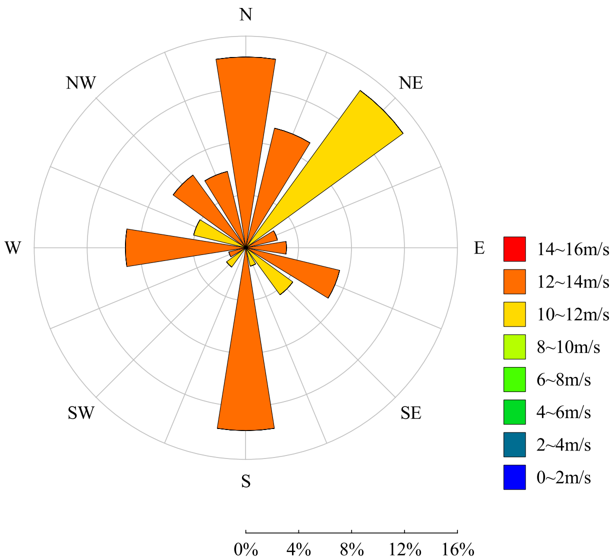


Fig. 4. The wind rose of Case 2.

values of Situation 3 decreases by 9.0% compared to Situation 1 and decreases by 2.2%–5.0% compared to Situation 2. Fig. 7(b)-(d) shows the optimized wind turbine layouts with 20 turbines for the three situations for Case 2. In the figures, the white circles represent the wind turbines at 50 m and the red ones represent the wind turbines at 78 m. The numbers in the circle represent the order of the turbines placed in the site. For Situation 2, as the hub height is large, the distances among the wind turbines should be enlarged to guarantee the safe distance condition. The wind farm can only reluctantly contain 20 wind turbines. Some wind turbines are placed in the bottom of the valley, which can only operate at low power output. For Situation 3, some wind turbines are replaced by the ones with lower hub height. There are more available positions for the wind turbines. Therefore, the turbines can be placed at the positions with high elevations and operate at high power output for Situation 3.

Fig. 8 shows the optimized results over complex terrain for various numbers of turbines. Compare Fig. 8 with Fig. 6, the curve of Situation 3 is much lower than other two situations for complex terrain. That is, the improvement of the optimized objective value is more obvious through using multiple hub height wind turbines over complex terrain. For complex terrain, the wind speed distribution at a specified height is not uniform due to the topology of the terrain. There are still area with large wind speeds at some parts of low heights. Therefore, the wind turbines with low hub height can still extract large power output. Meanwhile, the wind turbines are staggered in the height direction, which can reduce the speed deficit effect caused by the wake flow. Thus, the power outputs can be increased through adopting wind turbines with multiple hub heights as well as decreasing the objective value. In summary, for the wind farm over complex terrain, the profit can be increased more obviously when adopting wind turbines with multiple hub heights.

5. Conclusions

In this paper, the wind turbine layout optimization with multiple hub height wind turbines is studied using greedy algorithm. The wind speed profile with height in wind farm is characterized by the logarithmic law. The problems over flat terrain and complex terrain are considered, respectively. The wake flows of wind turbines are calculated by linear wake model over flat terrain, and by particle wake model over complex terrain. The power curve of wind turbine is modeled by the 3-order power law with consideration of power control mechanisms. The wind turbines with multiple hub heights are adopted to reduce the wake effect and extract more wind power from different heights. Three-dimensional grid system is introduced to identify the positions and the hub heights of wind turbines. Combining this grid system, greedy algorithm is used to

Table 3
Optimized results over flat terrain ($N_T = 22$).

	Case 1									Case 2		
Speed (m/s)	12			13			14			11–15		
Hub height (m)	50	78	50\78	50	78	50\78	50	78	50\78	50	78	50\78
Power output (MW)	8.390	9.985	9.986	10.67	12.70	12.70	13.33	14.43	14.45	8.559	–	10.37
Cost (M€)	14.72	15.64	15.60	14.72	15.64	15.60	14.72	15.64	15.05	14.72	–	15.51
Objective (€/W)	1.753	1.566	1.562	1.379	1.232	1.229	1.104	1.084	1.042	1.719	–	1.497

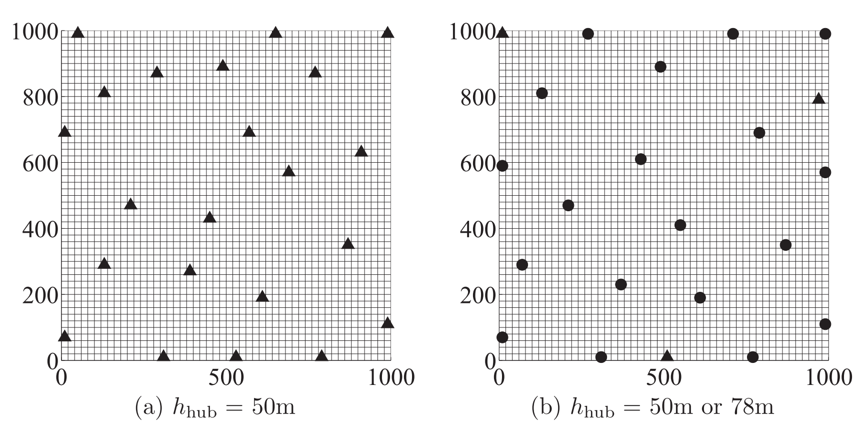


Fig. 5. Optimized layouts over flat terrain for Case 2 (N_T=22) (Black O: turbines with 78 m, Black Δ: turbines with 50 m).

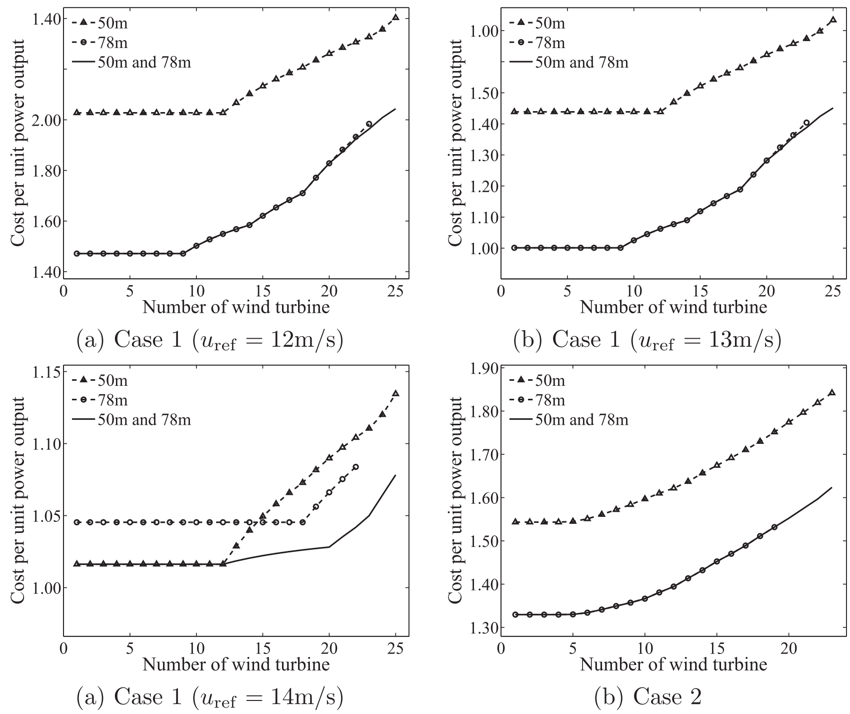


Fig. 6. Optimized curves over flat terrain for various wind cases.

solve the optimization problem with multiple hub height wind turbines. A test case is used to compare the developed greedy algorithm with genetic algorithm. The estimation expressions of the total computational costs for the two optimization methods are deduced. The results demonstrate that greedy algorithm can obtain better optimized result with much less computational costs compared to genetic algorithm.

Later the wind cases over flat terrain and complex terrain are introduced to further test the benefits of using wind turbines with different hub heights by three-dimensional greedy algorithm. The

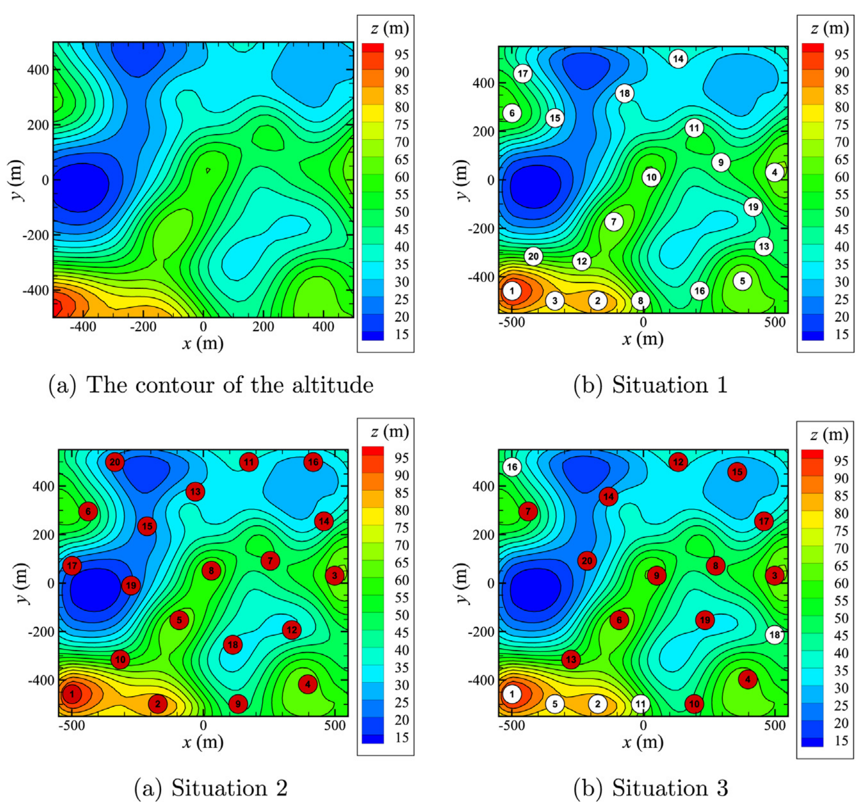
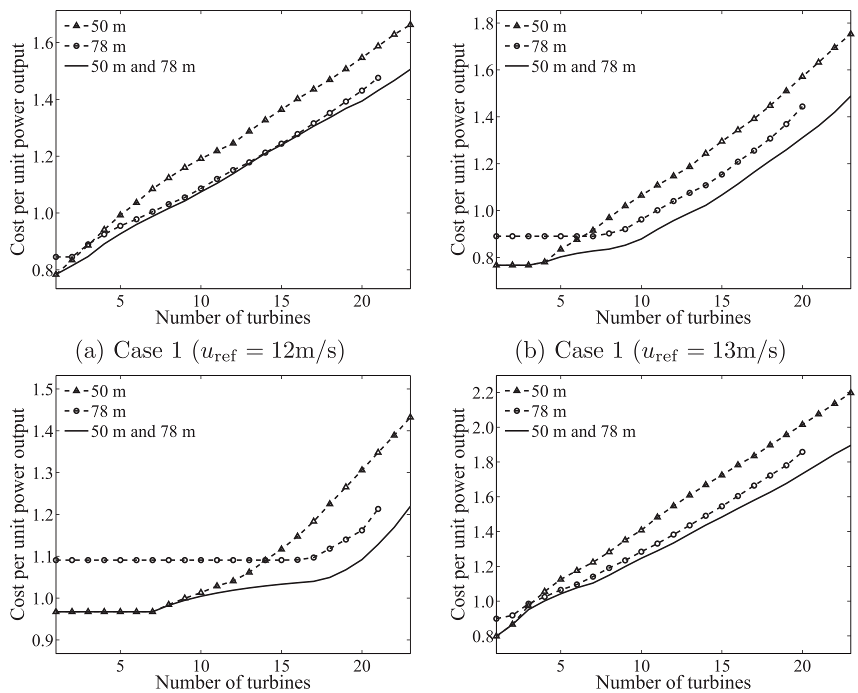


Fig. 7. Optimized layouts over complex terrain for Case 2 (N_T=20) (Red circles: turbines with 78 m, White circles: turbines with 50 m).

Table 4
Optimized results over complex terrain ($N_T = 20$).

	Case 1									Case 2		
Speed (m/s)	12			13			14			11–15		
Hub height (m)	50	78	50\78	50	78	50\78	50	78	50\78	50	78	50\78
Power output (MW)	7.659	8.718	8.814	9.655	10.76	11.12	11.60	13.15	13.31	8.322	9.299	9.525
Cost (M€)	13.38	14.22	14.05	13.38	14.22	13.97	13.38	14.22	13.92	13.38	14.22	13.97
Objective (€/W)	1.747	1.631	1.594	1.386	1.322	1.255	1.153	1.081	1.046	1.607	1.529	1.466



(a) Case 1 ($u_{ref} = 12$m/s) (b) Case 1 ($u_{ref} = 13$m/s)

(a) Case 1 ($u_{ref} = 14$m/s) (b) Case 2

Fig. 8. Optimized curves over complex terrain for various wind cases.

optimized results indicate that both the optimized power output and the relevant cost per unit power output can be improved remarkably, especially for the wind farm over complex terrain. Besides, more wind turbines can be arranged for a given wind farm for more wind energy capture through using multiple height wind turbines. It can be concluded that wind turbine layout can be designed with multiple hub height wind turbines for more benefits, and the three-dimensional greedy algorithm is an effective method for this wind turbine layout optimization problem.

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Nomenclature

Symbols

- a axial induction factor, 1
- A_{ij} actual influence area of the ith turbine by the jth turbine's wake flow, m^2
- c normalized particle concentration distribution, 1
- C_T thrust coefficient of wind turbine, 1
- Cost cost function of wind farm, €
- Cost_h cost per unit length of hub height of wind turbine, €
- Cost_b basic cost of wind turbine, €
- D rotor diameter of wind turbine, m
- d_F distance factor of wind turbine layout, 1
- D_{ij} horizontal distance of the ith and jth wind turbines
- h hub height of wind turbine, m
- h_{ref} referred height in logarithmic law, m
- N_d number of wind cases, 1
- N_g number of generations of genetic algorithm, 1
- N_{grid} number of grid cells for wind turbine positioning in greedy algorithm, 1
- N_p population size of genetic algorithm, 1
- N_T number of wind turbines, 1
- N_w number of wind turbine wake flow calculation times, 1
- $N_{w,layout}$ number of wind turbine wake flow calculation times for a wind turbine layout, 1
- $N_{w,GA}$ number of wind turbine wake flow calculation times of genetic algorithm, 1
- $N_{w,Greedy}$ number of wind turbine wake flow calculation times of greedy algorithm, 1
- P power output, W
- p_i probability of the ith wind case, 1
- P_e actual power output of wind turbine, W
- P_r rated power output of wind turbine, W

P_{tot}	total power output of wind farm, W
r	radius of wind rotor, m
R_1, R_2	uniform distributed random numbers within [0, 1], 1
R_{ij}	sum of the heights of the ith and jth wind turbines, m
R_w	radius of the wake region of wind turbine at a specified cross section, m
r_1	downstream rotor radius, m
u	wind speed, m/s
u'	wake influenced velocity, m/s
u_0	local wind speed without placing wind turbine, m/s
u_c	cut-in speed of wind turbine, m/s
u_f	cut-out speed of wind turbine, m/s
u_r	rated speed of wind turbine, m/s
u_{ref}	referred wind speed in logarithmic law, m/s
z_0	surface roughness of wind farm, m

Greek symbols

α	entrainment constant, 1
β	transformation constant of particle wake flow model, 1
λ	adjustment coefficient of greedy algorithm, 1
σ_d	diffusive intensity of particle wake flow model, 1
Δt	span of a time step of particle wake flow model, s

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