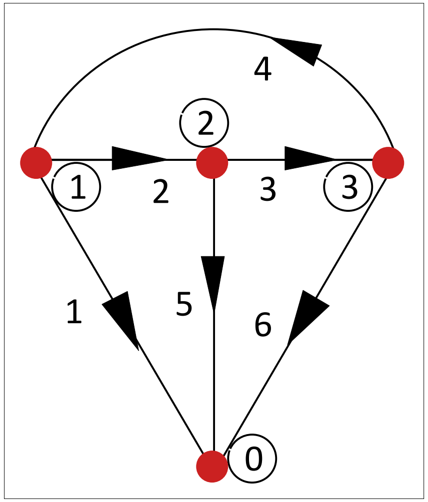
§2-10 特勒根定理

1. 定理陈述

(1) 特勒根功率定理：对于一个具有n个节点和b条支路的电路，假设各支路电流和电压取**关联参考方向**，并令$(I_1, I_2, \ldots, I_B)$，$(U_1, U_2, \ldots, U_B)$分别为b条支路的电流和电压，则对任何时间T，有：

$$\sum_{k=1}^{b} u_k i_k = 0$$



证明： 以 ⓪ 节点为参考节点，令u_{n1}、u_{n2}、u_{n3}分别为①、②、③的节点电压。由支路电压与节点电压的关系

则

$$\left.\begin{aligned}
u_1 &= u_{n1} \\
u_2 &= u_{n1} - u_{n2} \\
u_3 &= u_{n2} - u_{n3} \\
u_4 &= u_{n3} - u_{n1} \\
u_5 &= u_{n2} \\
u_6 &= u_{n3}
\end{aligned}\right\} \quad (2)$$

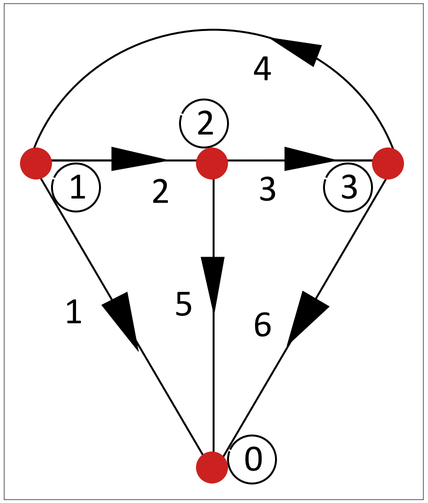
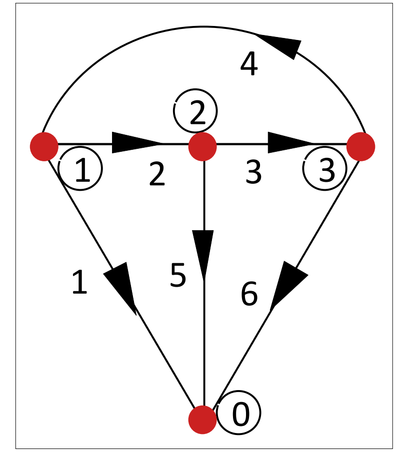


图1

对节点①、②、③，应用KCL，得

$$\left.\begin{aligned}
i_1 + i_2 - i_4 &= 0 \\
-i_2 + i_3 + i_5 &= 0 \\
-i_3 + i_4 + i_6 &= 0
\end{aligned}\right\} \quad (3)$$

而 $$\sum_{k=1}^{6} u_k i_k = u_1 i_1 + u_2 i_2 + u_3 i_3 + u_4 i_4 + u_5 i_5 + u_6 i_6$$



$$\sum_{k=1}^{6} u_k i_k = u_{n1} i_1 + (u_{n1} - u_{n2}) i_2 + (u_{n2} - u_{n3}) i_3$$

$$+ (-u_{n1} + u_{n3}) i_4 + u_{n2} i_5 + u_{n3} i_6$$

or $$\sum_{k=1}^{6} u_k i_k = u_{n1}(i_1 + i_2 - i_4) + u_{n2}(-i_2 + i_3 + i_5)$$

$$+ u_{n3}(-i_3 + i_4 + i_6)$$

上式括号中的电流分别为节点①. ②. ③
的电流之代数和，故引用 (3) 即有

$$\sum_{k=1}^{6} u_k i_k = 0$$

∴对任何具有N 个节点和 B条支路的电路，可以证明

$$\sum_{k=1}^{b} u_k i_k = 0$$

(2) 特勒根似功率定理：如果有两个具有N 个节点和 B条支路的电路，它们由不同的二端元件组成，但它们的拓扑图完全相同.

假设各支路的电压和电流取关联参考方向，并分别用$(I_1, I_2, \ldots, I_B)$，$(U_1, U_2, \ldots, U_B)$ 和 $(\hat{i}_1, \hat{i}_2, \ldots, \hat{i}_b), (\hat{u}_1, \hat{u}_2, \ldots, \hat{u}_b)$

来表示两者的 b条支路的电流和电压，则对任何时间 t, 有

$$\sum_{k=1}^{b} u_k \hat{i}_k = 0 \qquad (4)$$

$$\sum_{k=1}^{b} \hat{u}_k i_k = 0 \qquad (5)$$

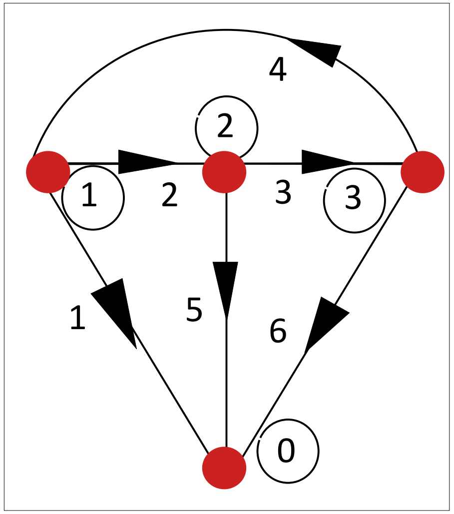


图1

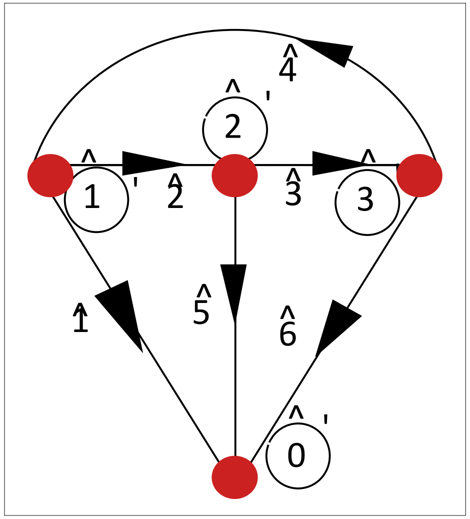


图2

证明：设两个电路如图1、图2，对图1电路，用KVL可以写出式(2)，对图2电路，应用KCL，有

$$\left.\begin{aligned} \hat{i}_1+\hat{i}_2-\hat{i}_4&=0 \\ -\hat{i}_2+\hat{i}_3+\hat{i}_5&=0 \\ -\hat{i}_3+\hat{i}_4+\hat{i}_6&=0 \end{aligned}\right\} \quad (6)$$

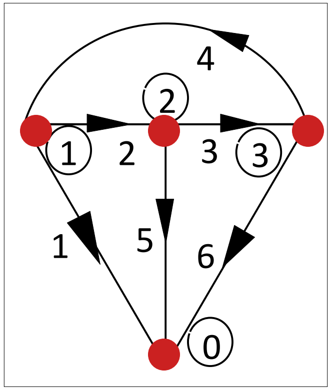


图1

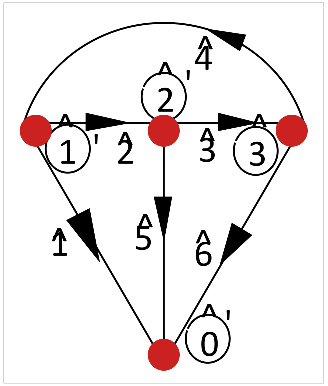


图2

利用式(2)可以得出

$$\sum_{k=1}^{6} u_k\hat{i}_k = u_1\hat{i}_1+u_2\hat{i}_2+u_3\hat{i}_3+u_4\hat{i}_4+u_5\hat{i}_5+u_6\hat{i}_6$$

$$= u_{n1}\hat{i}_1+(u_{n1}-u_{n2})\hat{i}_2+(u_{n2}-u_{n3})\hat{i}_3$$

$$+(-u_{n1}+u_{n3})\hat{i}_4+u_{n2}\hat{i}_5+u_{n3}\hat{i}_6$$

$$= u_{n1}(\hat{i}_1+\hat{i}_2-\hat{i}_4)+u_{n2}(-\hat{i}_2+\hat{i}_3+\hat{i}_5)$$

$$+u_{n3}(-\hat{i}_3+\hat{i}_4+\hat{i}_6)$$

$$\left.\begin{aligned} u_1&=u_{n1} \\ u_2&=u_{n1}-u_{n2} \\ u_3&=u_{n2}-u_{n3} \\ u_4&=u_{n3}-u_{n1} \\ u_5&=u_{n2} \\ u_6&=u_{n3} \end{aligned}\right\} \quad (2)$$

引用(6)式，即可得出

$$\sum_{k=1}^{6} u_k\hat{i}_k = 0 \quad \text{同理：} \quad \sum_{k=1}^{6} \hat{u}_k i_k = 0$$

此证明可以推广到任何具有N个节点和B条支路的两个电路，只要它们具有相同的拓扑图。

$$\sum_{k=1}^{b} u_k \hat{i}_k = 0 \quad \text{同理：} \quad \sum_{k=1}^{b} \hat{u}_k i_k = 0$$

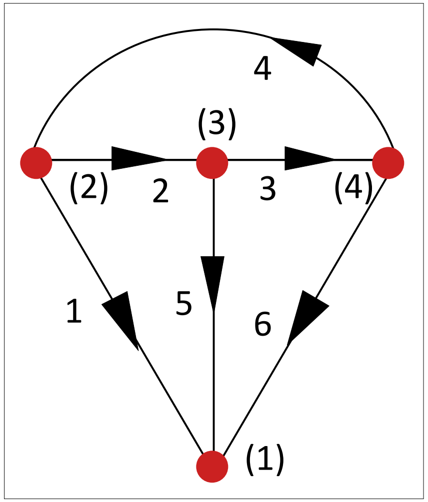
上式中的每一项，可以是一个电路的支路电压与另一个电路的支路电流所必须遵循的数学关系。它们具有功率之和的形式，所以又称为“似功率定理”。

另一种证明：

有向图G(n,b)，定义关联矩阵A_{nxb}

$$a_{ij}=\begin{cases}1 & \text{节点i与支路j有联系，电流流出节点}\\ -1 & \text{节点i与支路j有联系，电流流入节点}\\ 0 & \text{节点i与支路j无联系}\end{cases}$$

$$A=\begin{bmatrix}0 & 0 & -1 & 1 & 0 & 1\\ 0 & -1 & 1 & 0 & 1 & 0\\ 1 & 1 & 0 & -1 & 0 & 0\\ -1 & 0 & 0 & 0 & -1 & -1\end{bmatrix}_{n\times b}\quad\begin{matrix}(4)\\(3)\\(2)\\(1)\end{matrix}$$



支路电流I_{bx1}，支路电压U_{bx1}，节点电压V_{nx1}

$$KCL: AI=0$$

$$KVL: U=A^TV$$

各支路吸收功率之和：$U^T\mathrm{I}=\left(A^TV\right)^TI=V^TAI=0$

§2-11 互易定理

1. 定理陈述

对由线性电阻元件所组成的不含独立源和受控源的二端口网络N，互易定理具有下列三种形式：

(1) 短路-短路

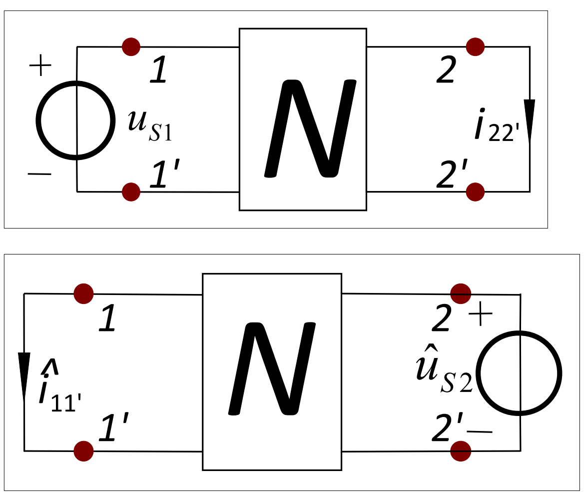


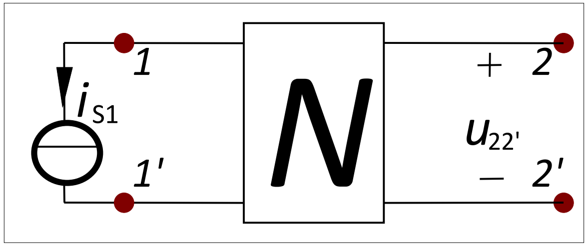
图1

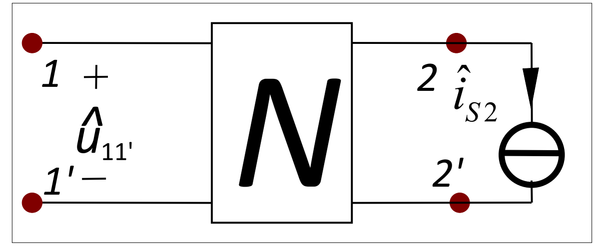
$$\frac{i_{22'}}{u_{S1}} = \frac{\hat{i}_{11'}}{\hat{u}_{S2}}$$

若 $u_{S1} = \hat{u}_{S2}$

则 $i_{22'} = \hat{i}_{11'}$

(2)



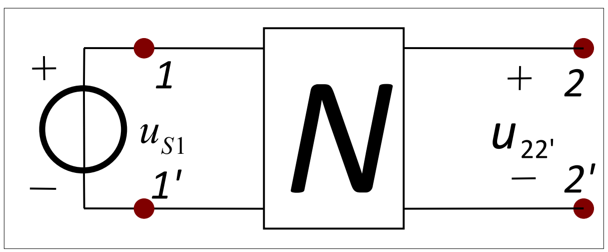


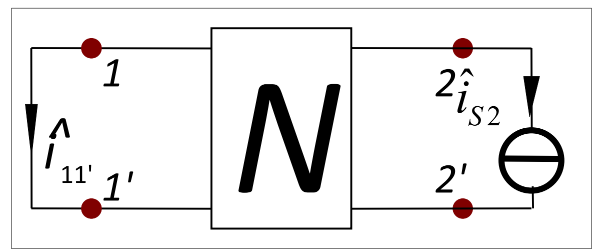
$$\frac{u_{22'}}{i_{S1}}=\frac{\hat{u}_{11'}}{\hat{i}_{S2}}$$

若 $i_{S1}=\hat{i}_{S2}$

则 $u_{22'}=\hat{u}_{11'}$

(3)

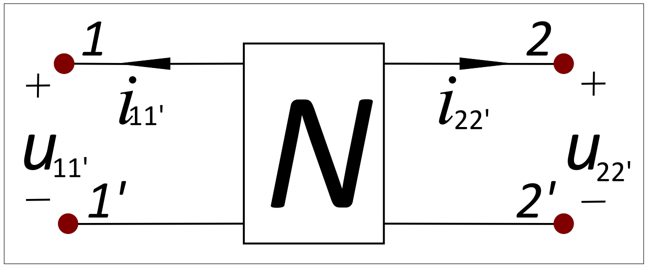




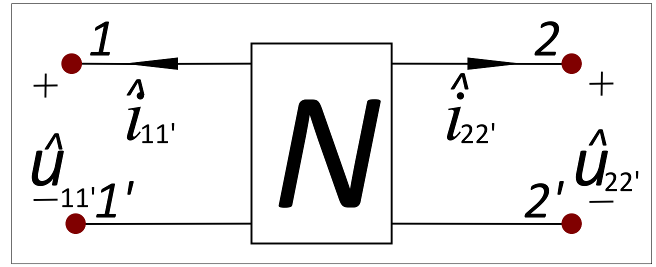
$$\frac{u_{22'}}{u_{S1}}=-\frac{\hat{i}_{11'}}{\hat{i}_{S2}}$$

数值满足 $\hat{i}_{S2}=u_{S1}$ 时 $\hat{i}_{11'}=-u_{22'}$ (数值)

2. 证明互易定理



a)

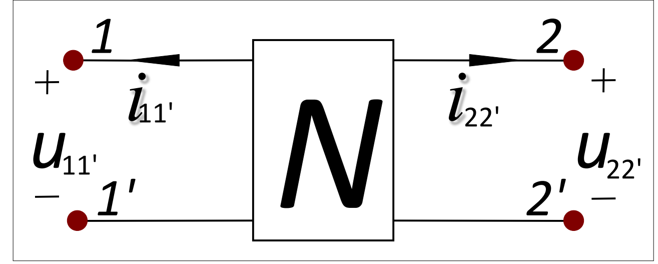


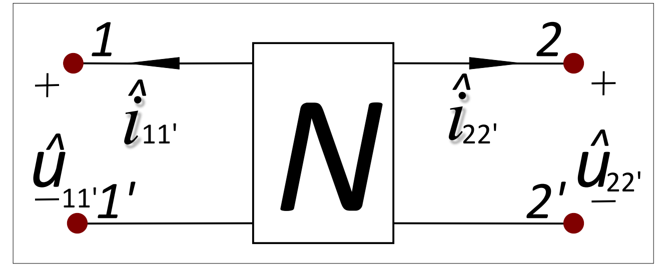
b)

N是b个线性电阻元件组成的无源网络。

图a) 中激励端口、响应端口的电压和电流分别为 $u_{11'}$ $u_{22'}$ $i_{11'}$ $i_{22'}$。N内部各支路的电压和电流分别为 $u_1, u_2 \ldots\ldots u_b$, $i_1, i_2 \ldots\ldots i_b$。

图b) 中激励端口、响应端口的电压和电流分别为 $\hat{u}_{11'}, \hat{u}_{22'}, \hat{i}_{11'}, \hat{i}_{22'}$。N内部各支路的电压和电流分别为 $\hat{u}_1, \hat{u}_2, \cdots, \hat{u}_b, \hat{i}_1, \hat{i}_2, \cdots, \hat{i}_b$。并设 u, i 为关联参考方向。





应用特勒根似功率定理

$$u_{11'}\hat{i}_{11'} + u_{22'}\hat{i}_{22'} + \sum_{k=1}^{b} u_k \hat{i}_k = 0 \quad (1a)$$

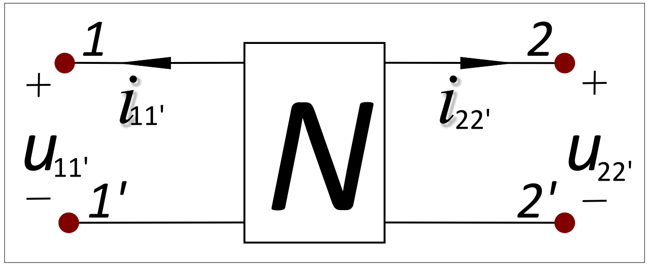
$$\hat{u}_{11'} i_{11'} + \hat{u}_{22'} i_{22'} + \sum_{k=1}^{b} \hat{u}_k i_k = 0 \quad (1b)$$

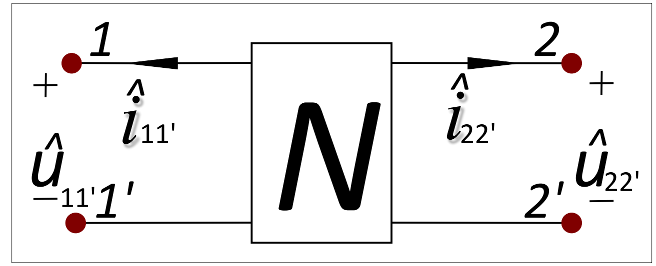
而 $\quad u_k = R_k i_k \qquad \hat{u}_k = R_k \hat{i}_k \qquad k = 1, 2, \cdots\cdots, b$

故 $$\sum_{k=1}^{b} u_k \hat{i}_k = \sum_{k=1}^{b} R_k i_k \hat{i}_k = \sum_{k=1}^{b} R_k \hat{i}_k i_k = \sum_{k=1}^{b} \hat{u}_k i_k$$

$\therefore$ 根据式(1)，应有

$$u_{11'}\hat{i}_{11'} + u_{22'}\hat{i}_{22'} = \hat{u}_{11'} i_{11'} + \hat{u}_{22'} i_{22'} \qquad (2)$$





若将u_S接入11'端,将22'端短路,则短路线中电流为$i_{22'}$,然后将$\hat{u}_{S2}$ 接入22'端,将11'端短路,则短路电流为$\hat{i}_{11'}$,在(2)式中,令

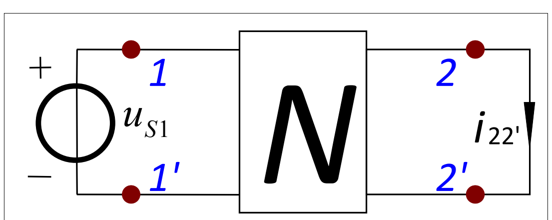
$$u_{11'} = u_{s1} \qquad u_{22'} = 0$$

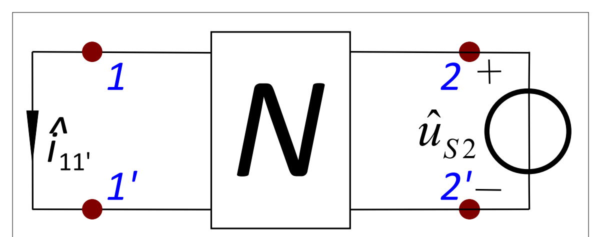
$$\hat{u}_{22'} = \hat{u}_{S2} \qquad \hat{u}_{11'} = 0$$

$$\boxed{u_{11'}\hat{i}_{11'} + u_{22'}\hat{i}_{22'} = \hat{u}_{11'}i_{11'} + \hat{u}_{22'}i_{22'} \qquad (2)}$$

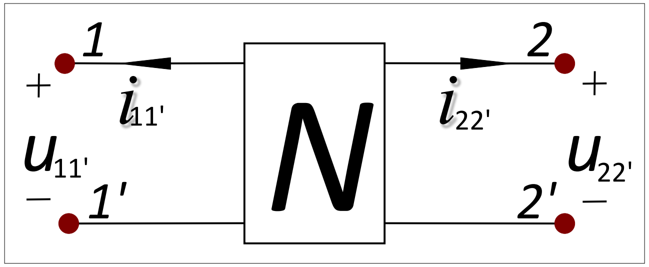
可得 $$u_{S1}\hat{i}_{11'} = \hat{u}_{S2}i_{22'}$$ 若$u_{S1} = \hat{u}_{S2}$ 则 $\hat{i}_{11'} = i_{22'}$

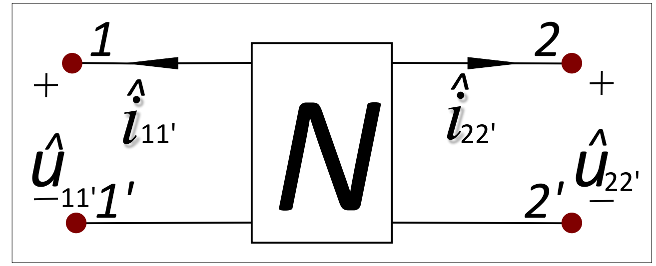
即激励电压源与短路端口互换,短路端口的响应电流不变,即证明了互易定理的第一种形式.





若将i_{S1}接入11'端,将22'端开路,则开路端中电流为$u_{22'}$,然后将$\hat{i}_{S2}$接入22'端,令11'端开路,则开路端电压为$u_{11'}$,在(2)式中,令





$$i_{11'} = -i_{S1} \qquad i_{22'} = 0$$

$$\hat{i}_{22'} = -\hat{i}_{S2} \qquad \hat{i}_{11'} = 0$$

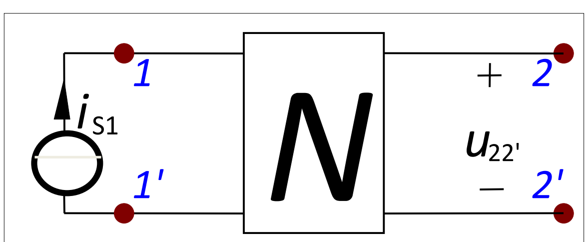
$$u_{11'}\hat{i}_{11'} + u_{22'}\hat{i}_{22'} = \hat{u}_{11'}i_{11'} + \hat{u}_{22'}i_{22'} \qquad (2)$$

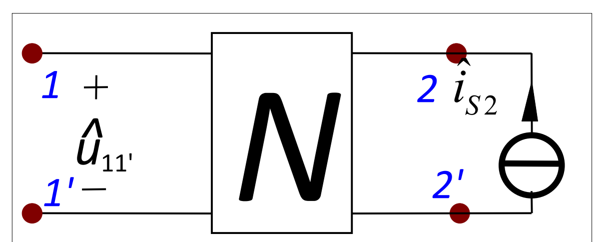
可得 $$-\hat{u}_{11'}i_{S1} = -u_{22'}\hat{i}_{S2}$$

若 $$i_{S1} = \hat{i}_{S2}$$

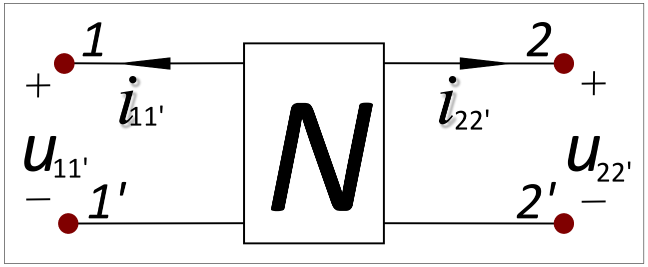
则 $$\hat{u}_{11'} = u_{22'}$$

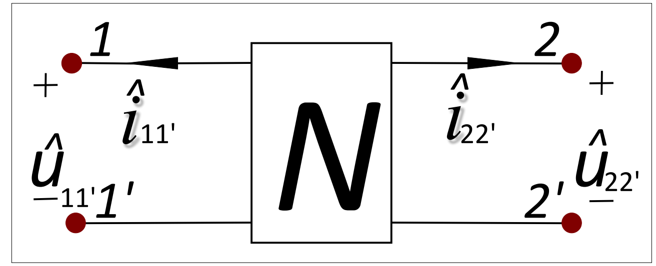
即激励电流源与开路端口互换,开路端口的响应电压不变,即证明了互易定理的第二种形式.





若将i_{S1}接入11'端，将22'端开路，则开路端中电流为$u_{22'}$，然后将$\hat{i}_{S2}$接入22'端，令11'端开路，则开路端电压为$u_{11'}$，在(2)式中，令





$$\hat{i}_{22'} = -\hat{i}_{S2} \qquad \hat{u}_{11'} = 0$$

$$u_{11'} = u_{S1} \qquad i_{22'} = 0$$

$$\boxed{u_{11'}\hat{i}_{11'} + u_{22'}\hat{i}_{22'} = \hat{u}_{11'}i_{11'} + \hat{u}_{22'}i_{22'} \qquad (2)}$$

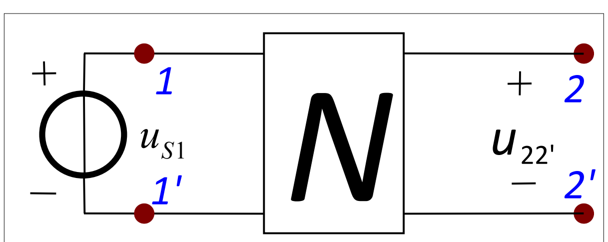
可得 $$u_{11'}\hat{i}_{11'} + u_{22'}\hat{i}_{22'} = 0$$

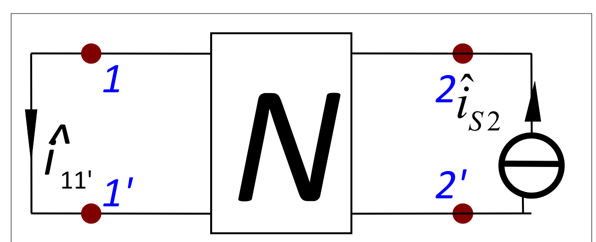
即 $$u_{11'}\hat{i}_{11'} = -u_{22'}\hat{i}_{22'}$$

若 u_{S1}(数值)=i_{S2}(数值)

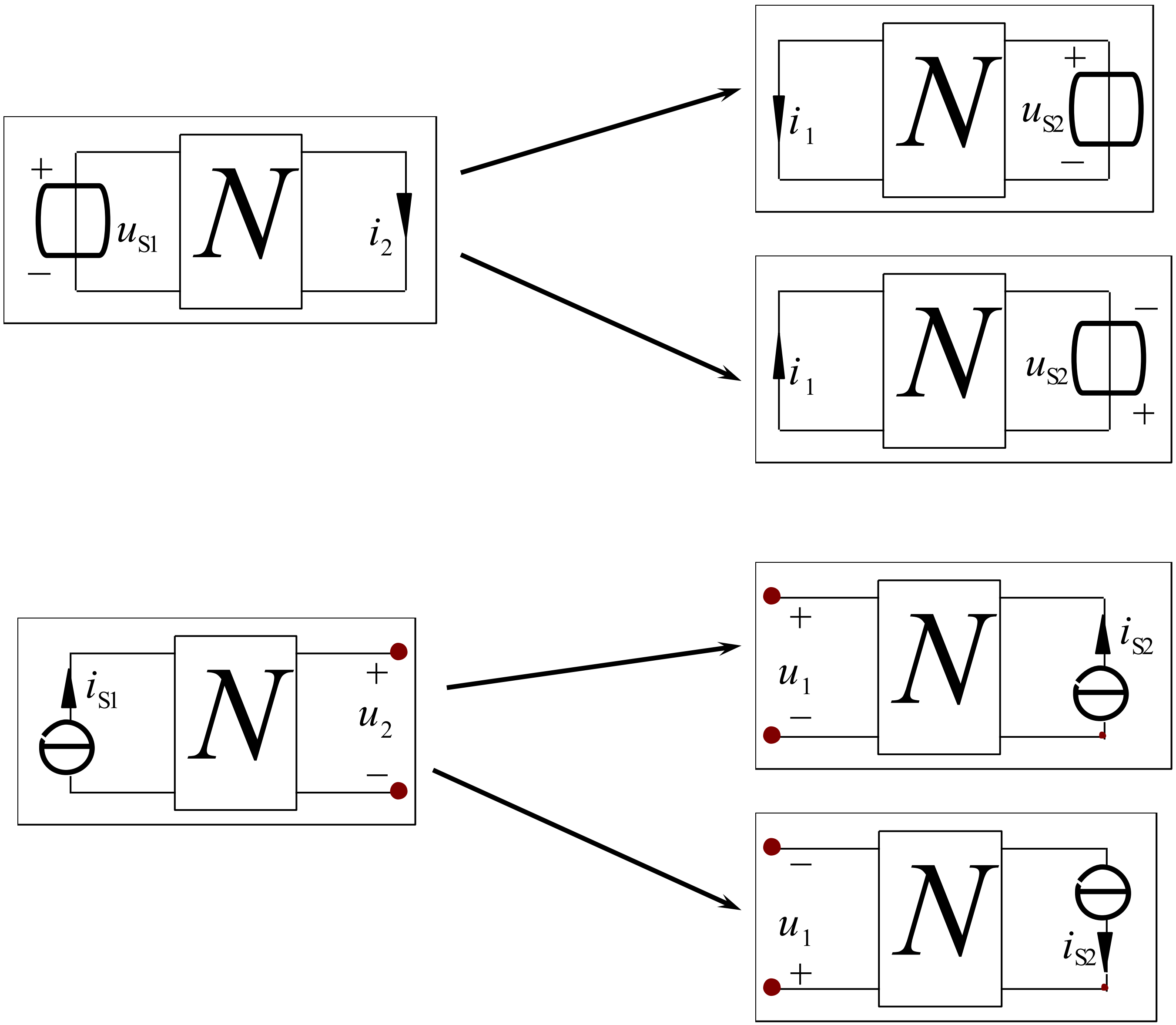
则 $$\hat{i}_{11'} = u_{22'}$$

即证明了互易定理的第三种形式.激励电压源以激励电流源取代并换位时，图中的开路电压与短路电流的数值相等。

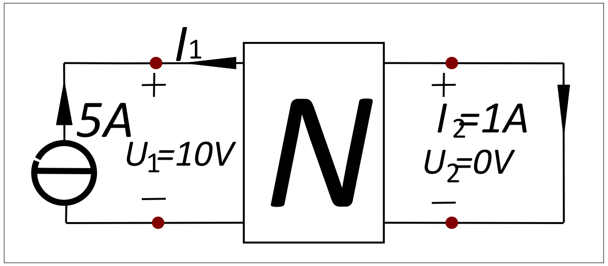




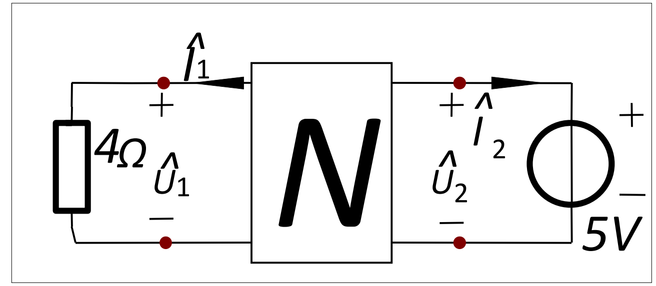
注意：互易前后，支路电流、电压的方向。



例：



(a)



(b)

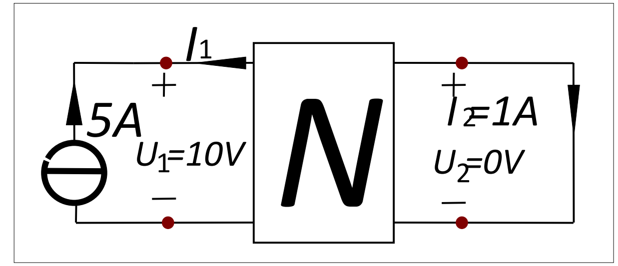
N具有一输入端口和一输出端口。当输入端口施加一个5A电流源激励而输出端短路时(a图)，U1=10V，I2=1A；而当输出端接一个5V电压源，输入端接一个4Ω电阻时(b图)

问： (1) $\hat{U}_1 = ?$

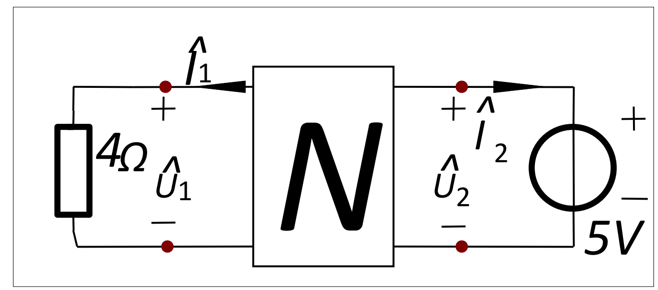
(2) 若5V电压源换成15V的电压源，则 $\hat{U}_1 =$ ？

解： (1) 根据式 2

$$u_{11'}\hat{i}_{11'}+u_{22'}\hat{i}_{22'}=\hat{u}_{11'}i_{11'}+\hat{u}_{22'}i_{22'} \qquad (2)$$



(a)



(b)

$$U_1\hat{I}_1+U_2\hat{I}_2=\hat{U}_1I_1+\hat{U}_2I_2$$

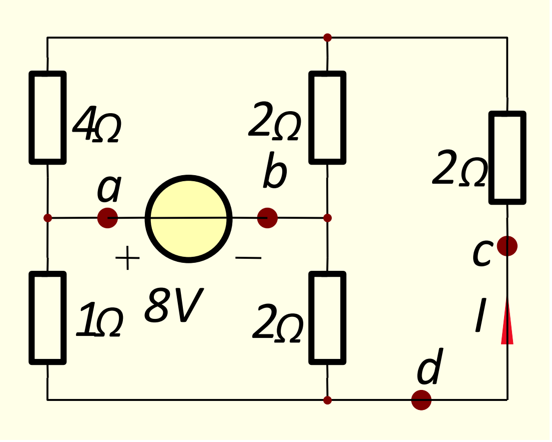
$$10\times\hat{U}_1/4+0\times\hat{I}_2=\hat{U}_1\times(-5)+5\times1$$

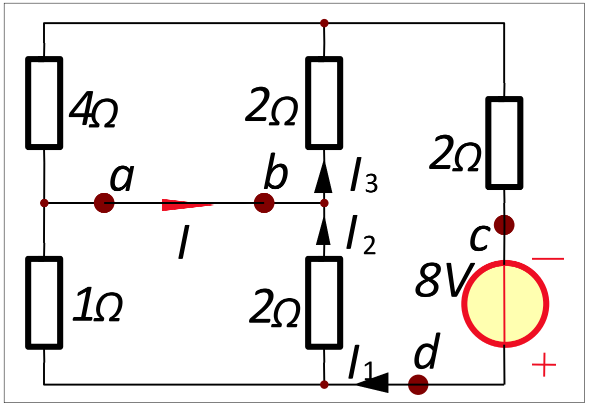
$$\hat{U}_1=2/3\,V$$

(2) b)图中的5V换成15V电压源，根据线性电路的齐次性

$$\hat{U}_1=\left(3\times\frac{2}{3}\right)V=2V$$

例：电路如图所示，求电流I





解：

$$I_1 = \frac{8}{4//2+1//2+2} = 2A$$

$$I_2 = I_1\frac{1}{1+2} = 2\times\frac{1}{3} = \frac{2}{3}A$$

$$I_3 = I_1\frac{4}{4+2} = 2\times\frac{2}{3} = 1\frac{1}{3}A$$

由KCL可得 $I = I_3 - I_2 = {}^{2}\!/_{3}\,A$

习题：

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