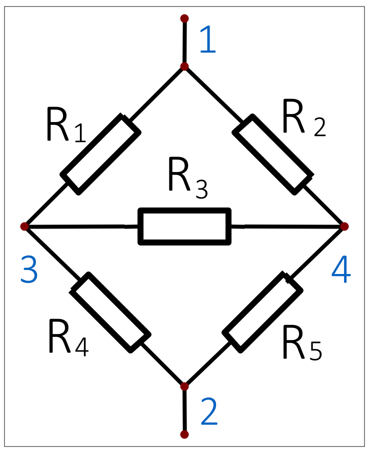
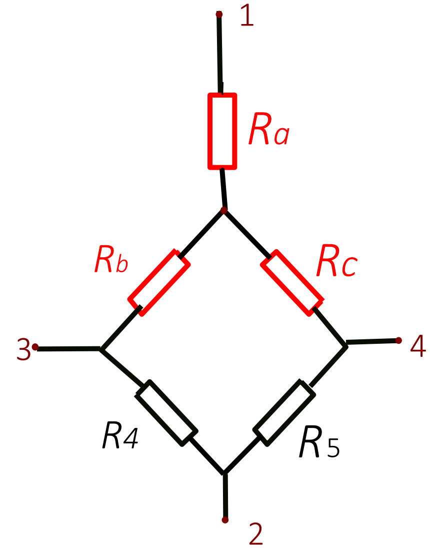
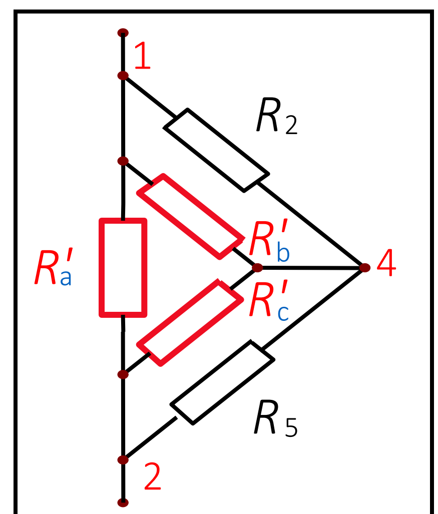
§ 2-3 星形 (Y) 和三角形 (△)网络的等效变换

(△) $R_1R_2R_3 \longrightarrow$ (Y) $R_aR_bR_c$

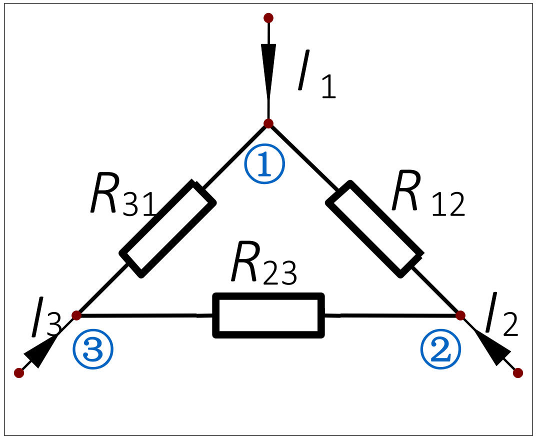


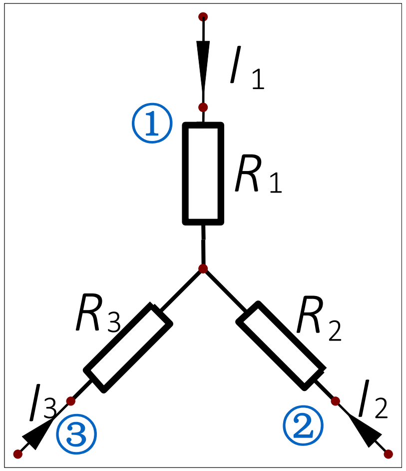


(Y) $R_1R_3R_4 \longrightarrow$ (△) $R_a'R_b'R_c'$



Y—△等效变换：要求Y、△的外部特性必须相同





从△形网络看

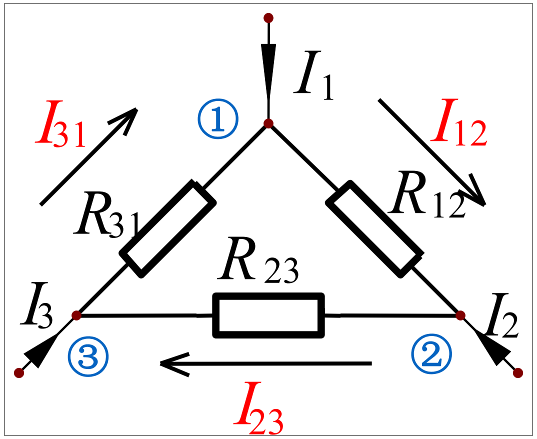
$$I_1 = I_{12} - I_{31} \qquad I_2 = I_{23} - I_{12} \qquad I_3 = I_{31} - I_{23}$$

但是

$$I_{12} = \frac{U_{12}}{R_{12}} \qquad I_{23} = \frac{U_{23}}{R_{23}} \qquad I_{31} = \frac{U_{31}}{R_{31}}$$

故有

$$\left.\begin{aligned} I_1 &= \frac{U_{12}}{R_{12}} - \frac{U_{31}}{R_{31}} \\ I_2 &= \frac{U_{23}}{R_{23}} - \frac{U_{12}}{R_{12}} \\ I_3 &= \frac{U_{31}}{R_{31}} - \frac{U_{23}}{R_{23}} \end{aligned}\right\} \quad (1)$$



从Y形网络看

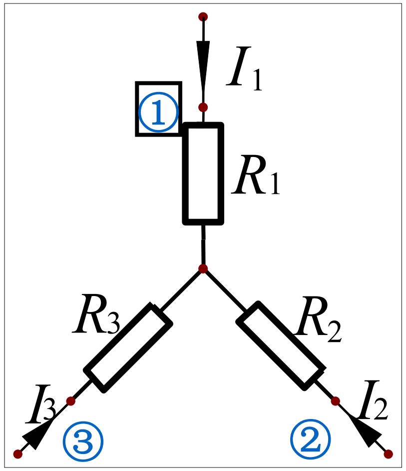
$$U_{12}=R_1I_1-R_2I_2 \qquad U_{23}=R_2I_2-R_3I_3 \qquad U_{31}=R_3I_3-R_1I_1$$

由于 $U_{12}+U_{23}+U_{31}=0$

∴上述3个方程彼此是不独立的。选其中两个，

再与 $I_1+I_2+I_3=0$ 联立，解出

$$\left.\begin{aligned} I_1&=\frac{U_{12}R_3-U_{31}R_2}{R_1R_2+R_2R_3+R_3R_1}\\ I_2&=\frac{U_{23}R_1-U_{12}R_3}{R_1R_2+R_2R_3+R_3R_1}\\ I_3&=\frac{U_{13}R_2-U_{23}R_1}{R_1R_2+R_2R_3+R_3R_1}\end{aligned}\right\}\quad(2)$$



比较(1)、(2)两式，

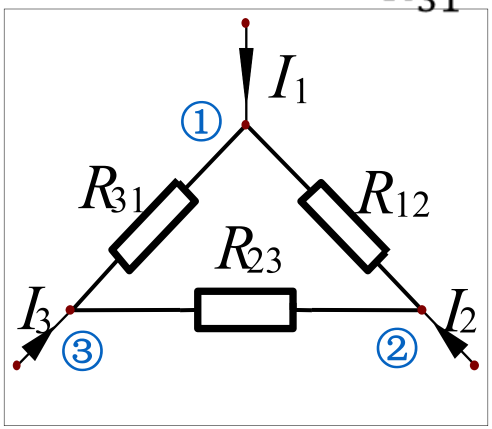
求△网络

(Y→△)

$$\left.\begin{aligned} R_{23} &= \frac{R_1R_2+R_2R_3+R_3R_1}{R_1} \\ R_{12} &= \frac{R_1R_2+R_2R_3+R_3R_1}{R_3} \\ R_{31} &= \frac{R_1R_2+R_2R_3+R_3R_1}{R_2} \end{aligned}\right\} \quad (3)$$

$$\left.\begin{aligned} I_1 &= \frac{U_{12}}{R_{12}}-\frac{U_{31}}{R_{31}} \\ I_2 &= \frac{U_{23}}{R_{23}}-\frac{U_{12}}{R_{12}} \\ I_3 &= \frac{U_{31}}{R_{31}}-\frac{U_{23}}{R_{23}} \end{aligned}\right\} \quad (1)$$

△形



$$\left.\begin{aligned} I_1 &= \frac{U_{12}R_3-U_{31}R_2}{R_1R_2+R_2R_3+R_3R_1} \\ I_2 &= \frac{U_{23}R_1-U_{12}R_3}{R_1R_2+R_2R_3+R_3R_1} \\ I_3 &= \frac{U_{13}R_2-U_{23}R_1}{R_1R_2+R_2R_3+R_3R_1} \end{aligned}\right\} \quad (2)$$

Y形

例如，将R_{12}、R_{31}代入到(1)的I_1中

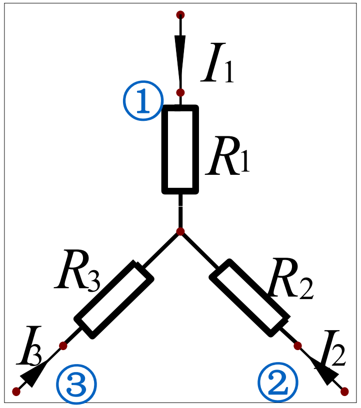
$$I_1=\frac{U_{12}}{\frac{R_1R_2+R_2R_3+R_3R_1}{R_3}}-\frac{U_{31}}{\frac{R_1R_2+R_2R_3+R_3R_1}{R_2}}$$

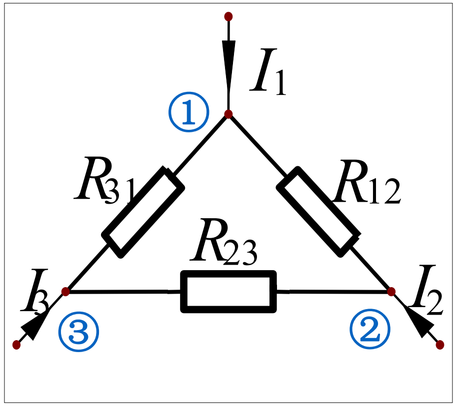
$$=\frac{U_{12}R_3-U_{31}R_2}{R_1R_2+R_2R_3+R_3R_1}$$

以上公式可归纳为

$$
\left.\begin{aligned}
R_1 &= \frac{R_{31}R_{12}}{R_{12}+R_{23}+R_{31}} \\
R_2 &= \frac{R_{12}R_{23}}{R_{12}+R_{23}+R_{31}} \\
R_3 &= \frac{R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}
\end{aligned}\right\} \quad (4)
$$

$$
\text{Y形电阻} = \frac{\triangle\text{形相邻电阻的乘积}}{\triangle\text{形电阻之和}}
$$

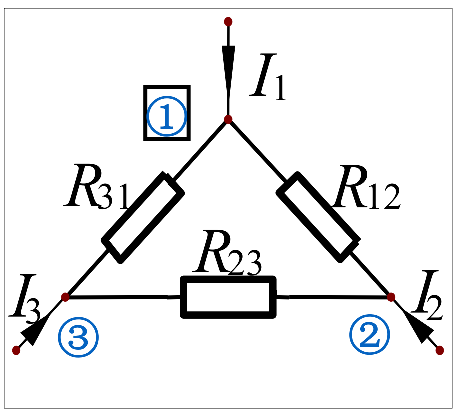


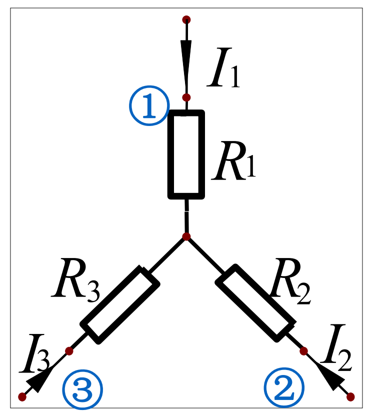


以上公式可归纳为

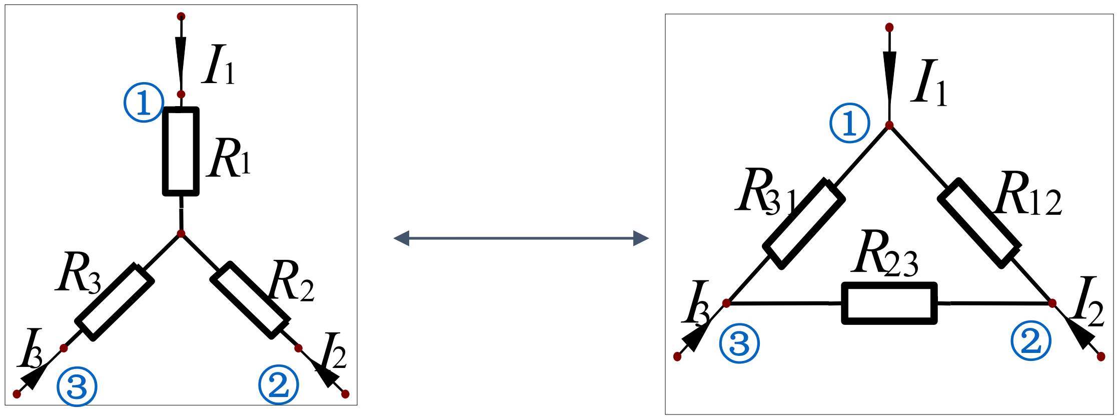
$$
\left.
\begin{aligned}
R_{23} &= \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \\
R_{12} &= \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \\
R_{31} &= \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}
\end{aligned}
\right\} \quad (3)
$$

$$\triangle\text{形电阻} = \frac{\text{Y形电阻两乘积之和}}{\text{Y形不相邻电阻}}$$





以上公式可归纳为



$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

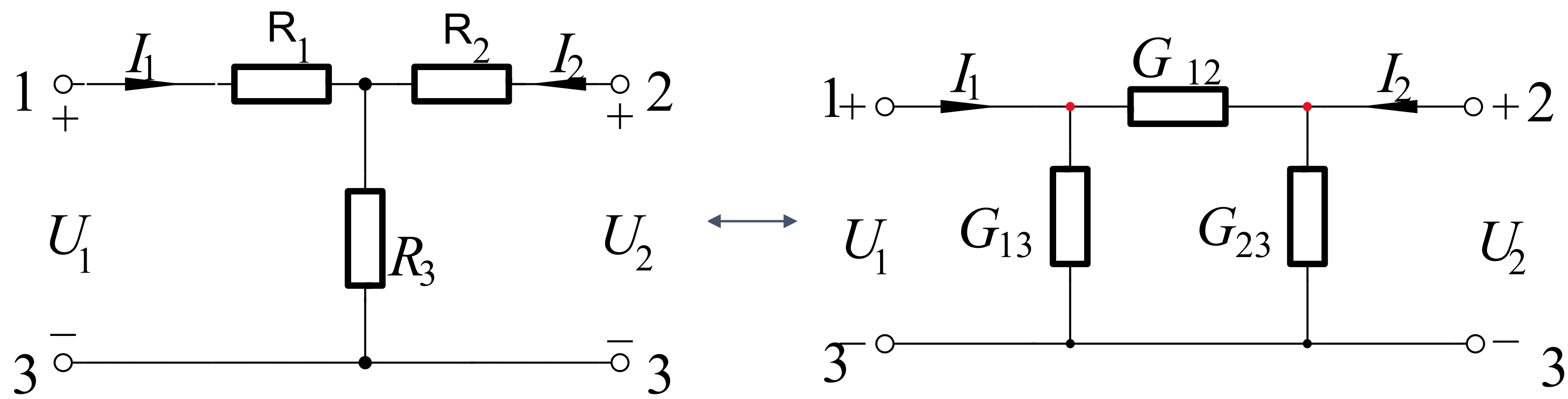
$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$

$$G_1 = G_{12} + G_{13} + \frac{G_{12} G_{13}}{G_{23}}$$

$$G_2 = G_{21} + G_{23} + \frac{G_{21} G_{23}}{G_{13}}$$

$$G_3 = G_{31} + G_{32} + \frac{G_{31} G_{32}}{G_{12}}$$

另一种证明：

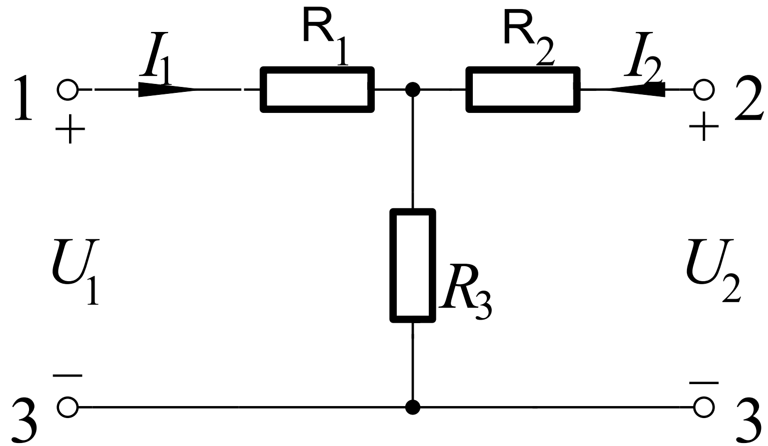


$$\begin{cases} R_1 I_1 + R_3 (I_1 + I_2) = U_1 \\ R_2 I_2 + R_3 (I_1 + I_2) = U_2 \end{cases}$$

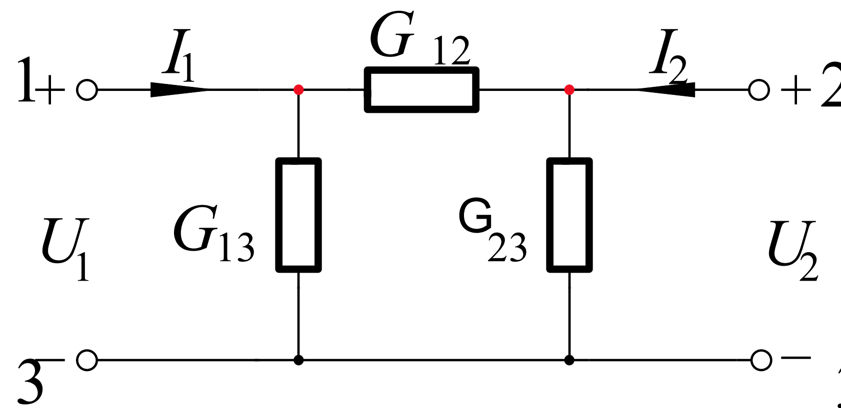
$$\begin{cases} G_{13} U_1 + G_{12} (U_1 - U_2) = I_1 \\ G_{23} U_2 + G_{12} (U_2 - U_1) = I_2 \end{cases}$$

$$\begin{cases} (R_1 + R_3) I_1 + R_3 I_2 = U_1 & (1) \\ R_3 I_1 + (R_2 + R_3) I_2 = U_2 & (2) \end{cases}$$

$$\begin{cases} (G_{13} + G_{12}) U_1 - G_{12} U_2 = I_1 & (3) \\ -G_{12} U_1 + (G_{12} + G_{23}) U_2 = I_2 & (4) \end{cases}$$



$\longleftrightarrow$



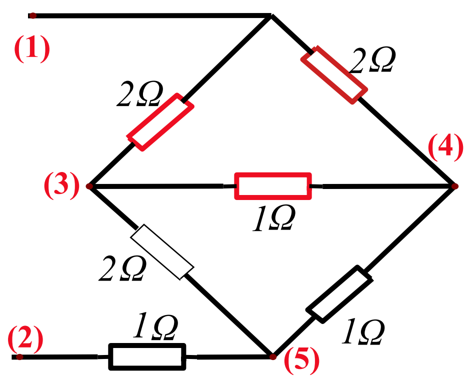
$$\begin{cases}(R_1+R_3)I_1+R_3I_2=U_1 & (1)\\ R_3I_1+(R_2+R_3)I_2=U_2 & (2)\end{cases}\qquad\begin{cases}(G_{13}+G_{12})U_1-G_{12}U_2=I_1 & (3)\\ -G_{12}U_1+(G_{12}+G_{23})U_2=I_2 & (4)\end{cases}$$

$(1)/R_3-(2)/(R_2+R_3)$ 得：

$$\left(\frac{R_1+R_3}{R_3}-\frac{R_3}{R_2+R_3}\right)I_1=\frac{U_1}{R_3}-\frac{U_2}{R_2+R_3}$$

$$I_1=\frac{R_2+R_3}{R_1R_2+R_1R_3+R_2R_3}U_1-\frac{R_3}{R_1R_2+R_1R_3+R_2R_3}U_2$$

例： 求图示桥形电路的总电阻R_{12}



(a)

$$Y\text{形电阻} = \frac{\triangle\text{形相邻电阻的乘积}}{\triangle\text{形电阻之和}}$$

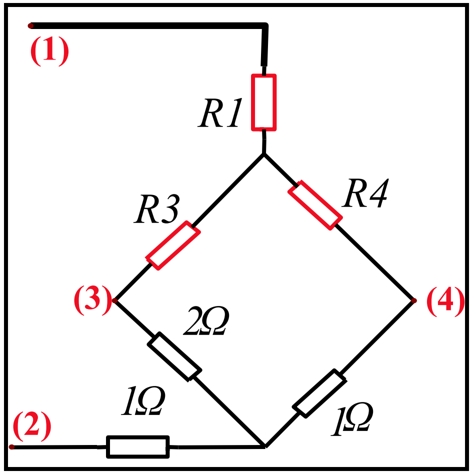
解： 将节点(1)(3)(4)内的△形电路用Y形电路替代，得到图(b)。

△→Y

$$R_1 = \frac{2\times 2}{2+1+2} = 0.8\Omega$$

$$R_3 = \frac{2\times 1}{2+1+2} = 0.4\Omega$$

$$R_4 = \frac{2\times 1}{2+1+2} = 0.4\Omega$$

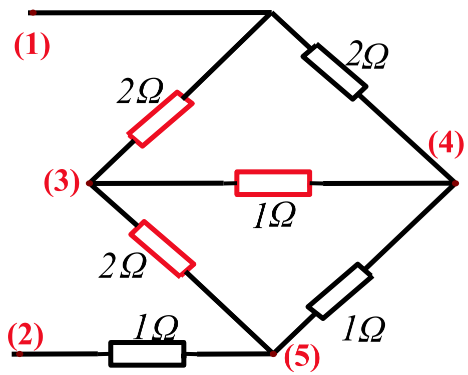


(b)

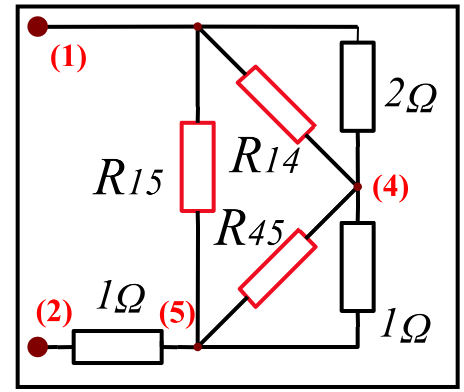
$$R_{12} = (R_3 + 2\Omega)//(R_4 + 1\Omega) + R_1 + 1\Omega = 2.4\Omega//1.4\Omega + 0.8\Omega + 1\Omega = 2.68\Omega$$

另一解法　将以节点(3)为公共点的Y形电路用△形电路替代，如图(c)。

Y→△



(a)



(c)

$$R_{14}=\frac{2\times 1+2\times 2+1\times 2}{2}=4\Omega$$

$$R_{15}=\frac{8}{1}=8\Omega$$

$$R_{45}=\frac{8}{2}=4\Omega$$

$$\triangle\text{形电阻}=\frac{\text{Y形电阻两乘积之和}}{\text{Y形不相邻电阻}}$$

$$R_{12}=(R_{14}//2+R_{45}//1)//R_{15}+1\Omega=\left({}^{4}/_{3}\,\Omega+{}^{4}/_{5}\,\Omega\right)//8\Omega+1\Omega=2.68\Omega$$

习题(星型－三角型)

2-6-1

2-6-2

2-6-3