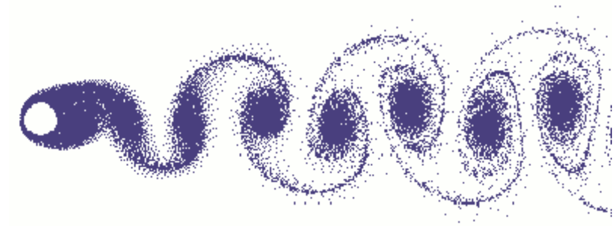
§6-3 电路的谐振



物理谐振：卡门涡街频率与大桥特征频率相同时发生谐振！

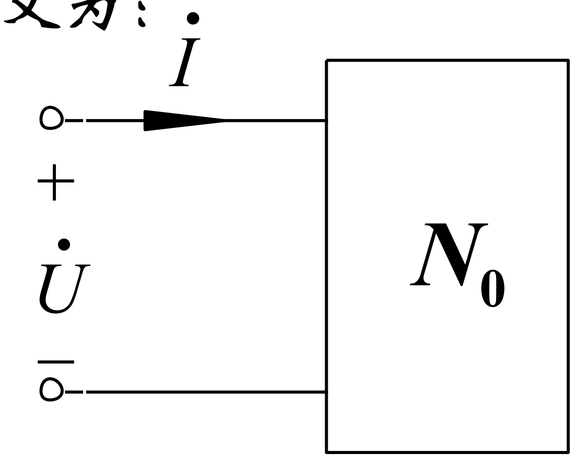
电路谐振：与物理谐振有什么关系？有什么好处及坏处？

电路谐振的定义：

任意无源单口网络，其端口阻抗和导纳定义为：

$$Z(\omega)=\frac{\dot{U}(\omega)}{\dot{I}(\omega)}=R+jX_L(\omega)-jX_C(\omega)$$

$$Y(\omega)=\frac{\dot{I}(\omega)}{\dot{U}(\omega)}=G+jB_C(\omega)-jB_L(\omega)$$



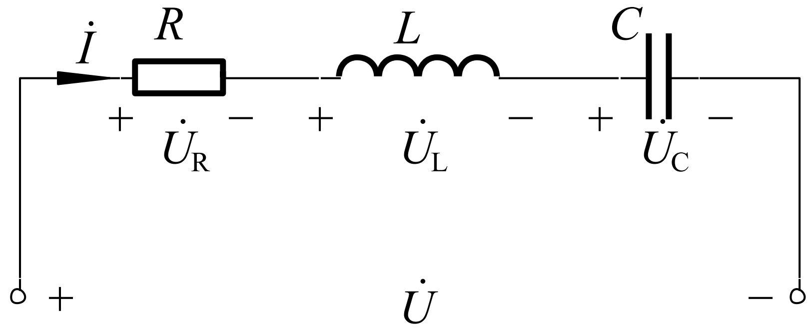
如果端口阻抗或导纳满足如下条件之一，称该网络发生谐振：

$X_L(\omega)=X_C(\omega)$ 发生串联谐振

$B_C(\omega)=B_L(\omega)$ 发生并联谐振

同一个系统是否只有一个谐振频率？

1.串联谐振



$$Z = R + jX$$

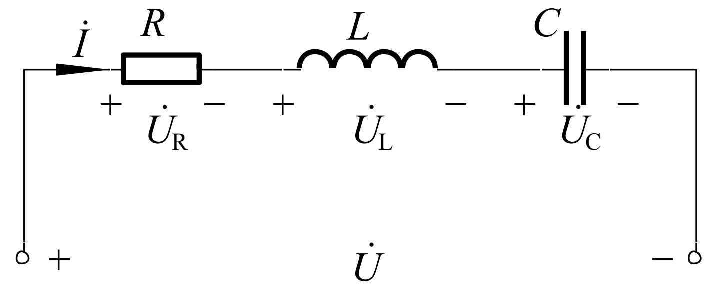
$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$X = \omega L - \dfrac{1}{\omega C} = 0$ 时,有: $\omega_0 L = \dfrac{1}{\omega_0 C}$

串联谐振的角频率: $\omega_0 = \dfrac{1}{\sqrt{LC}}$

定义特征阻抗: $\rho = \omega_0 L = \dfrac{1}{\omega_0 C} = \dfrac{1}{\sqrt{LC}} L = \sqrt{\dfrac{L}{C}}$

2.串联谐振的电压电流关系



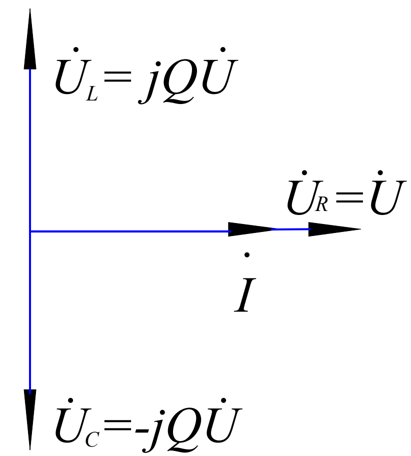
由KVL方程：　　$\dot{U}=\dot{U}_R+\dot{U}_L+\dot{U}_C$

串联谐振时，以外加电压 $\dot{U}$ 为参考相量：

$$\dot{U}_R=R\dot{I}=R\frac{\dot{U}}{R}=\dot{U}$$

$$\dot{U}_L=j\omega_0L\dot{I}=j\rho\frac{\dot{U}}{R}=jQ\dot{U}$$

$$\dot{U}_C=-j\frac{1}{\omega_0c}\dot{I}=-j\rho\frac{\dot{U}}{R}=-jQ\dot{U}$$



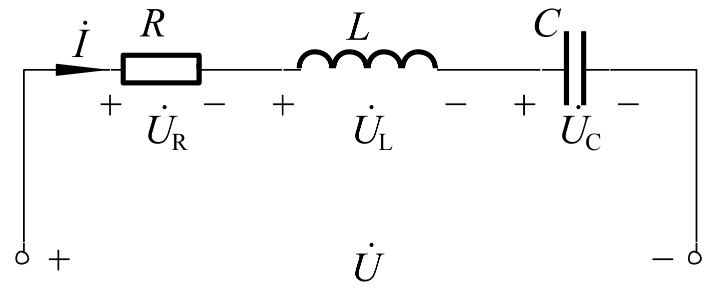
串联谐振时的
电压电流的相量图

式中Q为电路的品质因数：$Q=\frac{\rho}{R}$

$\dot{U}_L$与$\dot{U}_C$ 大小相等，相位相反，互相抵消，

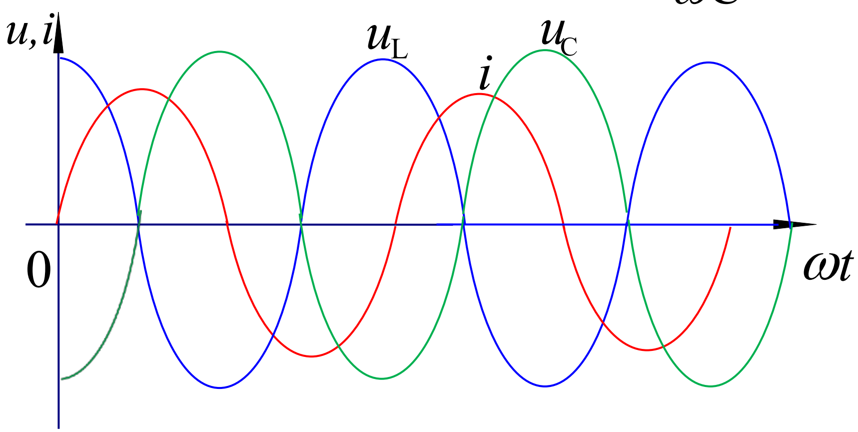
即　$\dot{U}_L+\dot{U}_C=0$　　$\dot{U}=\dot{U}_R$

3. 串联谐振的能量关系



$$i=\sqrt{2}I\sin\omega t$$

$$u_L=\sqrt{2}\omega LI\cos\omega t;\quad u_C=-\sqrt{2}\frac{1}{\omega C}I\cos\omega t$$



$$E_L=\frac{Li^2}{2}=LI^2\sin^2\omega t$$

$$E_C=\frac{Cu_C^2}{2}=\frac{1}{\omega^2 C}I^2\cos^2\omega t$$

第1/4 周期， $i\uparrow$ ， $|u_C|\downarrow$

电感磁场的能量 $E_L=\frac{Li^2}{2}\uparrow$

电容电场的能量 $E_C=\frac{Cu_C^2}{2}\downarrow$

第2个1/4周期正好相反

$E_L=\frac{Li^2}{2}\downarrow$

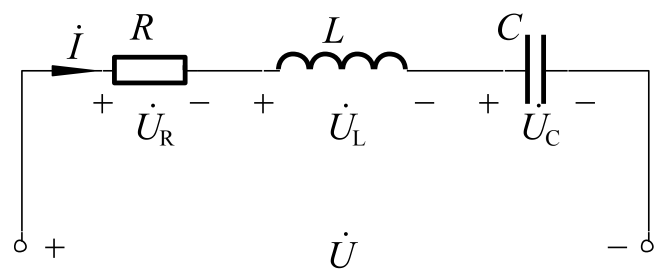
$E_C=\frac{Cu_C^2}{2}\uparrow$

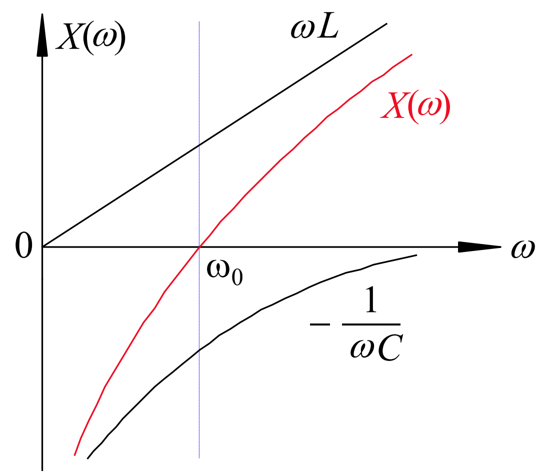
发生谐振时： $L=\frac{1}{\omega_0^2 C}$ 　电磁场总能量 $E=E_L+E_C=LI^2$

电容和电感不与电源发生能量交换！

4.串联谐振曲线（端电压固定）

(1)电抗X的频率特性



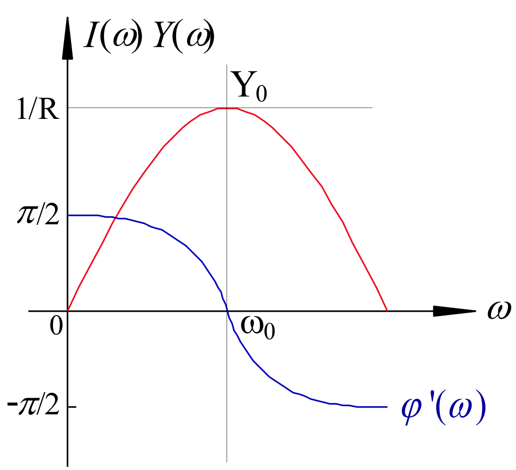


由X(ω) 的特性，可得出Y(ω)的曲线，该曲线成为谐振曲线

$$Y(\omega)=\frac{1}{Z(\omega)}=\frac{1}{R+j\left(\omega L-\frac{1}{\omega C}\right)}$$

$$=\frac{1}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}\angle arctg\frac{\frac{1}{\omega C}-\omega L}{R}$$

(2)导纳Y(I，U_R)的频率特性



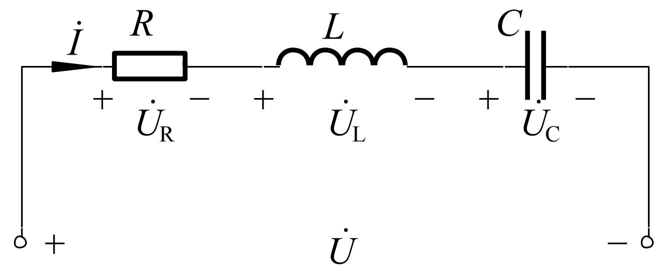
$\omega=0$：$\dot{I}$超前$\dot{U}$ 90° 电路呈容性

$\omega=\omega_0$：电路谐振 $\dot{U}$ $\dot{I}$ 同相

$\omega\to\infty$：$\dot{I}$ 滞后$\dot{U}$ 90°，电路呈感性

4.串联谐振曲线（端电压固定）

(3)导纳Y(I/U$_R$)频率特性的归一化表示

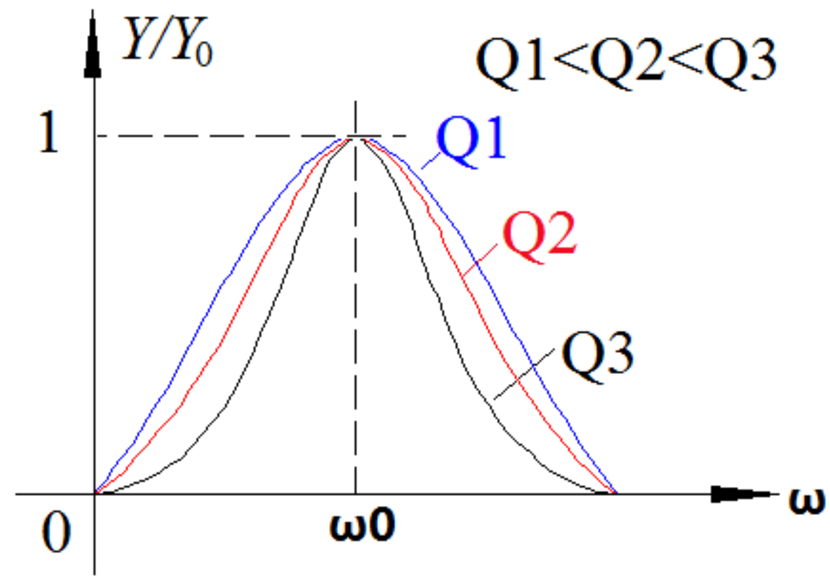


$$Y=\frac{1}{R+j\left(\omega L-\frac{1}{\omega C}\right)}=\frac{1}{R+jRQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}$$

$$\omega L-\frac{1}{\omega C}=L\left(\omega-\frac{\omega_0^2}{\omega}\right)=\omega_0 L\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)$$

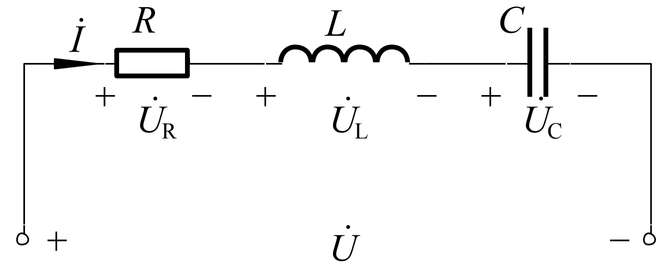
$$=RQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)$$

$$\because Q=\frac{\omega_0 L}{R}=\frac{1}{\omega_0 RC}$$



$$\frac{Y}{Y_0}=\frac{1}{R+jRQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}\bigg/\frac{1}{R}=\frac{1}{1+jQ\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}=\frac{\dot{U}_R}{\dot{U}}$$

4.串联谐振曲线（端电压固定）



(4) U_L 的频率特性

$$\dot{U}_L(\omega)=j\omega L\dot{I}=j\omega L\dot{U}Y(\omega)=\dot{U}\frac{j\omega L}{R+j(\omega L-\frac{1}{\omega C})}=\dot{U}\frac{jQ\frac{\omega}{\omega_0}}{1+jQ(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega})}$$

$$\left|\frac{\dot{U}_L}{\dot{U}}\right|^2=\frac{Q^2(\frac{\omega}{\omega_0})^2}{1+Q^2(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega})^2}=\frac{1}{\left((\frac{\omega_0}{\omega})^2-1+\frac{1}{2Q^2}\right)^2+\frac{1}{Q^2}-\frac{1}{4Q^4}}$$

当 $\omega=\dfrac{\omega_0}{\sqrt{1-\frac{1}{2Q^2}}}>\omega_0$ 时：

取极大值：$\dfrac{4Q^4}{4Q^2-1}$

(5) U_C 的频率特性

$$\dot{U}_C(\omega)=\frac{\dot{I}}{j\omega C}=\frac{\dot{U}Y(\omega)}{j\omega C}=\dot{U}\frac{\frac{1}{j\omega C}}{R+j(\omega L-\frac{1}{\omega C})}=\dot{U}\frac{-jQ\frac{\omega_0}{\omega}}{1+jQ(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega})}$$

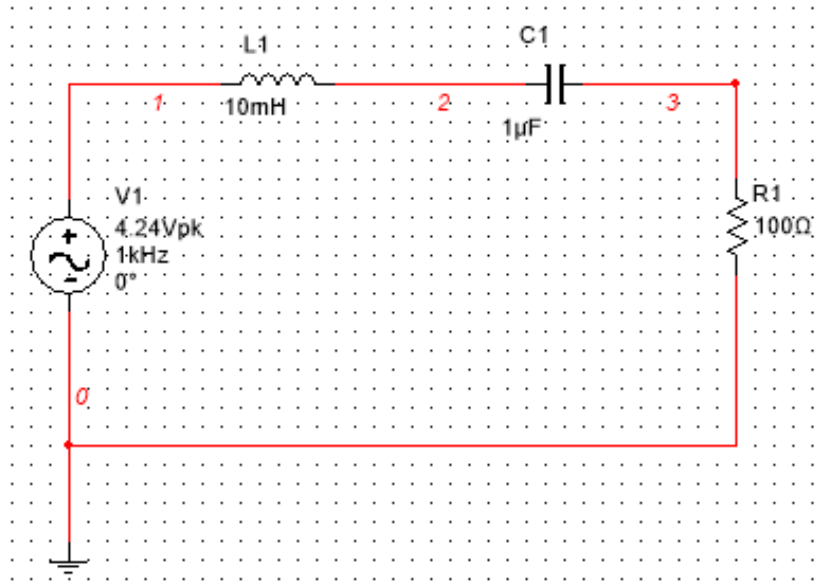
$$\left|\frac{\dot{U}_C}{\dot{U}}\right|^2=\frac{Q^2(\frac{\omega_0}{\omega})^2}{1+Q^2(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega})^2}=\frac{1}{\left((\frac{\omega}{\omega_0})^2-1+\frac{1}{2Q^2}\right)^2+\frac{1}{Q^2}-\frac{1}{4Q^4}}$$

当 $\omega=\omega_0\sqrt{1-\frac{1}{2Q^2}}<\omega_0$ 时：

取极大值：$\dfrac{4Q^4}{4Q^2-1}$

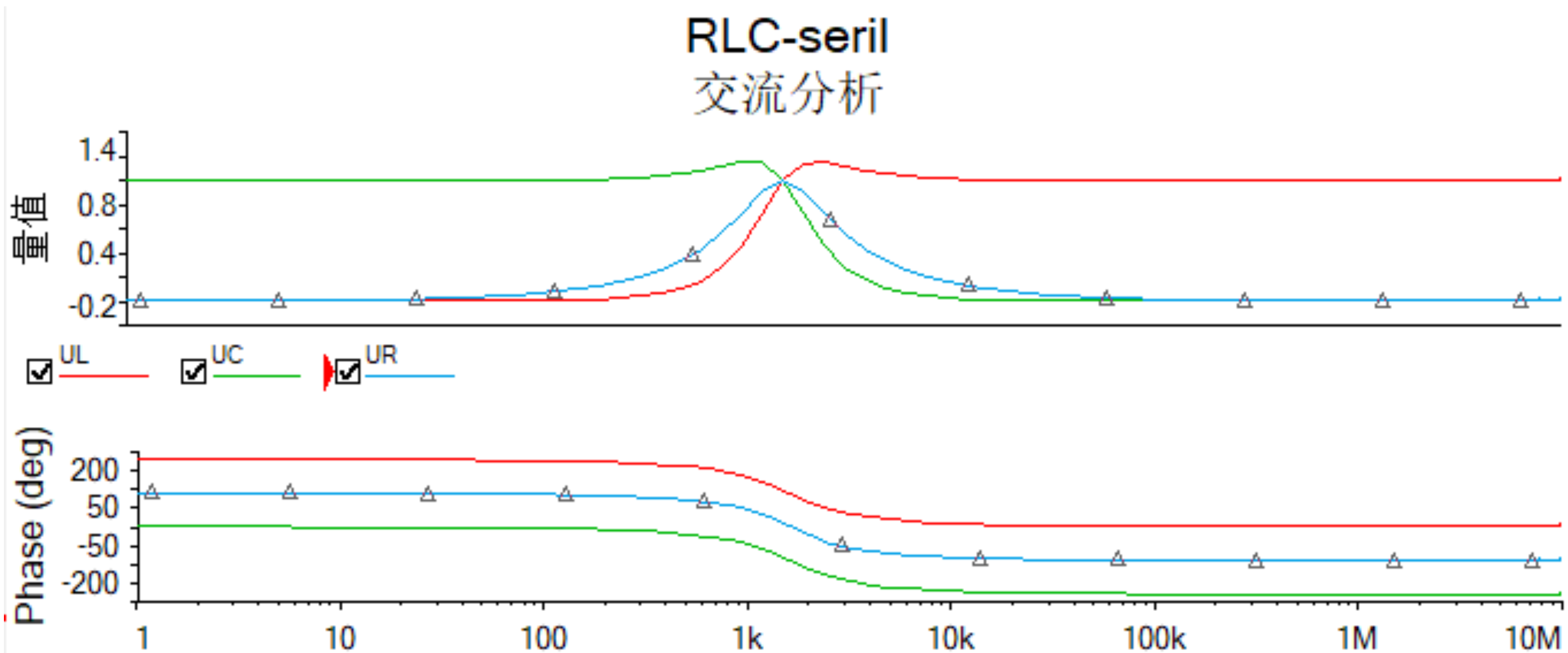
4.串联谐振曲线（端电压固定）

(6) 仿真实验　$L=10mH, C=1\mu F, R=100\Omega$



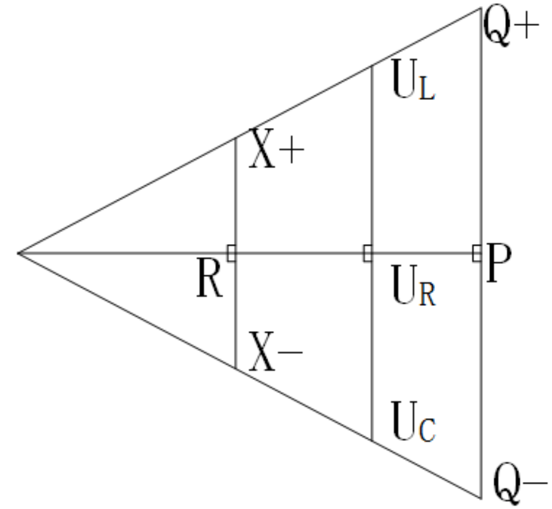
$$f_0=\frac{1}{2\pi\sqrt{LC}}=\frac{10^4}{2\pi}\approx 1.59kHz$$

$$Q=\frac{\omega_0 L}{R}=1$$



5.对"品质"的理解与应用

$$Q = \frac{\rho}{R} = \frac{U_L}{U_R} = \frac{Q+}{P}$$

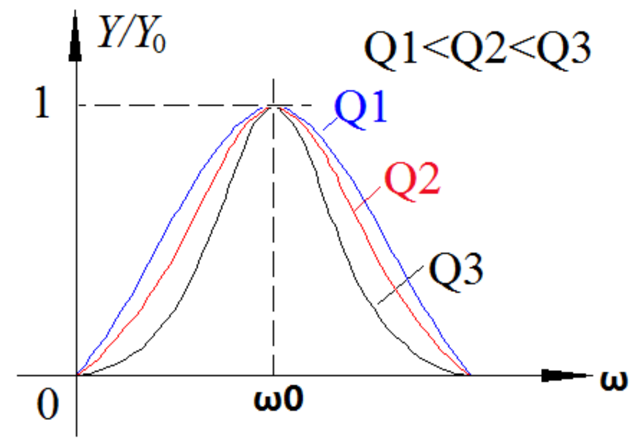


1、电压角度

$U_L >> U_R$ 用于小信号检测

2、能量角度

$Q_+ >> P$ 损耗能量远小于振荡能量



3、频率响应

接收机的选频特性

晶振的频率稳定性

滤波器的截频性能

串联与并联谐振电路的对偶关系

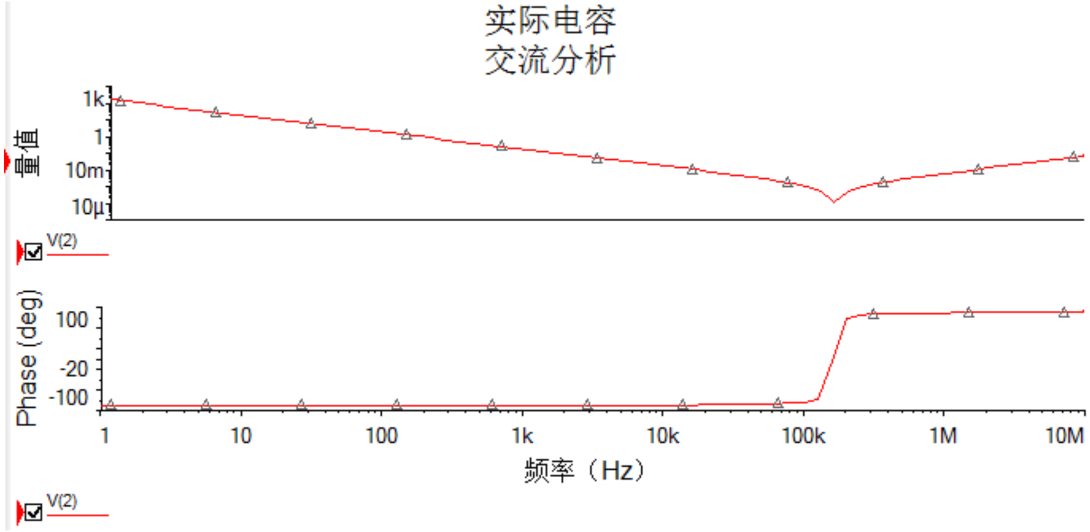
串联谐振电路	并联谐振电路
$\omega_0 L = \dfrac{1}{\omega_0 C}$	$\omega_0 C = \dfrac{1}{\omega_0 L}$
$\omega_0 = \dfrac{1}{\sqrt{LC}} \quad f_0 = \dfrac{1}{2\pi\sqrt{LC}}$	$\omega_0 = \dfrac{1}{\sqrt{CL}} \quad f_0 = \dfrac{1}{2\pi\sqrt{CL}}$
$Z = R + j\left(\omega L - \dfrac{1}{\omega C}\right)$	$Y = G + j\left(\omega C - \dfrac{1}{\omega L}\right)$
$Z(\omega_0) = R$	$Y(\omega_0) = G$
$\dot{U}_{L0} = j\omega L\dot{I}_0$	$\dot{I}_{C0} = j\omega C\dot{U}_0$
$\dot{U}_{C0} = -j\dfrac{1}{\omega C}\dot{I}_0$	$\dot{I}_{L0} = -j\dfrac{1}{\omega L}\dot{U}_0$
$Q = \dfrac{\omega_0 L}{R} = \dfrac{\sqrt{L/C}}{R}$	$Q = \dfrac{\omega_0 C}{G} = \dfrac{\sqrt{C/L}}{G}$
$\left\lvert\dfrac{Y}{Y_0}\right\rvert = \dfrac{1}{\sqrt{1+Q^2\left(\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}\right)^2}}$	$\left\lvert\dfrac{Z}{Z_0}\right\rvert = \dfrac{1}{\sqrt{1+Q^2\left(\dfrac{\omega}{\omega_0}-\dfrac{\omega_0}{\omega}\right)^2}}$

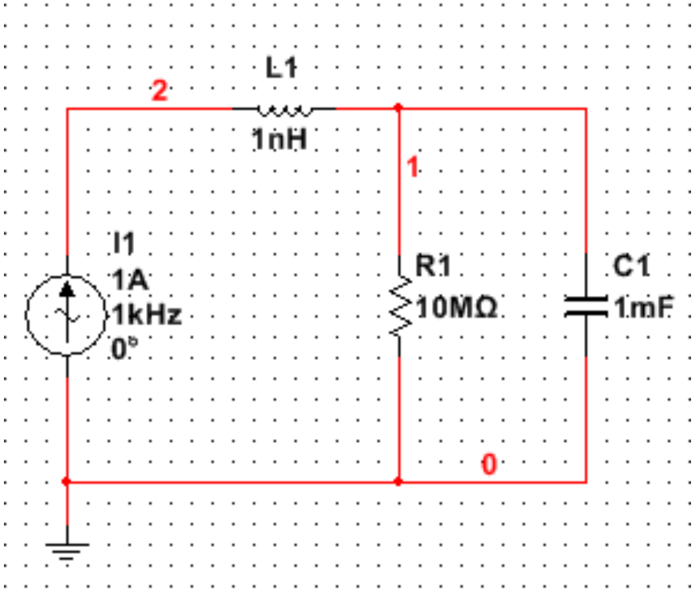
例：图示电源滤波电容为何需要几个不同容值电容并联？

$$Z = j\omega L + \frac{1}{G + j\omega C} = \frac{G}{G^2 + \omega^2 C^2} + j\omega\left(L - \frac{C}{G^2 + \omega^2 C^2}\right)$$

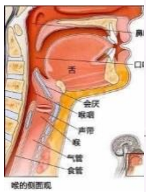
$$\therefore L - \frac{C}{G^2 + \omega^2 C^2} = 0$$ 时，

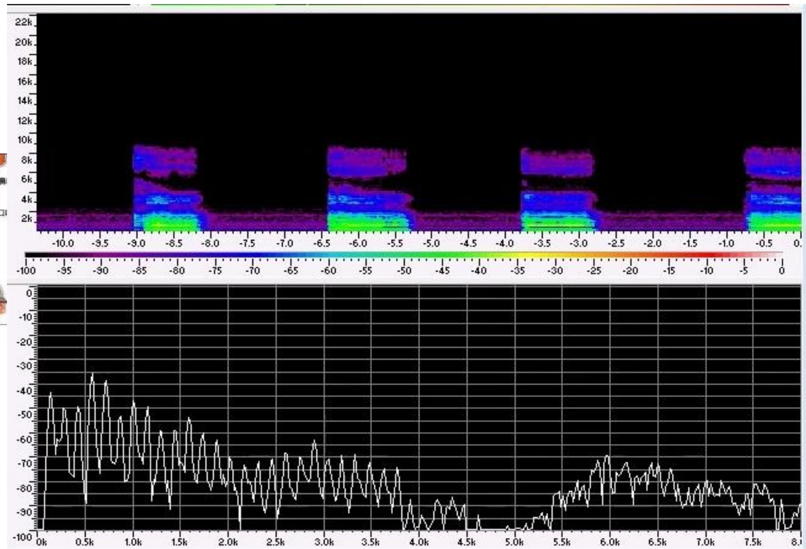
即：$\omega^2 = \frac{1}{LC} - \frac{G^2}{C^2}$ 时等效电抗为0

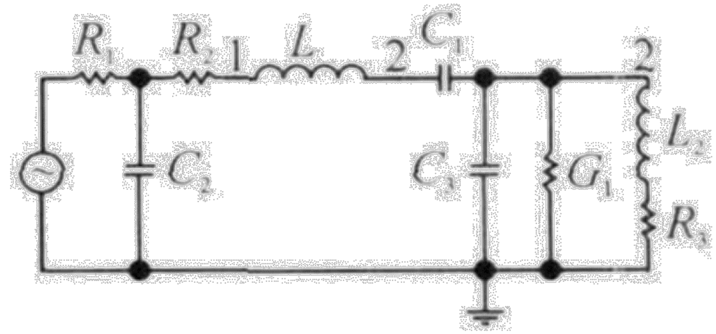




思考：物理谐振系统的电路模型







FMYB气体谐振腔电路模型

复杂系统可以有
多个谐振频率？

如何为更复杂的系统建立电路模型？
将在后续《信号与系统》等课程中学习到

作业：

6-20

6-22

6-23