§ 5-6 阻抗与导纳

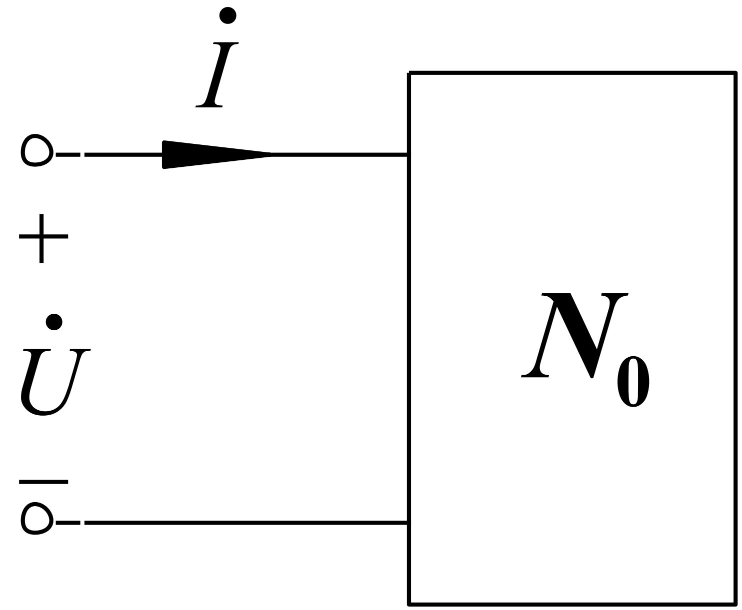
图中，N_0为正弦稳态电路中的无源二端网络

$$Z(j\omega)=\frac{\dot{U}}{\dot{I}} \quad (\Omega)$$

输入阻抗
(等效阻抗)

$$Y(j\omega)=\frac{1}{Z}=\frac{\dot{I}}{\dot{U}} \quad (S)$$

输入导纳
(等效导纳)



$$\left.\begin{aligned}\dot{U}&=Z\dot{I}\\ \dot{I}&=Y\dot{U}\end{aligned}\right\}$$ —欧姆定律的相量形式

$$Z_R = R\ ,\qquad\qquad Y_R = G$$

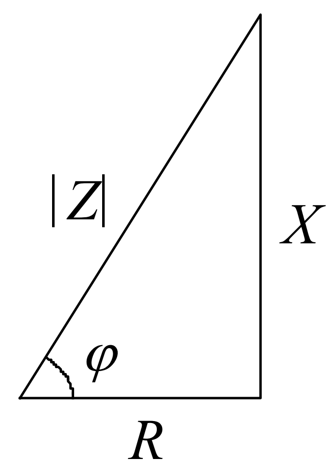
$$Z_C = \frac{1}{j\omega C} = -j\underbrace{X_C}_{\text{容抗}}\ (\Omega)\qquad\qquad Y_C = j\omega C = j\underbrace{B_C}_{\text{容纳}}\ (\mathrm{S})$$

$$Z_L = j\omega L = j\underbrace{X_L}_{\text{感抗}}\ (\Omega)\qquad\qquad Y_L = \frac{1}{j\omega L} = -j\underbrace{B_L}_{\text{感纳}}\ (S)$$

$$Z(j\omega) = \underbrace{R}_{\substack{\text{实部}\\(\text{电阻分量})}} + j\underbrace{X}_{\substack{\text{虚部}\\(\text{电抗分量})}} = \underbrace{|Z|}_{\frac{U}{I}(\text{阻抗的模})}\angle\varphi \longleftarrow \Psi_u - \Psi_i\ (\text{阻抗角})$$

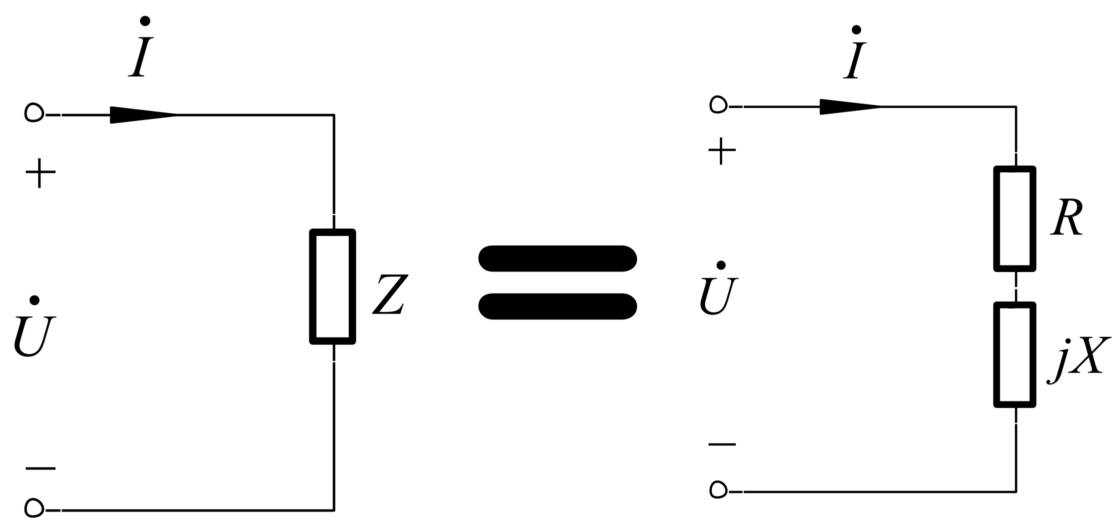
即 $$|Z| = \sqrt{R^2 + X^2}\qquad\qquad \varphi = arctg\frac{X}{R}$$

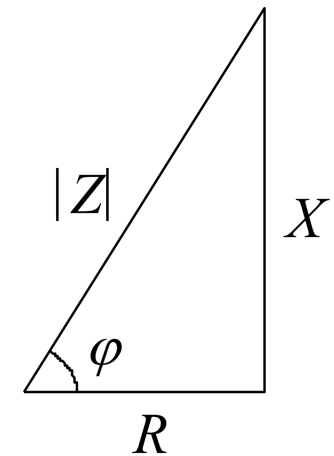
$$R = |Z|\cos\varphi\qquad\qquad X = |Z|\sin\varphi$$



阻抗三角形

$$Z(j\omega)=R+jX=|Z|\angle\varphi \qquad (1)$$





阻抗三角形

当X>0时，$\varphi>0$ $\dot{U}$ 超前于 $\dot{I}$ N_0呈感性(电抗等效为电感元件)

当X<0时,$\varphi<0$ $\dot{U}$ 滞后于 $\dot{I}$ N_0呈容性(电抗等效为电容元件)

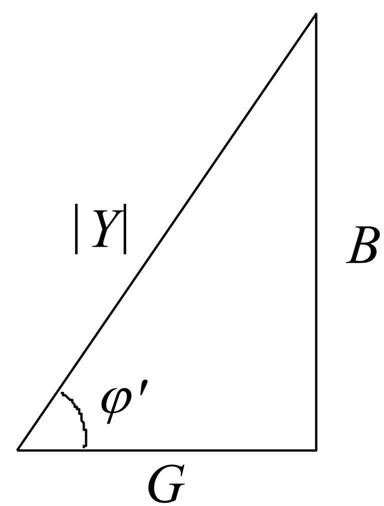
当X=0时,$\varphi=0$ $\dot{U}$ 与 $\dot{I}$ 同相 N_0呈阻性(电抗等效为电阻元件)

$$Y(j\omega)=G+jB=|Y|\angle\varphi' \longleftarrow \psi_i-\psi_u\text{(导纳角)}$$

实部（电导分量）　虚部（电纳分量）　$\dfrac{I}{U}$（导纳的模）

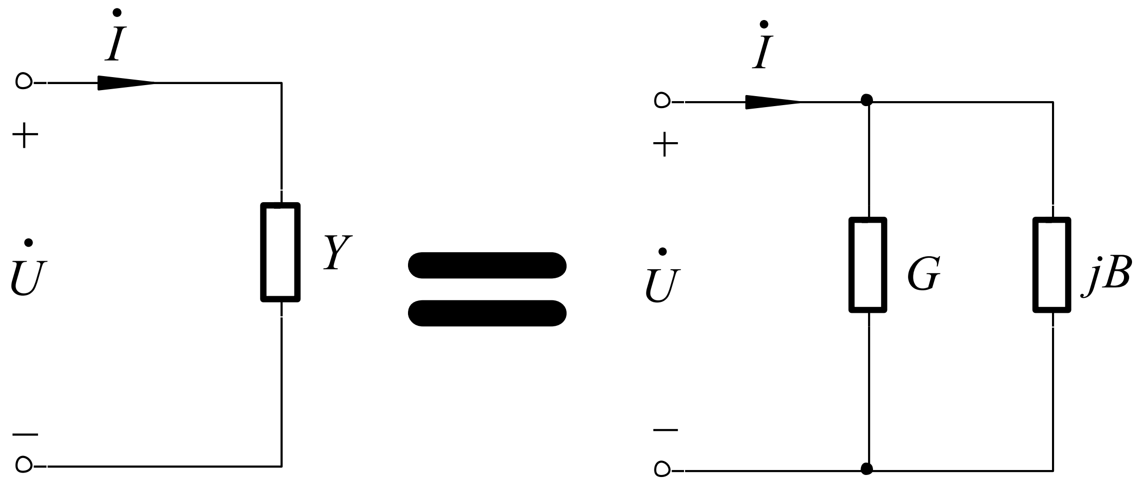
$$|Y|=\sqrt{G^2+B^2} \qquad \varphi'=arctg\frac{B}{G}$$

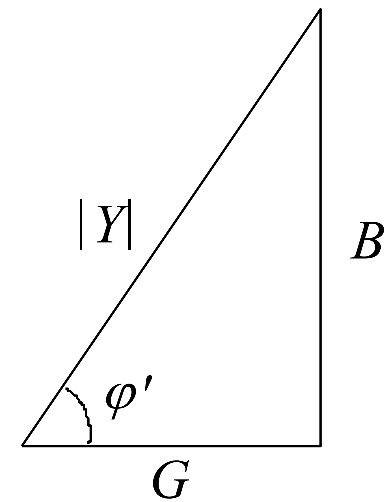
$$G=|Y|\cos\varphi' \qquad B=|Y|\sin\varphi'$$



导纳三角形

$$Y(j\omega)=G+jB=|Y|\angle\varphi' \qquad (2)$$





导纳三角形

当B>0时，$\varphi' > 0$ $\dot{I}$超前于$\dot{U}$ N_0呈容性(电纳等效为电容元件)

当B<0时，$\varphi' < 0$ $\dot{I}$滞后于$\dot{U}$ N_0呈感性(电纳等效为电感元件)

当B=0时，$\varphi' = 0$ $\dot{I}$与$\dot{U}$同相 N_0呈阻性(电纳等效为电阻元件)

§ 5-7 阻抗的串联与并联

1. 阻抗串联的电路

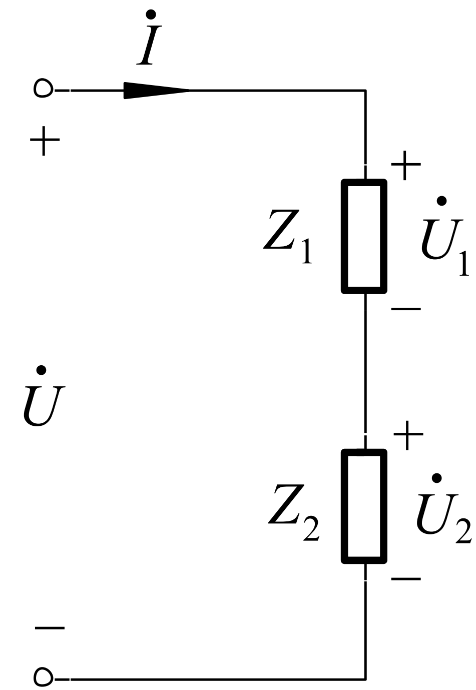
$$\dot{U} = \dot{U}_1 + \dot{U}_2$$

$$\dot{U}_1 = Z_1 \dot{I} \qquad \dot{U}_2 = Z_2 \dot{I}$$

于是有 $(Z_1 + Z_2)\dot{I} = \dot{U}$

$$\dot{I} = \frac{\dot{U}}{Z_1 + Z_2}$$

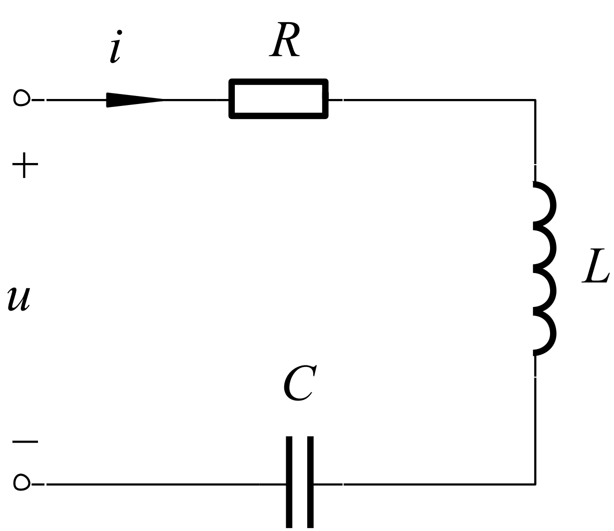
由此可得 $Z = Z_1 + Z_2$



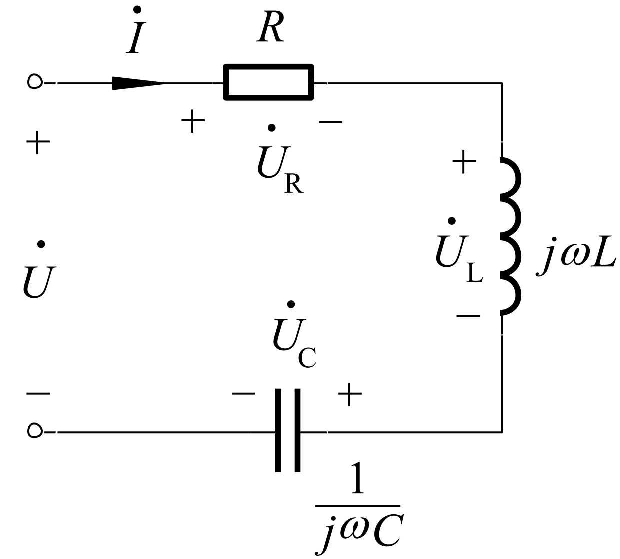
两个阻抗串联时的分压公式为

$$\dot{U}_1 = \frac{Z_1}{Z_1 + Z_2}\dot{U} \qquad \dot{U}_2 = \frac{Z_2}{Z_1 + Z_2}\dot{U}$$

例：



(a) R,L,C 串联电路



(b) 相量模型

图(a)表示一个R,L,C 串联电路，其相量模型如图(b)所示，

设电源电压为$\dot{U}=U\angle\psi_u$，电流相量为$\dot{I}=I\angle\psi_i$，

电阻、电感、电流的电压相量为:$\dot{U}_R$、$\dot{U}_L$、$\dot{U}_C$

由KVL和各元件方程，有
$$\dot{U}=\dot{U}_R+\dot{U}_L+\dot{U}_C$$
$$=R\dot{I}+j\omega L\dot{I}+\frac{1}{j\omega C}\dot{I}$$
$$=(R+j\omega L+\frac{1}{j\omega C})\dot{I}$$

将上式右端中 $\dot{I}$ 前的系数记为 $Z=|Z|\angle\varphi$，即
$$Z=\frac{\dot{U}}{\dot{I}}=R+j\omega L+\frac{1}{j\omega C}=R+j(\omega L-\frac{1}{\omega C})=R+j(X_L-X_C)$$

令 $X=X_L-X_C$

则有 $Z=\frac{\dot{U}}{\dot{I}}=R+jX$

$$Z=\sqrt{R^2+X^2}\angle arctg\frac{X}{R}=|Z|e^{j\varphi}=|Z|\angle\varphi$$

其中 $|Z|=\sqrt{R^2+X^2}$ $\varphi=arctg\frac{X}{R}$

$$|Z| = \frac{U}{I} \qquad\qquad \varphi = \psi_u - \psi_i$$

复阻抗 Z的模　　电压有效值与电流有效值之比　　复阻抗 Z 的辐角(电路的阻抗角)

由此可见，复阻抗Z决定了电压、电流有效值的大小和相位间的关系

若给定图中电源电压$\dot{U}$和各元件参数，可以求出

$$\dot{I} = \frac{\dot{U}}{Z} = \frac{U}{|Z|}\angle \psi_u - \varphi$$

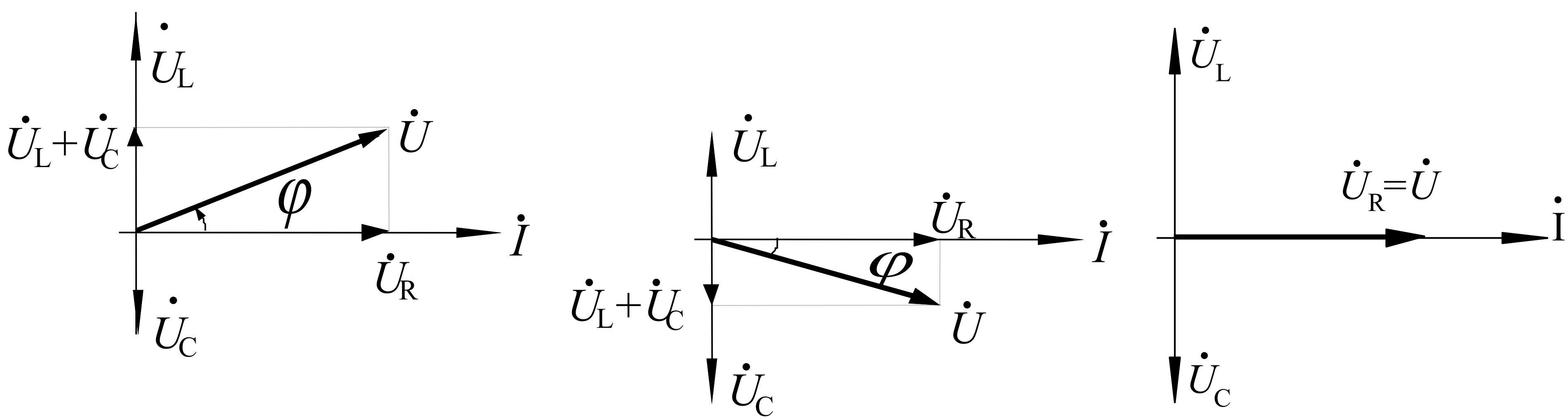
$$I = \frac{U}{|Z|} = \frac{U}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$\psi_i = \psi_u - \varphi = \psi_u - arctg\frac{X}{R}$$

若 $\omega L > \dfrac{1}{\omega C}$ 则电抗 X 为正值，$\varphi>0$，电流滞后于电压

若 $\omega L < \dfrac{1}{\omega C}$ 则电抗 X 为负值，$\varphi<0$，电流超前于电压

若 $\omega L = \dfrac{1}{\omega C}$ 则电抗 $X=0$ ，$\varphi=0$，电流和电压同相



(a) $\omega L>1/\omega C$　(b) $\omega L<1/\omega C$　(c) $\omega L=1/\omega C$

*RLC*串联电路中电压、电流相量图

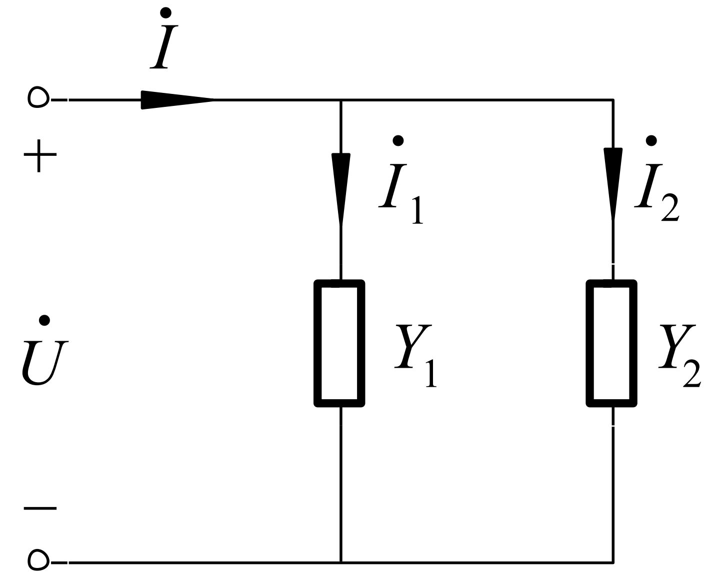
2.导纳并联的电路

$$\dot{I}_1 + \dot{I}_2 = \dot{I}$$

$$\dot{I}_1 = Y_1 \dot{U} \qquad \dot{I}_2 = Y_2 \dot{U}$$

于是有 $(Y_1 + Y_2)\dot{U} = \dot{I}$

$$\therefore \quad \dot{U} = \frac{\dot{I}}{Y_1 + Y_2}$$



由此可见 $Y = Y_1 + Y_2$

等效阻抗： $Z = \dfrac{1}{Y} = \dfrac{1}{Y_1 + Y_2} = \dfrac{Z_1 Z_2}{Z_1 + Z_2}$

两个导纳并联的分流公式为 $\dot{I}_1 = Y_1 \dot{U} = \dfrac{Y_1}{Y_1 + Y_2}\dot{I} = \dfrac{Z_2}{Z_1 + Z_2}\dot{I}$

$$\dot{I}_2 = Y_2 \dot{U} = \frac{Y_2}{Y_1 + Y_2}\dot{I} = \frac{Z_1}{Z_1 + Z_2}\dot{I}$$

例： 设电路中电阻、电容、电感的电流分别为$\dot{I}_R$，$\dot{I}_C$，$\dot{I}_L$

它们并联接至电流源 $\dot{I}_S = I_S\angle\psi_i$ 它们两端有同一电压 $\dot{U} = U\angle\psi_u$

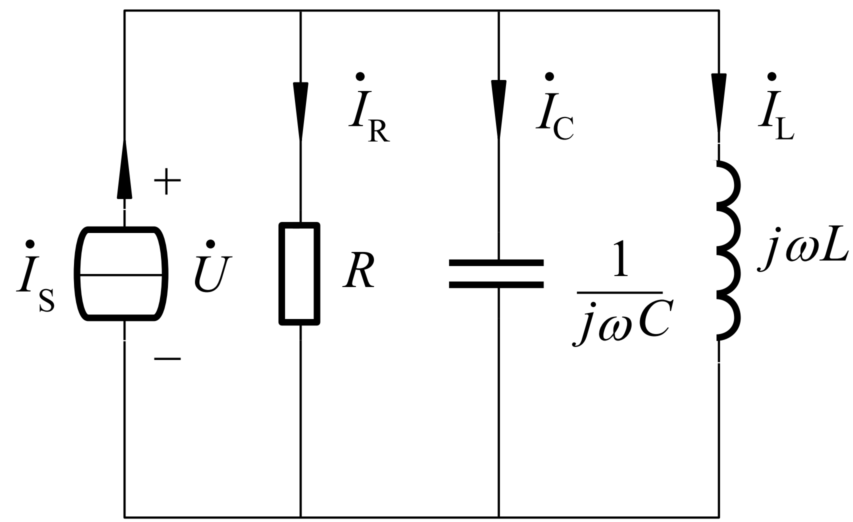
根据KCL

$$\dot{I}_S = \dot{I}_R + \dot{I}_C + \dot{I}_L$$

$$= \frac{\dot{U}}{R} + j\omega C\dot{U} + \frac{\dot{U}}{j\omega L}$$

$$= \underline{\left(\frac{1}{R} + j\omega C + \frac{1}{j\omega L}\right)}\dot{U}$$

Y（复导纳）



RLC并联电路相量模型

即 $Y=\dfrac{\dot{I}_S}{\dot{U}}=\dfrac{1}{R}+j(\omega C-\dfrac{1}{\omega L})$

$$=G+j(B_C-B_L)=G+jB$$

电导　　容纳　　感纳　　电纳

$$Y=\sqrt{G^2+B^2}\angle arctg\frac{B}{G}=|Y|e^{j\varphi'}=|Y|\angle\varphi'$$

其中 $|Y|=\sqrt{G^2+B^2}=\sqrt{G^2+(B_C-B_L)^2}$

复导纳的辐角（导纳角） $\varphi'=arctg\dfrac{B}{G}=\psi_i-\psi_u$

复导纳的模 $|Y|=\dfrac{I}{U}$

由此可见，复导纳Y决定了电流、电压有效值的大小和相位间的关系

若给定图中的 $\dot{I}_S$ 和各元件的参数值，可以得出

$$\dot{U} = \frac{\dot{I}_S}{Y} = \frac{I_S}{|Y|}\angle\psi_i - \varphi'$$

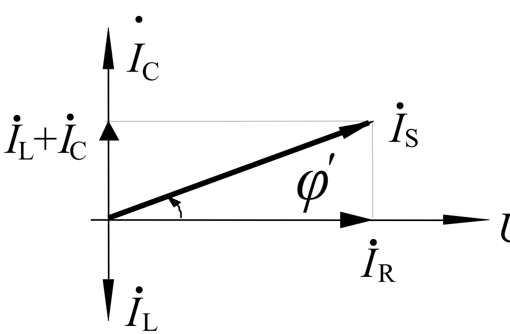
$$U = \frac{I_S}{\sqrt{G^2 + (B_C - B_L)^2}}$$

$$\psi_u = \psi_i - arctg\frac{B}{G}$$

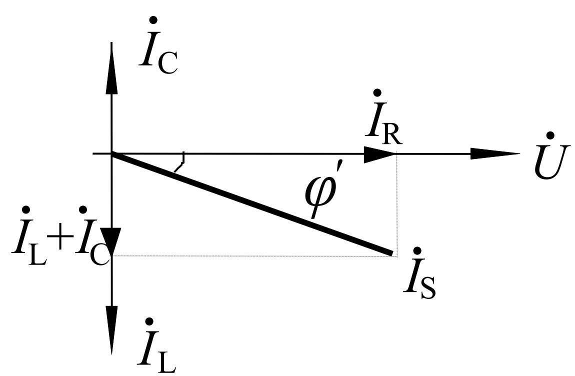
若 $\omega C > 1/\omega L$，则电纳B为正值　　$\varphi' > 0$,　$\dot{I}_S$ 超前于 $\dot{U}$

若 $\omega C < 1/\omega L$，则电纳B为负值　　$\varphi' < 0$,　$\dot{I}_S$ 滞后于 $\dot{U}$

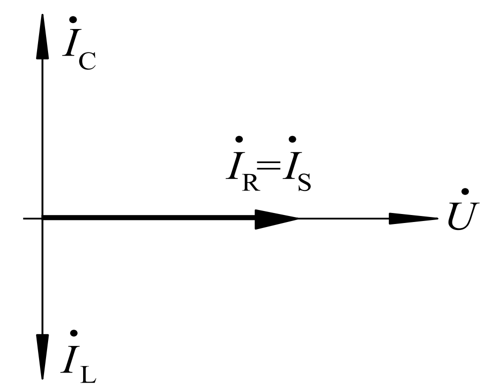
若 $\omega C = 1/\omega L$，则电纳B=0　　$\varphi' = 0$,　$\dot{I}_S$ 与 $\dot{U}$ 同相



(a) $\omega C>1/\omega L$ ($\varphi' > 0$)



(b) $\omega C<1/\omega L$ ($\varphi' < 0$)

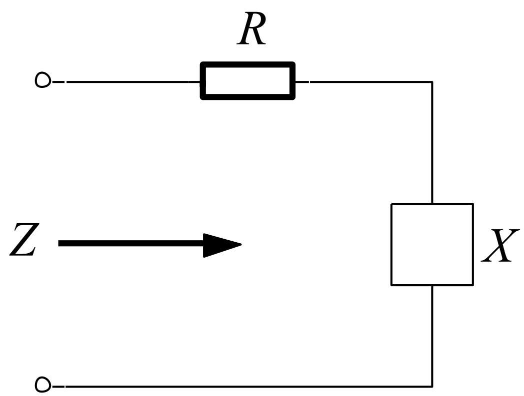


(c) $\omega C=1/\omega L$ ($\varphi' = 0$)

RLC 并联电路电压、电流相量图

§5-8 阻抗导纳的等效变换

已知阻抗Z=R+jX. 可以表示为电阻R和电抗X的串联



(a) 串联等效电路

$$\because \quad X=\omega L-\frac{1}{\omega C}$$

$\therefore$ 若$X>0$ 看作电感

若$X<0$ 看作电容

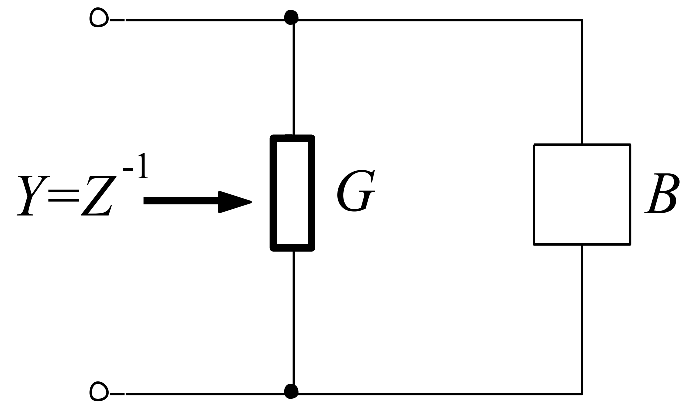
令导纳 $Y=\dfrac{1}{Z}=\dfrac{1}{R+jX}$

则
$$Y=\frac{(R-jX)}{(R+jX)(R-jX)}$$
$$=\frac{R}{R^2+X^2}-j\frac{X}{R^2+X^2}$$
$$=G+jB$$

$\because \quad B = \omega C - \dfrac{1}{\omega L}$

$\therefore$ 若 $B < 0$ 看作电感

若 $B > 0$ 看作电容



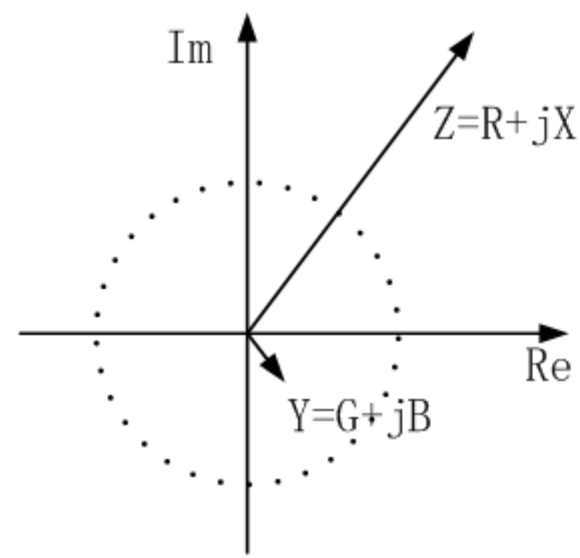
(b) 并联等效电路

若给定导纳 $Y = G + jB$

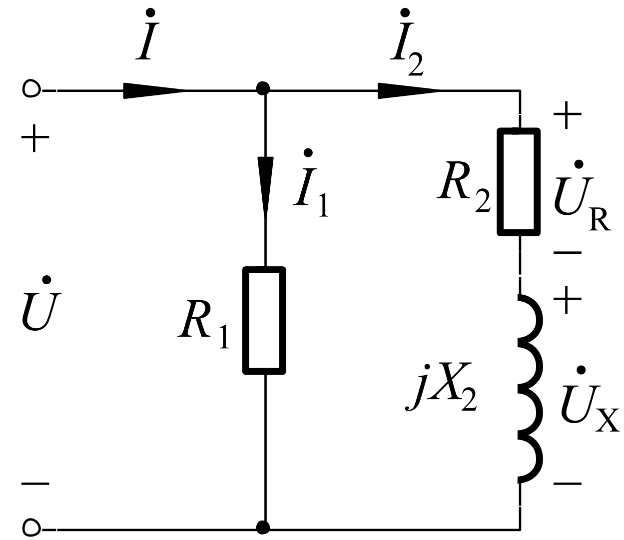
则 $Z = \dfrac{1}{Y} = \dfrac{1}{G + jB} = \dfrac{G - jB}{(G + jB)(G - jB)}$

$$= \frac{G}{G^2 + B^2} - j\frac{B}{G^2 + B^2}$$

$$= R + jX$$



例： 图中，已知 $I=\sqrt{3}A$ ，$I_1=I_2=1A$，$R_1=10\Omega$。求电感线圈的电阻R_2和感抗X_2。



解：由已知可得出$\dot{I}_1$和$\dot{I}_2$两个电流相量间夹角为60º

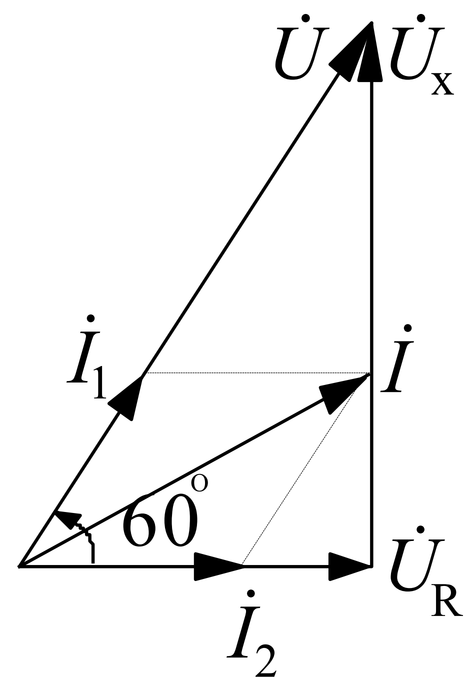
可得 $U_R = U\cos 60^\circ = 0.5U$

$$U_X = U\sin 60^\circ = 0.866U$$

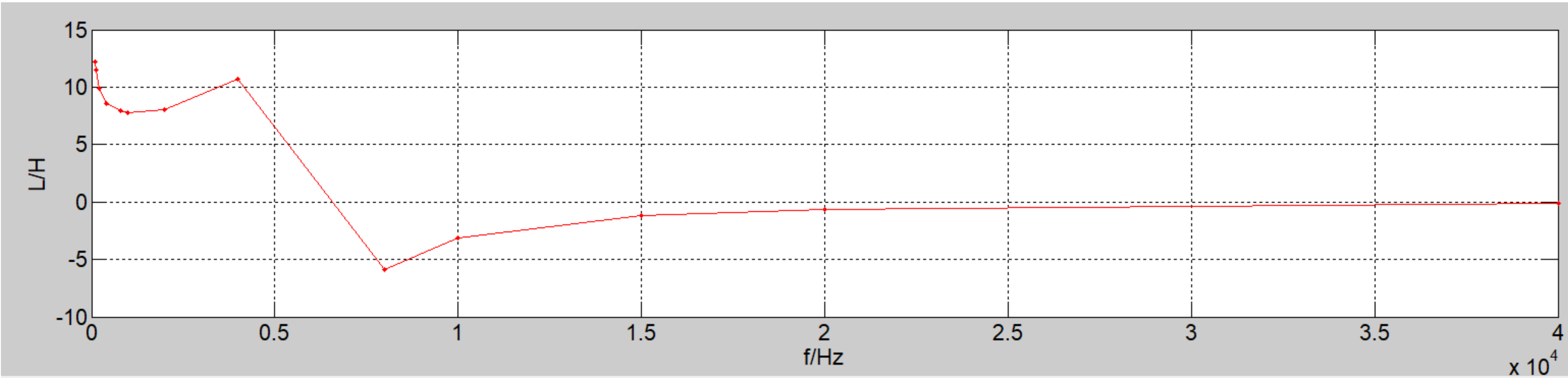
而 $U = R_1 I_1 = 10V$

则 $R_2 = \dfrac{U_R}{I_2} = \dfrac{0.5U}{1} = 5\Omega$

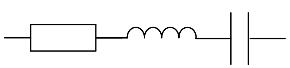
$$X_2 = \frac{U_X}{I_2} = \frac{0.866U}{1} = 8.66\Omega$$



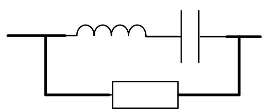
例：下图是某电感线圈在不同频率下的电感值测量结果，试分析电感值随频率变化的原因？

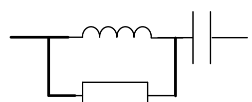


直流电阻5Ω

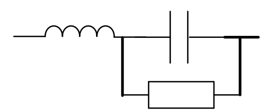


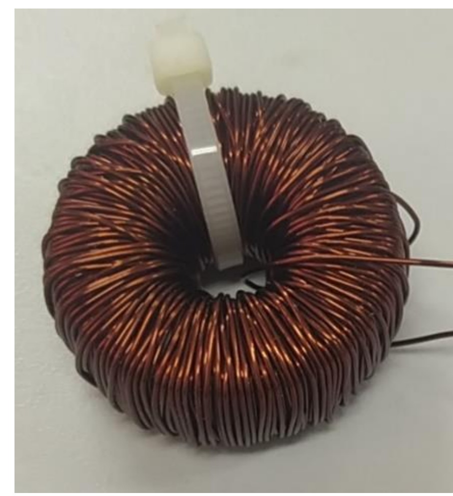
直流电阻无穷大

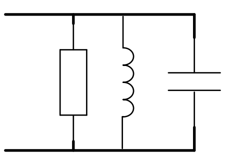




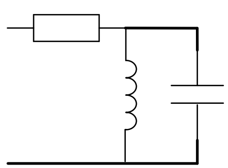
直流电阻无穷大

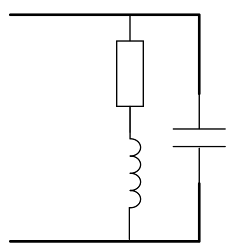


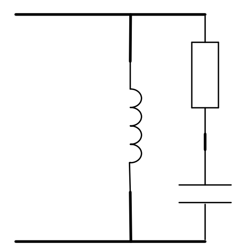




直流电阻为0

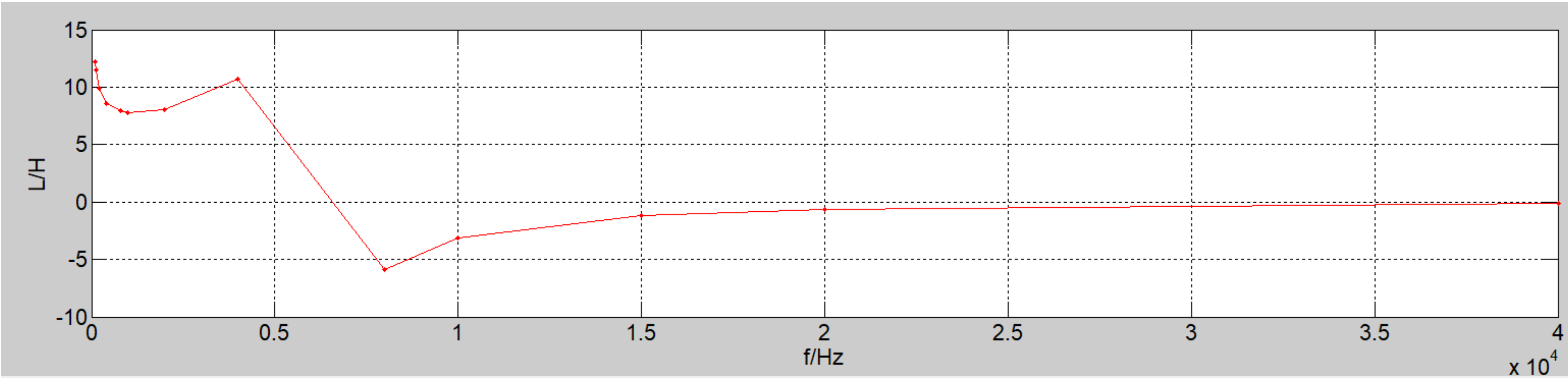


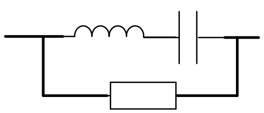




直流电阻为0

例：下图是某电感线圈在不同频率下的测量结果，试分析电感值随频率变化的原因？



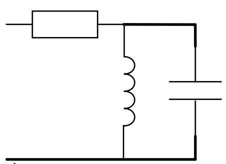


ω较小时为容性

$$Y = G + \frac{1}{j\omega L + \frac{1}{j\omega C}}$$

$$= G + j\omega \frac{C}{1-\omega^2 LC}$$

$$L_{req} = L\frac{b(\omega^2 - a)}{(\omega^2 - a)^2 + b\omega^2}$$

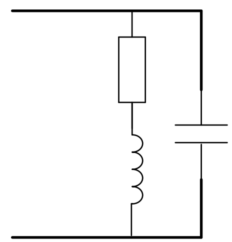


ω为某值时电流I=0

$$Z = R + \frac{1}{j\omega C + \frac{1}{j\omega L}}$$

$$= R + j\omega \frac{L}{1-\omega^2 LC}$$

$$L_{req} = \frac{L}{1-\omega^2 LC}$$

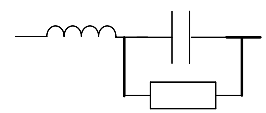


$$Y = j\omega C + \frac{1}{R + j\omega L}$$

$$= j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$= \frac{R}{R^2 + \omega^2 L^2} + j\omega\left(C - \frac{L}{R^2 + \omega^2 L^2}\right)$$

$$L_{req} = L\frac{a(a - b - \omega^2)}{(\omega^2 - a)^2 + b\omega^2}$$



ω趋近无穷时L>0

$$Z = j\omega L + \frac{1}{G + j\omega C}$$

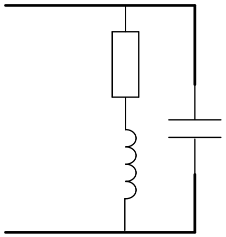
$$= j\omega L + \frac{G - j\omega C}{G^2 + \omega^2 C^2}$$

$$= \frac{G}{G^2 + \omega^2 C^2} + j\omega\left(L - \frac{C}{G^2 + \omega^2 C^2}\right)$$

$$L_{req} = L - \frac{C}{G^2 + \omega^2 C^2}$$

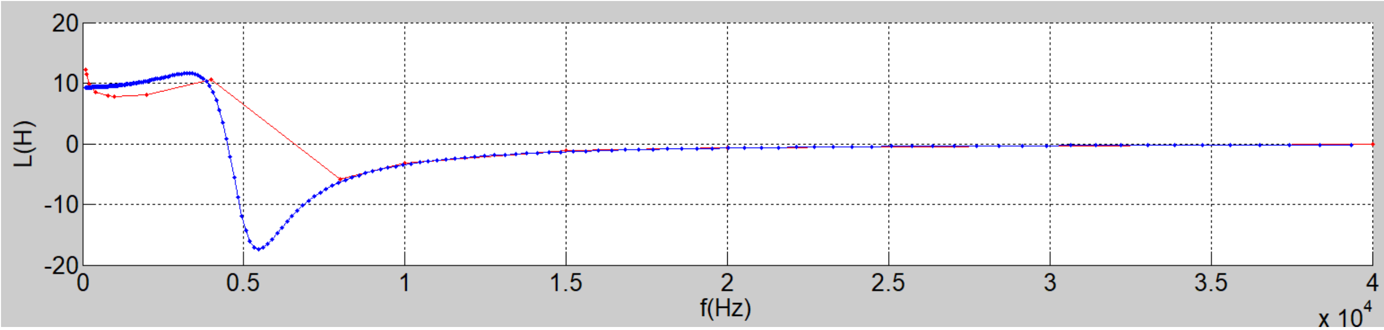
其中：$a = \frac{1}{LC}, b = \frac{1}{G^2 L^2} = \frac{R^2}{L^2}$

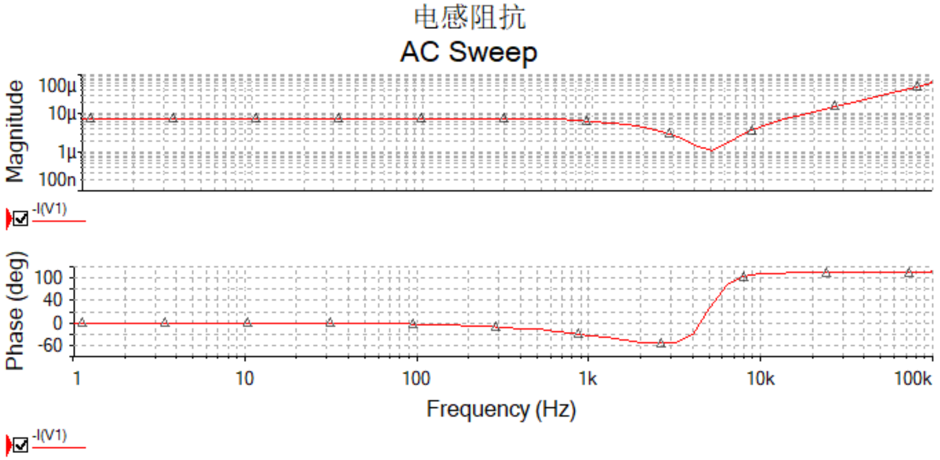
例：下图是某电感线圈在不同频率下的测量结果，试分析电感值随频率变化的原因？

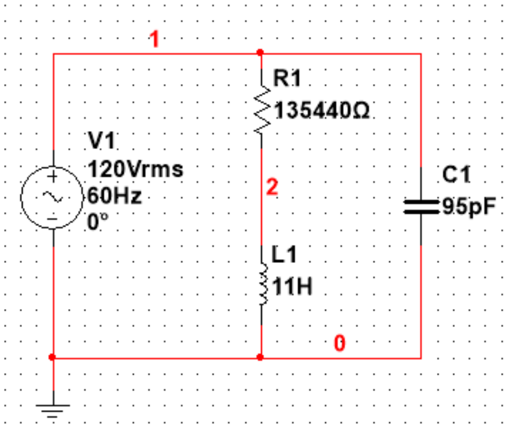


$$L_{req}=L\frac{a(a-b-\omega^2)}{(\omega^2-a)^2+b\omega^2}$$ 其中：$a=\frac{1}{LC},b=\frac{R^2}{L^2}$

Matlab参数拟合：$L=11.06H,R=135.44k\Omega,C=95.18pF$







作业：

5-15(b)

5-16

5-17

5-19

5-21