WATERLOO | ENGINEERING

Project Problem: Dynamics of a pool shot **(Deadline: 11th July'21)**

One of the best practical applications of the principle of kinetics of particles is modeling the collision of balls in the pool table. In this problem, you will formulate the motion of the colliding balls on a pool table and simulate using a computer program.

Consider a pool table with a set of 10 balls (colored, numbered 1 to 10) and a cue ball (white, numberless). The pool table is rectangular in shape and has the dimensions of $a \times b$. The mass and radius of the cue ball is m_c and r_c, respectively, while that of a numbered ball is m_n and r_n $(= r_c)$, respectively.

There are various sources of energy loss throughout the motion of the balls. In particular, we will consider the energy loss due to (a) the inelastic collision between the two bodies and (b) the friction between two bodies. The coefficient of restitution between the cue ball and a numbered ball is e_{cn}, between any two numbered balls is e_{nn}, and between any ball and the table cushion (edge) is e_{bc}. The kinetic friction coefficient between any two balls is μ_{bb}, between a ball and surface is μ_{bs}, and between a ball and the table cushion is μ_{bc}.

Rotation effects of the balls is neglected. Hence, the pool shots that induces top spin, back spin, side spin etc. in the balls need not be considered. Additionally, only the 2D motion of the balls is considered. Hence, any jump in the ball off the surface is also neglected.

1. As a first step, consider a system of two balls, a red (numbered) ball and the cue ball as shown in Fig. 1. Assume the red ball to be stationary and the cue ball approaching it at an offset distance e for the oblique central impact as shown in the Fig. 1. Determine the final velocity vectors of both the balls in terms of the known parameters. Assume the velocity of cue ball just before the collision as $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$. Consider the energy loss occurring both due to the inelastic collision and the friction.

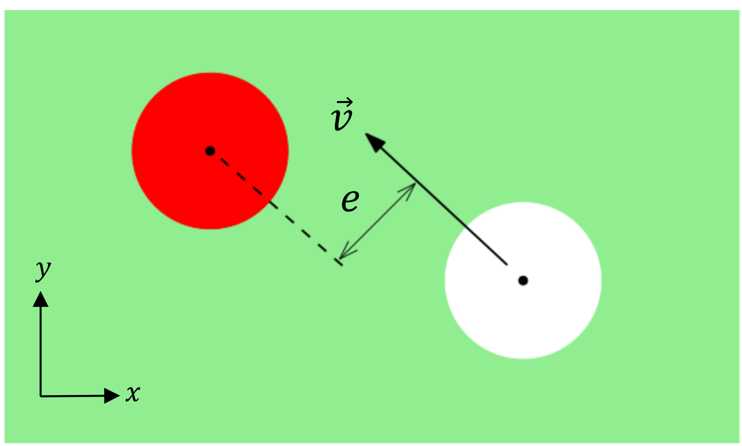

Fig. 1

2. Consider the system of all the numbered balls and the cue ball on a pool table as shown in the setup in Fig. 2. The initial coordinates (x_i, y_i) of the center of each ball with respect to the origin O will be given separately for each project group. The initial coordinates will be such that all the numbered balls will be placed to the left of the centerline AA', while the cue ball will be placed to the right of it. To begin the shot, the cue ball is impacted with the cue stick at an angle θ with the horizontal such that the initial speed is v. Using a MATLAB code, calculate the accurate positions and velocity of each ball from the instant the cue ball is hit from its initial position until all the balls come to the rest. As a final deliverable, you need to animate the entire shot (with minimalistic graphics) and export it as a video file for the following four initial conditions:
   i. A straight power shot: $v = 4\ m/s, \theta = 0°$
   ii. A fast and angled shot: $v = 2.5\ m/s, \theta = 15°$
   iii. A medium fast and angled shot: $v = 1.5\ m/s, \theta = 30°$
   iv. A medium fast shot on the nearest colored ball: $v = 1.5\ m/s$ (θ needs to be calculated by constructing a line that joins the center of the cue ball and the nearest colored ball)

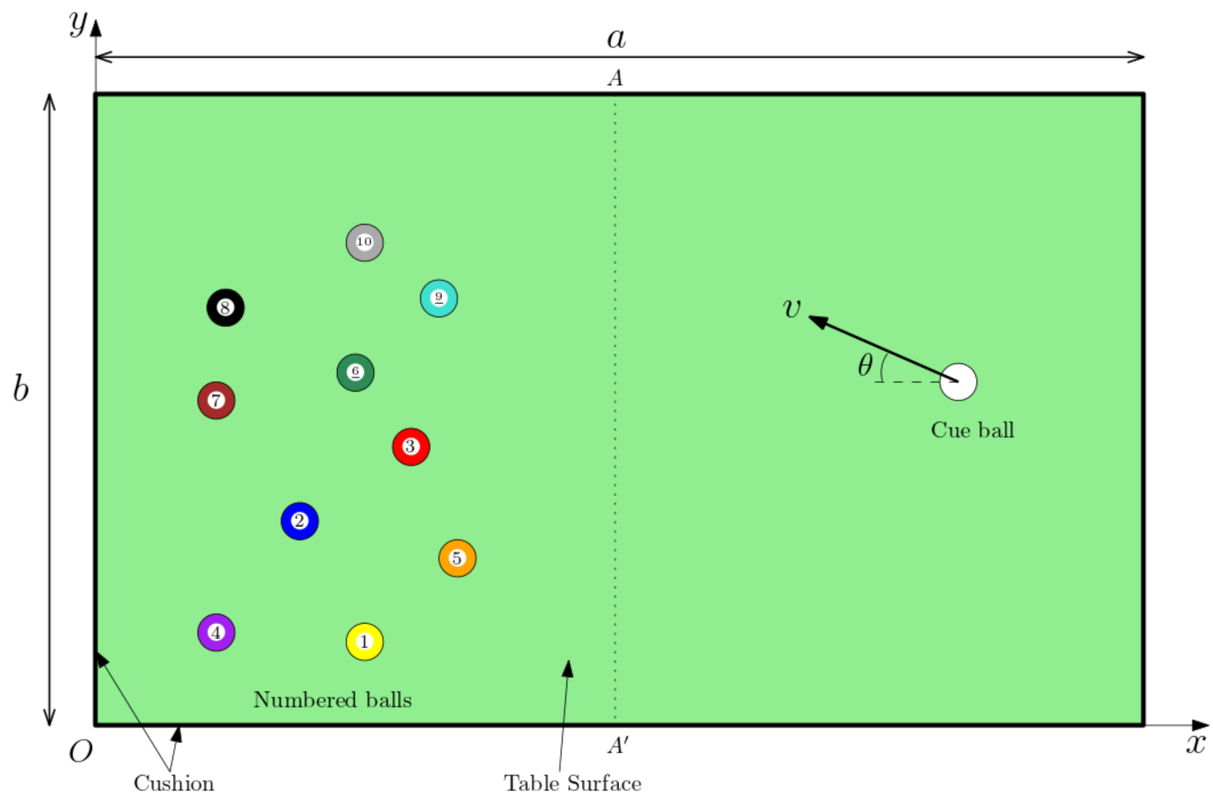

Fig. 2

Parameters: $a = 88''$, $b = 44''$, $m_c = 200\ g$, $m_n = 160\ g$, $r_c = r_n = 2.25''$, $e_{cn} = 0.95$, $e_{nn} = 0.9$, $e_{bc} = 0.7$, $\mu_{bb} = 0.05$, $\mu_{bs} = 0.1$, $\mu_{bc} = 0.2$. For any missing data, make reasonable assumptions to solve the problem and list them.

The mathematical formulations and the code used to solve the problem should also be submitted in the project report. Make sure that the code has proper comments.