Assignment 1

All assignment submissions are to be made on LEARN Quiz by 11:59 PM on the assigned due date. No extensions will be given, and assignments will not be accepted after the due date.

Problem Statement: A company developed a cheap sensor attached to a simple mechanical arm that can estimate the roughness of the surface (denoted by *x*) of different objects. This system is used in a packaging line where to detect different fruits and classify them for packaging. The measurement of roughness of 20 different apples and 20 different lemons using this system showed the measures below:

$$x_{apple} = \{2.56 \quad 3.01 \quad 3.22 \quad 3.44 \quad 3.62 \quad 3.03 \quad 2.40 \quad 2.70 \quad 2.58 \quad 3.94 \quad 2.76 \quad 3.30$$
$$2.92 \quad 3.36 \quad 2.69 \quad 2.44 \quad 2.43 \quad 3.20 \quad 2.93 \quad 2.92\}$$

$$x_{Lemon} = \{4.57 \quad 4.12 \quad 4.08 \quad 4.64 \quad 3.68 \quad 4.28 \quad 4.33 \quad 3.90 \quad 4.09 \quad 3.53 \quad 3.54 \quad 4.04$$
$$4.29 \quad 5.03 \quad 3.73 \quad 4.07 \quad 3.97 \quad 3.23 \quad 3.82 \quad 3.28\}$$

Build likelihood functions for roughness (*x*) separately for apples and lemons using above measurements. Consider that each likelihood functions can be represented by a **Normal distribution** in **both apples and lemons cases**;

for instance, $p(x|apple) = \frac{1}{\sqrt{2\pi}\sigma_1} e^{-0.5\left(\frac{x-\mu_1}{\sigma_1}\right)^2}$

Find the distribution for both likelihood functions, i.e., *p(x|Apple)* and *p(x|Lemon)*, in MATLAB or any other software.

Now, let's say we have **a pool of lemons and apples**, **including 2000 lemons** and **1000 apples**.

1. A fruit roughness was measured by the sensor which showed **x = 3.4;**
    a. **What is the likelihood of x=3.4 given it is an apple?**
    b. **What is the likelihood of x=3.4 given it is a lemon?**
    c. **What would be the decision of a max Likelihood classifier about this fruit?**

2. Now use the Bayes Rule, shown below, to **estimate the conditional probability of the fruit to be an apple or lemon given the measurements and prior knowledge? Compute P(Apple|x=3.4) and P(Lemon | x=3.4)**

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

    a. **What would be the decision of a Bayesian classifier about this fruit?**

3. As the performance in discriminating fruits was terrible, they decided to add another sensor that can detect the color. With that sensor they noticed that

P(Lemon|yellow) = 0.75, P(Apple|yellow) = 0.25,
P(Lemon | red) = 0.08, P(Apple|red) = 0.92
Now they measured a new fruit from the same pool, **its roughness x =3.4, and its color is detected yellow**. **a. What is the probability of this fruit being an apple?**
b. What is the probability of this fruit being a lemon?

Note that given each fruit the roughness and color measurements become independent.