Assignment-3

February 8, 2024

1 Assignment #3

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1.1 Problem Statement

Three 2D Anisotropic Magnetoresistor (AMR) sensors are used to measure the orientation of a permanent magnet. In an experiment, the magnet was rotated while its magnetic field direction was measured by three AMR sensors placed in close vicinity of each other.

See the data: **Assignmet3_data.mat**

Angle_new: indicates the orientation of the magnet (θ) in a 2D plane

$AMRij$: indicates ith AMR sensor jth axis

$i \in \{1,2,3\}, j \in \{x, y\};$

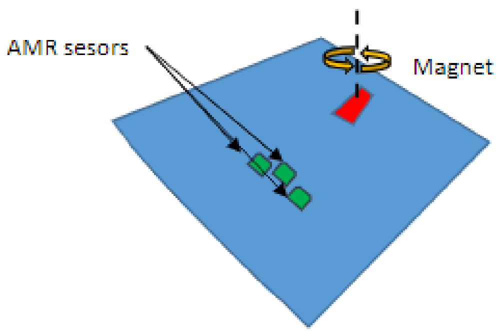


Figure 1. a schematic of AMR sensors and a parmanenet magnet.

For instance, $AMR2x$ includes the recordings of sensor 2's X channel and

$AMR3y$ includes the recordings of sensor 3's Y channel

A physical relation between the sensors channel measurements and orientation of the magent was obtained:

$$W_{i1}AMR_{ix} + W_{i0} = \sin(\theta)$$

$$W_{i3}AMR_{iy} + W_{i2} = \cos(\theta)$$

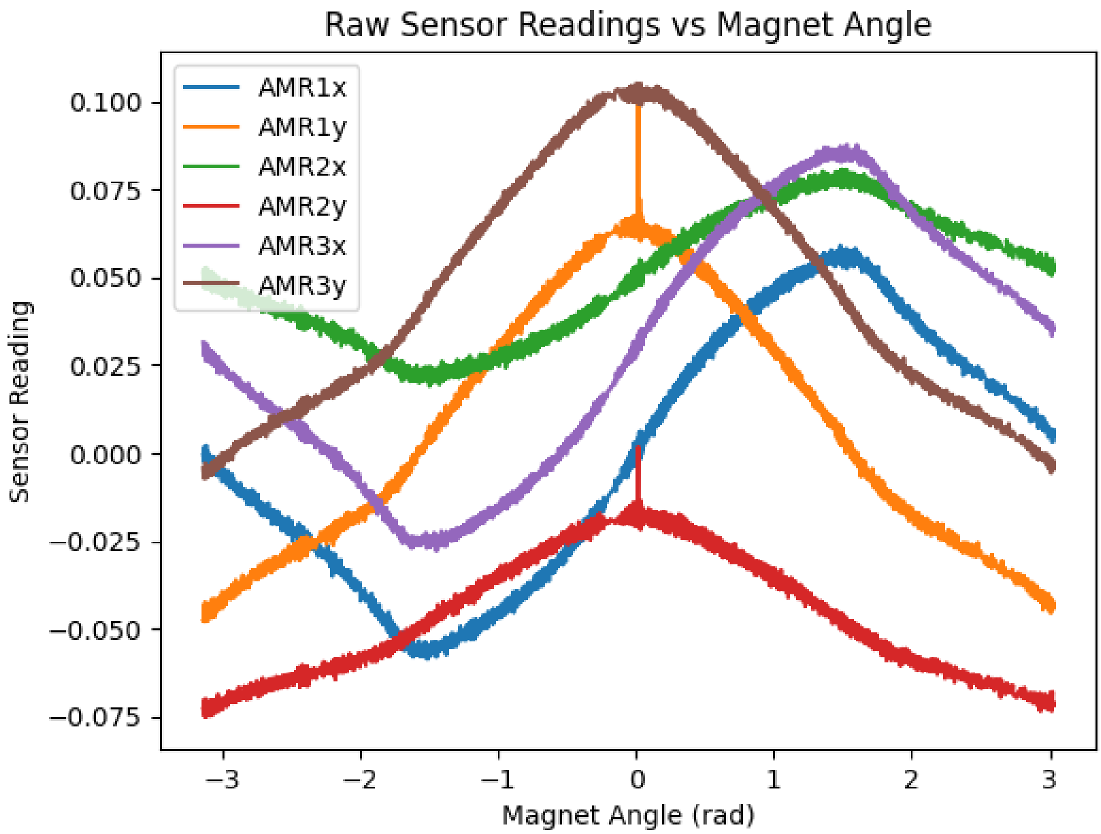
1.2 Part 1

Using ordinary least squares and linear regression, find W_S for each of the three sensors.
- *Note that you need to find 4 weights for each sensor.*
- *Also note that θ is not a linear function of sensor reading, but its* $\sin()$ *and* $\cos()$ *are linear functions of sensor readings.*

1.2.1 Imports and Data Parsing

[ ]:

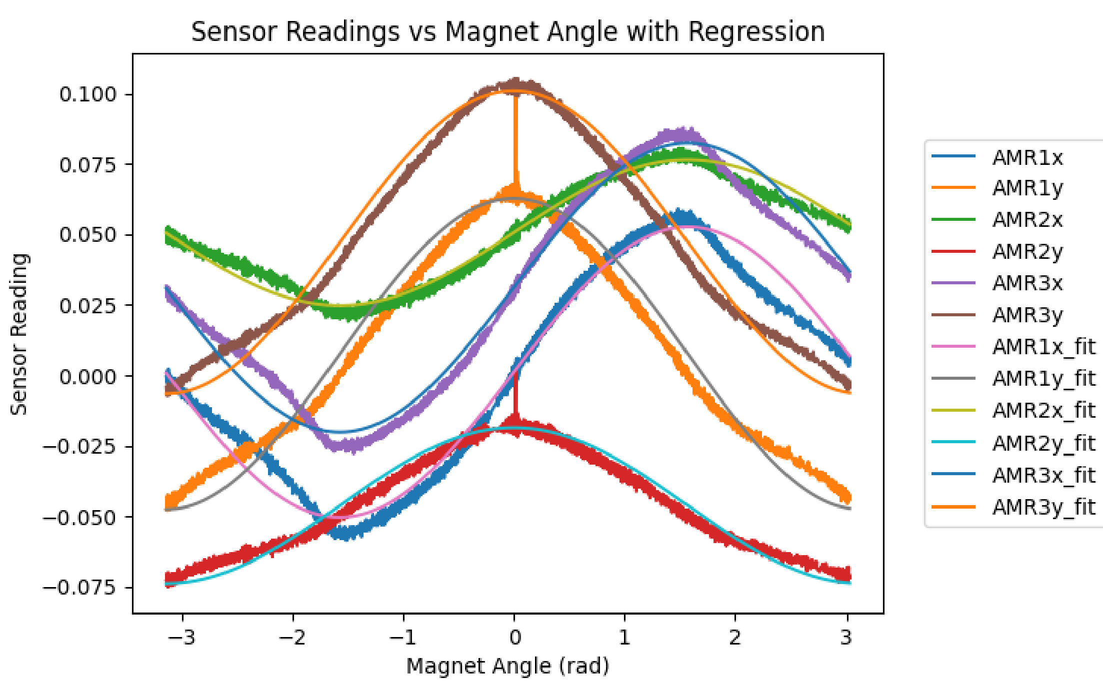
```
# Standard imports
import numpy as np
import pandas as pd
import scipy.io as sio
import statsmodels.api as sm
import matplotlib.pyplot as plt
from tabulate import tabulate

# Read data from matlab .mat file
mat = sio.loadmat('Assignment3_data.mat')

# Filter out useful data
keys = [
    'Angle_new',
    'AMR1x',
    'AMR1y',
    'AMR2x',
    'AMR2y',
    'AMR3x',
    'AMR3y'
]
sensors = ["AMR1", "AMR2", "AMR3"]
mat = dict([(key, mat[key].flatten()) for key in mat.keys() if key in keys])

# Rewrite data into pandas dataframe
df = pd.DataFrame(mat)
df.sort_values(by='Angle_new', inplace=True)

# Plot dataframe by Angle_new
for axis in keys[1:]:
    plt.plot(df['Angle_new'], df[axis], label=axis)
plt.title("Raw Sensor Readings vs Magnet Angle")
plt.xlabel("Magnet Angle (rad)")
plt.ylabel("Sensor Reading")
plt.legend()
plt.show()
```



1.2.2 Linear Regression

[ ]:
```
# Perform linear regression on each axis of each model
# x axis: us w_1 * r_ix + w_0 = sin(theta)
# y axis: us w_3 * r_iy + w_2 = cos(theta)
w = np.zeros((3, 4))
r_2 = np.zeros((3, 2))
for axis in keys[1:]:
    y = [np.sin(theta) for theta in df["Angle_new"]] if 'x' in axis else [np.
↪cos(theta) for theta in df["Angle_new"]]
    x = sm.add_constant(df[axis])
    result = sm.OLS(y, x).fit()
    w[int(axis[-2])-1, 0 if 'x' in axis else 2] = result.params['const']
    w[int(axis[-2])-1, 1 if 'x' in axis else 3] = result.params[axis]
    r_2[int(axis[-2])-1, 0 if 'x' in axis else 1] = result.rsquared

# Graph the raw data and the regression lines
x_func = lambda w, theta: (np.sin(theta) - w[0]) / w[1]
y_func = lambda w, theta: (np.cos(theta) - w[2]) / w[3]
for axis in keys[1:]:
```

```
    plt.plot(df['Angle_new'], df[axis], label=axis)
for axis in keys[1:]:
    if 'x' in axis:
        plt.plot(df['Angle_new'], x_func(w[int(axis[-2])-1,:], df['Angle_new']), label=axis+"_fit")
    else: # 'y' in axis
        plt.plot(df['Angle_new'], y_func(w[int(axis[-2])-1,:], df['Angle_new']), label=axis+"_fit")
plt.title("Sensor Readings vs Magnet Angle with Regression")
plt.xlabel("Magnet Angle (rad)")
plt.ylabel("Sensor Reading")
plt.legend(bbox_to_anchor=(1.04, 0.5), loc="center left")
plt.show()

# Print table of regression coefficients and R^2 values
print("Regression Coefficients")
props = pd.DataFrame(w, columns=["w_0", "w_1", "w_2", "w_3"]) # Add regression coefficients
props["R^2_x"] = r_2[:, 0] # Add X R^2 values
props["R^2_y"] = r_2[:, 1] # Add Y R^2 values
props["Sensor"] = list(dict.fromkeys((key[0:-1] for key in keys[1:]))) # Add sensor names
props = props[["Sensor", "w_0", "w_1", "R^2_x", "w_2", "w_3", "R^2_y"]] # Reorder columns
print(tabulate(props, headers='keys', tablefmt='psql', showindex=False, floatfmt=(".4f", ".4f", ".4f", ".4f", ".4f", ".4f")))
```



```
Regression Coefficients
+----------+---------+---------+---------+---------+---------+----------+
| Sensor   |     w_0 |     w_1 |   R^2_x |     w_2 |     w_3 |    R^2_y |
|----------+---------+---------+---------+---------+---------+----------|
| AMR1     | -0.0208 | 19.4271 |  0.9767 | -0.1365 | 18.1283 | 0.985877 |
| AMR2     | -1.9562 | 38.7100 |  0.9713 |  1.6744 | 36.1879 | 0.983693 |
| AMR3     | -0.6055 | 19.4923 |  0.9776 | -0.8763 | 18.6018 | 0.989171 |
+----------+---------+---------+---------+---------+---------+----------+
```

1.3 Part 2

Angle estimators can then be built based on each sensor model

$$\hat{\theta}_i = \text{atan2}\left(\frac{W_{i,1}AMR_{i,x} + W_{i,0}}{W_{i,3}AMR_{i,y} + W_{i,2}}\right)$$

2. Build estimation of magnet angles based on each sensor separately, $\hat{\theta}_1$, $\hat{\theta}_2$, and $\hat{\theta}_3$. Compute the error for each estimator:

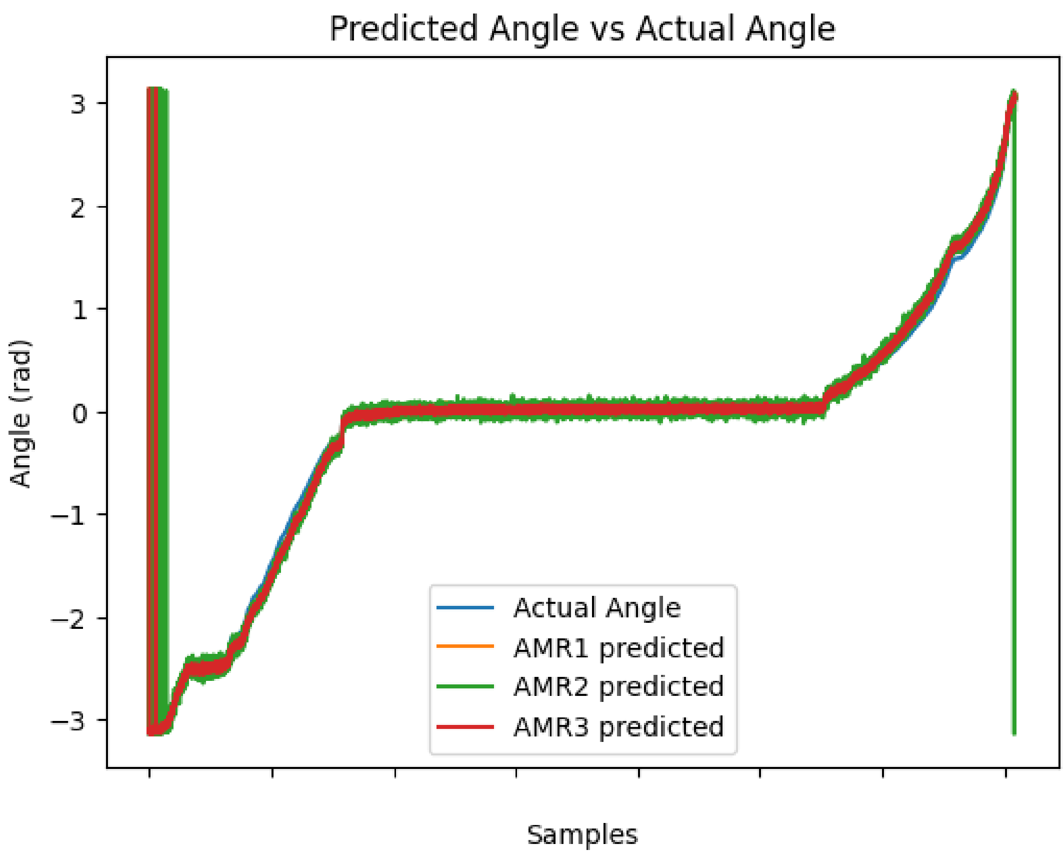
$$E_i = \theta - \hat{\theta}_i$$

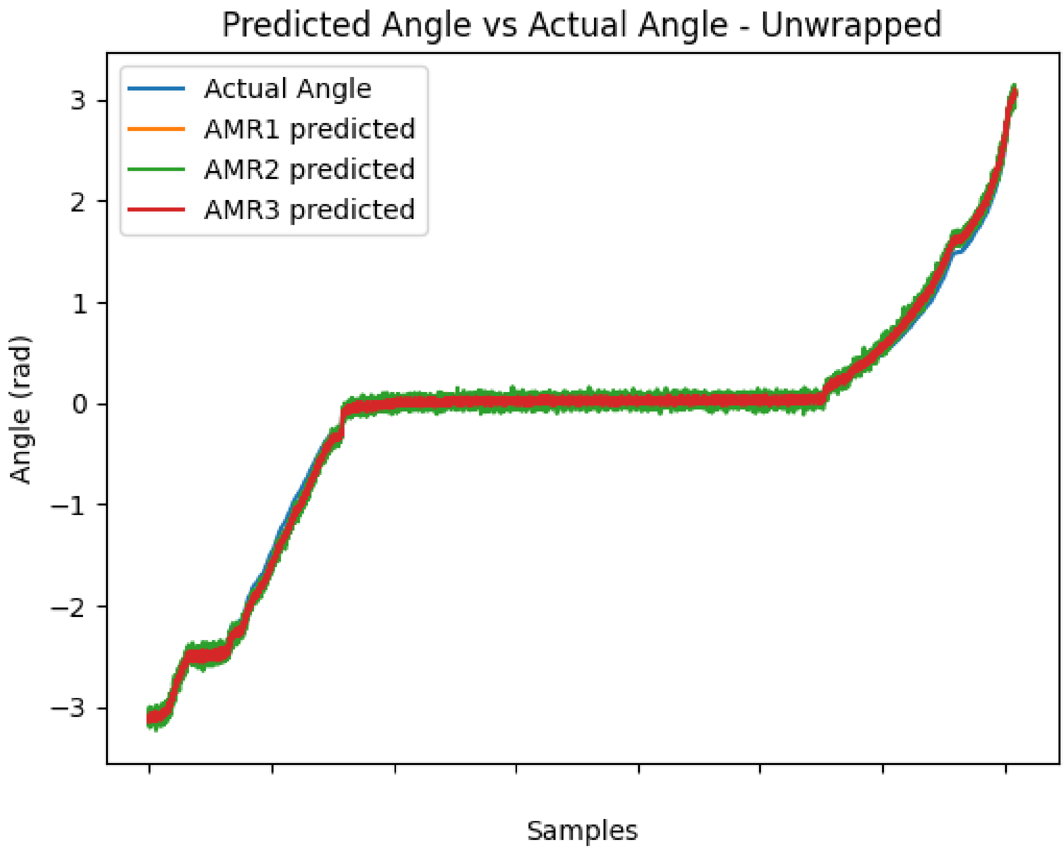
   Compute each estimator's mean error and variance of error.

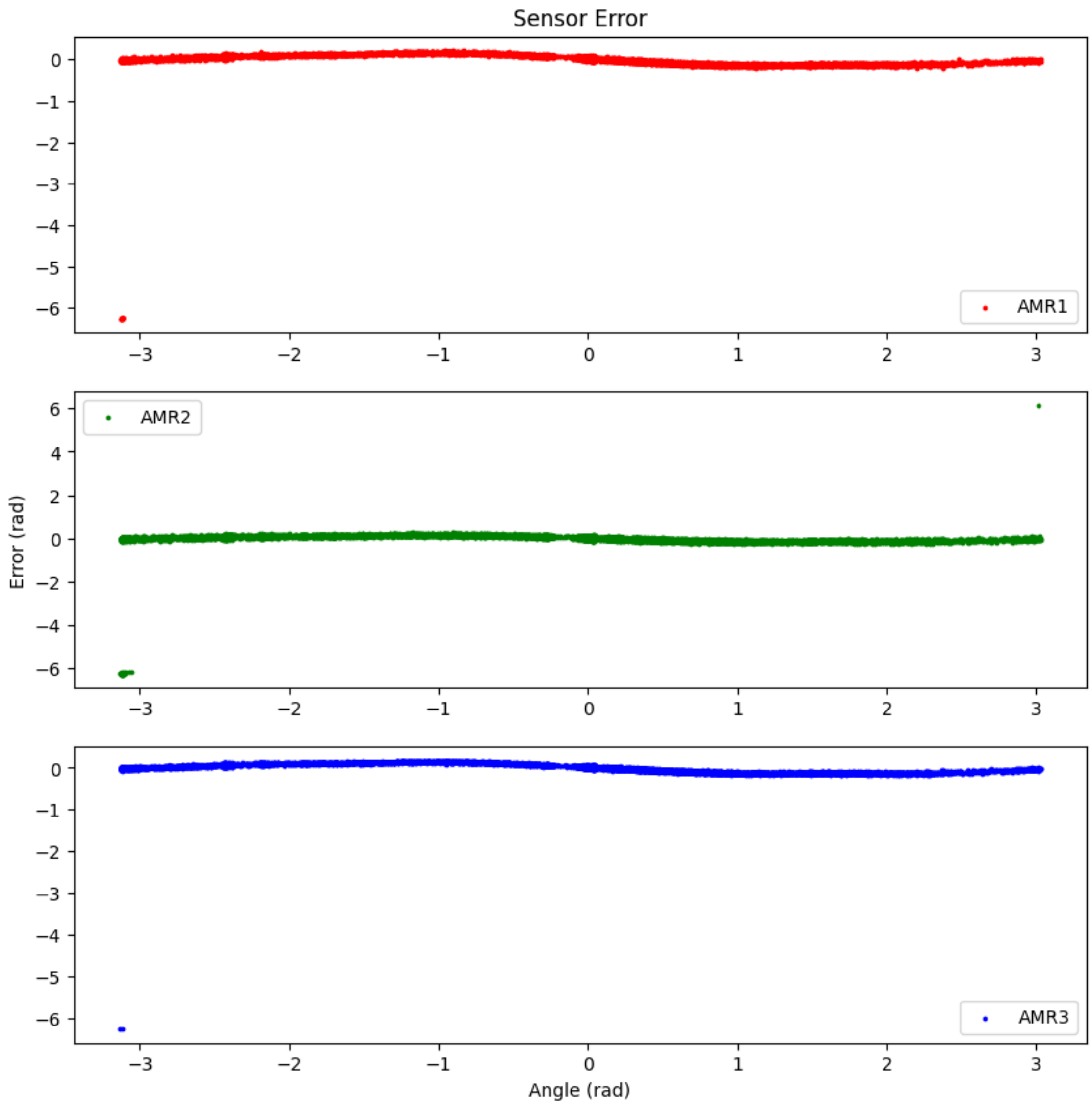
[ ]:
```python
# Use the regression coefficients to predict the angle
# and determine the error of each sensor
theta = lambda w, x, y: np.arctan2(x * w[1] + w[0], y * w[3] + w[2])

# Calculate the predicted angle for each sensor
for sensor in sensors:
    df[sensor+'_pred'] = theta(w[int(sensor[-1])-1, :], df[sensor+'x'],␣
 ↪df[sensor+'y'])

# Correct for radian wrap-around
for sensor in sensors:
    df[sensor+'_pred_unwrap'] = np.unwrap(df[sensor+'_pred'])
    if max(df[sensor+'_pred_unwrap']) > 2.1 * np.pi:
        df[sensor+'_pred_unwrap'] = df[sensor+'_pred_unwrap'] - 2 * np.pi

# Graph the predicted angle vs the actual angle
plt.plot(range(len(df['Angle_new'])), df['Angle_new'], label="Actual Angle")
for sensor in sensors:
    plt.plot(range(len(df[sensor+'_pred'])), df[sensor+'_pred'], label=sensor+"␣
 ↪predicted")
plt.title("Predicted Angle vs Actual Angle")
plt.xticks(color='w')
plt.xlabel("Samples")
plt.ylabel("Angle (rad)")
```

```
plt.legend()
plt.show()

# Graph the predicted angle vs the actual angle (unwrapped)
plt.plot(range(len(df['Angle_new'])), df['Angle_new'], label="Actual Angle")
for sensor in sensors:
    plt.plot(range(len(df[sensor+'_pred_unwrap'])), df[sensor+'_pred_unwrap'],␣
 ↪label=sensor+" predicted")
plt.title("Predicted Angle vs Actual Angle - Unwrapped")
plt.xticks(color='w')
plt.xlabel("Samples")
plt.ylabel("Angle (rad)")
plt.legend()
plt.show()

# Calculate error at each angle
for sensor in sensors:
    df[sensor+'_error'] = df['Angle_new'] - df[sensor+'_pred']
    df[sensor+'_error_unwrap'] = df['Angle_new'] - df[sensor+'_pred_unwrap']

# Graph the error of each sensor as a sub scatter plot
fig, axs = plt.subplots(3, 1, figsize=(10, 10))
for i, sensor in enumerate(sensors):
    # axs[i].hlines(y=[-0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3], xmin=-np.pi,␣
 ↪xmax=np.pi, colors='grey', linestyles='--', linewidth=0.5)
    axs[i].scatter(df['Angle_new'], df[sensor+'_error'], label=sensor, s=3,␣
 ↪color=['r', 'g', 'b'][i])
    axs[i].set_title("Sensor Error") if i == 0 else axs[i].set_title("")
    if i == 2: axs[i].set_xlabel("Angle (rad)")
    if i == 1: axs[i].set_ylabel("Error (rad)")
    # axs[i].set_ylim(-0.3, 0.3)
    axs[i].legend()
plt.show()

# Graph the error of each sensor as a sub scatter plot (unwrapped)
fig, axs = plt.subplots(3, 1, figsize=(10, 10))
for i, sensor in enumerate(sensors):
    axs[i].hlines(y=[-0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3], xmin=-np.pi, xmax=np.
 ↪pi, colors='grey', linestyles='--', linewidth=0.5)
    axs[i].scatter(df['Angle_new'], df[sensor+'_error_unwrap'], label=sensor,␣
 ↪s=3, color=['r', 'g', 'b'][i])
    axs[i].set_title("Sensor Error - Unwrapped") if i == 0 else axs[i].
 ↪set_title("")
    if i == 2: axs[i].set_xlabel("Angle (rad)")
    if i == 1: axs[i].set_ylabel("Error (rad)")
    axs[i].set_ylim(-0.3, 0.3)
```

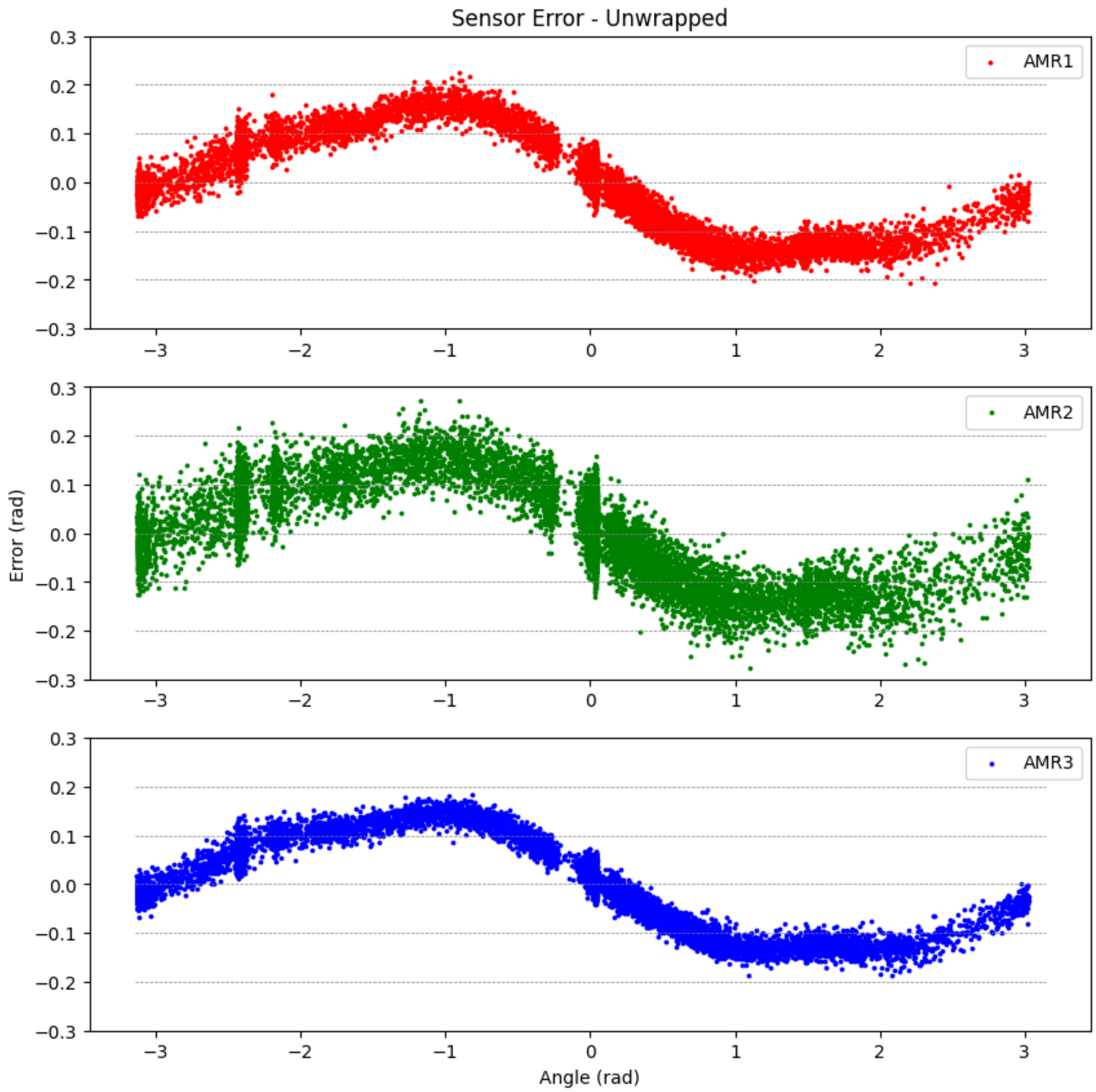
```
    axs[i].legend()
plt.show()

# Determine the mean error and variance of error for each sensor
props["Mean Err"] = [np.mean(np.abs(df[sensor+'_error'])) for sensor in sensors]
props["Mean Err Unwrp"] = [np.mean(np.abs(df[sensor+'_error_unwrap'])) for␣
 ↪sensor in sensors]
props["Var of Err"] = [np.var(df[sensor+'_error']) for sensor in sensors]
props["Var of Err Unwrp"] = [np.var(df[sensor+'_error_unwrap']) for sensor in␣
 ↪sensors]
print("Sensor Properties with Error")
display = props[["Sensor", "Mean Err", "Mean Err Unwrp", "Var of Err", "Var of␣
 ↪Err Unwrp"]]
print(tabulate(display, headers='keys', tablefmt='psql', showindex=False,␣
 ↪floatfmt=(".4f", ".4f", ".4f", ".4f", ".4f")))
```




Predicted Angle vs Actual Angle - Unwrapped
Actual Angle
AMR1 predicted
AMR2 predicted
AMR3 predicted
3
2
1
0
−1
−2
−3
Angle (rad)
Samples



Sensor Error
AMR1
AMR2
AMR3
Error (rad)
Angle (rad)



Sensor Properties with Error

Sensor	Mean Err	Mean Err Unwrp	Var of Err	Var of Err Unwrp
AMR1	0.0558	0.0520	0.0294	0.0051
AMR2	0.0786	0.0598	0.1244	0.0061
AMR3	0.0490	0.0483	0.0089	0.0044

1.3.1 IMPORTANT NOTE

Due to the boundary conditions of the angle measurements wrapping around at $\pm\pi$, there are cases where the predicted angle measurement wraps around the boundary. For example, if the true angle is $\pi * 0.999$ and the predicted angle is $pi * 1.001$, it will report as $\pi * -0.999$, resulting in an error of almost 2π. To avoid this, each measurement is inspected to prevent it from wrapping the boundary. This is done by checking if the difference between the true and predicted angle is greater than π (as it should never have that high of a prediction error) and if so, adding 2π to bring it to the correct error distance from the true angle.

This seems to be an error with the calculation done for the Quiz answers, as they are not entirely accurate!!!

1.4 Part 3

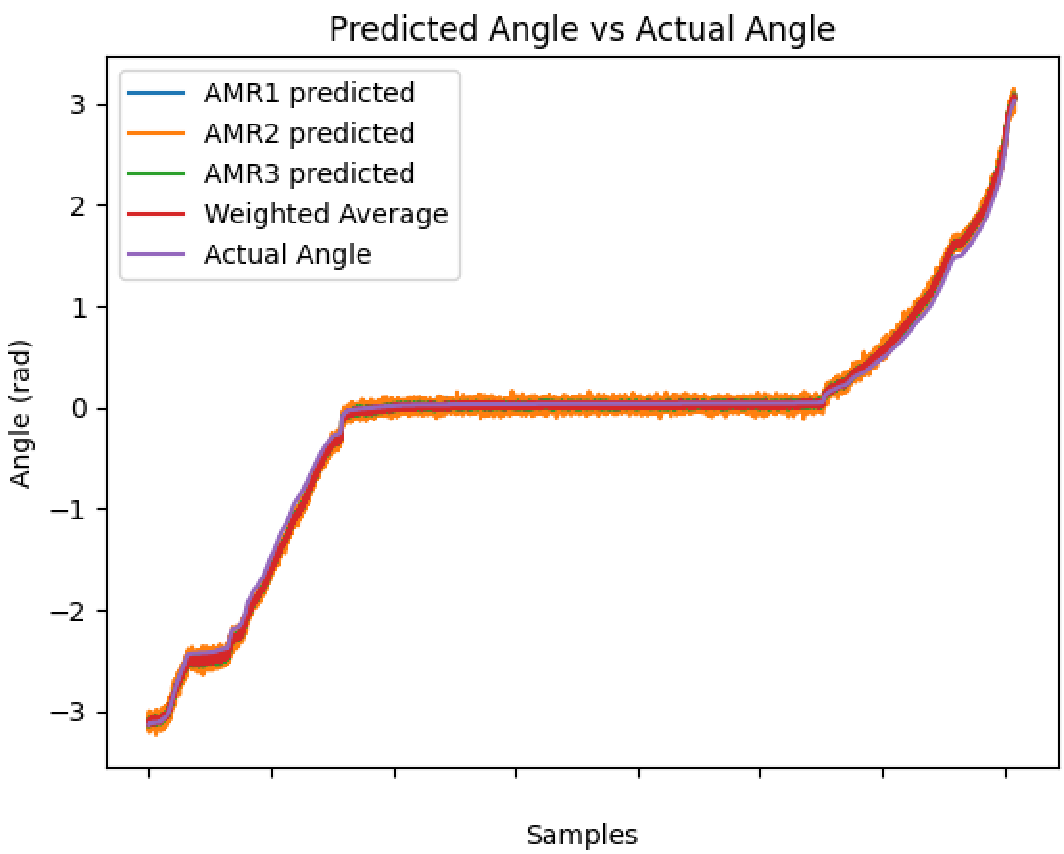
Now fuse the three estimators using weighted sum fusion (See Week 3, Lecture 8, recorded session 9) based on inverse of their error variance.

Sensor fusion equation:

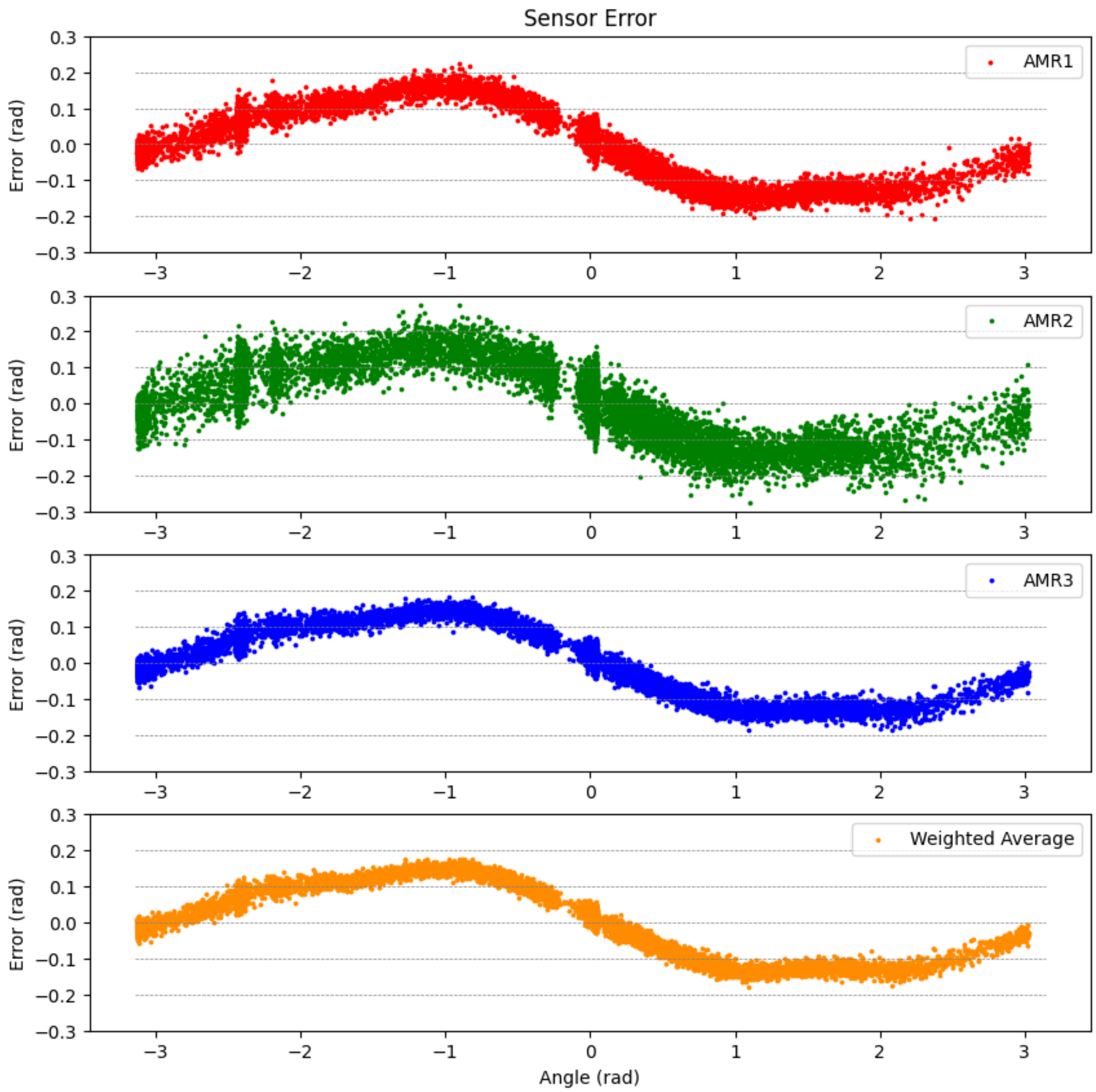
$$\hat{y}_{ws} = \frac{\frac{y_1}{{\sigma_1}^2} + \frac{y_2}{{\sigma_2}^2} + \cdots + \frac{y_n}{{\sigma_n}^2}}{\frac{1}{{\sigma_1}^2} + \frac{1}{{\sigma_2}^2} + \cdots + \frac{1}{{\sigma_n}^2}} \text{ Where: } \sigma_i^2 = \text{var}(\text{Error}_i)$$

[ ]:
```
# Create weighted average of sensor predictions
df['Angle_pred'] = (df['AMR1_pred_unwrap'] / props["Var of Err"][0] +␣
↪df['AMR2_pred_unwrap'] / props["Var of Err"][1] + df['AMR3_pred_unwrap'] /␣
↪props["Var of Err"][2]) / (1/props["Var of Err"][0] + 1/props["Var of␣
↪Err"][1] + 1/props["Var of Err"][2])

# Plot each sensors prediction and the weighted average
for sensor in sensors:
    plt.plot(range(len(df[sensor+'_pred_unwrap'])), df[sensor+'_pred_unwrap'],␣
↪label=sensor+" predicted")
plt.plot(range(len(df['Angle_pred'])), df['Angle_pred'], label="Weighted␣
↪Average")
plt.plot(range(len(df['Angle_new'])), df['Angle_new'], label="Actual Angle")
plt.title("Predicted Angle vs Actual Angle")
plt.xticks(color='w')
plt.xlabel("Samples")
plt.ylabel("Angle (rad)")
plt.legend()
plt.show()

# Calculate the error of the weighted average
df['Angle_error'] = df['Angle_new'] - df['Angle_pred']

# Graph the error of the weighted average and the error of each sensor
fig, axs = plt.subplots(4, 1, figsize=(10, 10))
for i, sensor in enumerate(sensors):
    axs[i].hlines(y=[-0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3], xmin=-np.pi, xmax=np.
↪pi, colors='grey', linestyles='--', linewidth=0.5)
    axs[i].scatter(df['Angle_new'], df[sensor+'_error'], label=sensor, s=3,␣
↪color=['r', 'g', 'b'][i])
```

```python
    axs[i].set_title("Sensor Error") if i == 0 else axs[i].set_title("")
    axs[i].set_ylabel("Error (rad)")
    axs[i].set_ylim(-0.3, 0.3)
    axs[i].legend()
axs[3].hlines(y=[-0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3], xmin=-np.pi, xmax=np.pi,␣
 ↪colors='grey', linestyles='--', linewidth=0.5)
axs[3].scatter(df['Angle_new'], df['Angle_error'], label="Weighted Average",␣
 ↪s=3, color='darkorange')
axs[3].set_ylabel("Error (rad)")
axs[3].set_ylim(-0.3, 0.3)
axs[3].set_xlabel("Angle (rad)")
axs[3].legend()
plt.show()

# Determine the mean error and variance of error for the weighted average
ws = pd.DataFrame({
    "Sensor": "Weighted Sum",
    "Mean Err Unwrp": np.mean(df['Angle_error']),
    "Var of Err Unwrp": np.var(df['Angle_error'])
}, index=[0])
props_ext = pd.concat([props, ws], ignore_index=True)
display = props_ext[["Sensor", "Mean Err Unwrp", "Var of Err Unwrp"]]
print("Sensor Properties with Error")
print(tabulate(display, headers='keys', tablefmt='psql', showindex=False,␣
 ↪floatfmt=(".4f", ".4f", ".4f", ".4f", ".4f")))
```



Predicted Angle vs Actual Angle
AMR1 predicted
AMR2 predicted
AMR3 predicted
Weighted Average
Actual Angle
Angle (rad)
3
2
1
0
−1
−2
−3
Samples



Sensor Properties with Error

Sensor	Mean Err Unwrp	Var of Err Unwrp
AMR1	0.0520	0.0051
AMR2	0.0598	0.0061
AMR3	0.0483	0.0044
Weighted Sum	0.0067	0.0045