空間資訊平差處理與分析
Adjustment and Analysis of Spatial Information
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Homework No.3
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1 Abstract

This report is based on the Home work No.3, also is continuation of Homework No.1, No.2. Compared to the Observation Only (O.O.) approach this tasks repeats the previous' content with the Indirect Observations (I.O.) approach. And try to compare the results obtained in HW02 and HW03 to see if they are identical.

The matlab code will attach in $4 and upload to github.

Contents

2 Adjustment by I.O. approach.

Repeat Homework No.2 but now with Indirect Observations (I.O.) approach. *Adjust the leveling network that you simulated in HW01 with Observations Only method.*

- *What are n, n_0, and r in this model?*
- *Compute the residuals and adgusted observables by a long-hand approach.*
- *Compute the residuals and adjusted observables by a matrix approah.*
- *What are the σ_0 and $\hat{\sigma_0}$ values?*
- *What will happen if you choose a different σ_0 value (for instance, double the value) in your adjustment computation.*
- *What will happen if you choose a different weight scheme (for instance, $w_i \propto 1/\sigma_i$)?*
- *Is there any assumption inherent in your computation?*

2.1 The leveling network

I have generated an level network in the Homework No.1 as Fig.1 (Same as Homework No.2), which is constructed by 13 station points, 4 loops. All station measurement is recorded clockwisely. And here is the Loops:

1. Loop I P1-P8-P7-P3-P12-P10-P1
2. Loop II P1-P6-P4-P11-P8-P1
3. Loop III P8-P11-P9-P13-P12-P3-P7-P8
4. Loop IV P13-P2-P5-P6-P1-P10-P12-P13

And the meansurement data is recorded as Table 1, the sufficient significant digit is [mm], in the other word. The data ΔH and $\Delta H'$ is recorded to 3 to decimal places, error is 2 (scientific notation). 'error' is corrected in this Homework.

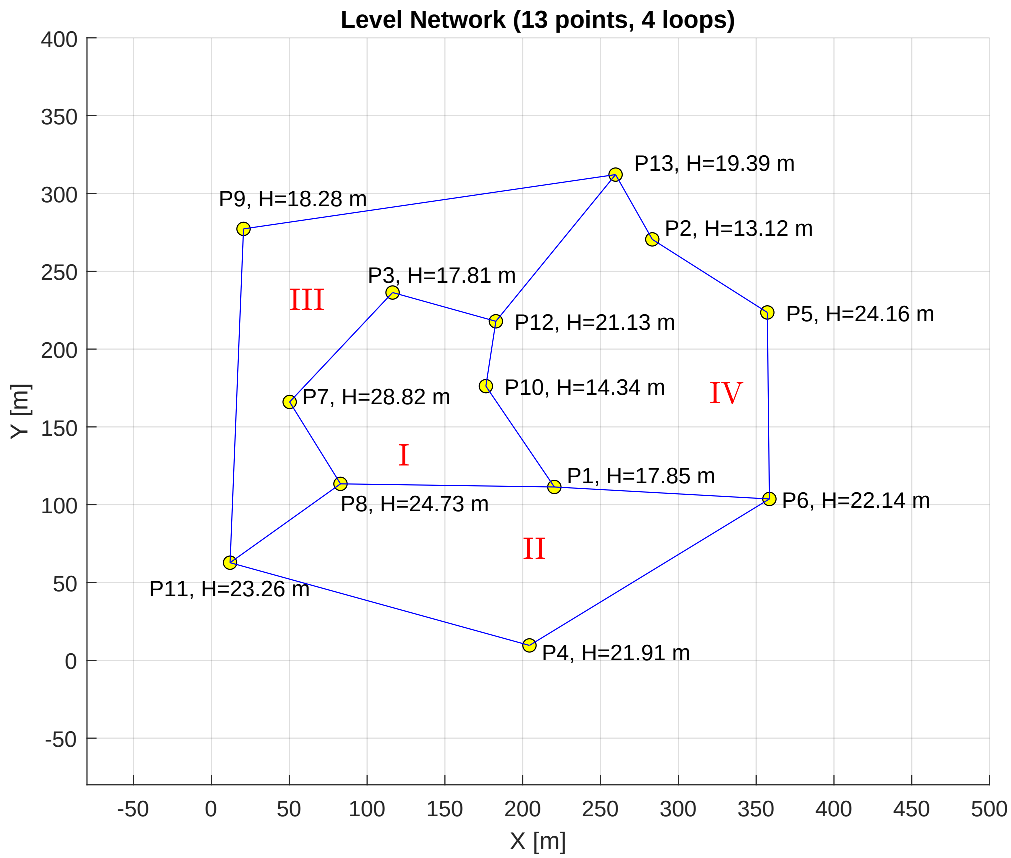

Figure 1: Level Network

Table 1: Measurement data of stations

Number	BS_Pt	FS_Pt	ΔH[m]	$\Delta H'$[m]	error [m]
1	1	8	6.880	6.878	1.74×10^{-3}
2	8	7	4.089	4.088	-5.38×10^{-4}
3	7	3	-11.006	-11.007	-2.74×10^{-4}
4	3	12	3.314	3.312	-1.47×10^{-3}
5	12	10	-6.784	-6.785	-6.05×10^{-4}
6	10	1	3.508	3.507	-3.02×10^{-4}
7	1	6	4.287	4.290	3.45×10^{-3}
8	6	4	-0.226	-0.225	1.22×10^{-3}
9	4	11	1.349	1.346	-2.96×10^{-3}
10	11	8	1.470	1.469	-1.21×10^{-3}
11	11	9	-4.976	-4.976	-2.02×10^{-4}
12	9	13	1.112	1.108	-4.16×10^{-3}
13	13	12	1.731	1.730	-9.43×10^{-4}
14	13	2	-6.279	-6.279	9.82×10^{-5}
15	2	5	11.041	11.038	-3.25×10^{-4}
16	5	6	-2.021	-2.021	-1.15×10^{-4}

2.2 My Answer to sub-question-1

For my leveling network:
Fixed elevation of station 1 $H_1 = 17.85[m]$Measurement number: $n = 16$, number of unknown $n_0 = u = 12$ (equals to the station number-1) and redundancy $r = n - u = 4$.

2.3 Residuals and adjusted observables

I.O. approach starts

Since the variables in the long-hand approach will using matrix to solve (too many variables), I write the long-hand approach and matrix approach as one section.

Note the difference $\Delta H_i'(= l)$ for line number i as l_i, corrected observatinon $\hat{l}_i = l_i + v_i$

Table 2: The relation of ΔH

Number	1	2	3	4	5	6	7	8
BS_Pt	1	8	7	3	12	10	1	6
FS_Pt	8	7	3	12	10	1	6	4
Number	9	10	11	12	13	14	15	16
BS_Pt	4	11	11	9	13	13	2	5
FS_Pt	11	8	9	13	12	2	5	6

Since the goal is to calculate the "relative height", I set the height $H_1 = 17.85m$ to the real height and introduce it into the calculation. So far, for each $\Delta H'(= l)$, I can write the equaiton

$$l = F(x); \quad \mathbf{v} = \mathbf{B}\hat{\mathbf{x}} + \mathbf{f}; \quad \hat{l} = l + v \tag{1}$$

where v is residuals, $\hat{\mathbf{x}}$ is the corrected height of stations, $\mathbf{B}$ is the coefficient matrix of $\mathbf{x}$, $\mathbf{f}$ is a column vector composed by l and H_1 for example:

$$l_1 + v_1 = \hat{H}_8 - H_1 \Leftrightarrow v_1 = [0 \ldots \underbrace{1}_{7-th} \ldots 0]\mathbf{x} + (-H_1 - v_1) \tag{2}$$

$$\mathbf{x} = \begin{bmatrix} \hat{H}_2 & \hat{H}_3 & \hat{H}_4 & \ldots & \hat{H}_{12} & \hat{H}_{13} \end{bmatrix}^T \tag{3}$$

At the same way:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_{15} \\ v_{16} \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} \hat{H}_2 \\ \hat{H}_3 \\ \hat{H}_4 \\ \vdots \\ \hat{H}_{15} \\ \hat{H}_{16} \end{bmatrix} + \begin{bmatrix} -6.880 - 17.85 \\ -4.089 \\ 11.006 \\ 3.314 \\ 6.784 \\ -3.508 + 17.85 \\ -4.287 - 17.85 \\ 0.226 \\ -1.349 \\ -1.470 \\ 4.976 \\ -1.112 \\ -1.731 \\ 6.279 \\ -11.041 \\ 2.021 \end{bmatrix} \tag{4}$$

$$\text{where } B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix} B_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Due to the constraint of my LaTeX compiler, TeXworks. I write as this form.
To compute the minimum of $\Phi = \sum v^2$

$$\begin{aligned} \Phi &= v_1^2 + \ldots + v_{16}^2 \\ &= (H_8 + l_1)^2 + \ldots + (-H_5 + H_6 + l_{16})^2 \end{aligned} \tag{5}$$

Which is hard to solve by hand rather than O.O. method.

So use the matrix method (theoritically equivalent to long-hand method)

$$\begin{aligned}\Phi &= \mathbf{v}^T\mathbf{w}\mathbf{v} \\ &= (\mathbf{B}\mathbf{x}+\mathbf{f})^T\mathbf{w}(\mathbf{B}\mathbf{x}+\mathbf{f}) \\ &= (\mathbf{x}^T\mathbf{B}^T\mathbf{w}+\mathbf{f}^T\mathbf{w})(\mathbf{B}\mathbf{x}+\mathbf{f}) \\ &= \mathbf{x}^T\mathbf{B}^T\mathbf{w}\mathbf{B}\mathbf{x}+2\mathbf{f}^T\mathbf{w}\mathbf{B}\mathbf{x}+\mathbf{f}^T\mathbf{w}\mathbf{f}\end{aligned} \tag{6}$$

take partial derivative to Φ by x to get where Φ_{min} exists

$$\begin{aligned}\frac{\partial\Phi}{\partial\mathbf{x}} &= 2\mathbf{x}^T\mathbf{B}^T\mathbf{B}+2\mathbf{f}^T\mathbf{B}^T\mathbf{w}\mathbf{B}=0 \\ &\Rightarrow \mathbf{x}^T\mathbf{B}^T\mathbf{w}\mathbf{B} = -\mathbf{f}^T\mathbf{w}\mathbf{B} \\ &\Rightarrow \mathbf{B}^T\mathbf{w}\mathbf{B}\mathbf{x} = -\mathbf{B}^T\mathbf{w}\mathbf{f}\end{aligned} \tag{7}$$

Thus, we have $\mathbf{x} = (\mathbf{B}^T\mathbf{w}\mathbf{B})^{-1}(-\mathbf{B}^T\mathbf{w}\mathbf{f})$
And here for weight matrix $\mathbf{w}$ (an diagonal matrix), I have two sets:

1. equal weight $w_{i,j=i} = 1$

   the result is equivalent to long-hand method. Beacuse the long-hand approach is based on the equal weight assumption.

2. unequal weight $w_{i,j=i} = 1/\sigma_i^2$, $\sigma_i = \sigma_0\sqrt{L_i}$

   where the priori standard deviation $\sigma_0 = 7mm/\sqrt{km}$, L_i is the distance between two stations.

Table 3: Elevation H and Residual v after matrix method
(same weight, equal long-hand's result)

No.	H[m] real	H[m] calculated	No.	l [m] ($\Delta H'$)	v [m]	$\hat{l}$ [m]
1	17.85	[17.85]	1	6.88	1.61×10^{-3}	6.880
2	13.12	13.12	2	4.088	7.04×10^{-4}	4.089
3	17.81	17.81	3	-11.007	7.04×10^{-4}	-11.006
4	21.91	21.91	4	3.312	7.04×10^{-4}	3.314
5	24.16	24.16	5	-6.785	6.04×10^{-4}	-6.784
6	22.14	22.14	6	3.507	6.04×10^{-4}	3.508
7	28.82	28.82	7	4.290	-1.01×10^{-3}	4.289
8	24.73	24.73	8	-0.225	6.48×10^{-4}	-0.225
9	18.28	18.29	9	1.346	6.48×10^{-4}	1.347
10	14.34	14.34	10	1.469	-9.09×10^{-4}	1.468
11	23.26	23.26	11	-4.976	1.56×10^{-3}	-4.975
12	21.13	21.13	12	1.108	1.56×10^{-3}	1.109
13	19.39	19.40	13	1.730	-1.00×10^{-4}	1.730
			14	-6.279	1.66×10^{-3}	-6.277
			15	11.038	1.66×10^{-3}	11.040
			16	-2.021	1.66×10^{-3}	-2.020

Table 4: Elevation H and Residual v after matrix method
(same weight, equal long-hand's result)

No.	H[m] real	H[m] calculated	No.	l [m] ($\Delta H'$)	v [m]	$\hat{l}$ [m]
1	17.85	[17.85]	1	6.878	2.05×10^{-3}	6.880
2	13.12	13.12	2	4.008	6.15×10^{-4}	4.089
3	17.81	17.81	3	-11.007	9.58×10^{-4}	-11.006
4	21.91	21.91	4	3.312	6.84×10^{-4}	3.313
5	24.16	24.16	5	-6.785	2.18×10^{-4}	-6.784
6	22.14	22.14	6	3.507	4.05×10^{-4}	3.508
7	28.82	28.82	7	4.290	-1.35×10^{-3}	4.289
8	24.73	24.73	8	-0.225	7.62×10^{-4}	-0.224
9	18.28	18.29	9	1.346	8.42×10^{-4}	1.347
10	14.34	14.34	10	1.469	-4.36×10^{-4}	1.469
11	23.26	23.26	11	-4.976	1.98×10^{-3}	-4.974
12	21.13	21.13	12	1.108	2.23×10^{-3}	1.110
13	19.39	19.40	13	1.730	-5.78×10^{-4}	1.729
			14	-6.279	6.69×10^{-3}	-6.288
			15	11.038	1.22×10^{-3}	11.039
			16	-2.021	1.67×10^{-3}	-2.020

2.4 $\sigma_0, \hat{\sigma_0}$

Professor said in class that $\hat{\sigma_0}$ does not to calculate this assgnment

For the $2.3, the matrix method I used. the priori standard deviation $\sigma_0 = 7mm/\sqrt{km}$

2.5 Computation with different σ_0 choice

Then, what will happen while we choose different σ_0 ? I tried two different case:

$$\sigma_0 = \begin{cases} 2\times 7mm/\sqrt{km} & \text{case I} \\ 0.5\times 7mm/\sqrt{km} & \text{case II} \end{cases} \tag{8}$$

After computation, we have the result: Table 5.

Table 5: Adjustment by matrix method with different σ_0

		Case I			Case II		
	l [m]$\Delta H'$	$\hat{x}$ [m]	v [m]	$\hat{l}$ [m]	$\hat{x}$ [m]	v [m]	$\hat{l}$ [m]
1	6.878	[17.85]	2.05×10^{-3}	6.880	[17.85]	2.05×10^{-3}	6.880
2	4.088	13.12	6.15×10^{-4}	4.089	13.12	6.15×10^{-4}	4.089
3	−11.007	17.81	9.58×10^{-4}	−11.006	17.81	9.58×10^{-4}	−11.006
4	3.312	21.91	6.84×10^{-4}	3.313	21.91	6.84×10^{-4}	3.313
5	−6.785	24.16	2.18×10^{-4}	−6.784	24.16	2.18×10^{-4}	−6.784
6	3.507	22.14	4.05×10^{-4}	3.508	22.14	4.05×10^{-4}	3.508
7	4.290	28.82	-1.35×10^{-3}	4.289	28.82	-1.35×10^{-3}	4.289
8	−0.225	24.73	7.62×10^{-4}	−0.224	24.73	7.62×10^{-4}	−0.224
9	1.346	18.29	8.42×10^{-4}	1.347	18.29	8.42×10^{-4}	1.347
10	1.469	14.34	-4.36×10^{-4}	1.469	14.34	-4.36×10^{-4}	1.469
11	−4.976	23.26	1.98×10^{-3}	−4.974	23.26	1.98×10^{-3}	−4.974
12	1.108	21.13	2.23×10^{-3}	1.110	21.13	2.23×10^{-3}	1.110
13	1.730	19.40	-5.78×10^{-4}	1.729	19.40	-5.78×10^{-4}	1.729
14	−6.279		6.69×10^{-4}	−6.278		6.69×10^{-4}	−6.278
15	11.038		1.22×10^{-3}	11.039		1.22×10^{-3}	11.039
16	−2.021		1.67×10^{-3}	−2.020		1.67×10^{-3}	−2.020

From Table 5, we can see that enlarging or reducing σ_0 by the same proportion does not change the result.

2.6 Adjustment by different weight scheme W

Also could compute with different weight scheme $w_{i,j=i}$. Here also tried two different case:

$$w_{i,j=i} = \begin{cases} 1/\sigma_i & \text{case I} \\ e^{-100\sigma_i} & \text{case II} \end{cases} \quad w_{i,j\neq i} = 0 \tag{9}$$

For these computation, $\sigma_i = \sigma_0\sqrt{L_i}$, where $\sigma_0 = 7mm/\sqrt{km}$, L_i is the distance between two stations. Case I is same weight, on the other hand, $W = I$.
After computation, we have the result: Table 6.

Table 6: Adjustment by matrix method with different $w_{i,j}$

		Case I			Case II		
	l [m]$\Delta H'$	$\hat{x}$ [m]	v [m]	$\hat{l}$ [m]	$\hat{x}$ [m]	v [m]	$\hat{l}$ [m]
1	6.878	[17.85]	1.84×10^{-3}	6.880	[17.85]	1.65×10^{-3}	6.880
2	4.088	13.12	6.78×10^{-4}	4.089	13.12	7.07×10^{-4}	4.089
3	-11.007	17.81	8.47×10^{-4}	-11.006	17.81	7.38×10^{-4}	-11.006
4	3.312	21.91	7.15×10^{-4}	3.313	21.91	7.14×10^{-4}	3.313
5	-6.785	24.16	3.61×10^{-4}	-6.784	24.16	5.42×10^{-4}	-6.784
6	3.507	22.14	4.92×10^{-4}	3.508	22.14	5.71×10^{-4}	3.508
7	4.290	28.82	-1.19×10^{-3}	4.289	28.82	-1.05×10^{-3}	4.289
8	-0.225	24.73	7.10×10^{-4}	-0.224	24.73	6.57×10^{-4}	-0.224
9	1.346	18.29	7.46×10^{-4}	1.347	18.29	6.67×10^{-4}	1.347
10	1.469	14.34	-6.60×10^{-4}	1.469	14.34	-8.45×10^{-4}	1.468
11	-4.976	23.26	1.81×10^{-3}	-4.974	23.26	1.63×10^{-3}	-4.976
12	1.108	21.13	1.92×10^{-3}	1.110	21.13	1.66×10^{-3}	1.110
13	1.730	19.40	-3.38×10^{-4}	1.729	19.40	-1.59×10^{-4}	1.730
14	-6.279		1.07×10^{-3}	-6.278		1.52×10^{-4}	-6.277
15	11.038		1.44×10^{-3}	11.040		1.60×10^{-3}	11.040
16	-2.021		1.69×10^{-3}	-2.020		1.66×10^{-3}	-2.020

The result with various weight scheme are different (Including $w_i = 1/\sigma_i^2$).

3 Comparision of Results

Can sort the results with same weight scheme W and prior standard deviation σ_0 computed by two method (and matrix approach) as Table 7. below.

Table 7: Results of O.O. and I.O. method

		Observation Only		Indirect Observation	
	$l[m](\Delta H)$	v[m]	$\hat{l}$[m]	v[m]	$\hat{l}$[m]
1	6.878	1.67×10^{-3}	6.880	2.05×10^{-3}	6.880
2	4.088	8.44×10^{-4}	4.089	6.15×10^{-4}	4.089
3	−11.007	1.32×10^{-3}	−11.005	9.58×10^{-4}	−11.006
4	3.312	9.39×10^{-4}	3.313	6.84×10^{-4}	3.313
5	−6.785	7.81×10^{-4}	−6.784	2.18×10^{-4}	−6.784
6	3.507	1.45×10^{-3}	3.509	4.05×10^{-4}	3.508
7	4.290	8.48×10^{-4}	4.291	-1.35×10^{-3}	4.289
8	−0.225	9.49×10^{-4}	−0.224	7.62×10^{-4}	−0.224
9	1.346	1.05×10^{-3}	1.347	8.42×10^{-4}	1.347
10	1.469	7.86×10^{-4}	1.470	-4.36×10^{-4}	1.469
11	−4.976	8.08×10^{-4}	−4.975	1.98×10^{-3}	−4.974
12	1.108	9.09×10^{-4}	1.109	2.23×10^{-3}	1.110
13	1.730	5.96×10^{-4}	1.730	-5.78×10^{-4}	1.729
14	−6.279	-5.41×10^{-5}	−6.279	6.69×10^{-4}	−6.278
15	11.038	-9.90×10^{-5}	11.038	1.22×10^{-3}	11.039
16	−2.021	-1.35×10^{-4}	−2.021	1.67×10^{-3}	−2.020

The results of $\hat{l}$ using the I.O. (Indirect Observation) method and the O.O. (Observation Only) method are similar, with a maximum difference of 0.002 m which is allowable (for me). The difference between the two methods lies in the computation process, which results in different residuals v.

$$O.O. \quad \begin{cases} F(\mathbf{l}+\mathbf{v}) = 0 \\ \mathbf{v}^T\mathbf{w}\mathbf{v} = min \end{cases} \quad \Rightarrow \text{compute } \mathbf{v}, \quad \hat{\mathbf{l}} = \mathbf{l}+\mathbf{v} \tag{10}$$

$$I.O. \quad \begin{cases} \mathbf{l}+\mathbf{v} = F(\mathbf{x}) = 0 \\ \mathbf{v}^T\mathbf{w}\mathbf{v} = min \end{cases} \quad \Rightarrow \text{compute } \mathbf{x}, \quad \mathbf{v} = \mathbf{B}\mathbf{x}+\mathbf{f}, \quad \hat{\mathbf{l}} = \mathbf{l}+\mathbf{v} \tag{11}$$

4 Appendix

Can view and download code by visit this Github website: https://github.com/Awecean/Adjustment-and-Analysis-of-Spatial-Information/tree/main/HW2.

4.1 Code of Problem 1

```
% AASI HW3
% Po-Tao, Lin | B11501037 |
% Date: 2025/10/02

clear; close all; clc;



%% Part 1: generate network
rng(3);  % fix random seed (convinience to regenerate)

nPts = 13;
coords = 400*rand(nPts,2);       % XY-location [m]
elev   = 20 + 5*randn(nPts,1);   % elevation [m]

% network (13 points, 4 loops)
edges = [1 8; 8 7; 7 3; 3 12; 12 10; 10 1;  % loop 1: [1 8 7 3 12 10]
         1 6; 6 4; 4 11; 11 8;              % loop 2: [1 6 4 11 8]
         11 9; 9 13; 13 12;                 % loop 3
         13 2; 2 5; 5 6;];                  % loop 4

nEdges = size(edges,1);

%% Part 2 true level difference
true_dh = elev(edges(:,2)) - elev(edges(:,1));

%% Part 3 add random error (7mm/sqrt(km))
dist_km = sqrt(sum((coords(edges(:,2),:) - coords(edges(:,1),:)).^2,2))/1000; % distance [km]
sigma = 0.007.* sqrt(dist_km); %7mm/sqrt(km)
rand_err = sigma .* randn(nEdges,1);

meas_dh = true_dh + rand_err;

%% Part 4 compute
H1 = 17.85;
H_diff = zeros(16,1);
```

```
H_diff([1,6,7]) = [-H1,H1,-H1];
f = -meas_dh+H_diff;
B = [0,0,0,0,0,0,1,0,0,0,0,0;
    0,0,0,0,0,1,-1,0,0,0,0,0;
    0,1,0,0,0,-1,0,0,0,0,0,0;
    0,-1,0,0,0,0,0,0,0,0,1,0;
    0,0,0,0,0,0,0,0,1,0,-1,0;
    0,0,0,0,0,0,0,0,-1,0,0,0;
    0,0,0,0,1,0,0,0, 0,0,0,0;
    0,0,1,0,-1,0,0,0,0,0,0,0;
    0,0,-1,0,0,0,0,0,0,1,0,0;
    0,0,0,0,0,0,1,0,0,-1,0,0;
    0,0,0,0,0,0,0,1,0,-1,0,0;
    0,0,0,0,0,0,0,-1,0,0,0,1;
    0,0,0,0,0,0,0,0,0,0,1,-1;
    1,0,0,0,0,0,0,0,0,0,0,-1;
    -1,0,0,1,0,0,0,0,0,0,0,0;
    0,0,0,-1,1,0,0,0,0,0,0,0
];
%% Part 4-1 equal weight
W = diag(ones(1,16));              %w, weight matrix
x = inv((B')*W*B)*(-B')*W*f;     %x, elevation of H_2 to H_13
v = B*x+f;
%% Part 4-2 unequal weight

sigma_0 = 0.007.* sqrt(dist_km); %7mm/sqrt(km)
sigma_i = sigma_0;
W = diag(1./sigma_i.^2);          %w, weight matrix
x = inv((B')*W*B)*(-B')*W*f;     %x, elevation of H_2 to H_13
v = B*x+f;

%% Part 5 different sigma 0
% 5-1 ->14mm/sqrt(km)
sigma_0 = 0.014.* sqrt(dist_km); %14mm/sqrt(km)
sigma_i = sigma_0;
W = diag(1./sigma_i.^2);          %w, weight matrix
x1 = inv((B')*W*B)*(-B')*W*f;     %x, elevation of H_2 to H_13
v1 = B*x1+f;
% 5-2->3.5mm/sqrt(km)
sigma_0 = 0.0035.* sqrt(dist_km); %3.5mm/sqrt(km)
sigma_i = sigma_0;
W = diag(1./sigma_i.^2);          %w, weight matrix
x2 = inv((B')*W*B)*(-B')*W*f;     %x, elevation of H_2 to H_13
v2 = B*x2+f;
%% Part 6 different weight scheme
```

```
sigma_0 = 0.007.* sqrt(dist_km); %7mm/sqrt(km)
sigma_i = sigma_0;
% 6-1->wi = 1=1/sigma_i
W = diag(1./sigma_i);
x3 = inv((B')*W*B)*(-B')*W*f;     %x, elevation of H_2 to H_13
v3 = B*x3+f;
% 6-2->wi = exp(-100sigma_i)
W = diag(exp(-100*sigma_i));
x4 = inv((B')*W*B)*(-B')*W*f;     %x, elevation of H_2 to H_13
v4 = B*x4+f;
```