CS132 Homework 3

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Question 1.

$$\begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{bmatrix} \xrightarrow{R3+R2} \begin{bmatrix} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 0 & 10 & 8 & 4 \end{bmatrix} \xrightarrow{R2+2R1} \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 10 & 8 & 4 \end{bmatrix} \xrightarrow{R3-2R2} \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

No, b is not a linear combination of a_1, a_2, and a_3.

Question 2.

$$\begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & -3 \end{bmatrix} \xrightarrow{R3-8R2} \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 0 & 37 \end{bmatrix} \xrightarrow{R3\div-2} \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ 1 & 0 & -18.5 \end{bmatrix}$$

We can then turn this matrix into a set of equations:

$$\begin{aligned} 1x - 3y &= h \\ 0x + 1y &= -5 \\ 1x + 0y &= -18.5 \end{aligned} \tag{1}$$

Here, the second and third equations tell us that x = -18.5 and y = -5. Therefore, if we plug in these numbers into the first equation, we get:

$$\begin{aligned} 1(-18.5) - 3(-5) &= h \\ -18.5 + 15 &= h \\ -3.5 &= h \end{aligned} \tag{2}$$

Hence, h = -3.5

Question 3.

(a) Part a

$\begin{bmatrix} 27.6 \end{bmatrix} x_1 + \begin{bmatrix} 30.2 \end{bmatrix} x_2 =$ Millions of Btu of heat the plant produced

(b) Part b

$x_1 \begin{bmatrix} 27.6 \\ 3100 \\ 250 \end{bmatrix} + x_2 \begin{bmatrix} 30.2 \\ 6400 \\ 360 \end{bmatrix} =$ Total things produced

(c) Part c

$$\begin{bmatrix} 27.6 & 30.2 & 162 \\ 3100 & 6400 & 23610 \\ 250 & 360 & 1623 \end{bmatrix} \xrightarrow{OnePythonProgramLater} \begin{bmatrix} 1 & 0 & 3.9 \\ 0 & 1 & 1.8 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence, 3.9 tons of anthracite coal and 1.8 tons of bituminous coal were burned.

```
>>> runfile('C:/Users/Carlos/Desktop/Class Notes/CS132/Hws/HW2/hw2GECode.py', wdir=
Users/Carlos/Desktop/Class Notes/CS132/Hws/HW2')

>>> W = np.array([[27.6,30.2,162],[3100,6400,23610],[250,360,1623]])

>>> W=forwardElimination(W)

>>> W = backsubstitution(W)

>>> print(W)
[[1.  0.  3.9]
 [0.  1.  1.8]
 [0.  0.  0. ]]

>>>
```

Python Proof:

Question 4.

The answer is undefined since the number of rows in the second matrix do not match the number of columns for the matrix on the left.

Question 5.

$$\begin{bmatrix} 8+3-4 \\ 5+1+2 \end{bmatrix} \rightarrow \begin{bmatrix} 7 \\ 8 \end{bmatrix}$$

Question 6.

$$-2 \begin{bmatrix} 7 \\ 2 \\ 9 \\ -3 \end{bmatrix} - 5 \begin{bmatrix} -3 \\ 1 \\ -6 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ 12 \\ -4 \end{bmatrix}$$

Question 7.

$$\begin{bmatrix} 4 & -4 & -5 & 3 \\ -2 & 5 & 4 & 0 \end{bmatrix} \begin{bmatrix} z1 \\ z2 \\ z3 \\ z4 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$$

Question 8.

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ -3 & -1 & 2 & 1 \\ 0 & 5 & 3 & -1 \end{bmatrix} \xrightarrow{R2+3R1} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 5 & 3 & -1 \end{bmatrix} \xrightarrow{R3-R2} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 0 & -2 & -2 \end{bmatrix} \xrightarrow{R3\div-2} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R2-5R3}$$
$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 0 & -4 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R1-R3} \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 5 & 0 & -4 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R2\div 5} \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & \frac{-4}{5} \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R1-2R2} \begin{bmatrix} 1 & 0 & 0 & \frac{3}{5} \\ 0 & 1 & 0 & \frac{-4}{5} \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Question 9.

Let's make an augmented matrix to check:

$$\begin{bmatrix} 5 & 8 & 7 & 2 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 2 \end{bmatrix} \xrightarrow{R1+7R2} \begin{bmatrix} 5 & 15 & 0 & -19 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 2 \end{bmatrix} \xrightarrow{R1\div 5} \begin{bmatrix} 1 & 3 & 0 & \frac{-19}{5} \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 2 \end{bmatrix} \xrightarrow{R3-R1} \begin{bmatrix} 1 & 3 & 0 & \frac{-19}{5} \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & \frac{29}{5} \end{bmatrix}$$

u can't be in the span of A since the system above is inconsistent.

Question 10.

$$\begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix} \xrightarrow{R4+2R1} \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ 0 & -2 & -2 & 3 \end{bmatrix} \xrightarrow{R3-R1} \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & -1 & -1 & 5 \\ 0 & -2 & -2 & 3 \end{bmatrix} \xrightarrow{R3+R2} \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & -2 & 3 \end{bmatrix}$$
$$\xrightarrow{R4+2R2} \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

Not only does the equation $Bx = y$ not have a solution since the system is inconsistent, but the columns in B also don't span R^4 since there aren't four different pivots.

Question 11.

The martix B only has 3 pivots. Hence, not all vectors in R^4 can be written as a linear combination of the columns in B. However, the columns do span R^3 because again, B has 3 pivots.

Question 12.

$$\begin{bmatrix} 8 & 11 & -6 & -7 & 13 \\ -7 & -8 & 5 & 6 & -9 \\ 11 & 7 & -7 & -9 & -6 \\ -3 & 4 & 1 & 8 & 7 \end{bmatrix} \xrightarrow{One Python Program Later} \begin{bmatrix} 1 & 0 & 0 & 0 & -8.475 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 1 & -2.4 \end{bmatrix}$$

The columns do span R^4 since the matrix that they form has 4 pivots.

Python Proof:

```
>>> Q= np.array([[8,11,-6,-7,13],[-7,-8,5,6,-9],[11,7,-7,-9,-6],[-3,4,1,8,7]])

>>> Q=forwardElimination(Q)

>>> Q = backsubstitution(Q)

>>> print(Q)
[[ 1.     0.     0.     0.    -8.475]
 [ 0.     1.     0.     0.     2.   ]
 [ 0.     0.     1.     0.    -7.   ]
 [ 0.     0.     0.     1.    -2.4  ]]

>>>
```