CS132 Homework 2

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Question 1.

a.)Reduced Echelon Form
b.)Echelon Form
c.)Echelon Form
d.)Echelon Form

Question 2.

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix} \xrightarrow{R2-3R1} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 5 & 7 & 9 & 1 \end{bmatrix} \xrightarrow{R3-5R1} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -34 \end{bmatrix} \xrightarrow{R2\div -4} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & -8 & -16 & -34 \end{bmatrix}$$

$$\xrightarrow{R3+8R2} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & -10 \end{bmatrix}$$

The system is not consistent. Hence, there is no solution. Also, the pivot columns are column 1 and 2.

Question 3.

$$\begin{bmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{bmatrix} \xrightarrow{R2-2R1} \begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & -1 & 0 & -4 \end{bmatrix} \xrightarrow{R2\div -1} \begin{bmatrix} 1 & 4 & 0 & 7 \\ 0 & 1 & 0 & 4 \end{bmatrix} \xrightarrow{R1-4R3} \begin{bmatrix} 1 & 0 & 0 & -9 \\ 0 & 1 & 0 & 4 \end{bmatrix}$$

$x_1 = -9$
$x_2 = 4$
but there are infinitely many solutions since x_3 is a free variable.

Question 4.

$\begin{bmatrix} a & \text{bt} & \text{ct}^2 & \text{f(t)} \end{bmatrix}$
This an then be expanded with the points given into:

$$\begin{bmatrix} 1a & 1b & 1c & -1 \\ 1a & 2b & 4c & 3 \\ 1a & 3b & 9c & 13 \end{bmatrix}$$

or better written:

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 3 \\ 1 & 3 & 9 & 13 \end{bmatrix} \xrightarrow{R3-R2} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 4 & 3 \\ 0 & 1 & 5 & 10 \end{bmatrix} \xrightarrow{R2-R1} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 3 & 4 \\ 0 & 1 & 5 & 10 \end{bmatrix} \xrightarrow{R3-R2} \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 2 & 6 \end{bmatrix} \xrightarrow{R3\div 2}$$

$$\begin{bmatrix}1&1&1&-1\\0&1&3&4\\0&0&1&3\end{bmatrix}\xrightarrow{R2-3R3}\begin{bmatrix}1&1&1&-1\\0&1&0&1\\0&0&1&3\end{bmatrix}\xrightarrow{R1-R2}\begin{bmatrix}1&0&1&-2\\0&1&0&1\\0&0&1&3\end{bmatrix}\xrightarrow{R1-R3}\begin{bmatrix}1&0&0&-5\\0&1&0&1\\0&0&1&3\end{bmatrix}$$

Hence, the equation should be $f(t) = -5 + t + 3t^2$

Question 5.

We can turn the equation $a + bx + cy + x^2 + y^2 = 0$ into a matrix: $\begin{bmatrix}1a&5b&5c&50\\1a&4b&6c&52\\1a&6b&2c&40\end{bmatrix}$ or better written:

$$\begin{bmatrix}1&5&5&50\\1&4&6&52\\1&6&2&40\end{bmatrix}\xrightarrow{R3-R1}\begin{bmatrix}1&5&5&50\\1&4&6&52\\0&1&-3&-10\end{bmatrix}\xrightarrow{R2-R1}\begin{bmatrix}1&5&5&50\\0&-1&1&2\\0&1&-3&-10\end{bmatrix}\xrightarrow{R3+R2}\begin{bmatrix}1&5&5&50\\0&-1&1&2\\0&0&-2&-8\end{bmatrix}$$

$$\xrightarrow{R3\div-2}\begin{bmatrix}1&5&5&50\\0&-1&1&2\\0&0&1&4\end{bmatrix}\xrightarrow{R1+5R2}\begin{bmatrix}1&0&10&60\\0&-1&1&2\\0&0&1&4\end{bmatrix}\xrightarrow{R2\div-1}\begin{bmatrix}1&0&10&60\\0&1&-1&-2\\0&0&1&4\end{bmatrix}\xrightarrow{R2+R3}\begin{bmatrix}1&0&10&60\\0&1&0&2\\0&0&1&4\end{bmatrix}$$

$$\xrightarrow{R1-10R3}\begin{bmatrix}1&0&0&-20\\0&1&0&-2\\0&0&1&-4\end{bmatrix}$$

From here, we get the equation:

$$\begin{aligned}-20-2x-4y+x^2+y^2&=0\\-2x-4y+x^2+y^2&=20\\x^2-2x+y^2-4y&=20\\x^2-2x+1+y^2-4y+4&=20+1+4\\(x-1)^2+(y-2)^2&=25\end{aligned}\tag{1}$$

Hence, the radius of the circle is 5 and its center is at (1,2)

Question 6.

The following are the solutions to the coding problems:

1

$x_1 = -3$
$x_2 = 4.9$
$x_3 = -7.6$

2

$x_1 = 1$
$x_2 = 5$
$x_3 = 3$
$x_4 = 9$
$x_5 = 7$
$x_6 = 8$

3
The system is inconsistent and hence has no solutions.

4
$x_1 = 2$
$x_2 = -2$
$x_3 = 4$
$x_4 = -4$
$x_5 = 3$
x_6 is a free variable so we have infinitely many solutions

5
$x_1 = 2 + 1.05x_4 + 2.41x_5$
$x_2 = -2 - 2.07x_4 - 4.12x_5$
$x_3 = 4 + 1.90 + 4.17$
where x_4 and x_5 can be any real numbers.

6 $x_1 = -15$
$x_2 = 22$
$x_3 = -81$
x_4, x_5, x_6, x_7, x_8 are all free variables. Hence, there are infinite solutions.