CS132 Homework 6

Carlos Lopez ()

April 2, 2020

Question 1.

$$\begin{bmatrix} v_1 & v_2 & v_3 & u \end{bmatrix} = \begin{bmatrix} 1 & 4 & 5 & -4 \\ -2 & -7 & -8 & 10 \\ 4 & 9 & 6 & -7 \\ 3 & 7 & 5 & -5 \end{bmatrix} \xrightarrow{R2+2R1} \dots \xrightarrow{R3-4R1} \dots \xrightarrow{R4-3R1} \dots \xrightarrow{R4+5R2} \begin{bmatrix} 1 & 4 & 5 & -4 \\ 0 & 1 & 2 & 2 \\ 0 & -7 & -14 & 9 \\ 0 & 0 & 0 & 17 \end{bmatrix}$$

$\boxed{\text{Since the system is inconsistent } (0 \neq 17)\text{, u is not in the subspace of } R^4 \text{ generated by } v_1, v_2, v_3.}$

Question 2.

$$\begin{bmatrix} v_1 & v_2 & v_3 & p \end{bmatrix} = \begin{bmatrix} -3 & -2 & 0 & 1 \\ 0 & 2 & -6 & 14 \\ 6 & 3 & 3 & -9 \end{bmatrix} \xrightarrow{R3+2R1} \dots \xrightarrow{R3+.5R2} \dots \xrightarrow{R3+.5R2} \dots \xrightarrow{R1\div -3} \dots \xrightarrow{R2\div 2}$$

$$\begin{bmatrix} 1 & \frac{2}{3} & 0 & \frac{-1}{3} \\ 0 & 1 & -3 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Since the system [Ap] is consistent, p is in the subspace.}}$

Question 3.

$$\text{Au} = \begin{bmatrix} -3 & -2 & 0 \\ 0 & 2 & -6 \\ 6 & 3 & 3 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{\text{Since the multiplication equals 0, yes u is in Nul A.}}$

Question 4.

$\boxed{\begin{array}{l} \text{p=3} \\ \text{q=4} \end{array}}$

For p, we know that "the null space of a mxn matrix A is in subspace R^n," and since A in this case is a 4x3 matrix, we know that Nul A is a subspace of R^3 due to the solutions of Ax=0 needing to have 3 entries to be able to match the columns of A.

Similarly for q, we know that "the column space of a mxn matrix A is a subspace of R^m," so

This is because the solution of Ax = 0 must have 3 entries since it must match the columns of A. In addition, q should equal 4 each column vector in A has 4 entries.

Question 5.

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 5 & 7 & 0 \\ -5 & -1 & 0 & 0 \\ 2 & 7 & 11 & 0 \end{bmatrix} \xrightarrow{R2-4R1} \dots \xrightarrow{R3+5R1} \dots \xrightarrow{R4-2R1} \dots \xrightarrow{R2\div-3} \dots \xrightarrow{R3-9R2} \dots \xrightarrow{R4-3R2} \dots \xrightarrow{R1-2R2}$$
$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} & 0 \\ 0 & 1 & \frac{5}{3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence, we get the general solution of:

$x_1 = (\frac{1}{3})x_3$
x$_2 = -(\frac{-5}{3})x_3$
$x_3 =$ free variable
$x_4 =$ free variable

As long as we set either x_3 or x_4 to a non-zero value, we'll get an answer, so let's make $x_3 = 6$, and we get the following nonzero vector for Nul A:

$$\boxed{\begin{bmatrix} 2 \\ -10 \\ 6 \end{bmatrix}}$$

And for Col A, we can just select any column in A. Let us just select the first one in this case:

$$\boxed{\begin{bmatrix} 1 \\ 4 \\ -5 \\ 2 \end{bmatrix}}$$

Question 6.

$\boxed{\text{No. Since the vectors are linearly dependent, they cannot be a basis for a subspace}}$

(You can easily see that $v_1 = -2v_2$)

Question 7.

By theorem 8 in section 1.7, we can see that since there are more vectors in the set than entries in each vector, the vectors are linearly dependent. Hence, $\boxed{\text{the vectors can't be a basis for any subspace.}}$

Question 8.

For the basis for Col A we get its first and third columns since the pivots for the reduced version of the matrix are in these columns. Hence, the basis for Col A is: $\boxed{\begin{bmatrix}-3\\2\\3\end{bmatrix} \text{ and } \begin{bmatrix}-2\\4\\-2\end{bmatrix}}$

In addition, we can get the basis for Nul A by getting the reduced-augmented matrix for A and turning it into its equation form:

$$\begin{bmatrix}1 & -3 & 6 & 9 & 0\\0 & 0 & 4 & 5 & 0\\0 & 0 & 0 & 0 & 0\end{bmatrix} \xrightarrow{equation form} \begin{bmatrix}\mathrm{x}_1 - 3x_2 + 1.5x_4 = 0\\ \mathrm{x}_3 + 1.25x_4 = 0\\ 0 = 0\end{bmatrix}$$

We can then solve for the variables in parametric vector form:

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}3x_2 - 1.5x_4\\x_2\\-1.25x_4\\x_4\end{bmatrix} = x_2\begin{bmatrix}3\\1\\0\\0\end{bmatrix} + x_4\begin{bmatrix}-1.5\\0\\-1.25\\1\end{bmatrix}$$

Therefore, we get that the basis for Nul A is: $\boxed{\begin{bmatrix}3\\1\\0\\0\end{bmatrix} \text{ and } \begin{bmatrix}-1.5\\0\\-1.25\\1\end{bmatrix}}$

Question 9.

For the basis for Col A we get its first, second, and fourth columns since the pivots for the reduced version of the matrix are in these columns. Hence, the basis for Col A is: $\boxed{\begin{bmatrix}3\\-2\\-5\\-2\end{bmatrix}, \begin{bmatrix}-1\\2\\9\\6\end{bmatrix}, \text{ and } \begin{bmatrix}3\\7\\3\\3\end{bmatrix}}$

In addition, we can get the basis for Nul A by getting the reduced-augmented matrix for A and turning it into its equation form:

$$\begin{bmatrix}3 & -1 & 7 & 0 & 6 & 0\\0 & 2 & 4 & 0 & 3 & 0\\0 & 0 & 0 & 1 & 1 & 0\\0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \xrightarrow{R2\div 2} \dots \xrightarrow{R1+R2} \dots \xrightarrow{R1\div 3} \begin{bmatrix}1 & 0 & 3 & 0 & 2.5 & 0\\0 & 1 & 2 & 0 & 1.5 & 0\\0 & 0 & 0 & 1 & 1 & 0\\0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \xrightarrow{equation form} \begin{bmatrix}\mathrm{x}_1 + 3x_3 + 2.5x_5 = 0\\ \mathrm{x}_2 + 2x_3 + 1.5x_5 = 0\\ \mathrm{x}_4 + x_5 = 0\\ 0 = 0\end{bmatrix}$$

We can then solve for the variables in parametric vector form:

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix} = \begin{bmatrix}-3x_3 - 2.5x_5\\-2x_3 - 1.5x_5\\x_3\\-x_5\\x_5\end{bmatrix} = x_3\begin{bmatrix}-3\\-2\\1\\0\\0\end{bmatrix} + x_5\begin{bmatrix}-2.5\\-1.5\\0\\-1\\1\end{bmatrix}$$

Therefore, we get that the basis for Nul A is: $\boxed{\begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} -2.5 \\ -1.5 \\ 0 \\ -1 \\ 1 \end{bmatrix}}$

Question 10.

$$\boxed{\begin{array}{l} False \\ False \\ False \\ False \\ True \end{array}}$$

a.)
R^2 is not a subset of R^3 because vectors in R^3 have 3 entries while vectors in R^2 only have 2 entries.

b.)
It's the number of free variables in the equation that equals the dimension of Nul A not the number of variables.

c.)
Nope, you could for example span R^3 with infinitely many vectors, but R^3 would still only have 3 dimensions.

d.)
S would need exactly n vectors to be a basis for V. However, S could have more than n vectors which would make the vectors linearly dependent.

e.)
R^3 needs 3 linearly independent vectors that span all of R^3 to form its basis. Hence, the only subspace of R^3 can be itself.

Question 11.

Following the same steps from 8 and 9 we can see that:

Bases for Col A: $\boxed{\begin{array}{c} \begin{bmatrix} 1 \\ 1 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \text{ and } \begin{bmatrix} 5 \\ 5 \\ 1 \\ 1 \end{bmatrix} \\ \text{Dimension of 3} \end{array}}$ (Due to the pivots being in those **3** columns)

Then for the basis of Nul A:

$$\begin{bmatrix}1 & -2 & 9 & 5 & 4\\0 & 1 & -3 & 0 & -7\\0 & 0 & 0 & 1 & -2\\0 & 0 & 0 & 0 & 0\end{bmatrix} \xrightarrow{R1-5R3} \ldots \xrightarrow{R1+2R2} \begin{bmatrix}1 & 0 & 3 & 0 & 0\\0 & 1 & -3 & 0 & -7\\0 & 0 & 0 & 1 & -2\\0 & 0 & 0 & 0 & 0\end{bmatrix} \xrightarrow{equationform} \begin{bmatrix}x_1 + 3x_3 = 0\\x_2 - 3x_3 - 7x_5 = 0\\x_4 - 2x_5 = 0\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix} = \begin{bmatrix}-3x_3\\3x_3 + 7x_5\\x_3\\2x_5\\x_5\end{bmatrix} = \text{x}_3\begin{bmatrix}-3\\3\\1\\0\\0\end{bmatrix} + x_5\begin{bmatrix}0\\7\\0\\2\\1\end{bmatrix}$$

Basis for Nul A: $\boxed{\begin{array}{c}\begin{bmatrix}-3\\3\\1\\0\\0\end{bmatrix} \text{ and } \begin{bmatrix}0\\7\\0\\2\\1\end{bmatrix}\\ \text{Dimension of 2}\end{array}}$

(It's 2 dimensional since there's two vectors)

Question 12.

Following the same steps from the previous problem we can see that:

Bases for Col A: $\boxed{\begin{array}{c}\begin{bmatrix}1\\5\\4\\-2\end{bmatrix}, \begin{bmatrix}-4\\-9\\-9\\5\end{bmatrix}, \text{ and } \begin{bmatrix}3\\8\\7\\-6\end{bmatrix}\\ \text{Dimension of 3}\end{array}}$ (Due to the pivots being in those **3** columns)

Then for the basis of Nul A:

$$\begin{bmatrix}1 & 2 & -4 & 3 & 3\\0 & 0 & 1 & -2 & 0\\0 & 0 & 0 & 0 & -5\\0 & 0 & 0 & 0 & 0\end{bmatrix} \xrightarrow{R5\div -5} \ldots \xrightarrow{R1-3R3} \ldots \xrightarrow{R1+4R2} \begin{bmatrix}1 & 2 & 0 & -5 & 0\\0 & 0 & 1 & -2 & 0\\0 & 0 & 0 & 0 & 1\\0 & 0 & 0 & 0 & 0\end{bmatrix} \xrightarrow{equationform} \begin{bmatrix}x_1 + 2x_2 - 5x_4 = 0\\x_3 - 2x_4 = 0\\x_5 = 0\end{bmatrix}$$

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix} = \begin{bmatrix}-2x_2 + 5x_4\\x_2\\2x_4\\x_4\\0\end{bmatrix} = \text{x}_2\begin{bmatrix}-2\\1\\0\\0\\0\end{bmatrix} + x_4\begin{bmatrix}5\\0\\2\\1\\0\end{bmatrix}$$

Basis for Nul A: $\boxed{\begin{matrix} \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 5 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} \\ \text{Dimension of 2} \end{matrix}}$

(It's 2 dimensional since there's two vectors)

Question 13.

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ -1 & -3 & 2 & 4 & -8 \\ -2 & -1 & -6 & -7 & 9 \\ 5 & 6 & 8 & 7 & -5 \end{bmatrix} \xrightarrow{R2+R1} \cdots \xrightarrow{R3+2R1} \cdots \xrightarrow{R4-5R1} \begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ 0 & -1 & 2 & 3 & -5 \\ 0 & 3 & -6 & -9 & 15 \\ 0 & -4 & 8 & 12 & -20 \end{bmatrix} \xrightarrow{R3+3R2} \cdots$$

$$\xrightarrow{R4-4R2} \begin{bmatrix} 1 & 2 & 0 & -1 & 3 \\ 0 & -1 & 2 & 3 & -5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since there are two pivots, one on the first column and the other in the second column, the basis for H is:

$\boxed{\begin{matrix} \begin{bmatrix} 1 \\ -1 \\ -2 \\ 5 \end{bmatrix} \text{ and } \begin{bmatrix} 2 \\ -3 \\ -1 \\ 6 \end{bmatrix} \\ \text{Dimension of 2} \end{matrix}}$

Question 14.

The Rank Theorem states that:

rank A + dim Nul A = n

Hence, the rank of A will equal 5 - (dim Nul A). Therefore, since the null space is three-dimensional, the rank of A will be $\boxed{2}$

Question 15.

A rank 1 matrix has a one dimensional column space, and every column is a multiple of some fixed vector. Hence, to make our desired matrix in R^4, we can just use any nonzero vector with 4 entries (1,1,1,1 in my case) for one column and use multiples of this vector for the other two columns in the matrix. this gives us the following matrix:

$$\boxed{\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix}}$$

Question 16.

Again, via the Rank Theorem:

rank A + dim Nul A = n

Therefore,

rank A = n - dim Nul A = 6 - 5 = 1

Then, by extension,

rank A = $\boxed{\text{dim Col A = 1}}$

Question 17.

Again, via the Rank Theorem:

rank A + dim Nul A = n

Therefore, dim Nul A = n - max rank A = 4 - 4 = 0.

Hence, the smallest possible dimension of Nul A is $\boxed{0}$

Question 18.

Let us consider the system as Ax = b, where A is a 6 x 8 matrix. By the given info the dimension of Nul A = 2 (since it equals the amount of free variables) and n = 8.

Hence, by the Rank Theorem:

rank A + dim Nul A = n

Therefore, rank A = n - dim Nul A = 8 - 2 = 6

Hence, due to dim Col A = rank A = 6, we can see that Col A is a subspace of R^6 which means Col A = R^6.

This signifies that Ax=b has a solution for every b and by extension:
$\boxed{\textbf{it is not possible} \text{ to change constants on the right sides of the equations to make the system inconsistent.}}$