Gravity

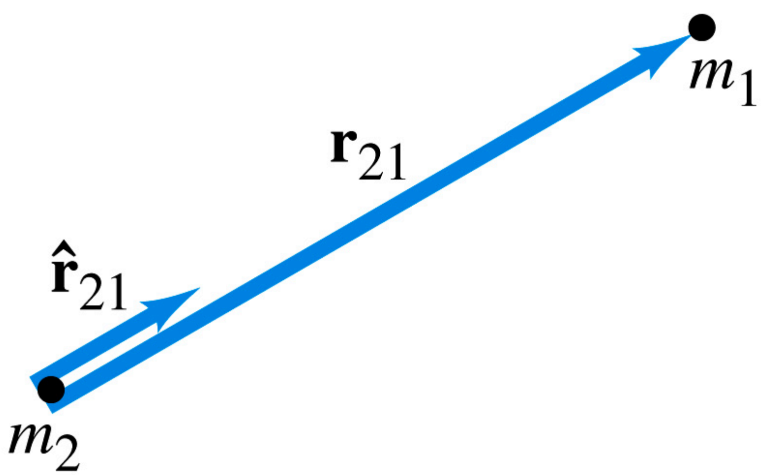
- Newton's law of gravitation
  Why gravitational acceleration?
- Kepler's law of planetary motion
  How to determine the distance and mass of planets
- Geosynchronous Satellite (cell phone, weather)
  Non-geosynchronous satellites (?)

Newton's law of gravity

Any two objects attract each other with a force that is proportional to the product of their masses, and inversely proportional to the square of the distance between them. The force acts along the line joining the two objects.

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

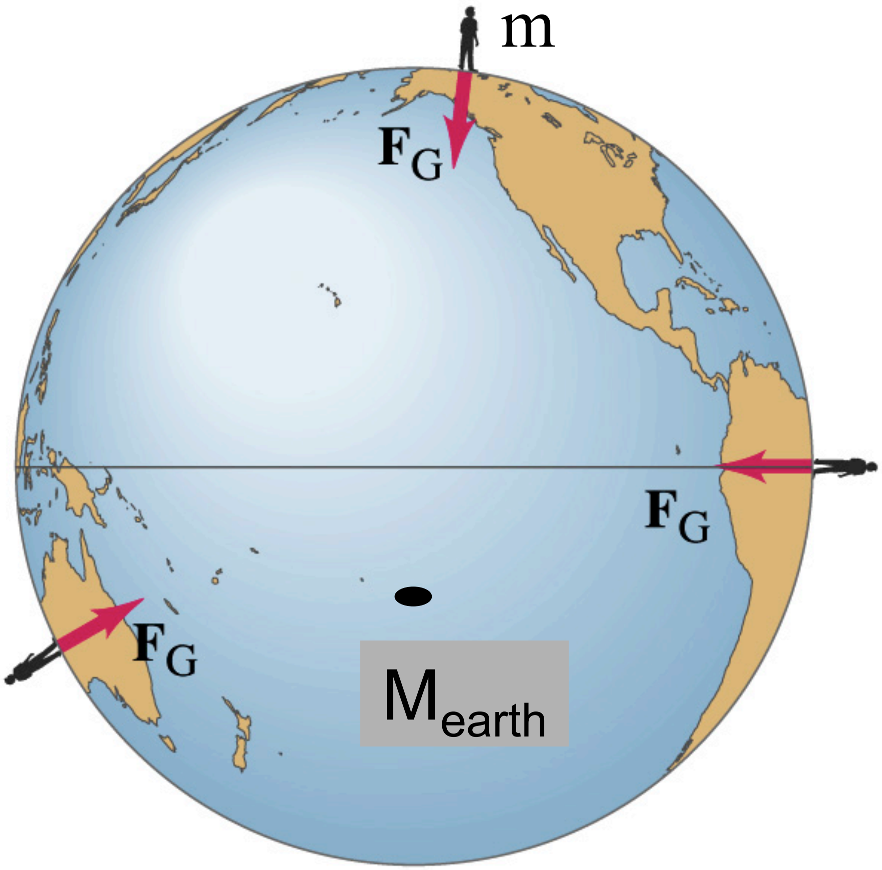
G = 6.67 x 10^{-11} N.m^2/kg^2, gravitation constant



$$\vec{F}_{12} = -G\frac{m_1 m_2}{r_{12}^2}\hat{r}_{12}$$

Force acting on m_1 exerted by m_2.

Gravitational Force & Gravitational Acceleration (g)



$$mg = G\frac{M_{earth}m}{r^2} \quad \Rightarrow \quad g = G\frac{M_{earth}}{r^2}$$

r= radius of earth + height = $r_E + h$

$$g = \frac{GM_E}{R_E^2} = \frac{(6.67\times10^{-11}\ \mathrm{N\ m^2/kg^2})\times(5.97\times10^{24}\ \mathrm{kg})}{(6.38\times10^{6}\ \mathrm{m})^2} = 9.8\ \mathrm{m/s^2}$$

Estimate Moon's Mass

Using the fact that the gravitational field at the surface of the Earth is about six times larger than that at the surface of the Moon, and the fact that the Earth's radius is about four times the Moon's radius, determine how the mass of the Earth compares to the mass of the Moon.

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$$g = \frac{Gm}{r^2}$$

Earth's mass: $$M_E = \frac{gR_E^2}{G}$$

Moon's mass: $$M_M = \frac{g_M R_M^2}{G} = \frac{\left(\frac{1}{6}g\right)\left(\frac{R_E}{4}\right)^2}{G} = \frac{1}{96}\frac{gR_E^2}{G}$$

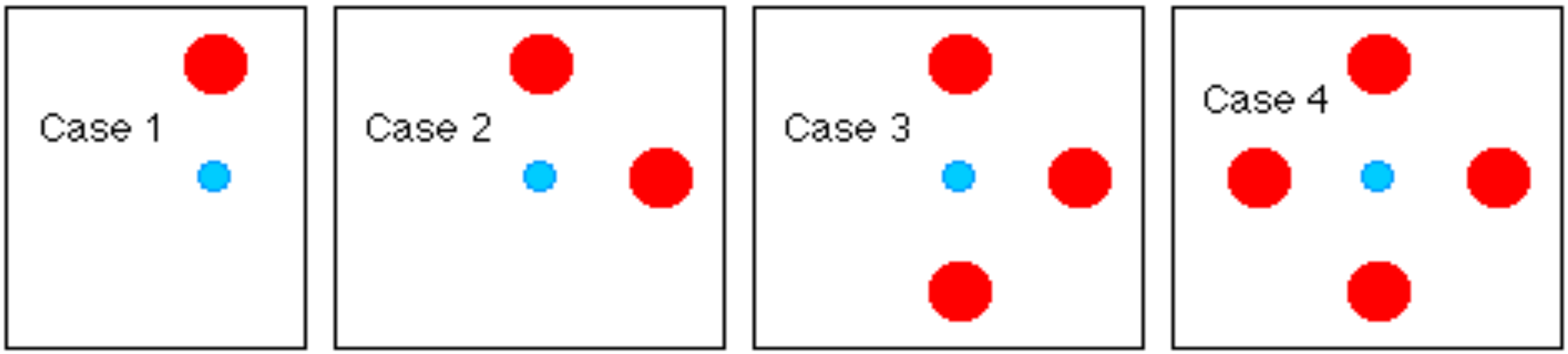
Superposition

If an object experiences multiple forces, we can use:

The principle of superposition - the net force acting on an object is the vector sum of the individual forces acting on that object.

Rank these situations

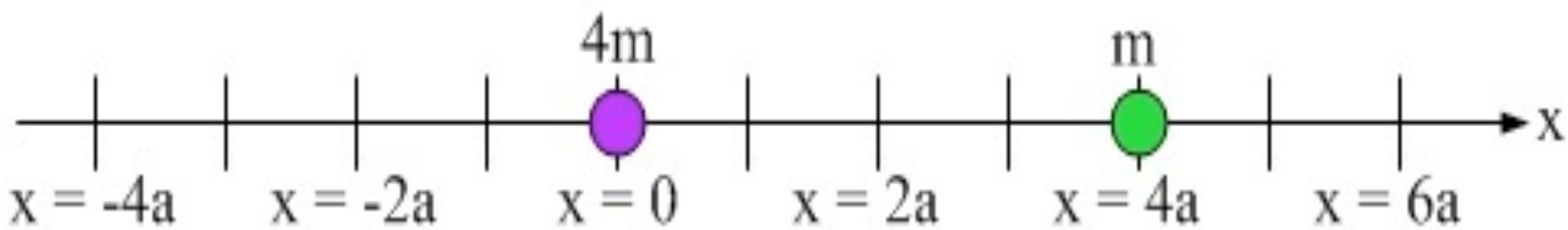
A small blue ball has one or more large red balls placed near it. The red balls are all the same mass and the same distance from the blue one. Rank the different cases based on the net gravitational force experienced by the blue ball due to the neighboring red ball(s).



1. 1=3>2>4
2. 3>2>1>4
3. 4>3>2>1
4. 2>1=3>4
5. 4>1=2=3

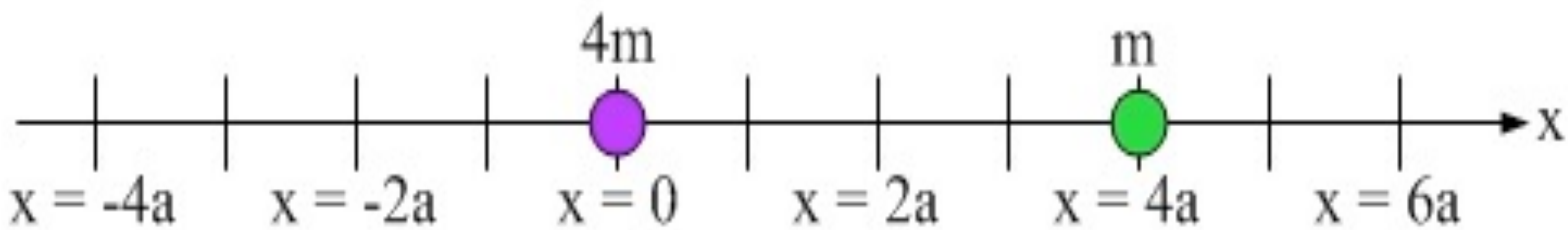
Worksheet – a one-dimensional situation

Ball *A*, with a mass *4m*, is placed on the *x*-axis at *x* = 0. Ball *B*, which has a mass *m*, is placed on the *x*-axis at *x* = +*4a*. Where would you **<u>place ball C</u>**, which also has a mass *m*, so that **<u>ball *A*</u>** feels no net force because of the other balls? Is this even possible?



Worksheet – a one-dimensional situation

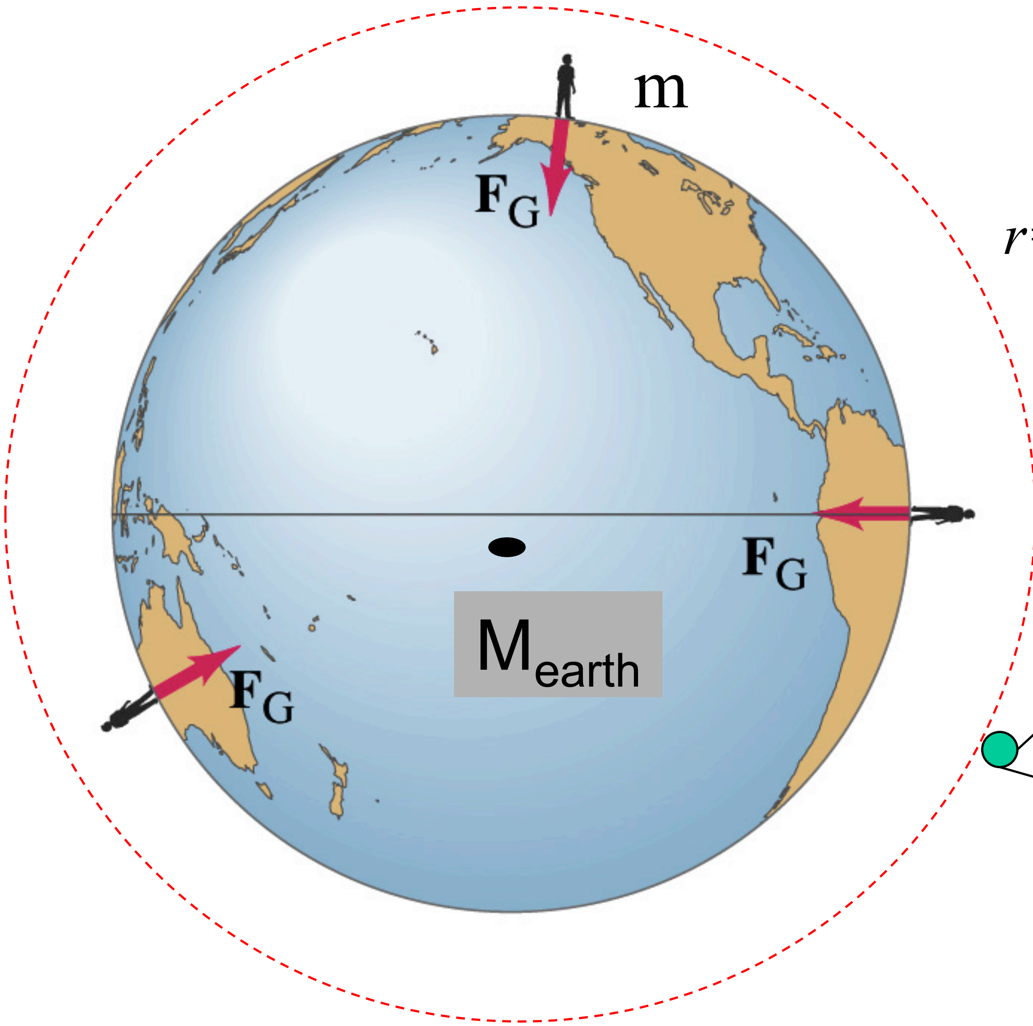
Ball *A*, with a mass 4*m*, is placed on the *x*-axis at *x* = 0. Ball *B*, which has a mass *m*, is placed on the *x*-axis at *x* = +4*a*. Where would you **RE-POSITION ball C**, which also has a mass *m*, so that ***ball B*** feels no net force because of the other balls? Is this even possible?



Gravitational Force & Orbital Motion

Net force (due to gravity) = mass x centripetal acceleration

$$\frac{m_{sat}v^2}{r} = G\frac{M_{earth}m_{sat}}{r^2} \quad \Rightarrow \quad v^2 = \frac{GM_{earth}}{r}$$



r= radius of earth + height = $r_E + h$

Geo-Synchronous Orbit

Geo-Synchronous: In Sync with the Earth, time taken for the satellite to go around its orbit once, T, is the same as the time the Earth takes to go around its own axis once, 24 hours.

$$v = \frac{2\pi r}{T}; \qquad T = 24 \text{ hours} = 24.\ 3600 \text{ s} = 86400 \text{ s}$$

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$$\left(\frac{2\pi r}{T}\right)^2 = \frac{GM_{earth}}{r} \quad \Rightarrow \quad \frac{r^2 4\pi^2}{T^2} = \frac{GM_{earth}}{r}$$

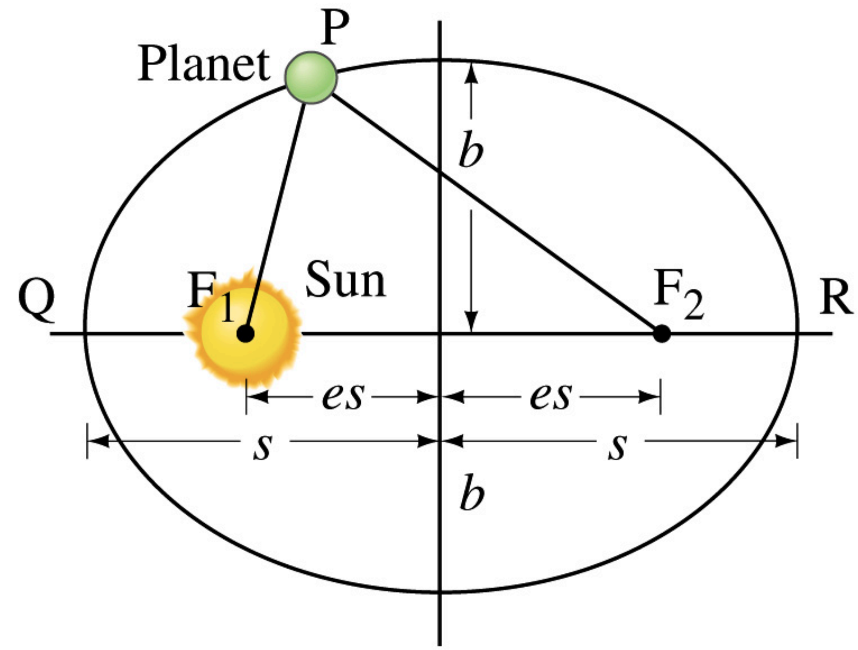
$$r^3 = \left(\frac{GM_{earth}}{4\pi^2}\right) T^2$$

For Geo-Synchronous Satellites, T is fixed, & so is r (= 42300 km)

Kepler's Laws of Planetary Motion

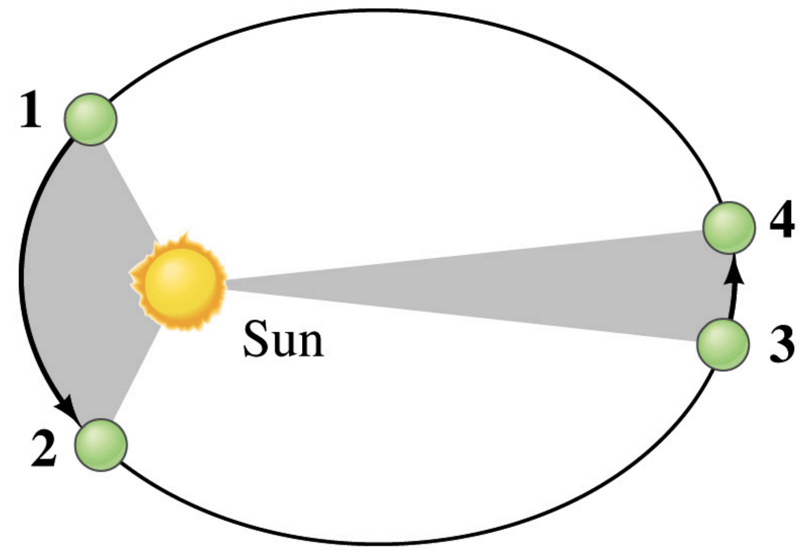
Law 1:

Each planet goes around the Sun in an elliptical path with the Sun at one focus.



Law 2:

The planet sweeps out equal areas in equal periods of time.

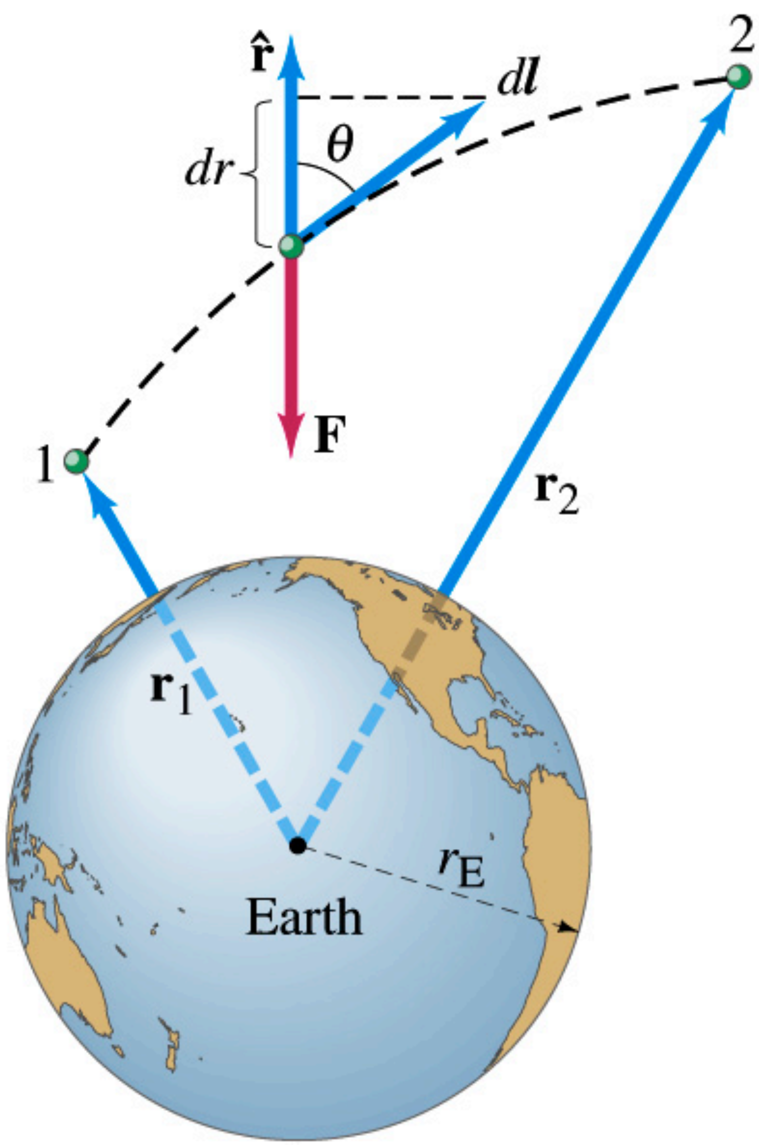


Law 3:

The ratio of the squares of the period of ANY TWO PLANETS is equal to the ratio of the cubes of their semi-major axes.

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{s_1}{s_3}\right)^3$$

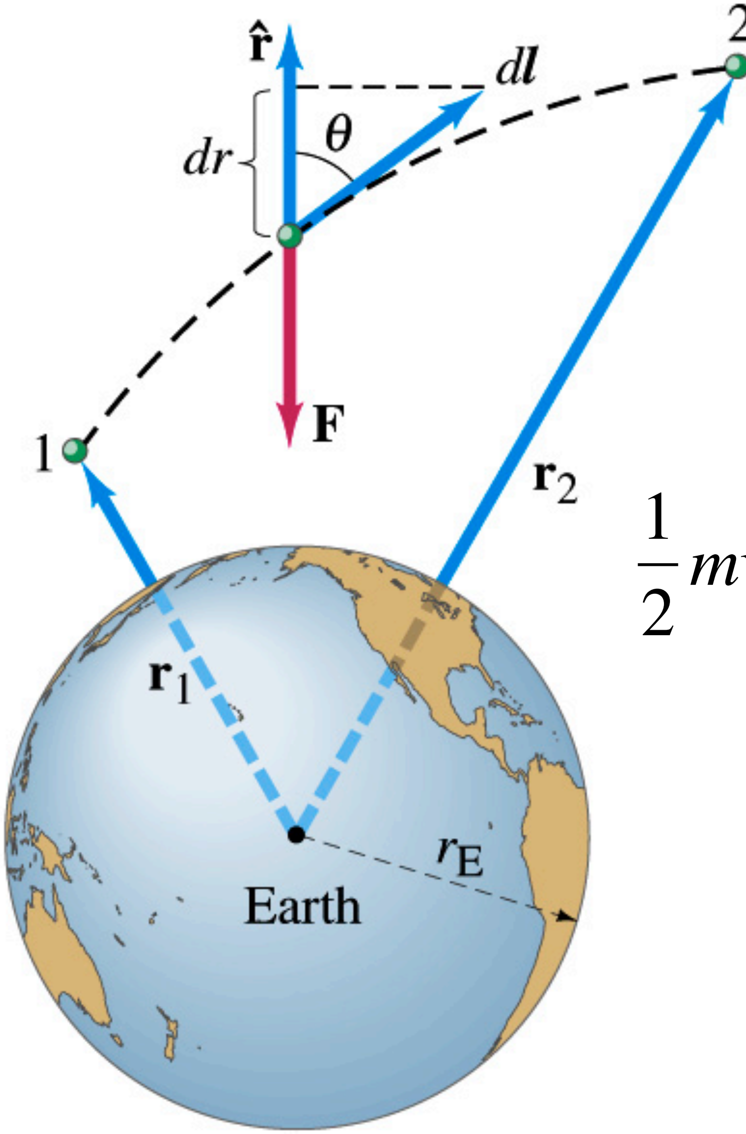
Gravitational Potential Energy



Total energy of a particle of mass m, feeling only the force of gravity, since gravity is a conserved force.

$$\boxed{U(r) = -\frac{GmM_E}{r}}$$

Total Energy = Kinetic Energy + Gravitational Potential Energy



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$$\frac{1}{2}mv_1^2 - G\frac{mM_E}{r_1} = \frac{1}{2}mv_2^2 - G\frac{mM_E}{r_2} = \text{constant}$$

Escape Velocity

An object thrown into the air from the Earth will return back, unless the speed is so high that the total energy is positive. (Total negative energy represents a bound system.)

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Escape velocity: The minimum initial velocity needed to escape the Earth's gravity is escape velocity.

$$\frac{1}{2}mv_{escape}^2 - G\frac{mM_E}{r_E} = 0$$

$$v_{escape} = \sqrt{\frac{2GM_E}{r_E}} = 11.2 \times 10^3 m/s$$

Escape Velocity and Orbit Velocity

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For the orbit velocity, centripetal force is equal to gravitational force:

$$\frac{m v_{orbit}^2}{r_E} = G\frac{mM_E}{r_E^2}$$

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$$\boxed{v_{escape} = \sqrt{2}v_{orbit}}$$