CS132 Homework 1

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Question 1.

$$\begin{bmatrix} 2 & 4 & -4 \\ 5 & 7 & 11 \end{bmatrix} \xrightarrow{R1 \div 2} \begin{bmatrix} 1 & 2 & -2 \\ 5 & 7 & 11 \end{bmatrix} \xrightarrow{R2 - 5R1} \begin{bmatrix} 1 & 2 & -2 \\ 0 & -3 & 21 \end{bmatrix} \xrightarrow{R2 \div -3} \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -7 \end{bmatrix}$$

Hence, $x_2 = -7$

Afterwards, we can solve via substitution:

$$\begin{aligned} x_1 + 2(-7) &= -2 \\ x_1 - 14 &= -2 \\ x_1 &= 12 \end{aligned} \tag{1}$$

Therefore, $x_1 = 12$ and $x_2 = -7$

Question 2.

Sure they do. Here's how to find that point:

$$\begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 2 & 3 & 0 & 0 \end{bmatrix} \xrightarrow{R3 - R1} \begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 0 & -1 & -4 & -4 \end{bmatrix} \xrightarrow{R3 + R2} \begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & -6 & -6 \end{bmatrix} \xrightarrow{R3 \div -6} \begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\xrightarrow{R2 + 2R3} \begin{bmatrix} 2 & 4 & 4 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R1 - 4R2} \begin{bmatrix} 2 & 0 & 4 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R1 - 4R3} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{R1 \div 2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

As you can see, there is a solution for this system of equations when $x_1 = 0$, $x_2 = 0$, and $x_3 = 1$

Hence, the planes have a common point of intersection at $(0, 0, 1)$

Question 3.

$$\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{bmatrix} \xrightarrow{R2 + 2R1} \begin{bmatrix} 1 & h & -3 \\ 0 & 4 + 2h & 0 \end{bmatrix}$$

This system will be consistent for any value of h since the value at the bottom row and last column ended up being a zero.

Question 4.

$$\begin{bmatrix} 1 & -3 & 8 & g \\ 0 & 4 & -15 & h \\ -3 & 5 & -9 & k \end{bmatrix} \xrightarrow{R3+3R1} \begin{bmatrix} 1 & -3 & 8 & g \\ 0 & 4 & -15 & h \\ 0 & -4 & 15 & k+3g \end{bmatrix} \xrightarrow{R3+R2} \begin{bmatrix} 1 & -3 & 8 & g \\ 0 & 4 & -15 & h \\ 0 & 0 & 0 & k+3g+h \end{bmatrix}$$

Therefore, the equation the question is looking for is: $k + 3g + h = 0$

Question 5.

$$\begin{bmatrix} a & b & f \\ c & d & g \end{bmatrix} \xrightarrow{R1 \div a} \begin{bmatrix} 1 & \frac{b}{a} & \frac{f}{a} \\ c & d & g \end{bmatrix} \xrightarrow{R2-(c)R1} \begin{bmatrix} 1 & \frac{b}{a} & \frac{f}{a} \\ 0 & d - \frac{cb}{a} & g - \frac{cf}{a} \end{bmatrix}$$

Due to the matrix above, we can see that $d - \frac{cb}{a} \neq 0$. Hence, the system is cosistent for all values f and g.

In addition, since $a \neq 0$ you could multiply both sides of the previous equation and also say that $ad - bc \neq 0$.

Question 6.

We first define the equations for every T_X point:

$T_1 = (10 + 20 + T_2 + T_4)/4$
$T_2 = (T_1 + 20 + 40 + T_3)/4$
$T_3 = (T_4 + T_2 + 40 + 30)/4$
$T_4 = (10 + T_1 + T_3 + 30)/4$

Then we turn these equations into the form described in the problem($4T_1 - T_2 - T_4 = 30$):

$4T_1 - T_2 + 0T_3 - T_4 = 30$
$-T_1 + 4T_2 - T_3 + 0T_4 = 60$
$0T_1 - T_2 + 4T_3 - T_4 = 70$
$-T_1 + 0T_2 - T_3 + 4T_4 = 40$

Afterwards, we can use the equations to come up with a matrix

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 30 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ -1 & 0 & -1 & 4 & 40 \end{bmatrix}$$

We can then solve this matrix in the next problem.

Question 7.

Using the matrix from the previous problem:

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 30 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ -1 & 0 & -1 & 4 & 40 \end{bmatrix} \xrightarrow{R1 \Longleftrightarrow R4} \begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix} \xrightarrow{R1 \div -1} \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix}$$

$$\xrightarrow{R4+4R2} \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & 15 & -4 & -1 & 270 \end{bmatrix} \xrightarrow{R4+R3} \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & 14 & 0 & -2 & 340 \end{bmatrix} \xrightarrow{R2+R1} \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 4 & 0 & -4 & 20 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & 14 & 0 & -2 & 340 \end{bmatrix}$$

$$\xrightarrow{R2 \div 4} \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & 14 & 0 & -2 & 340 \end{bmatrix} \xrightarrow{R3+R2} \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & 0 & 4 & -2 & 75 \\ 0 & 14 & 0 & -2 & 340 \end{bmatrix} \xrightarrow{R4-14R2} \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & 0 & 4 & -2 & 75 \\ 0 & 0 & 0 & 12 & 270 \end{bmatrix} \xrightarrow{R3 \div 4}$$

$$\begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & 0 & 1 & -0.5 & 18.75 \\ 0 & 0 & 0 & 12 & 270 \end{bmatrix} \xrightarrow{R4 \div 12} \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & 0 & -1 & 5 \\ 0 & 0 & 1 & -0.5 & 18.75 \\ 0 & 0 & 0 & 1 & 22.5 \end{bmatrix}$$

After that process, we can turn the matrix back into the temperature equations and solve for each variable:

$$T_4 = 22.5 \tag{2}$$

$$\begin{aligned} T_3 - .5T_4 &= 18.75 \\ T_3 - .5(22.5) &= 18.75 \\ T_3 - 11.25 &= 18.75 \\ T_3 &= 30 \end{aligned} \tag{3}$$

$$\begin{aligned} T_2 - 1T_4 &= 5 \\ T_2 - 1(22.5) &= 5 \\ T_2 - 22.5 &= 5 \\ T_2 &= 27.5 \end{aligned} \tag{4}$$

$$\begin{aligned} T_1 + T_3 - 4T_4 &= -40 \\ T_1 + 30 - 4(22.5) &= -40 \\ T_1 + 30 - 90 &= -40 \\ T_1 &= 20 \end{aligned} \tag{5}$$

So to reiterate, $T_1 = 20$, $T_2 = 27.5$, $T_3 = 30$, and $T_4 = 22.5$

Question 8.

Python was successfully installed in the machine.

Question 9.

(a) Part a
I got 1.8

(b) Part b
No, the correct result should be 1.9714...

(c) Part c
The computer stores the result of this operation as a float due to the limited storage space of the computer. The computer hence approximates the actual answer. However, you can see that the approximation and the actual answer are fairly different numbers.