[CSCI-GA 1180 Summer 2022] Math Techniques for Computer Science Applications

Module A: Linear Algebra

- 1. **Vectors**: definition of vectors, application of vectors, basic operations, dot product, vectors in MATLAB
- 2. **Matrices**: definition of matrices, application of matrices, basic operations, multiplying a matrix and a vector and its applications, linear transformation, composition of linear transformations, matrix multiplication, properties of matrix multiplication, convolution as matrix multiplication, matrices in MATLAB
- 3. **Vector spaces**: span, subspaces, coordinates, bases, linear independence, orthogonal and orthonormal basis, operations on vector subspaces, complementary subspace, orthogonal complement of a subspace, linear transformation, null space, image space, column space, row space, rank, system of linear equations, null space and rank in MATLAB
- 4. **Algorithms**: Solving a system of linear equations, Row-echelon form, Gaussian elimination, inverse of a matrix, computing inverse, ill-conditioned matrices
- Geometry: arrows, coordinate systems, geometric calculations, projections, geometric transformations, translations, rotations, reflections, homogeneous coordinates, rigid motion, similarity transformation, affine transformation, determinants, calculation of determinants
- 6. Change of Basis and Singular Value Decomposition (SVD), SVD for lossy compression, condition number

Module B: Probability and Statistics

- 7. **Probability Theory:** interpretations of probability theory, finite sample spaces, axioms of probability, conditional probability, the likelihood interpretation, relation between likelihood and sample space probability, Baye's law, independence, random variables, Naïve Bayes classification
- 8. **Numerical Random Variables:** marginal distribution, expected value, decision theory, decision trees, variance and standard deviation, random variables over infinite set of integers, discrete distributions (Bernoulli, Binomial, Zipf), continuous random variables, continuous distributions (uniform, Gaussian), central limit theorem
- 9. **Markov Models:** transition probability matrix, Chapman-Kolmogorov equations, stochastic matrix, classification of states, recurrent, transient, irreducible chain, periodicity, stationary distribution, uniqueness, convergence, PageRank and link analysis, Hidden Markov models, Viterbi algorithm, Part of speech tagging
- 10. **Statistical Inference and Confident Intervals:** point estimation, biased and unbiased estimators, point estimators for mean and variance, interval estimation and confidence intervals, chi-squared distribution, *t*-distribution, confidence intervals for mean and variance of normal random variables, error function, hypothesis testing and statistical significance

- 11. **Monte Carlo methods:** finding area, generating distributions, counting, sums, expected values, integrals, probabilistic problems, generating random shuffles, Fisher-Yates shuffles, Markov Chain Monte Carlo (MCMC), Metropolis-Hastings algorithm, pseudorandom numbers, resampling
- 12. **Information and Entropy:** information theory, entropy, conditional entropy, mutual information, coding theorems, Huffman and block coding, entropy of numeric and continuous random variables, principle of maximum entropy
- 13. **Maximum Likelihood Estimation:** sampling, uniform distribution, Gaussian distribution with known and unknown variance, least square estimates, Principal component analysis (PCA) and its applications

General Information

• Lecture: Wednesdays 6:20pm-8:50pm (In-person)

Loc: BOBS Room: LL138First class on 05/25/2022

• Instructor: Parijat Dube

Grading: Programming Assignments (25%)

Problem Sets (25%) Mid Term (20%) Final Exam (20%) Quizzes (10%)