Q2

insertion sort:

10, 5, 7, 9, 8, 3

5, 10, 7, 9, 8, 3

5, 7, 10, 9, 8, 3

5, 7, 9, 10, 8, 3

5, 7, 8, 9, 10, 3

3, 5, 7, 8, 9, 10 done

Quick sort: (method 1)

{10, 5, 7, 9, 8, 3} partition around 9

{5, 7, 8, 3} partition around 7 {10}

{5, 3} partition around 3 {8}

{ } 5

3, 5, 7, 8, 9, 10. done

Quick sort (method 2)

10 5 7 9 8 3 $x \leftarrow 10$ pivot

i j

10 5 7 9 8 3

i j

10 5 7 9 8 3

i j

10 5 7 9 8 3

i j

10 5 7 9 8 3

i j   loop ends

$\underbrace{5 \quad 7 \quad 9 \quad 8 \quad 3}_{\le \text{pivot}}$ 10

$x \leftarrow 5$ pivot

i j

3 5 7 9 8 $\boxed{10}$

i j $7 \leftarrow$ pivot

3 $\boxed{5}$ $\boxed{7}$ 9 8 $\boxed{10}$

i j $9 \leftarrow$ pivot

3 5 7 8 9 10

Q3

1. $n+3 \in \Omega(n)$ True

$0 \leq cn \leq n$

$n_0 = 0$

2. $n+3 \in O(n^2)$ True

$0 \leq n \leq cn^2$

$cn \geq 3$ $n \geq \frac{3}{c}$

$n_0 = \frac{3}{c}$

3. $n+3 \in \Theta(n^2)$ False

$0 \leq c_1 n^2 \leq n \leq c_2 n^2$

$0 \leq c_1 n \leq 1 \leq c_2 n$

$\frac{1}{c_2} \leq n \leq \frac{1}{c_1}$

4. $2^{n+1} \in O(n+1)$ False

$2^{n+1} \leq c(n+1)$

$2^{n+1} \leq cn$

$2^n \leq c$

5. $2^{n+1} \in \Theta(2^n)$ True

$0 \leq c_1 2^n \leq 2^{n+1} \leq c_2 2^n$

$c_1 \leq 2 \leq c_2$

$n_0 = 2$

Q4

1. $T(n) = 8T(\frac{n}{2}) + n$

$a=8$, $b=2$

$n^{\log_b a} = n^{\log_2 8} = n^3$

$f(n) = n = O(n^{3-\varepsilon})$ for some $\varepsilon > 0$

$\Rightarrow$ case 1: $T(n) = \Theta(n^3)$

2. $T(n) = 8T(\frac{n}{2}) + n^2$

$a=8$, $b=2$, $f(n) = n^2$ $n^{\log_b a} = n^3$ $\Rightarrow$ case 1:
$T(n) = \Theta(n^3)$

3. $T(n) = 8T(\frac{n}{2}) + n^3$

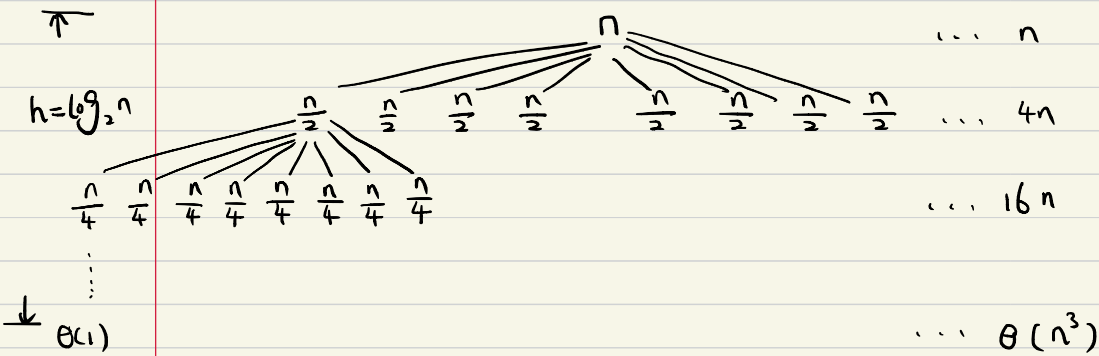
$a=8$. $b=2$, $f(n) = n^3$ $n^{\log_b a} = n^3$ $\Rightarrow$ case 2:
$T(n) = \Theta(n^3 \cdot \lg n)$

4. $T(n) = 8T(\frac{n}{2}) + n^4$

$a=8$, $b=2$, $f(n) = n^4$ $n^{\log_b a} = n^3$ $\Rightarrow$ case 3:
$T(n) = \Theta(n^4)$

Q5

$T(n) = 8T(\frac{n}{2}) + n$



\# leaves $a^{\log_b n} = n^{\log_b a} = n^3$

$$T_n = \underbrace{(1 + 4 + 16 + \cdots + 4^{k-1})}_{\log_2 n} n + \Theta(n^3)$$

$$= O(n^3)$$

Substitution method :

- guess $T(n) = O(n^3)$
- Assume $T(k) \le ck^3$ for $k < n$

$T(n) = 8T(\frac{n}{2}) + n \le 8 \cdot c(\frac{n}{2})^3 + n = cn^3 + n$

$= \underbrace{cn^3}_{\text{desired}} - \underbrace{(-n)}_{\text{residual}}$

- assume $T(k) \le C_1 k^3 - C_2 k$ for $k < n$

$T(n) = 8T(\frac{n}{2}) + n \le 8[C_1(\frac{n}{2})^3 - C_2(\frac{n}{2})] + n = C_1 n^3 - 4C_2 n + n$

$= \underbrace{(C_1 n^3 - C_2 n)}_{\text{desired}} - \underbrace{(3C_2 - 1)n}_{\text{residual}}$

base case $T(1) = \Theta(1) \le C_1 - C_2$ if c is chosen sufficiently large with respect to C_2