

EXPERIMENT OF SIMPLE VIBRATION

1. PURPOSE

The purpose of the experiment is to show free vibration and damped vibration on a system having one degree of freedom and to investigate the relationship between the basic vibration parameters.

2. THEORY

2.1. Free Vibrations

Consider a system including a body of mass m moving along only the vertical direction, which is supported by a spring of stiffness coefficient k and a negligible mass (Figure 1a).

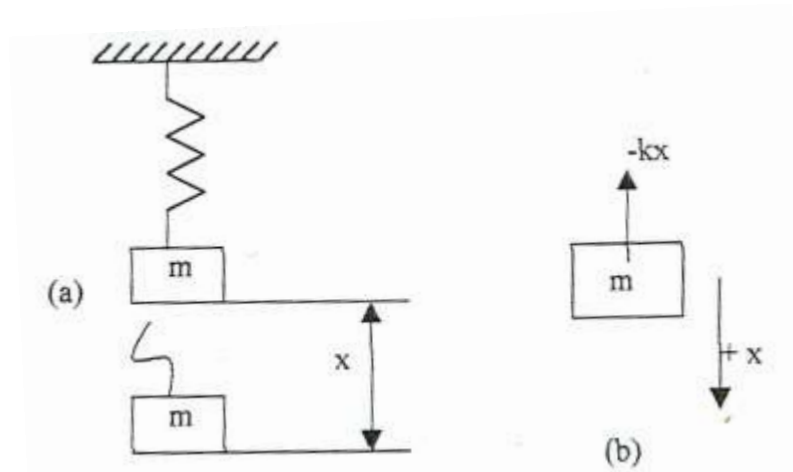


Figure 1. A system having one degree of freedom

Let the mass m be given a downward displacement from the equilibrium position and then released. At some time t , the mass will be at a distance x from the equilibrium position.

The net force on the mass is the spring force $(-kx)$ which will tend to restore it to its equilibrium position. Therefore, the equation of the motion by Newton's second law (Figure 1b).

$$m \frac{d^2 x}{dt^2} = -kx \quad (1)$$

or

$$\frac{d^2 x}{dt^2} + \omega_n^2 x = 0 \quad (2)$$

where

$$\omega_n^2 = \frac{k}{m} \quad (3)$$

Solution of equation VIII.2 with the initial conditions

$$x(t=0) = 0 \quad (4a)$$

and

$$\frac{dx}{dt}(t=0) = 0 \quad (4b)$$

is

$$x(t) = x_0 \cos \omega_n t \quad (5)$$

The motion can be seen in figure (2)

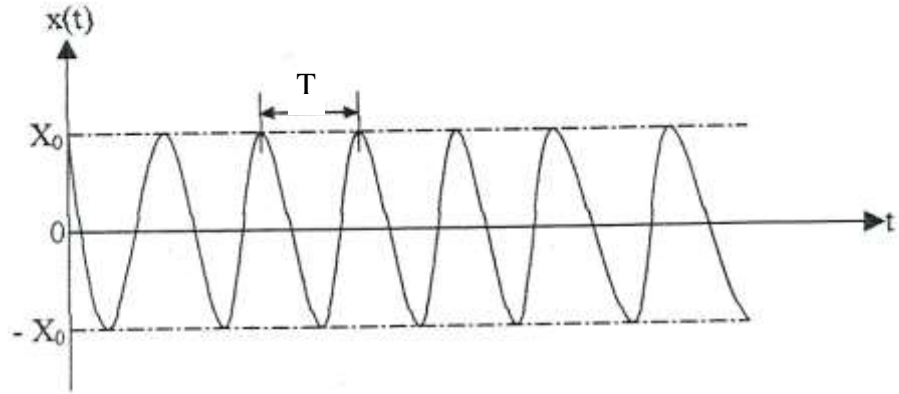


Figure 2 The motion of free vibration of spring mass system

The period T of the motion is determined from Figure 2. From equation 2 frequency f is given as

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (6a)$$

or

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta s}} \quad (6b)$$

where

$$\Delta s = \frac{mg}{k} \quad (7)$$

2.2. Damped Vibrations

Consider a system including a body of mass m hung by a spring of stiffness coefficient k and negligible mass. The mass is also attached by a perforated piston to an oil-cylinder whose resistance is proportional to the velocity of the mass (Figure 3a).

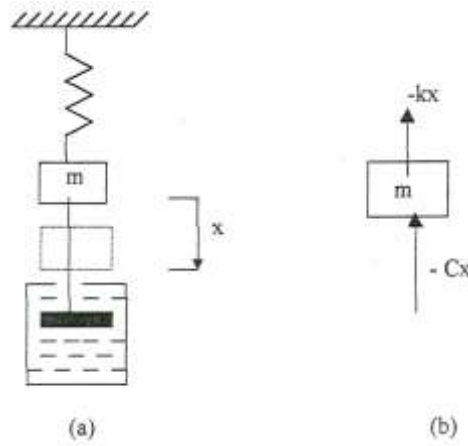


Figure 3. Oil-cylinder with a perforated piston.

Let the mass m be given a downward displacement from the equilibrium position and then released. To determine the equation of motion, a free body diagram of the mass at a distance x from the equilibrium position at some time t is shown in figure 3b.

Net forces on the mass are the damping force $(-Cdx/dt)$ which will tend to restore its equilibrium position, the spring force and the inertia force. C is the damping coefficient of the oil. From the Newton's second law

$$m \frac{d^2 x}{dt^2} = -kx - C \frac{dx}{dt} \quad (8)$$

or

$$\frac{d^2 x}{dt^2} + 2\omega_n r \frac{dx}{dt} + \omega_n^2 x = 0 \quad (9)$$

where

$$\omega_n^2 = \frac{k}{m} \quad \text{and} \quad r = \frac{C}{2\omega_n m} = \frac{C}{2\sqrt{km}} \quad (10)$$

r is called the damping factor (ratio) in equation 9

Solution of the equation of motion is dependent on the damping ratio. If $r > 1$, a non-periodic movement which approaches the equilibrium position is obtained (Figure 4).

$$r > 1 \quad x(t) = Ae^{(-r + \sqrt{r^2 - 1})\omega_n t} + Be^{(-r - \sqrt{r^2 - 1})\omega_n t} \quad (11)$$

A and B are constants which are dependent on the initial conditions of motion. If $r = 1$, the motion corresponding to the critical damping ratio is approaching the equilibrium position with time.

$$r = 1 \quad x(t) = (A + Bt)e^{-r\omega_n t} \quad (12)$$

If $r < 1$, a motion which has a period T and a decreasing amplitude with time t is obtained.

$$r < 1 \quad x(t) = e^{-r\omega_n t} (A \cos \sqrt{1 - r^2} \omega_n t + B \sin \sqrt{1 - r^2} \omega_n t) \quad (13)$$

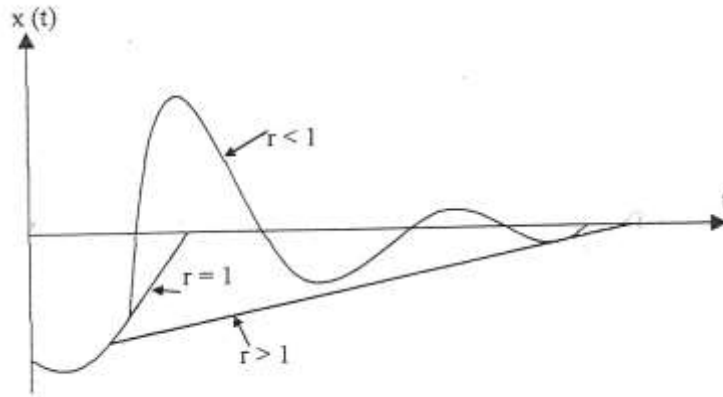


Figure 4. The motion of damped free vibration of spring-mass system

The following relation is found between two consecutive wave widths, as can be seen from equation 13.

$$x_{n+1} = x_n e^{-r\omega_n t} \quad (14)$$

The *logarithmic decrement* is defined as follows to calculate the damping ratio.

$$\delta = \log_e \frac{x_n}{x_{n+1}} \quad (15a)$$

where x_n : *nth*. maximum amplitude during vibration; x_{n+1} : is the next maximum.

If we consider the drop in amplitude in n successive cycles then the log decrement is given by

$$\delta = \frac{1}{n} \log_e \frac{x_0}{x_n} \quad (15b)$$

where x_0 : vibration amplitude of initial cycle; x_n : vibration amplitude of n . cycle.

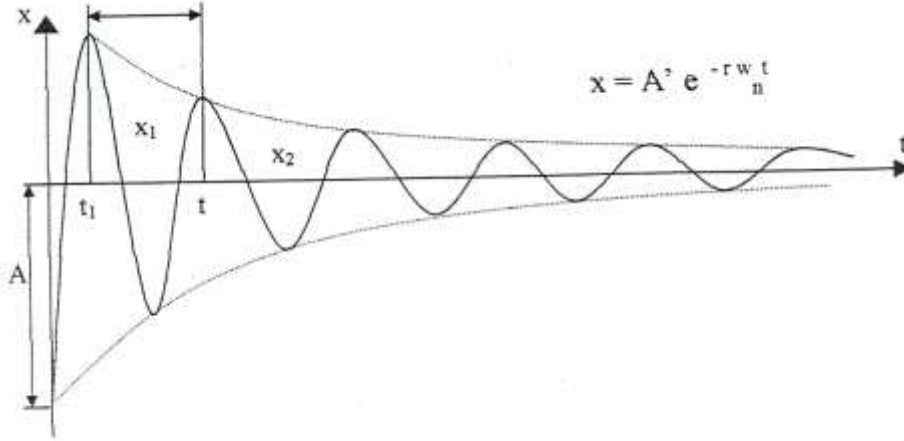


Figure5. Damped vibrations with damping ratio less than 1.

$$\delta = r \omega_n t \quad (16)$$

Vibration period T is

$$T = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n \sqrt{1-r^2}} \quad (17)$$

where ω_d is the damped natural frequency. Note that time interval between two consecutive peaks in the graphic is equal to period, T . Because of this, time t between x_1 and x_2 cycles must be equal to period T . So $t=T$. The damping ratio is calculated as follows

$$\delta = r \omega_n t = \frac{2\pi r}{\sqrt{1-r^2}} \quad (18)$$

or

$$r = \frac{\delta / 2\pi}{1 + (\delta / 2\pi)^2} \quad (19)$$

for small damping we have $r \cong \frac{\delta}{2\pi}$. So the damping ratio,

$$r = \frac{1}{2\pi} \log_e \frac{x_n}{x_{n+1}} \quad (20)$$

3. EXPERIMENTAL APPARATUS

The experimental rig basically consists of a frame free to move vertically on roller guides, removable weights and a drawing pen attached to frame. The movable frame is attached to the fixed frame. A recording pen is also attached to the fixed frame. A paper rolling system driven by an electric motor and a perforated piston in an oil-cylinder are also attached to the fixed frame. The mass of the moving frame is approximately 1.7 kg. There are some mass blocks that will be added on the movable frame during experiment. Various springs, whose spring constants are to be determined experimentally, will be available. The drawing pen records the spring vibrations on the paper strip moving at a speed 0.02 m/s.



(a) Experimental rig



(b) Different springs and removable weights



(c) Plotter



(d) Frame



(e) Dashpot

Figure. Experimental rig system

4. EXPERIMENTS

4.1. Free Vibration Tests

Below steps are performed for each one of the springs:

- Elongation values are measured and recorded from a reference point while the frame is empty and carrying 1, 2 and 3 kg masses respectively. These data are going to be used to calculate the spring coefficient.
- Vibration amplitude and frequency are determined on the recording surface while the frame is empty, and while it has a 1, 2, 3 kg mass respectively.

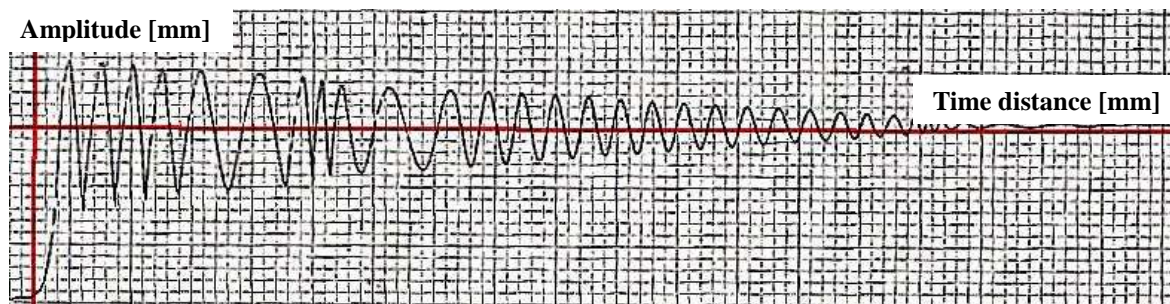


Figure. An example vibration amplitude plotting while $F=15.7\text{ N}$

4.2. Damped Vibration Tests

Below steps are repeated for each spring after the oil-cylinder is charged with oil and the piston is connected with the frame.

- The portable frame is released from a certain height and the amplitude-time curves are determined on the recording surface when the frame is empty and when additional masses are attached respectively. Also for each mass added, tests are performed again in the 1/4 turn of the adjusting nut of pistons' holes.

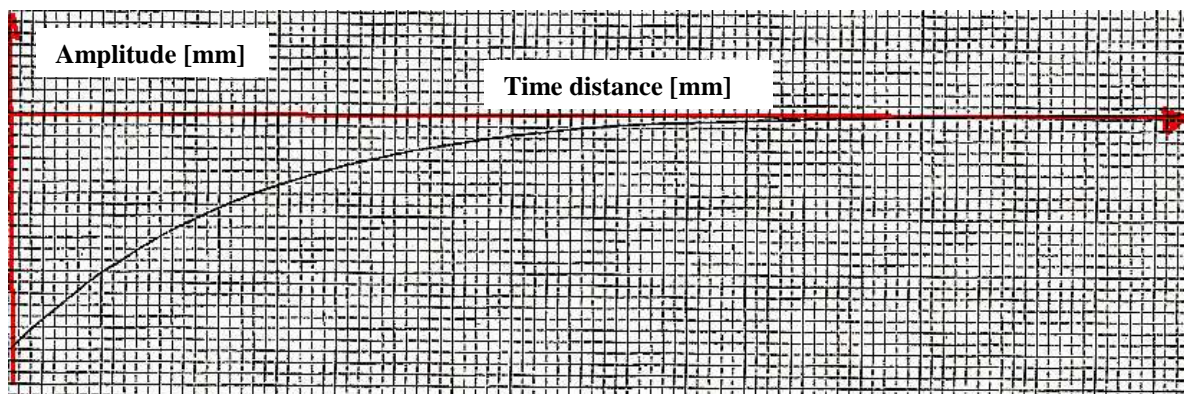


Figure. An example vibration amplitude plotting from damped vibration tests

- Attach a suitable mass on the frame and record the motion after the frame is released from an initial displacement.

5. GRAPHS

- The relations between the force and elongation are read on the scaled paper by using displacement values resulting from static loads. The *spring coefficients* are calculated from the curve for each one of the springs.
- The *experimental period* and *frequency* values are determined from the vibration plotting of each mass-spring system. Also the *theoretic period* and *frequency* values also will be calculated using Equation 6 a, b for the same mass-spring systems.

Using these results, the whole data are recorded on the Table 1.

Table 1. Sample table for free vibration test.

Spring coefficient k (kN/m)	Mass m (kg)	Period (s)		Frequency (Hz)		Static displacement (cm)
		Test	Theory	Test	Theory	

Below graphics will be obtained from the experimental data tabulated above.

- Plot change in *frequency* as a function of *mass* for each spring.(test and theory)
 - Plot the change in *frequency* as a function *spring constant* for each spring.
 - Plot the curve with the *static displacements* on the abscissa and *the frequencies* on the ordinate axes.
- c) The velocities, which correspond to the each piston-hole-adjusting nut positions, are calculated using the vibration plotting of each mass-spring-dashpot system. These values can be recorded in Table 2.

Table 2. Sample table for experimental data

	Adjusting nut position				
	Closed	1/4 turn	2/4 turn	3/4 turn	4/4 turn
Force	Piston velocities (cm/s)				

	Adjusting nut position				
	Closed	1/4 turn	2/4 turn	3/4 turn	4/4 turn
Force	Damping coefficient of oil-cylinder C				

From the experimental data and calculated data in the tables, below graphics will be obtained:

- 1) Change in *piston velocity* as a function of the *adjusting nut position*
- 2) Change in *force* with respect to *piston velocity*.
- 3) Change in the *damping coefficient* of oil-cylinder with respect to the *adjusting nut position*.
- 4) Calculate the *damping ratio* of any mass-spring system.

6. OBSERVATIONS AND DISCUSSIONS

Observations and results from figures in section “5.Graphs” will be discussed. While discussing, it will be useful to look for answers to the following questions:

- a) What is the relationship between the force and displacement?
- b) Frequency of free vibration changes with respect to the mass and spring coefficient. Compare the frequencies determined from the theory with those determined experimentally. Which error sources exist? What are the other differences between theoretical and experimental results? How are they explained? What is the relationship between frequency and static displacement? Does it give the same relation regardless of the mass and spring chosen?
- c) What is the changing in the piston velocity with respect to the adjusting nut position? How are the force and velocity related? Is it in accordance with the assumptions used in the motion? Which differences are observed between the theory and experiment? What are the error sources and how can they be decreased? What are your observations on the damped-free vibration experiment?
- d) Search other techniques to measure vibrations.
- e) What are the advantage and disadvantages of the vibrations of mechanical parts? Explain briefly.

EXAMPLE CALCULATIONS

Mass & elongation data are in the Figure and Table.

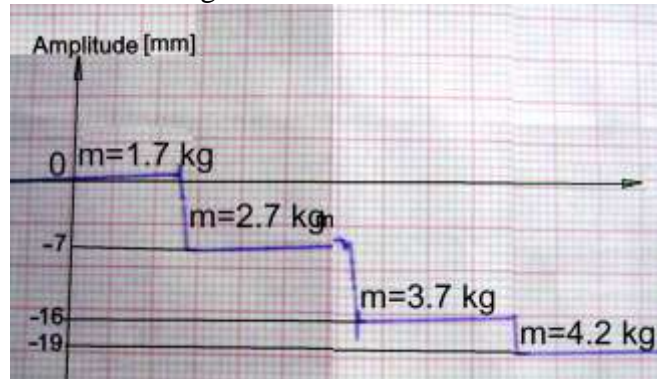


Figure. *mass* versus Δx relation

System's Total Mass m (kg)	1.7	2.7	3.7	4.2
Elongation (mm)	0	7	16	19

$$k_1 = \frac{(2.7-1.7).9,81}{(7-0).10^{-3}} = 1401.43 \text{ N/m} \quad k_2 = \frac{(3.7-2.7).9,81}{(16-7).10^{-3}} = 1090 \text{ N/m} \quad k_3 = \frac{(4.2-3.7).9,81}{(19-16).10^{-3}} = 1635 \text{ N/m}$$

$$k_{mean} = \frac{\sum_{i=1}^n k_i}{n} = 1375.5 \text{ N/m}$$

Obtaining Period & Frequency (Experimentally)

Amplitude & time plotting for free vibration experiment is in the Figure.

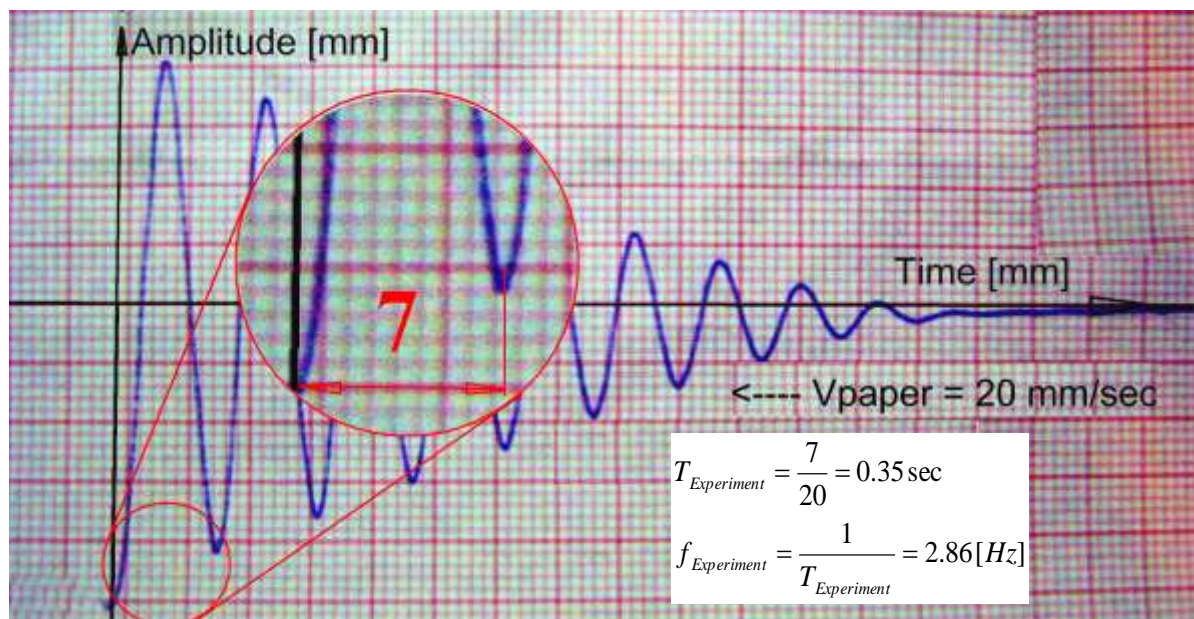


Figure . $F=15.7\text{ N}$ and $m=2.7\text{ kg}$

Obtaining Period & Frequency (Theoretically)

$$f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{1375.5}{2.7}} = 3.59 \text{ Hz} \quad T = \frac{1}{f} = 0.28 \text{ sec}$$

Obtaining Static Displacement Δs (mm) $\Delta s = \frac{mg}{k} = \frac{1.7 * 9.81}{1122.1} = 0.014m$

Experimentally calculating the **damping ratio r**

Damping ratio r is calculated experimentally using amplitudes of the *consecutive two peaks* and the formula of $r = \frac{1}{2\pi} \log_e \frac{x_n}{x_{n+1}}$ where r : damping ratio, x_n : *nth.* maximum amplitude during vibration; x_{n+1} is the next maximum.

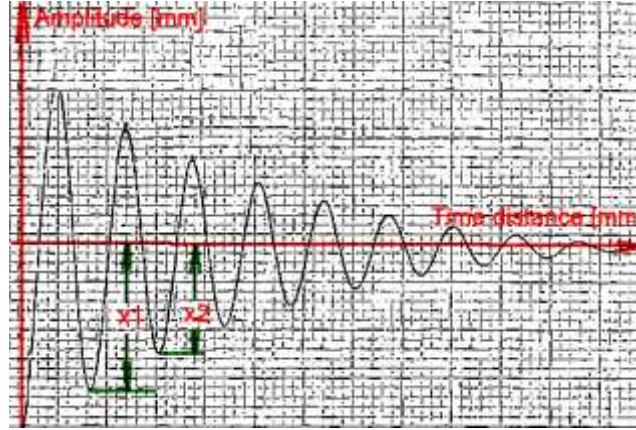


Figure $m=1.7$ kg and full open position of the whole holes.

$$r = \frac{1}{2\pi} \ln \frac{-15}{-11.5} = 0,0423$$

To calculate the damping ratio r of an **over damped** vibrating system.

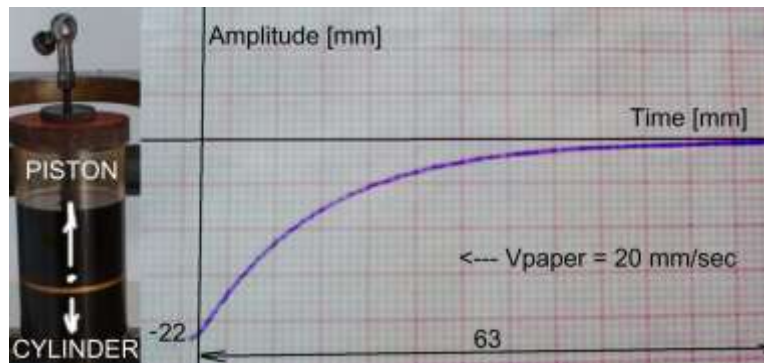


Figure $m=1.7$ kg and full closed position of the whole holes, $k=1090$ N/m.

We can use $r = \frac{C}{2\sqrt{km}}$. We need to determine C before to calculate. For this, we can

use $C = -\frac{mg + kx}{v}$, where, m =mass, k =spring coefficient, v =piston velocity. Before to calculate,

we need to also determine piston velocity V_{Piston} first.

$$V_{Piston} = \frac{X_{Piston}}{t} = \frac{-0.022}{63/20} = -6.984 * 10^{-3} \text{ m/s}$$

$$C = -\frac{mg + kx}{v_{Piston}} = -\frac{1.7 * 9.81 + 1090 * 0.015}{-6.984 * 10^{-3}} = 4729 \text{ Ns/m}$$

$$\text{So; } r = \frac{C}{2\sqrt{km}} = \frac{4729}{2\sqrt{1090 * 1.7}} = 54.93$$