W8 Experiment Design Guided by Estimates of Uncertainty

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INTRODUCTION

- This workshop is about performing an uncertainty analysis before sensors have been selected, before an apparatus has been built, and before you have any data. This analysis is done in the *design phase* of an experimental program.

- The purpose of this type of analysis is to inform your design decisions based on evidence that realistic bounds on the *expected uncertainty in the result* can be met. When the analysis is complete, you will have:
  - determined the feasibility of a proposed design before prototyping and shaking-down,
  - determined design features of the apparatus,
  - selected instrumentation with systematic uncertainties appropriate to each measurand, and
  - developed an uncertainty budget for design.

ERROR PROPAGATION

- Consider a general case in which an experimental result R is a function of n measured variables (measurand) x_i ,

$$R = R\left(x_1, x_2, \ldots, x_n\right) \qquad (1)$$

- Assumption:
  - R is continuous and differentiable w.r.t x_i
  - each x_i is independent one other
  - each uncertainty of x_i is independent one another

- Then uncertainty of R including random uncertainty is given by

$$w_R{}^2 = \left(\frac{\partial R}{\partial x_1}\right)^2 w_{x_1}^2 + \cdots + \left(\frac{\partial R}{\partial x_n}\right)^2 w_{x_n}^2 + w_{R,rand}^2 \qquad (2)$$

RSSC- Root-Sum-Square contributions

- Contributions of individual measurands and random uncertainty to the total uncertainty in the resultant

$$w_R{}^2 = \Big(\underbrace{\frac{\partial R}{\partial x_1} w_{x_1}}_{RSSC}\Big)^2 + \cdots + \left(\frac{\partial R}{\partial x_n} w_{x_n}\right)^2 + \left(w_{R,rand}^2\right)^2 \qquad (3)$$

$$w_R = \sqrt{\sum (RSSC)^2}$$

To compare individual contributions in percentage form, divide every term in (3) by R^2

$$\left(\frac{w_R}{R}\right)^2 = \left(\frac{1}{R}\frac{\partial R}{\partial x_1} w_{x_1}\right)^2 + \cdots + \left(\frac{1}{R}\frac{\partial R}{\partial x_n} w_{x_n}\right)^2 + \left(\frac{w_{R,\mathbf{rand}}}{R}\right)^2 \qquad (4)$$

$$\left(\frac{w_R}{R}\right) = \sqrt{\sum (RSSC\ \%)^2}$$

UMF-Uncertainty Magnification Factor Form

- Multiplying each term by $(x_i/x_i)^2$ on RHS of (4) gives relative uncertainties for all measurands,

$$\left(\frac{w_R}{R}\right)^2 = \Big(\underbrace{\frac{x_1}{R}\frac{\partial R}{\partial x_1}}_{UMF}\Big)^2 \Big(\frac{w_{x_1}}{\underbrace{x_1}_{\text{Relative uncertainty in measurand}}}\Big)^2 + \cdots + \left(\frac{x_n}{R}\frac{\partial R}{\partial x_n}\right)^2 \left(\frac{w_{x_n}}{x_n}\right)^2 + \left(\frac{w_{R,\mathbf{rand}}}{R}\right)^2 \qquad (5)$$

The coefficients of the relative uncertainties, before squaring, are called *UMF*s.

UMFs play a role in the design phase of an experiment, helping us determine which meaurands might require special care.

$$RSSC_i\% = UMF_i \times \frac{w_{x_i}}{x_i} \qquad (6)$$

UPC- Uncertainty Percentage Contribution Form

- Another non-dimensional forms of uncertainty relationship is in terms of *UPCs*. To obtain this form, divide all terms on both sides of (5) by the relative uncertainty term,

$$\frac{\left(\frac{w_R}{R}\right)^2}{\left(\frac{w_R}{R}\right)^2} = \frac{\left(\frac{x_1}{R}\frac{\partial R}{\partial x_1}\right)^2\left(\frac{w_{x_1}}{x_1}\right)^2}{\left(\frac{w_R}{R}\right)^2} + \cdots + \frac{\left(\frac{x_n}{R}\frac{\partial R}{\partial x_n}\right)^2\left(\frac{w_{x_n}}{x_n}\right)^2}{\left(\frac{w_R}{R}\right)^2} + \frac{\left(\frac{w_{R,\mathbf{rand}}}{R}\right)^2}{\left(\frac{w_R}{R}\right)^2} \qquad (7)$$

$$1 = \sum \frac{(RSSC\%)^2}{\left(\frac{w_R}{R}\right)^2} = \sum \frac{\left(UMF\,\frac{w_{x_i}}{x_i}\right)^2}{\left(\frac{w_R}{R}\right)^2}$$