Línuleg algebra A
Verkefnablað vegna umræðutíma 2

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Skilverkefni fyrir 29.09.21: Þessi verkefni eru einstaklingsverkefni. Lausnum verkefnanna á að skila miðvikudaginn 29. september í gegnum Canvaskerfið.

1. Látum A, B og C vera mengi.

(a) Sannið eða afsannið: Ef $A \setminus C \subseteq B \setminus C$ þá er $A \subseteq B$.

(b) Sannið eða afsannið: Ef $A \subseteq B$ þá er $A \setminus C \subseteq B \setminus C$.

2. Verkefni 4.3.3 á bls. 65

Let $A \subseteq \mathbb{R}$, and let $x \in \mathbb{R}$. We say that the point x is *far away* from the set A if and only if:

$\exists d > 0$: No element of A belongs to the set $[x - d, x]$.

Equivalently, $A \cap [x - d, x] = \emptyset$. if this does not happen, we say that x is *close* to A

(a) Draw a picture of a set A and an element x such that x is *far away* from A.

(b) Draw a picture of a set A and an element x such that x is *close* to A.

(c) Compute the definition of "x is *close* to A". [So negate "x is far away from A".]

(d) Let $A = \{1, 2, 3\}$. Show that $x = 4$ is *far away* from A by using definitions.

(e) Let $A = \{1, 2, 3\}$. Show that $x = 1$ is *close* to A by using definitions.

(f) Show that if $x \in A$, then x is *close* to A.

(g) Let A be the open interval (a, b). Is the end-point a *far away* from A? What about the end point b

3. Sannið formlega að ef A, B og C er mengi þá er $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

4. Verkefni 6.2.4 og 6.2.5 á bls. 104–105.

Here are three incorrect proofs of Theorem 6.5. Explain why each fails.

(a) Let $x = \mathcal{P}(A)$. Then $x \in A$. Since $A \subseteq B$, we have $x \in B$. Therefore $x \in \mathcal{P}(B)$, and so $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

(b) Let $A = \{1, 2\}$ and $B = \{1, 2, 3\}$. Then $\mathcal{P}(A) = \{\emptyset, \{1\}, \{2\}, A\}$, and $\mathcal{P}(B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, B\}$. Thus $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

(c) Let $x \in A$. Since $A \subseteq B$, we have $x \in B$. Since $x \in A$ and $x \in B$, we have $\{x\} \in \mathcal{P}(A)$, and $\{x\} \in \mathcal{P}(B)$.