Set 36

Data and Statistics

These are the sections you must complete in this set:

- ✓ **Topic Questions**
- ✓ **Problem Solving**
- ✓ **Challenge Questions**

What is statistics ?

Statistics is a branch of Mathematics that is focused on the collection and analysis of data and facts. The data may relate to any entity such as a country, a population, the environment, a school, the economy, and so forth. Statistics is composed of many components including such tasks as:

- the collection and organization of data relating to an investigation
- the statistical calculations and analysis of the data
- the summary, presentation and outcome of the investigation.

Types of statistical data

Data is a collection of facts, such as numbers, words, measurements, observations, and so forth. The data collected can be classified as follows:

- **Numerical** (or quantitative) data that provides information about the quantity or size. This type of data can be further described as:
  - ➢ **Discrete** data that has a finite numerical value such as a counted number.
    For example, number of students, number of cars or goals scored.
  - ➢ **Continuous** data that may not be a whole number such as a measured value.
    For example, the height of students, time in a race or the length of an object.
- **Categorical** (or qualitative) data that provides information about categories or groups. This type of data can be further described as:
  - ➢ **Nominal** data that is a named category such as a colours, suburbs or types of pets.
  - ➢ **Ordinal** data where an order is implied, such as days of the week or postcodes.

Analysing the data

After collecting and organising the data, we need to analyse and find out more information about the data by calculating values such as the mean, median, mode and range.

- The **mean** or **average** of a set of values is the sum of all the values divided by the number of values. This is calculated using the formula:

$$Mean = \frac{Sum\ of\ all\ values}{Number\ of\ values}$$

- The **median** is the middle value when the data has been arranged in numerical order. If there is an odd number of values, then the median is simply the middle value. For an even number of values, the median is the average of the two middle data values.
- The **mode** is the value that appears most often in a set of data. Note that some sets of data have more than one mode, or no mode at all; that is, there is no value which occurs multiple times as all values occur once only.
- The **range** is the difference between the largest and smallest value in a set of data.

Example: For the data values 12, 9, 8, 6, 5, 5, 4 find the mean, median, mode and range.

$$Mean = \frac{Sum\ of\ all\ values}{Number\ of\ values} = \frac{49}{7} = 7$$

The data in already in numerical order. There is an odd number of values.
The median will occur at the middle value, namely, the 4th value.

∴ Median = 6.

Range = largest value – smallest value = 12 – 4 = 8.

Example: For the set of data values { 6 , 2 , 4 , 3 , 4 , 5 , 4 , 5 , 1 , 5 }, find the mean, median, mode and range.

$$Mean = \frac{Sum\ of\ all\ values}{Number\ of\ values} = \frac{39}{10} = 3.9$$

Arranging the data in numerical order gives: 1 , 2 , 3 , 4 , 4 , 4 , 5 , 5 , 5 , 6
The median will occur between the 5th and 6th values. Both of these are 4.
∴ the median is 4.

There are two modes for this set of data, namely 4 and 5. Both occur three times.

Range = largest value – smallest value = 6 – 1 = 5.

Topic Questions

For each of the following sets of data, find the mean, median, mode and range:

1. 4 , 5 , 5 , 7 , 7 , 8 , 8 , 8 , 9 , 9

Mean: 7 Median: 7.5

Mode: 8 Range: 5

2. 2 , 3 , 3 , 1 , 4 , 2 , 5 , 7 , 5 , 9 , 3

Mean: 4 Median: 3

Mode: 3 Range: 8

3. 19 , 18 , 15 , 21 , 14 , 19 , 13 , 20 , 23

Mean: 18 Median: 19

Mode: 19 Range: 10

4. 56 , 60 , 68 , 49 , 65 , 87 , 67 , 56 , 72 , 70

Mean: 65 Median: 66

Mode: 56 Range: 38

5. Pedro has calculated his mean score for history to be 89 %, based on five tests. If he scores 95 % in the sixth test, what will his new mean score be ?

Answer: 90 %

6. Below are the height measurements (in cm) for 20 students in Year 6:

145, 152, 148, 152, 163, 148, 165, 158, 159, 162,
157, 167, 156, 148, 156, 152, 154, 133, 141, 154.

(a) What is the range of the heights ? *Answer:* 34 cm

(b) What is the mean height of the class ? *Answer:* 153.5 cm

(c) What is the mode for the heights ? *Answer:* 148 cm and 152 cm

(d) What is the median height of the class ? *Answer:* 154 cm

Problem Solving

1. Fiona bought a new phone and paid $765 after a 15 % discount. What was the original price of the phone ?

Answer: $900

2. Alex needs to pour a concrete slab for a shed on his farm. The slab is 6 metres long, 4 metres wide and 30 cm deep. How many cubic metres of concrete is needed ?

Answer: 7.2 m^3

3. Marcus wants to buy enough tiles to cover an area that has dimensions 8 metres by 7 metres. If the tiles cost $48 per square metre, how much will Marcus have to pay for the tiles ?

Answer: $2 688

4. Samantha drives 820 km on a trip. For the first 4 hours, she drove at 80 km/h and then drove at 100 km/h for the rest of the trip. Find the time taken to complete the whole trip.

Answer: 9 hours

5. A tap was dripping at a rate of one millilitre every two seconds. If the tap was left dripping for 2 hours and 30 minutes, find the amount of water in litres that was wasted.

Answer: 4.5 litres

6. Harry is 10 years old at present. His mother is four times his age and three years younger than his father. Find his father's age when Harry is 15 years old

Answer: 48 years old

7. Alex, Ben and Caroline won $130 000 in a lottery. The jackpot will be divided among them based on their share of the money they put towards the ticket. Thus, Alex should get 35 % of the jackpot and Caroline gets $\frac{1}{4}$ of the winnings. How much money should Ben get ?

Answer: $52 000

Challenge Questions

1. Sally is going on a holiday and packs 6 tops, 4 pairs of pants and 3 pairs of shoes. She wants to wear a different outfit each day. How many different combinations are possible ?

Answer: 72

2. A 200 km bike ride around the bay began with riders completing $\frac{3}{8}$ of the distance on the first day. If they covered 25 % of the distance the next day, how far do they have to travel on the third day to finish the ride ?

Answer: 75 kilometres

3. Increase 750 kg by 10 % and then decrease the result by 10 %. What is the new weight ?

Answer: 742.5 kg

4. If 55 % of a number is 165, what is the number ?

Answer: 300

5. Peter and Rita received a $2 450 prize from a competition they entered together. If Rita gets the larger share and they agree to divide it in the ratio 3 : 4, how much will each receive.

Answer: Peter receives $1 050 and Rita receives $1 400.

6. The pages of Jack's book are numbered from 1. The page numbers have a total of 555 digits. How many pages has the book ?

Answer: 221 pages

7. What are the next two characters in this pattern ?

5 H 7 L 11 P 17 ? ?

Answer: T and 25

Set 37

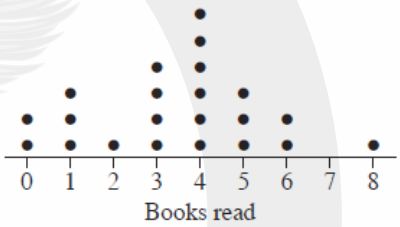
Dot Plots and Stem-and-Leaf Plots

These are the sections you must complete in this set:

- ✓ **Topic Questions**
- ✓ **Problem Solving**
- ✓ **Challenge Questions**

Dot plots

A **dot plot** is a graphical display of data using dots where each piece of data or value is represented by a single dot.



Dot plots consist of a horizontal axis that is labelled and evenly scaled, and each data value is represented by a dot. If a value is repeated, a second dot is placed directly above the previous one.

Once all values have been recorded, the data points, if neatly drawn and evenly spaced, resemble columns placed over a number line, as shown in the diagram above.

Example: Display the following data in a dot plot.

1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5,
6, 1, 2, 2, 3, 3, 3, 4, 4, 5

Stem-and-leaf plots

The data values in a **stem-and-leaf plot** are made up of two parts: a stem and a leaf. For example, the value 98 is made up of a ten's part (the stem) and a unit's part (the leaf), and is written as:

Stem	Leaf
9	8

Key: 9 | 8 = 98

Remember that you should always include a key when presenting a stem-and-leaf plot.

Data in stem-and-leaf plots can be analysed to obtain the median, mode and range. It is important that the number values in the leaf section are arranged in ascending order.

Example: Display the following data in a stem-and-leaf plot. Find the median and range.

30 , 25, 11, 2, 18, 29, 43, 36, 4,

11, 24, 1, 4, 9, 16, 38, 7, 13, 27

0	1 2 4 4 7 9
1	1 1 3 6 8
2	4 5 7 9
3	0 6 8
4	3

Key: 2 | 4 = 24

Ensure the leaf section is in ascending order.

Remember to include a key.

There are 19 data values.

The median will be the 10th value. $\therefore$ median = 16.

Range = largest value – smallest value = 43 – 1 = 42.

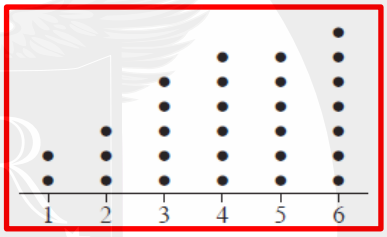
Topic Questions

1. Display the following data in a dot plot.

1, 2, 3, 4, 5, 6, 6, 5, 4, 3,

2, 1, 2, 3, 4, 5, 6, 6, 5, 4,

3, 3, 4, 5, 6, 6, 5, 4, 6



2. For the data in the previous question, find the median, mode and the range.

Answer: Median = 4 , mode = 6 and range = 5

3. Find the mean, rounded to the nearest whole number.

Answer: Mean = 4

4. Display the following data in a stem-and-leaf plot.

75, 88, 99, 69, 70, 84,

96, 72, 85, 91, 87, 93,

87, 94, 97, 90

6	9
7	0 2 5
8	4 5 7 7 8
9	0 1 3 4 6 7 9

5. For the data in the previous question, find the median, mode and the range.

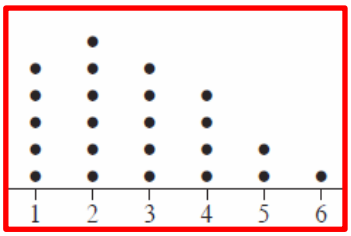
Answer: Median = 87.5 , mode = 87 and range = 30

6. Find the mean, rounded to the nearest whole number.

Answer: Mean = 87

7. Display the following data in a dot plot.

1, 2, 3, 4, 5, 6, 5, 4, 3,
2, 1, 1, 2, 3, 4, 4, 3, 2,
1, 1, 2, 3, 2



8. For the data in the previous question, find the median, mode and the range.

Answer: Median = 3 , mode = 2 and range = 5

9. Find the mean, rounded to one decimal place.

Answer: Mean = 2.8

10. Display the following data in a stem-and-leaf plot.

138, 157, 147, 149, 165, 153,
122, 130, 141, 146, 123, 168,
156, 135, 146, 137, 143, 154

12	2 3
13	0 5 7 8
14	1 3 6 6 7 9
15	3 4 6 7
16	5 8

11. For the data in the previous question, find the median, mode and the range.

Answer: Median = 146 , mode = 146 and range = 46

12. Find the mean, rounded to the nearest whole number.

Answer: Mean = 145

13. Display the following data in a stem-and-leaf plot.

7 20 15 36 18 12 9 24 25
45 16 29 28 19 25 30 36 38
40 5 32 27 42 22 30

Stem	Leaf
0	5 7 9
1	2 5 6 8 9
2	0 2 4 5 5 7 8 9
3	0 0 2 6 6 8
4	0 2 5

14. For the data in the previous question, find the median, mode and the range.

Answer: Median = 25 , range = 40 and mode = 25, 30 and 36.

15. Find the mean, rounded to one decimal place.

Answer: Mean = 25.2

Problem Solving

1. In a class, 45 % of the students are girls. If the number of girls in the class is 36, how many students are there in the class altogether ?

Answer: 80 students

2. Samantha is baking a large vanilla slice cake for a function. The cake will be 90 cm long, 60 cm wide and 10 cm high. If the cake is to be shared equally among 120 attendees, what volume of cake will each person receive ?

Answer: 450 cm^3

3. Solve the equation $x + 2 = 2x + 4$.

Answer: $x = -2$

4. Convert 63 000 seconds into hours and minutes.

Answer: 17 hours 30 minutes

5. On a school test, students receive 4 points for every question answered correctly and lose 7 points for every question answered incorrectly. On one particular test, Terry answered 87 questions correctly and 46 questions incorrectly. What was his score for the test ?

Answer: 26

6. The price of 5 boxes of chocolates is \$65. Find the cost of two dozen boxes of chocolates.

Answer: \$312

7. Cathy works 8 hours a day, 5 days a week at a local supermarket. Her daily wage is \$178.

(a) Calculate her hourly rate.

Answer: \$22.25 per hour

(b) How much she is paid per week ?

Answer: \$890 per week

Challenge Questions

1. If the side length of a cube is 6 cm, find the volume and the total surface area of the cube.

Answer: Volume = 343 cm^3 and total surface area = 343 cm^2.

2. A hula-hoop has a circumference of 2.2 metres. If you spin it 995 times, how far will the hula-hoop spin ? Give your answer in kilometres.

Answer: 2.189 kilometres

3. What is 25 % of 50 % of 60 ?

Answer: 7.5

4. Find the missing number: 20 24 38 ? 96

Answer: 62

5. At a carwash, it takes 2 hours for 10 people to wash 50 cars. If two people have called in sick today, how long will it take the remaining staff to wash the 50 cars ?

Answer: 2 hours 30 minutes

6. Karen had a birthday party and 85 % of her friends were able to come to the party. How many friends came to the party if there were 9 friends who were unable to attend ?

Answer: 51 friends attended the party.

7. In a company of 350 employees, 40 % are married and 55 % of the married workers have children. How many employees in the company have children ?

Answer: 77 employees have children.

8. What is the smallest number between 100 and 200 that is divisible by 3, 4 and 7 ?

Answer: 168

🎯 Set 38

Column Graphs

These are the sections you must complete in this set:

- ✓ **Topic Questions**
- ✓ **Problem Solving**
- ✓ **Challenge Questions**

Why use column graphs ?

A **column graph** is a simple way of displaying and comparing statistical data. These types of graphs are also referred to as bar charts or bar graphs.

Regardless of the name you prefer to use, these types of graphs may be either horizontal or vertical. However, to differentiate between the two, a common naming convention uses **column** graphs to represent vertical orientation while **bar** graphs represent horizontal orientation.

In this set, column graphs will refer to vertical orientation and bar graphs will be horizontal.

Similar to a dot plot, these graphs use rectangular columns to represent the number of times a value occurs. This number is called the **frequency**. The larger the frequency, the taller the column. The highest column will represent the **mode** of the data.

When constructing a column graph, follow these rules:

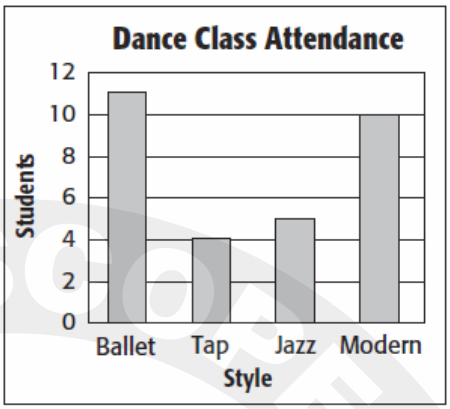
- give your graph a title.
- label axes clearly and scale them evenly.
- the vertical axis will represent the frequency.
- ensure your columns are the same width.
- have an even gap between each column.
- begin the first column half a unit (that is, half the column width) from the vertical axis.
- use colours to highlight the different columns.

Often, the data for the column graph will be given in a table.

Remember, if the data is numerical, it will be possible to calculate the **mean** and the **range**.

Example: At a dance school, 11 students take ballet, 4 students take tap dance, 5 students take jazz and 10 students take modern dance. Create both a table and a column graph to represent this data.

Dance Classes	
Style	**Students**
Ballet	11
Tap	4
Jazz	5
Modern	10



Topic Questions

The table below shows the hair colour of students in my class.

Colour	Students
Red	2
Brown	12
Blonde	4
Black	9

1. How many students are in my class ?

Answer: 27

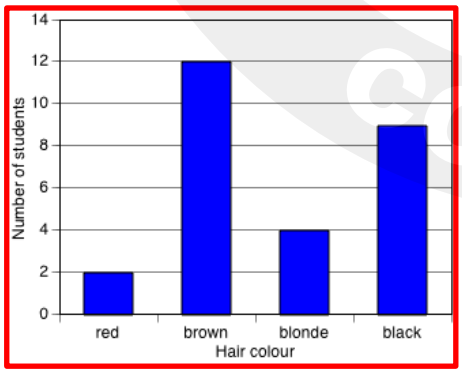
2. What is the most common hair colour ?

Answer: Brown

3. What is the least common hair colour ?

Answer: Red

4. Construct a column graph to represent the data in the table.



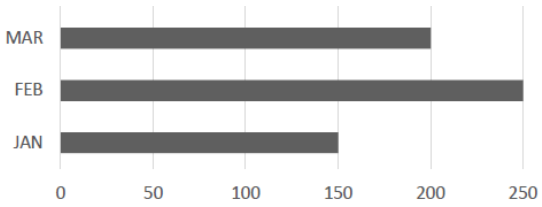
The bar chart on the right shows the number of sales per month for three months.

5. How many sales were there in February ?

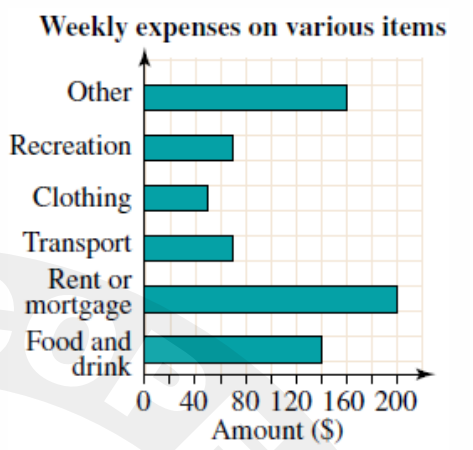
Answer: 250

6. What was the average sales per month ?

Answer: 200



The bar chart below shows the weekly expenses for a particular household.



7. How much is spent on clothing each week ?

Answer: $50

8. What is the largest expense for the week ?

Answer: Rent or Mortgage $200

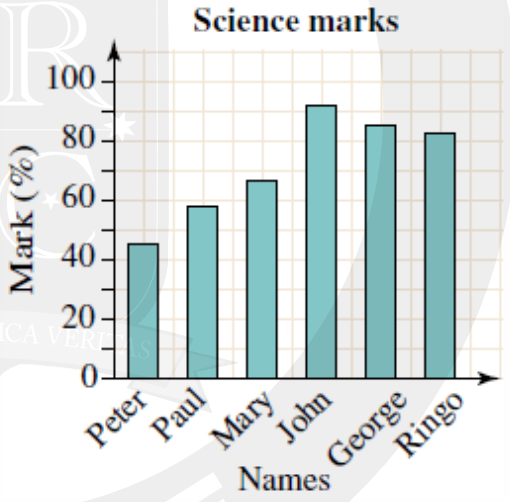
9. How much is spent on food and drink each week ?

Answer: $140

10. What is the total of the expenses each week ?

Answer: $690

The column graph below shows the marks for a test for six students.



11. Who got the highest mark ?

Answer: John

12. Rank the students from highest to lowest ?

John, George, Ringo, Mary, Paul, Peter

13. What is the approximate range of the marks ?

Answer: 45

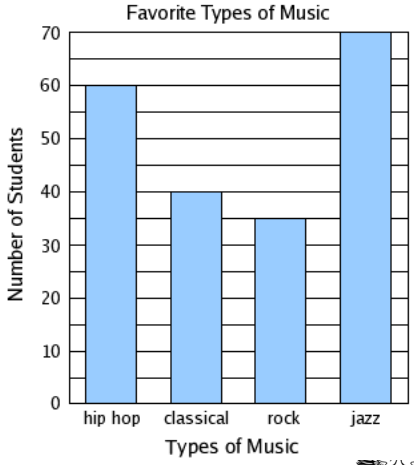
14. What is the approximate median mark ?

Answer: 75

15. What is the approximate mean mark ?

Answer: 72

The column graph below shows the music preferences of students at a dance school.



16. What is the most popular type of music ?

Answer: Jazz

17. What is the least popular ?

Answer: Rock

18. How many students are there at the school ?

Answer: 205

Problem Solving

1. Samantha is monitoring the bottles of water she drinks. On Monday she drinks $3\frac{1}{2}$ bottles, on Tuesday $4\frac{1}{3}$ bottles and on Wednesday $2\frac{1}{4}$ bottles. If a bottle contains 600 mL, what is the total number litres of water that she consumes ?

Answer: 6.05 litres

2. Ken, Len and Jen race around a large park. Ken finishes first, taking $8\frac{1}{4}$ minutes. Jen takes $9\frac{3}{4}$ minutes and Len was third taking $10\frac{1}{2}$ minutes. After he finished, how many seconds did Ken have to wait until Len finished ?

Answer: 135 seconds

3. A family bought 3 pizzas for dinner. They ate three-quarters of one of the pizzas, two-thirds of another and five-eighths of the remaining pizza. What fraction of a pizza was not eaten?

Answer: $\frac{23}{24}$ of a pizza was not eaten.

4. A cruise ship has 3000 passengers. There are women, men and children on the ship. Three in every eight passengers are men and three in every five passengers are women. How many children are on the cruise?

Answer: 75 children

5. Three-eighths of the students in a group cannot swim. There are eight more swimmers than non-swimmers. How many students are in the group?

Answer: 32 students

6. Ben grows lemons in his backyard which he would like to sell. He sold one-third on the first day. He sold one-third of the remaining lemons on the second day. He still had 20 lemons remaining. How many lemons did he have originally for sale?

Answer: 45 lemons

Challenge Questions

1. Arrange the following in ascending order:

$\frac{23}{36}$, $\frac{10}{17}$, $\frac{18}{23}$, $\frac{8}{15}$

$\frac{8}{15}$, $\frac{10}{17}$, $\frac{23}{36}$, $\frac{18}{23}$

2. A fitness club has 570 members in total. Two in every three of the members are female. What is the number of male members in the fitness club ?

Answer: 190 males

3. Every weekday Joseph walks to and from school using the same route. The total distance covered each day is $3\frac{1}{2}$ kilometres. So far this term he has walked a total of 70 kilometres. For how many weeks has Jose walked to school ?

Answer: 4 weeks

4. A third of the students in a class have blonde hair. A quarter of the blonde-haired students do not have blue eyes. What fraction of the students have blonde hair and blue eyes ?

Answer: $\frac{1}{4}$ of the students have blonde hair and blue eyes.

5. What is 15 minutes as a fraction of three hours ?

Answer: $\frac{1}{12}$

6. What number must be added to 17, 23, and 15 to get an average of 17 ?

Answer: 13

7. There are 15 light poles in a straight line next to a road. Each light pole is placed 25 metres apart. What is the distance between the fourth pole and the twelfth pole ?

Answer: 200 metres