⦿ Set 41

Chance and Probability

These are the sections you must complete in this set:

- ✓ **Topic Questions**
- ✓ **Problem Solving**
- ✓ **Challenge Questions**

What is chance ?

Chance is about expectation. It is the likelihood or **probability** of a particular event occurring.

In everyday life, we often describe the chance that an event will or will not happen by using certain words. For example, if we said , 'There's a good chance of rain tomorrow', what we are really doing is predicting how likely it is for that event to happen. When we speak about the **chance** of something occurring, the words that we tend to use are not exact.

By using mathematics, precise values can be given to events in the form of a **probability**, which is a value ranging from and including 0 (for impossible events) to 1 (for events that are certain). A value of 0.5 is given to something that was equally likely or unlikely to happen.

Some common words could be placed at approximate places on the following probability scale.
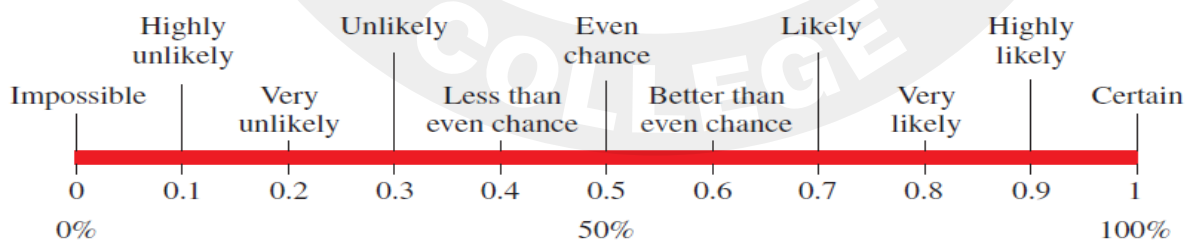
Calculating probability

Prior to calculating the probability of a certain event, we need to know all the possible outcomes or results that can happen. This list of all possible outcomes is called the **sample space**.

The sample space lists all possible outcomes in a pair of curled brackets { }.

Example: A coin is tossed twice. List the sample space for this event.

Sample space = { HH, HT, TH, TT }.

There is a total of 4 possible outcomes for this event.

Probability is best expressed as a simplified fraction and is calculated using the rule:

$$\text{Probability of an event} = \frac{\text{Number of successful outcomes}}{\text{Total number of possible outcomes}}$$

Example: If a coin is tossed twice, what is the probability of getting one head and one tail.

First, list the sample space for this event. Sample space = { HH, HT, TH, TT }.

∴ There is a total of 4 possible outcomes for this event.

Next, determine which are the successful outcomes, that is, one head and one tail.

There are 2 successful outcomes, namely HT and TH.

∴ Probability of one head and one tail $= \frac{2}{4} = \frac{1}{2}$

Commonly, this is abbreviated as follows:

Pr (one head and one tail) $= \frac{1}{2}$

Example: A die is rolled once, and a fair coin is tossed once.

List all the possible outcomes, namely the sample space, for this experiment.

Sample space = { 1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T }.

There are $6 \times 2 = 12$ possible outcomes in the sample space.

Example: A box contains 20 counters, numbered 1 to 20. If one counter is selected from the box at random, find the probability that the number is less than 9.

Total number of possible outcomes = 20

There are 8 successful ways of selecting a counter less than 9.

∴ Pr (less than 9) $= \frac{8}{20} = \frac{2}{5}$

Example: A fair die is rolled once. What is the probability of getting a prime number.

Sample space = { 1, 2, 3, 4, 5, 6 }.

There are 6 possible outcomes in the sample space.

There are three prime numbers, namely, 2, 3 and 5.

∴ Pr (prime number) $= \frac{3}{6} = \frac{1}{2}$

Topic Questions

1. Classify each event as impossible, unlikely, an even chance, likely or certain.
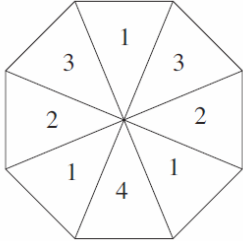
(a) Choosing a green ball from a bag containing 4 red and 4 green balls.   Even chance

(b) Rolling a total of 13 using two normal 6-sided die.   Impossible

(c) The year 2024 will be a leap year.   Certain

(d) You will grow 30 cm taller in the next six months.   Unlikely

(e) Rolling a fair die and obtaining a number less than 6.   Likely

(f) Tossing a fair coin and getting a head.   Even chance

2. A letter is chosen at random from the alphabet. What is the probability that the letter is:

(a) R ? $\frac{1}{26}$ **(b)** A or Z ? $\frac{1}{13}$

(c) a vowel ? $\frac{5}{26}$ **(d)** a consonant ? $\frac{21}{26}$

3. A fair die is thrown. What is the probability of rolling:

(a) a 6 ? $\frac{1}{6}$ **(b)** an even number ? $\frac{1}{2}$

(c) a prime number ? $\frac{1}{2}$ **(d)** a number greater than 2 ? $\frac{2}{3}$

4. A bag has 4 red, 1 green and 5 blue marbles. What is the probability of selecting:

(a) a red marble ? $\frac{2}{5}$ **(b)** a green marble ? $\frac{1}{10}$

(c) a red or blue marble ? $\frac{9}{10}$ **(d)** not a green marble ? $\frac{9}{10}$

**5.** The spinner on the right is spun once.

(a) What number is most likely to be spun ?   1

(b) What number is least likely to be spun ?   4

6. What is the probability of spinning the following numbers:

(a) 4 ? $\frac{1}{8}$ **(b)** 5 ?   0

(c) 1 ? $\frac{3}{8}$ **(d)** 2 or 3 ? $\frac{1}{2}$

(e) a number > 1 ? $\frac{5}{8}$ **(f)** a number < 6 ?   1

Problem Solving

1. The Venn diagram on the right represents the number of dog and cat owners in a class. How many students are there in the class altogether ?
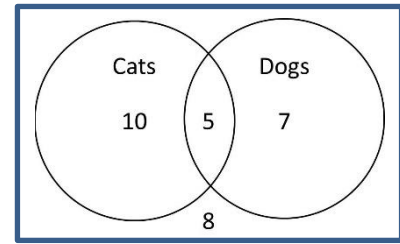
   Answer: 30 students

2. Jason and Anna have 240 marbles altogether. After Anna gave Jason five marbles, Jason now has three times as many marbles as Anna does. How many marbles did Anna initially have before giving Jason the five marbles ?

   Answer: Anna had 65 marbles initially.

3. What is the square of 1.4 ?   *Answer:* E

   A 1.54   **B** 2.88   **C** 19.6   **D** 1.88   **E** 1.96

4. Find the missing number: 13   18   24   ?   39   48

   Answer: 31

5. On a school test, students receive two points for every question answered correctly and lose three points for each incorrect answer. On one test, Terry answered 77 questions correctly and 46 questions incorrectly. What was his score for the test ?

   Answer: 16

6. Sammy had a sleep at 4:35 pm. When she woke it was 20:26. How long did she sleep for ?

   Answer: 3 hours 51 minutes

7. Sally works 7 hours a day, 5 days a week at a local shop. Her daily wage is $173.25.

   (a) Calculate her hourly rate.

   Answer: $24.75 per hour

   (b) How much she is paid per week if she also receives a meal allowance of $8.25 per day ?

   Answer: $907.50 per week

Challenge Questions

1. If the side length of a cube is 8 cm, find the volume and the total surface area of the cube.

   Answer: Volume = 512 cm³ and total surface area = 384 cm².

2. In a netball team, the heights of the players are 168 cm, 150 cm, 177 cm, 155 cm, 146 cm, 151 cm and 159cm. What is the average height of the team ? Give your answer in metres.

   Answer: 1.58 metres

3. When a container is 55 % full it holds 38.5 litres. What is its full capacity ?

   Answer: 70 litres

4. A water tap fills a 75 litre container at a rate of 1.5 litres every 20 seconds. How long will it take to fill the container ? Give the answer in minutes and seconds.

   Answer: 16 minutes 40 seconds

5. Tom drives 275 kilometres each week. He drives his car to and from work five days a week. He also drives an additional 140 kilometres on the weekend. How far is it to Tom's work ?

   Answer: 13.5 kilometres

6. Melanie has 10 coins that total $7.20. She has no 5 cent coins but at least one of every other type of coin. What coins does Melanie have ?

   Answer: Two $2, one $1, three 50 cent, three 20 cent and one 10 cent.

7. The product of two whole numbers is 96 and their sum is 28. What are the two numbers ?

   Answer: 4 and 24

8. A jar has a capacity of 0.95 litres. If it is only 80 % full, how many millilitres is this ?

   Answer: 760 millilitres

🎯 Set 42

More Probability

These are the sections you must complete in this set:

- ✓ **Topic Questions**
- ✓ **Problem Solving**
- ✓ **Challenge Questions**

Total probability

As seen in the previous set, probabilities are expressed as fractions and can range from a value of 0 (an impossible event) to a value of 1 (a certain event). Mathematically, we define the probability of a particular event as:

$$Pr\ (Event)\ =\ \frac{number\ of\ successful\ outcomes}{total\ number\ of\ possible\ outcomes} \qquad \text{where} \qquad 0\ \leq\ Pr\ (Event)\ \leq\ 1$$

Two of the fundamental laws of probability are:

- all probabilities must lie between 0 and 1 inclusive, and

- all probabilities for an event must sum to 1.

Example: When rolling a fair dice, the probability of rolling any number is the same, namely:

$$Pr\ (1)\ =\ Pr\ (2)\ =\ Pr\ (3)\ =\ Pr\ (4)\ =\ Pr\ (5)\ =\ Pr\ (6)\ =\ \frac{1}{6}$$

∴ The sum of all probabilities for this event is equal to 1.

Complementary events

Two events are **complementary** if the sum of their probabilities is 1. In simple terms, two events are complementary if they are opposites of each other.

Example: When tossing a coin, if the event is "tossing a head" then its complement event is "tossing a tail". In probability terms, this would be expressed as:

$$Pr\ (Head)\ =\ \frac{1}{2} \qquad \text{and} \qquad Pr\ (Tail)\ =\ \frac{1}{2}$$

Since they are complementary events, $Pr\ (Head)\ +\ Pr\ (Tail)\ =\ 1$.

In probability, if an event is defined as E then its **complement** is denoted by E'.

Using the laws of probability, if two events E and E' are complementary then:

$$Pr\,(E)\; +\; Pr\,(E'\,)\; =\; 1$$

or $\quad Pr\,(E'\,)\; =\; 1\; -\; Pr\,(E).$

Example: When rolling a fair dice, what is the probability of not rolling a 6.

"Rolling a 6" and "not rolling a 6" are complementary events.

Since $\;\; Pr\,(\text{rolling a 6})\; =\; \dfrac{1}{6}$

$\therefore\; Pr\,(\text{not rolling a 6})\; =\; 1\; -\; Pr\,(\text{rolling a 6})\; =\; 1 - \dfrac{1}{6}\; =\; \dfrac{5}{6}$

Independent events

Two events are **independent** if the result of the second event is not affected by the result of the first event. For example, when rolling a fair dice, each roll is independent and is not influenced by the results of previous rolls. The probabilities will not change from one roll to the next.

When two or more events are independent, the probability of the combined events occurring is the product of the probabilities of the individual events. Mathematically, if two events are defined as A and B, then the probability of both occurring is defined as:

$$Pr\,(A \text{ and } B)\; =\; Pr\,(A) \times\; Pr\,(B)$$

Example: A fair coin is tossed three times. What is the probability of getting 3 heads ?

Each toss of the coin is independent. The probability of a head does not change.

$\therefore\; Pr\,(\text{tossing a head})\; =\; \dfrac{1}{2}$

$\therefore\; Pr\,(\text{tossing 3 heads})\; =\; \dfrac{1}{2} \times \dfrac{1}{2} \times \dfrac{1}{2}\; =\; \dfrac{1}{8}$

Example: A bag contains 5 red marbles, 3 green marbles and 2 blue marbles. One marble is drawn from the bag but not replaced. Another marble is drawn from the bag. What is the probability that the first marble is blue and the second green ?

Because the first marble is not replaced, the total number of marbles is reduced for the second draw, but the two events are still considered to be independent.

$\therefore\; Pr\,(\text{blue first})\; =\; \dfrac{2}{10}\quad$ and $\quad Pr\,(\text{green second})\; =\; \dfrac{3}{9}$

$\therefore\; Pr\,(\text{blue then green})\; =\; Pr\,(\text{blue}) \times Pr\,(\text{green})\; =\; \dfrac{2}{10} \times \dfrac{3}{9}\; =\; \dfrac{6}{90}\; =\; \dfrac{1}{15}$

Topic Questions

1. State the complementary event for each of the following:

(a) It will rain today. It will not rain today

(b) Rolling a 1 on a normal 6-sided die. Not rolling a 1

(c) Today is the weekend. Today is a weekday

(d) Tossing a fair coin and getting a tail. Tossing a tail

(e) My next class is English. My next class is not English

(f) My teacher is a female. My teacher is a male

2. If the probability of a train arriving on time is $\frac{21}{32}$, what is the probability that it will not arrive on time ?

Answer: $\frac{11}{32}$

3. The probability of winning a competition is 0.002. What is the probability of not winning ?

Answer: 0.998

4. A fair coin is tossed 4 times. What is the probability of tossing 4 tails ?

Answer: $\frac{1}{16}$

5. The coin is tossed again. What is the probability of tossing another tail ?

Answer: $\frac{1}{2}$

A box contains 5 red , 4 green and 3 blue tokens. Two tokens are taken randomly from the box.

6. What is the probability that the first token is blue and the second green ?

Answer: $\frac{3}{12} \times \frac{4}{11} = \frac{1}{11}$

7. What is the probability that the first token is green and the second blue ?

Answer: $\frac{4}{12} \times \frac{3}{11} = \frac{1}{11}$

8. What is the probability that both tokens are not red ?

Answer: $\frac{7}{12} \times \frac{6}{11} = \frac{7}{22}$

Problem Solving

1. At a large city gymnasium, 42 % of the members are female. If the number of females is 231, how many members are there at the gymnasium altogether ?

   Answer: 550 members

2. Evaluate the following expression: $\frac{4}{7} \times \left(\frac{1}{4} + 3 \times \frac{1}{6} \right)$

   Answer: $\frac{3}{7}$

3. In a year level of 96 students, 30 play a musical instrument. What percentage do not play a musical instrument ?

   Answer: 68.75 %

4. Solve the equation $3x + 5 = 2x + 1$.

   Answer: $x = -4$

5. Convert 33 900 seconds into hours and minutes.

   Answer: 9 hours 25 minutes

6. Ben has a length of rope that is 13 metres long. He wants to cut it into even pieces that are each 200 millimetres in length. How many smaller lengths of rope will he have ?

   Answer: 65

7. At a cinema, a movie runs for 1 hour 30 minutes. If the movie is shown for 9 hours non-stop, how many times was it played ?

   Answer: 6

8. A sightseeing cruise along the Yarra River went for 1 hour and 10 minutes, travelling a distance of 14 kilometres. What was the average speed of the cruise ?

   A   7 km/h   **B**   8 km/h   **C**   10 km/h   **D**   11 km/h   **E**   12 km/h

   Answer: E

Challenge Questions

1. In a packet of 40 pens, three-eighths are red and 40 % are blue. If two-thirds of the remaining pens are black, how many black pens are there ?

    Answer: 6 black pens

2. Adam was driving at 54 km/h. What is this speed in metres per second ?

    Answer: 15 m/s

3. In a town of 1 700 people, 85 % own a car. How many people do not own a car ?

    Answer: 255

4. Kylie has an average of 77 % after completing four tests. She would like to increase her average to 80 %. What percentage will she need in the next test to achieve this ?

    Answer: 92 %

5. There are 45 students in a class.   30 play tennis, 19 play netball and 2 students do not play any sport. How many students play both tennis and netball ?

    Answer: 6 students play both tennis and netball

6. Sam decides to stack 81 blocks in the shape of a square-based pyramid. If each block has a height of 12.5 centimetres, how tall will the pyramid be ?

    Answer: 75 centimetres

7. In a bag, there are 16 red, 8 yellow, 5 black and 11 green jellybeans. In percentage, what is the chance that I will randomly pick a black jellybean ?

    Answer: 12.5 %

8. What is the smallest number between 100 and 200 that is divisible by 3, 4, 5 and 9 ?

    Answer: 180