Report LAB 4 : Navigation with IMU and Magnetometer

Introduction

Objective: To build a navigation stack using two different sensors – GPS & IMU, understand their relative strengths + drawbacks, and get an introduction to sensor fusion.

Sensors

Equipment: IMU- VectorNav VN-100 IMU and GPS- GNSS puck

Data Files

All the data files are uploaded by Varun Raghavendra. User_name: raghavendra.va

Analysis of the data

1. Estimate the heading (yaw)

A. Magnetometer Calibration:

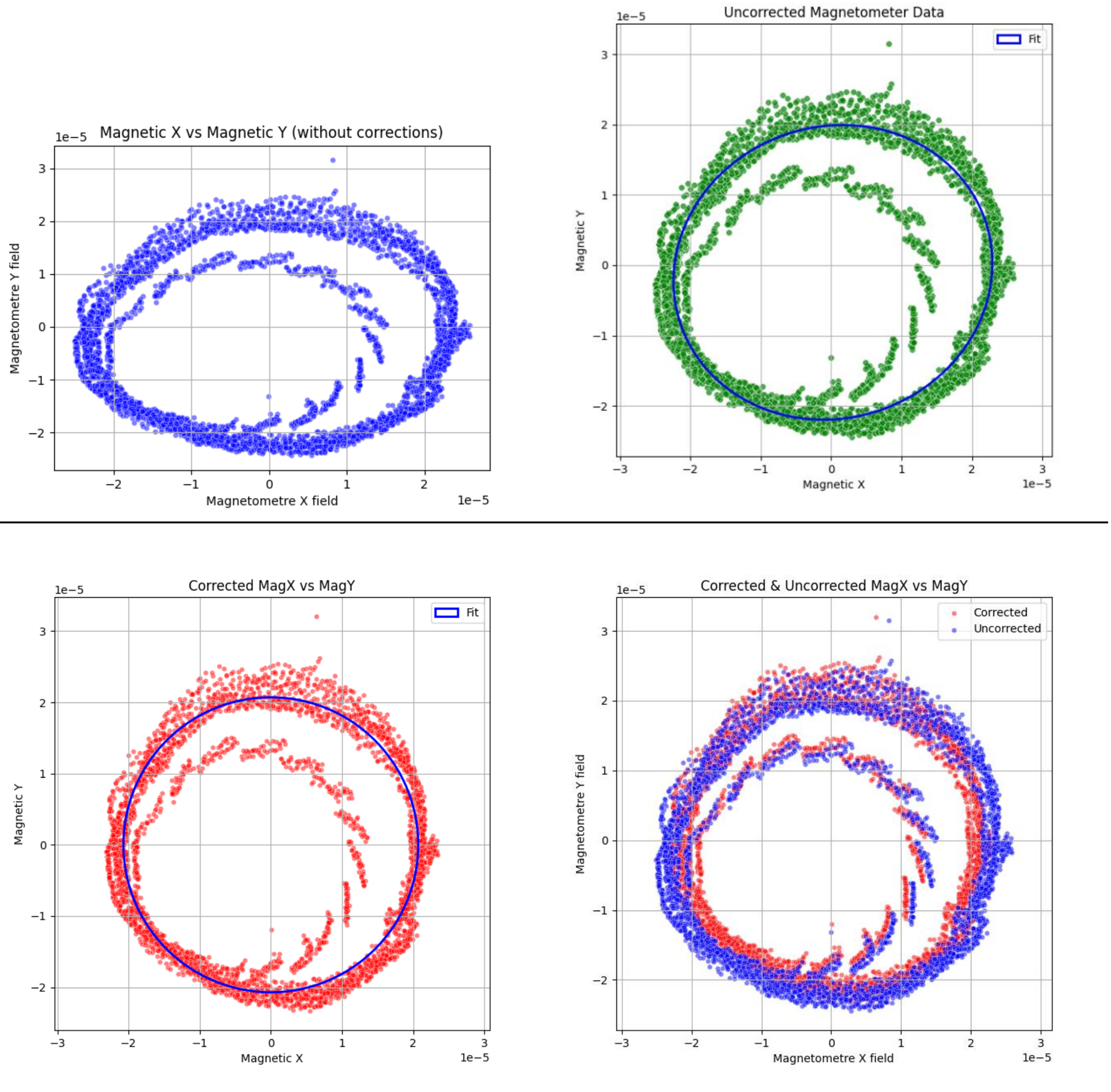

Figure.1(top-left): Magnetic.X vs Magnetic.Y without any calibrations and fitting

Figure.2(top-right): Uncorrected Magnetic.X v/s Magnetic.Y with fitting in an ellipse

Figure.3(bottom-left): Corrected Magnetic.X v/s Magnetic.Y with fitting in an ellipse

Figure.4(bottom-right): Comparison of corrected and uncorrected Magnetic.X v/s Magnetic.Y

Q) How did you calibrate the magnetometer from the data you collected? What were the sources of distortion present, and how do you know?

Solution. In this lab, the magnetometer was calibrated using the data collected during a circular motion. The calibration process aimed to correct both hard iron and soft iron distortions, which are common sources of errors in magnetometer readings. Calibration process is as follows:

Data Collection:

- Magnetometer data (Magnetic.x, Magnetic.y, Magnetic.z) was collected during a circular motion to ensure a wide range of readings.
- Data was selected from a specific range to focus on the circular motion period.

Plotting Raw Data:

- The raw Magnetic.x vs Magnetic.y data was plotted to visualize the distortions.
- Initially, the raw data forms an ellipse due to distortions.
- Figure 1 and Figure 2 are the raw data plots with and without ellipse fitting respectively.

Elliptical Fit:

- An elliptical fit was applied to the raw data to identify the center, width, height, and orientation of the distorted ellipse.
- The 'fit_ellipse' function calculates these parameters by solving a least-squares problem.

Hard Iron Correction:

- Hard iron distortions cause a shift in the magnetometer readings. These were corrected by subtracting the center of the ellipse from the magnetometer data.
- The center coordinates of the ellipse were determined from the elliptical fit.

Soft Iron Correction:

- Soft iron distortions affect the shape and orientation of the magnetic field data, causing the ellipse to be stretched and rotated.
- These were corrected by applying a rotation and scaling transformation to the data.

Plotting Corrected Data:

- The corrected magnetometer data was plotted to verify that the distortions were successfully removed.
- After calibration, the data should form a circle, indicating that the distortions have been corrected.
- Figure 3 is the plot after soft and hard iron corrections are made.

Sources of Distortion:

1. Hard Iron Distortion:
    - Caused by permanent magnetic fields from objects like magnets or magnetized metal pieces in the environment or the device itself, shifting the magnetometer readings uniformly in all directions.
    - This was identified by the offset in the center of the ellipse formed by the raw magnetometer data.
2. Soft Iron Distortion:
    - Caused by ferromagnetic materials like iron or nickel in the vicinity that distort the magnetic field, affecting the shape and orientation of the magnetometer readings.
    - This was identified by the elliptical shape of the data, which indicated stretching and rotation of the magnetic field data.

Identification of Distortions:

- The presence of an offset center and an elliptical shape in the raw magnetometer data indicated hard iron and soft iron distortions, respectively.
- The elliptical fit provided parameters (center, width, height, orientation) that were used to correct these distortions.
- After applying the corrections, the magnetometer data formed a circle, confirming that the distortions were successfully removed.

By following this calibration process, the magnetometer data was corrected for both hard iron and soft iron distortions, resulting in more accurate yaw estimates and reliable navigation data. Figure 4 shows the difference in the corresponding readings.

B. Sensor Fusion:

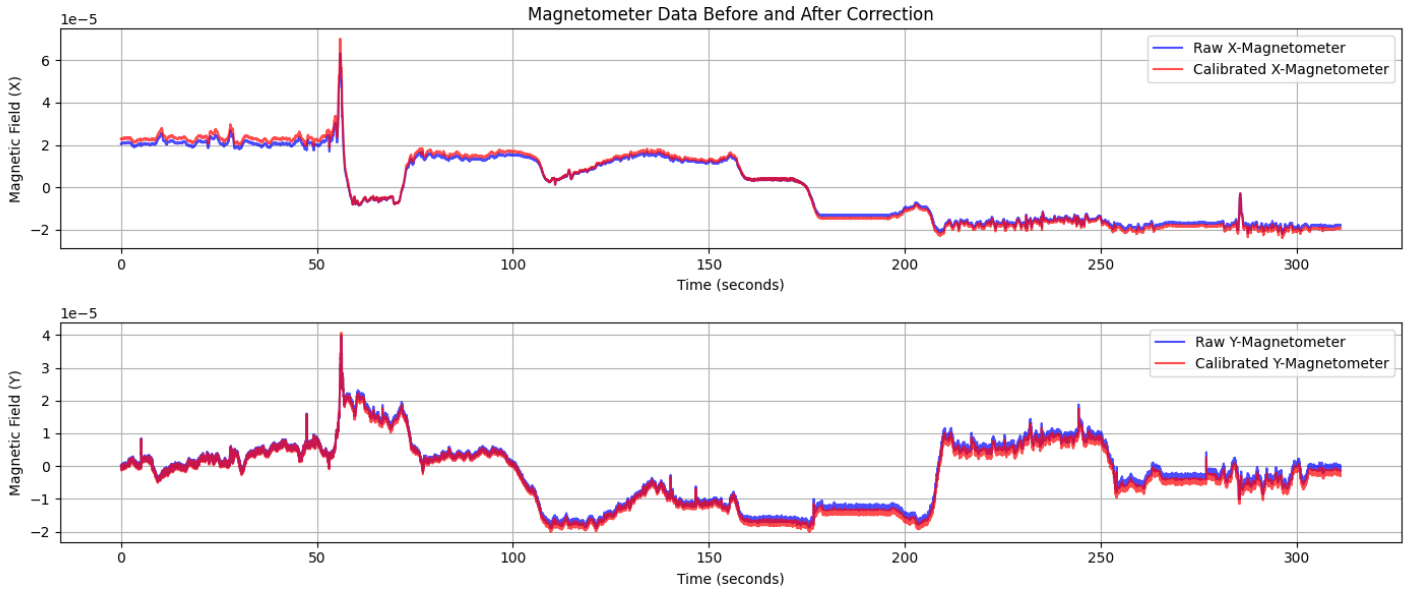

Figure 5: The plot above shows the time-series of magnetometer data before and after the correction. The red plots are after the calibrations are made and the blue plot are pre-calibrated.

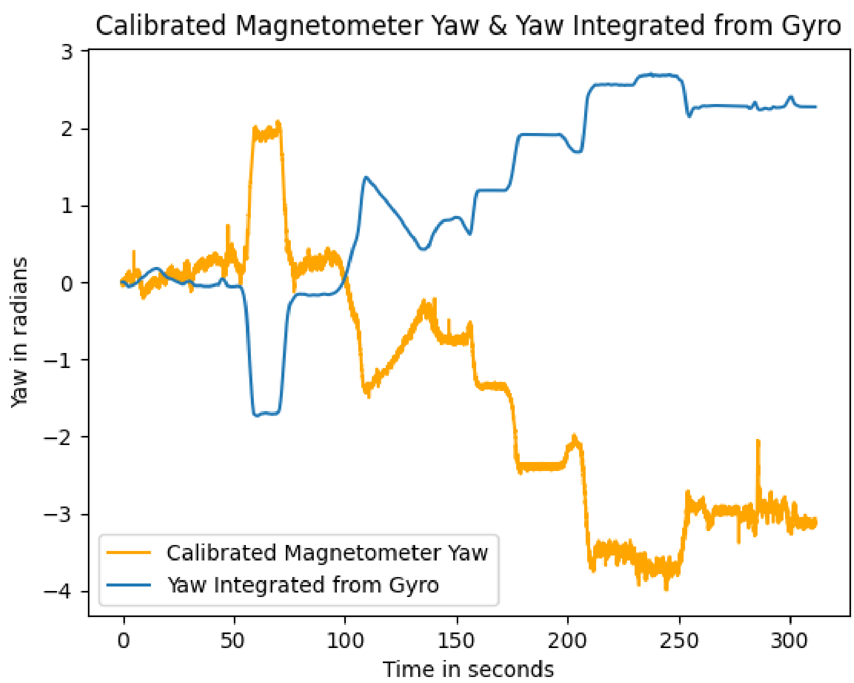

Figure 6: *The plot of Yaw from Calibrated Magnetometer and Yaw integrated from the Gyroscope of the IMU sensor. The plot shows a comparison between the two. We observe that the two graphs are near-mirror images about x-axis.*

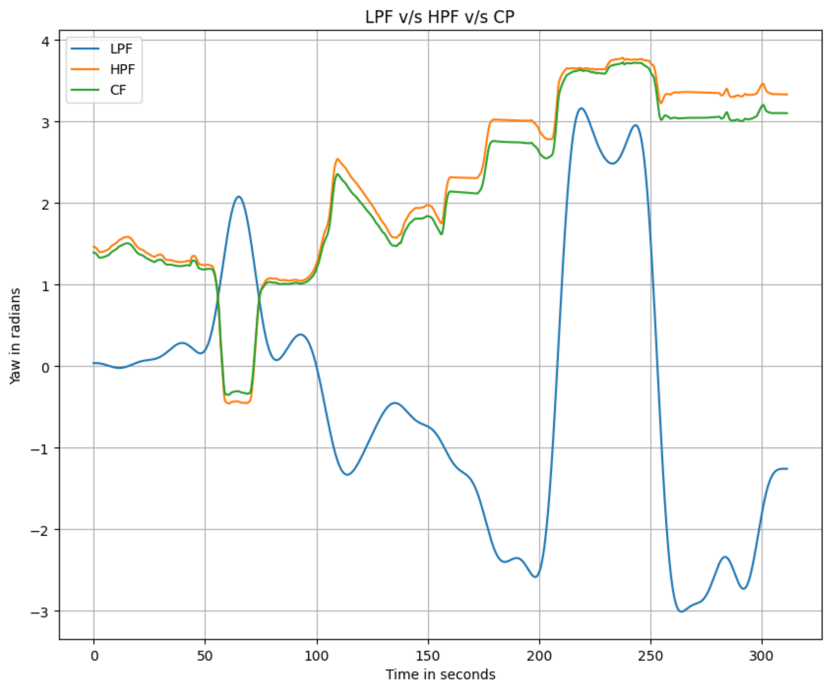

Figure 7: *This plot has 3 yaw - data plotted against time. Yaw through Low-pass filter(blue), High-pass filter(orange) and Complimentary filter(green). The complimentary filter is a combination of low-pass filter and high-pass filter with weight value of 0.95. Thus the plot of yaw of complimentary filter runs nearly with the high-pass filter.*

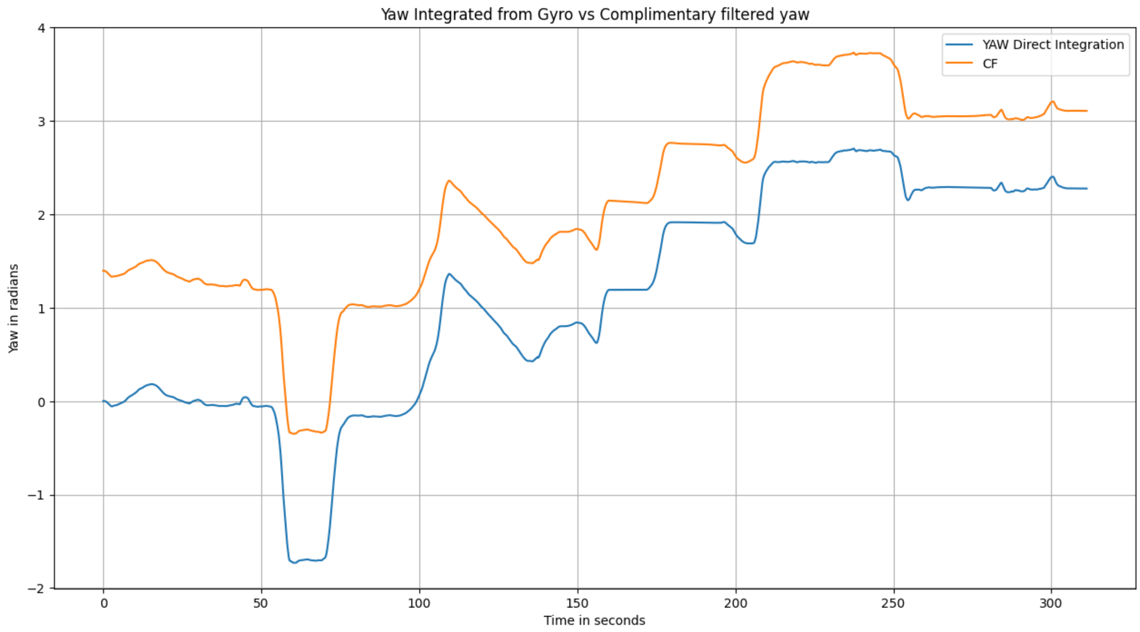

Figure 8: This plot shows Yaw Integrated from Gyro(blue) and Yaw from Complimentary filter(orange). As observed, the yaw from complimentary filter follows the trend of the yaw data collected from the IMU with a near-constant offset.

Q) How did you use a complementary filter to develop a combined estimate of yaw? What components of the filter were present, and what cutoff frequencies did you use?

Solution. In this lab, a complementary filter was employed to combine yaw estimates from the magnetometer and the gyroscope to obtain a more accurate and reliable estimate of the yaw angle. The complementary filter effectively merges the low-frequency data from the magnetometer with the high-frequency data from the gyroscope. The magnetometer provides a stable reference for orientation, but it is susceptible to high-frequency noise. Conversely, the gyroscope data is more reliable over short time intervals but tends to drift over time due to integration errors.

Raw Yaw Calculation:

- The raw yaw was initially calculated from the corrected magnetometer readings using the arctangent function. This provides an initial estimate of the yaw angle based on the magnetic field data.

Low-Pass Filtering on Magnetometer Data:

- A low-pass filter was applied to the yaw angle derived from the magnetometer. The low-pass filter helps to retain the low-frequency components of the signal, effectively removing high-frequency noise.
- Cutoff Frequency: 0.05 Hz
- Filter Order: 4

High-Pass Filtering on Gyroscope Data:

- The yaw rate from the gyroscope was integrated to obtain the yaw angle. To remove the low-frequency drift inherent in the gyroscope data, a high-pass filter was applied. This filter retains the high-frequency components of the signal, effectively removing low-frequency drift.

- Cutoff Frequency: 0.00001 Hz
- Filter Order: 4

Combining Filters Using a Complementary Filter:

- The complementary filter combines the filtered magnetometer and gyroscope data. The filter uses a weighting factor, alpha, to balance the contributions of each data source. Typically, alpha is chosen to be close to 1 to favour the reliable low-frequency components from the magnetometer.
- Combines the low-pass filtered magnetometer yaw data with the high-pass filtered gyroscope yaw data.
- Weighting Factor: 0.95 (chosen to balance the two sources of data)

This approach leverages the strengths of both sensors, resulting in a more accurate and robust yaw estimation suitable for navigation and trajectory estimation tasks.

Q) Which estimate or estimates for yaw would you trust for navigation? Why?

Solution. Based on the codes provided and the analysis conducted so far, the most trustworthy estimate for yaw for navigation purposes is the *calibrated yaw derived from the magnetometer data*.

1. Calibration and Correction:
   - The calibrated yaw is obtained after applying hard iron and soft iron corrections to the raw magnetometer data. This involves fitting an ellipse to the raw data to identify and correct for distortions caused by permanent and temporary magnetic fields in the environment.
   - The corrections ensure that the magnetometer readings are aligned correctly, and the data represents true magnetic north.
2. Handling Distortions:
   - Hard Iron Distortion: This is corrected by subtracting the center of the ellipse fitted to the magnetometer data, which accounts for fixed magnetic fields that shift the readings.
   - Soft Iron Distortion: This is corrected by applying rotation and scaling transformations, which address the stretching and rotation of the magnetic field data caused by surrounding ferromagnetic materials.
3. Stability:
   - Magnetometer data, once calibrated, provides a stable and reliable reference for yaw, as it is aligned with the Earth's magnetic field. This stability is crucial for long-term navigation, where drift and noise in gyroscope data can accumulate over time, leading to significant errors.
4. Comparison with Raw Data:
   - The comparison plot of raw yaw versus calibrated yaw clearly shows that the calibrated yaw data is more consistent and free from the distortions present in the raw data.
   - The raw yaw data, on the other hand, shows significant variability and drift due to the uncorrected distortions, making it less reliable for accurate navigation.
5. From the plot, it's evident that the calibrated yaw data is smoother and more consistent, which indicates the effectiveness of the calibration process in correcting distortions.

Thus for navigation, the *calibrated yaw from the magnetometer* is the most trustworthy estimate. The calibration process ensures that the data is free from distortions and accurately represents the magnetic north, providing a stable and reliable reference for navigation. In contrast, the raw yaw data contains distortions that can lead to inaccurate navigation.

2. Estimate the forward velocity

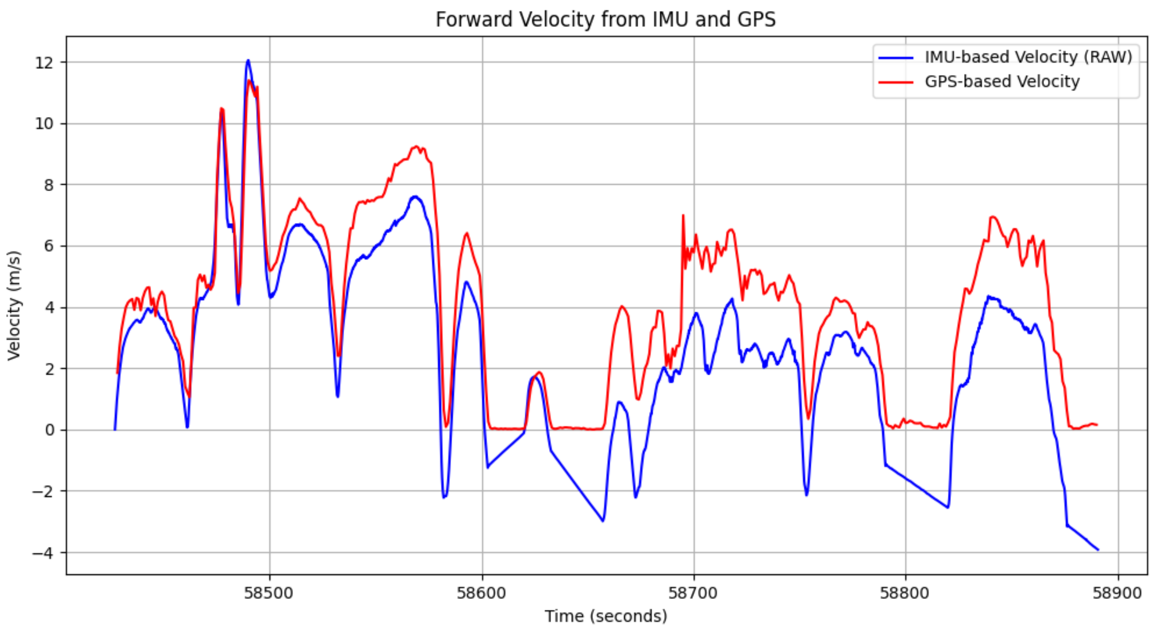

Figure 9: Velocity estimate from the GPS with Velocity estimate from accelerometer before adjustment

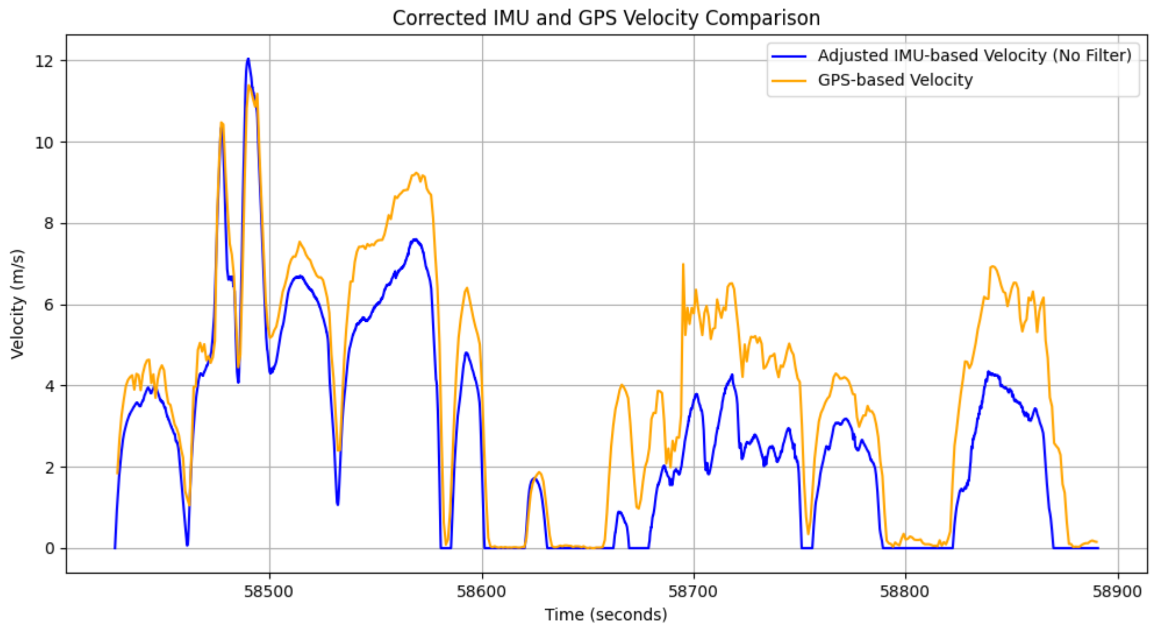

Figure 10: Velocity estimate from the GPS with Velocity estimate from accelerometer after adjustment

Q) **What adjustments did you make to the forward velocity estimate, and why?**

Solution. Adjustments to the Forward Velocity Estimate

1. Mean Removal from Acceleration Data:
   - Adjustment: The mean of the acceleration data ("Linear.x") was removed.
   - Reason: This step is crucial to correct for any static offset in the accelerometer data, ensuring that the mean acceleration is centered around zero. Without this correction, the integration of acceleration to compute velocity would accumulate errors, resulting in an incorrect velocity estimate.

Zeroing Negative Velocities:

- Adjustment: Any negative velocities in the IMU-based velocity were zeroed out.
- Reason: In the context of forward velocity, negative velocities can indicate either reverse motion or noise/errors in the IMU data. Given the nature of the experiment, which likely focuses on forward motion, negative values are considered erroneous and are set to zero to prevent them from skewing the results.

▪ What discrepancies are present in the velocity estimate between accel and GPS. Why?

Discrepancies in the Velocity Estimate Between Accelerometer and GPS

1. Sensor Noise and Drift:
   - Discrepancy: IMU-based velocity tends to show more noise and drift over time compared to GPS-based velocity.
   - Reason: IMU sensors, particularly accelerometers, are prone to noise and drift. The integration process amplifies these issues, leading to inaccuracies in the velocity estimate. Over time, even small errors in acceleration can accumulate significantly, causing the IMU-based velocity to diverge from the true velocity.
2. Sampling Rate Differences:
   - Discrepancy: There may be slight temporal misalignments or differences in the granularity of the data due to different sampling rates of the IMU and GPS sensors.
   - Reason: GPS data typically has a lower sampling rate compared to IMU data. This difference can lead to discrepancies when comparing the two datasets directly, as the GPS data may not capture all the rapid changes in velocity that the IMU data does.
3. Environmental Factors Affecting GPS:
   - Discrepancy: GPS-based velocity may show sudden changes or noise due to signal loss or multipath effects.
   - Reason: GPS signals can be affected by environmental factors such as buildings, trees, or atmospheric conditions. These factors can cause sudden jumps or noise in the GPS position data, which directly affects the velocity calculation derived from GPS coordinates.
4. Accumulation of Errors in IMU Integration:

- Discrepancy: IMU-based velocity may drift over time, showing a systematic deviation from the GPS-based velocity.
- Reason: The process of integrating acceleration to obtain velocity is inherently prone to error accumulation. Even after mean removal, small inaccuracies in acceleration readings can integrate into larger errors in velocity, leading to drift over time.

Conclusion

The adjustments made to the IMU-based velocity estimate, such as mean removal and zeroing negative velocities, are essential for improving the accuracy of the velocity estimate. However, discrepancies between IMU-based and GPS-based velocities remain due to inherent limitations and characteristics of the sensors, such as noise, drift, sampling rate differences, and environmental factors affecting GPS signals. Understanding and addressing these discrepancies are crucial for enhancing the reliability of velocity estimates in navigation applications.

3. Dead Reckoning with IMU

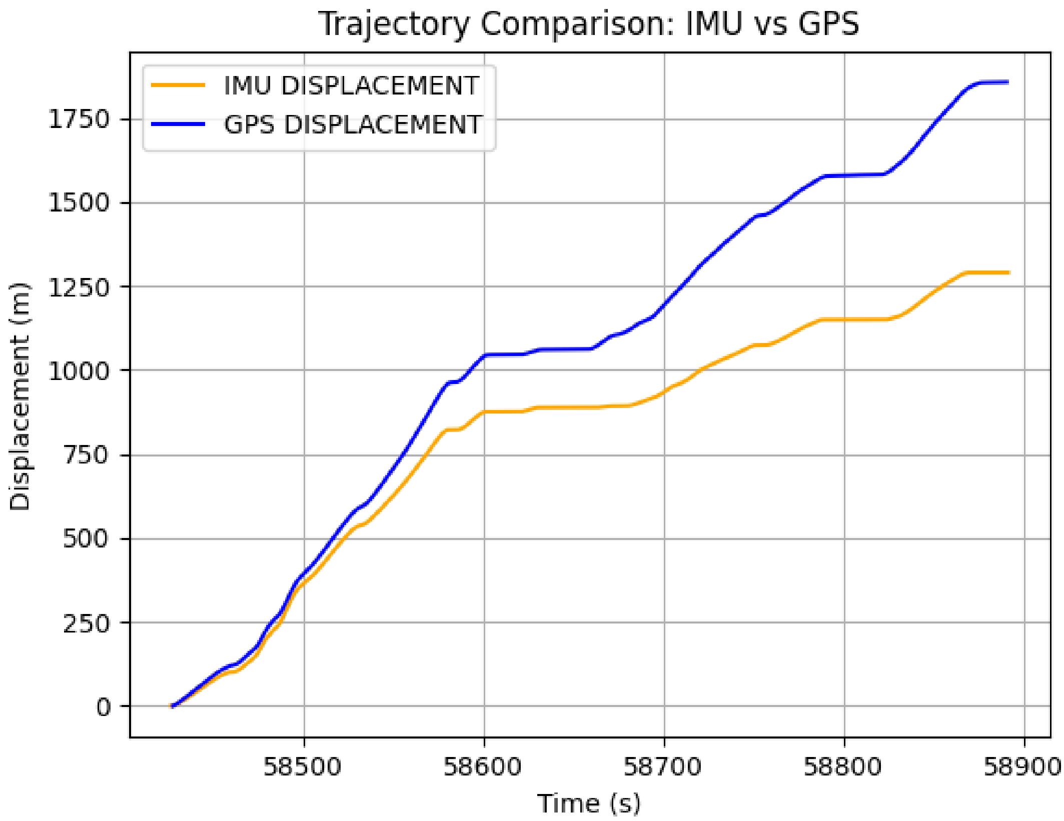

Figure 11: This plot compares the difference in displacement produced by the IMU and GPS sensor. This tells us that IMU shows lesser displacement and thus there exists noise and bias that disrupts its functioning.

Q) Compute ωX and compare it to $yobs$. How well do they agree? If there is a difference, what is it due to?

Solution. To compute ωX and compare it to y_obs , we use the following steps:

- Compute x_dot by integrating x_dotdot (which is the adjusted linear acceleration along the x-axis).
- Compute y_dotdot as ωx_dot , where ω is the yaw rate.
- Compare y_dotdot with y_obs .

Comparison and Explanation:

- Agreement: The plot shows how well y_obs matches with ωX.
- Differences: Any discrepancies between y_obs and ωX could be due to:
    - Sensor noise in the IMU data.
    - Calibration errors or residual biases in the accelerometer or gyroscope data.
    - External disturbances affecting the accelerometer readings but not captured by the gyroscope.

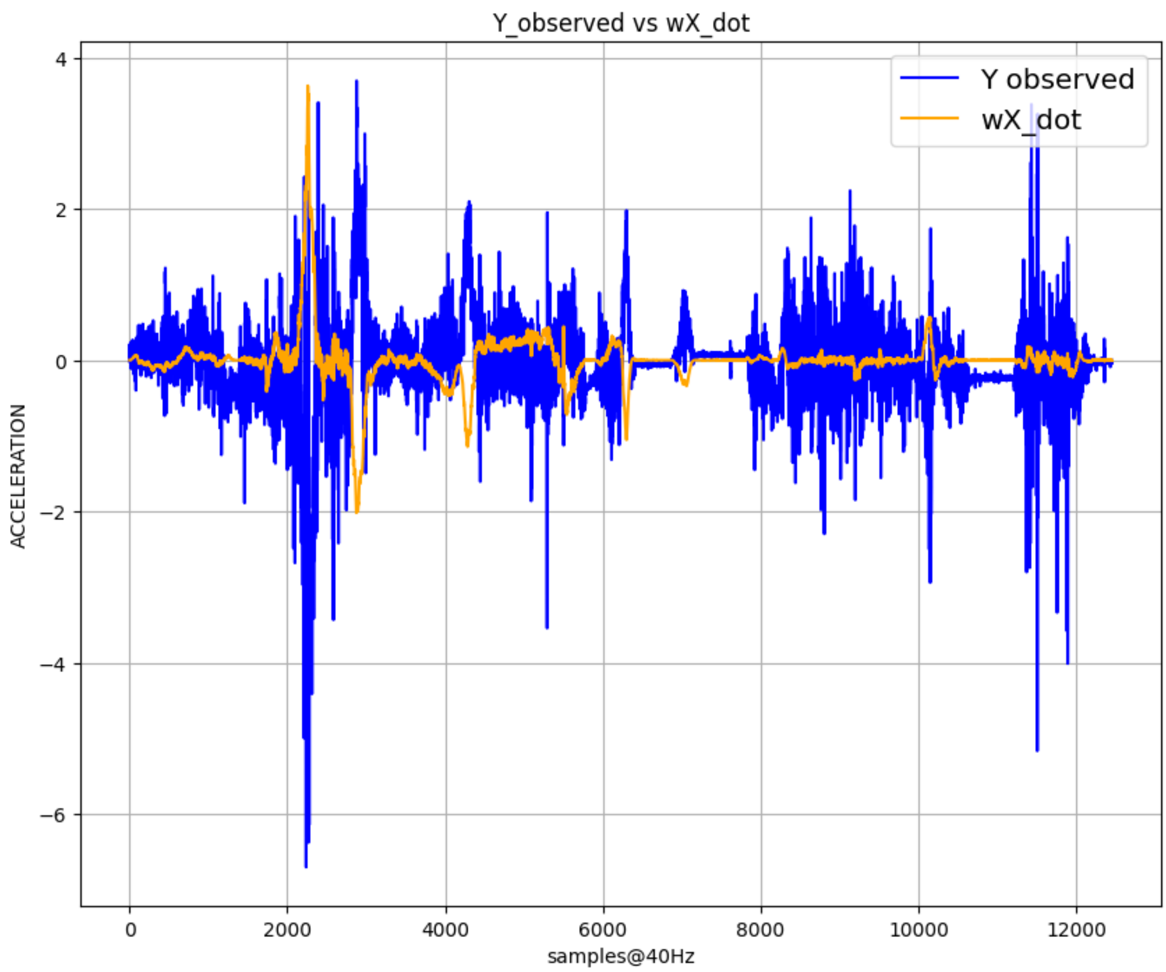

*Figure 12: w*X_dotdot with Y_obs. The wX_dotdot has a lot of noise and in order to get it*

Q) **Estimate the trajectory of the vehicle (Xe, Xn) from inertial data and compare with GPS by plotting them together. (adjust heading so that the first straight line from both are oriented in the same direction).**

Solution.

1. Data Collection and Initial Setup
    - Data Files: The IMU and GPS data files were loaded and combined with timestamps derived by merging seconds and nanoseconds for precise time tracking.
    - IMU Forward Acceleration: The forward acceleration (Linear.x) from the IMU was used to compute the vehicle's forward velocity. Any static offsets were removed by subtracting the mean, and a low-pass filter was applied with a cutoff frequency of 10 Hz to smooth high-frequency noise.
2. Velocity Calculation
    - IMU-Based Velocity: The forward velocity was computed by integrating the filtered forward acceleration using the cumulative trapezoidal method.
    - GPS-Based Velocity: GPS velocity was derived from the UTM coordinates by calculating the distance between consecutive points and dividing it by the time intervals between GPS measurements.
3. Yaw Calibration and Adjustment
    - Magnetometer Calibration: Hard and soft iron corrections, as well as a rotation adjustment (ϕ=0.327\phi = 0.327ϕ=0.327 radians with an additional offset of 1.5 radians), were applied to the magnetometer's x and y components. This provided an accurate yaw angle that was used to determine the vehicle's heading at each time step.
    - IMU Forward Velocity Adjustment: Adjusted the IMU forward velocity to ensure all negative values were set to zero, reflecting realistic motion constraints where reverse motion was not expected in this context.
2. Trajectory Estimation
    - Easting and Northing Components: Using the corrected yaw, the IMU-derived forward velocity was split into Easting (Ve) and Northing (Vn) components through rotation. These components were scaled by the computed scaling factor (1.1032) to match GPS coordinate scales.
    - Trajectory Integration: The Easting and Northing components were integrated over time to produce the trajectory (Xe, Xn). Both IMU and GPS starting points were aligned by setting their initial positions to (0,0).
3. Comparison and Plotting
    - Plot Overview: The IMU-based trajectory was compared with the GPS trajectory by plotting both on the same axes. The IMU trajectory closely follows the GPS path, with minor deviations possibly due to accumulated drift over time from integration.
4. Scaling Factor
    - Adjustment: A scaling factor of 1.1032 was applied to the IMU trajectory to ensure the paths matched in scale, accounting for any discrepancy in the IMU measurement range and GPS-derived distances.

Conclusion: The IMU trajectory closely approximated the GPS trajectory after calibration and scaling adjustments, although slight divergence was observed, which may be attributed to integration drift. Aligning the initial points and applying the scaling factor significantly improved the overlap between the two paths.

Figure 13: The trajectory estimations of IMU in comparison to the GPS trajectory. This shows that due to presence of bias and noise, which couldn't be resolved totally, the path of IMU is shortened. In order to remove this, IMU data must be parsed through low and high pass filters in order to remove the bias at specific spot.

Q) Estimate xc and explain your calculations (bonus up to 100%)

Date : / /

Estimation of x_c :

$$\ddot{x}_{obs} = \ddot{X} - \omega Y - \omega^2 x_c$$

& $$\ddot{Y}_{obs} = \ddot{Y} + \omega \dot{X} + \dot{\omega} x_c$$

Taking $Y = 0 \Rightarrow \dot{Y} = 0 \Rightarrow \ddot{Y} = 0$

$$\Rightarrow \ddot{x}_{obs} = \ddot{x} - \omega^2 x_c \quad — ①$$

$$\ddot{Y}_{obs} = \omega \dot{x} + x_c \dot{\omega} \quad — ②$$

from ①,

$$\ddot{x} = \ddot{x}_{obs} + \omega^2 x_c$$

from ②,

$$\dot{x} = \frac{\ddot{Y}_{obs} - \dot{\omega} x_c}{\omega}$$

Differenciate:

$$\ddot{x} = \frac{(\dddot{y}_{obs} - \ddot{\omega} x_c)\omega - \dot{\omega}(\ddot{y}_{obs} - \dot{\omega} x_c)}{\omega^2}$$

Comparing with ①,

$$\omega \ddot{\omega} x_c + \omega^4 x_c - \dot{\omega}^2 x_c = \omega \dddot{y}_{obs} - \dot{\omega} \ddot{y}_{obs} - \omega^2 \ddot{x}_{obs}$$

$$\boxed{x_c = \frac{\omega \dddot{y}_{obs} - \dot{\omega} \ddot{y}_{obs} - \omega^2 \ddot{x}_{obs}}{\ddot{\omega}\omega - \dot{\omega}^2 + \omega^4}}$$