STRING MANIPULATION, GUESS-and-CHECK, APPROXIMATIONS, BISECTION

(download slides and .py files l·yŘ follow along!)

6.0001 LECTURE 3

LAST TIME

- strings
- branching – if/elif/else
- while loops
- for loops

TODAY

- string manipulation
- guess and check algorithms
- approximate solutions
- bisection method

STRINGS

- think of as a **sequence** of case sensitive characters
- can compare strings with ==, >, < etc.
- `len()` is a function used to retrieve the **length** of the string in the parentheses

```
s = "abc"
```

`len(s)` → evaluates to 3

STRINGS

方括号

- square brackets used to perform **indexing** into a string to get the value at a certain index/position

```
s = "abc"
```

index: 0 1 2 ← indexing always starts at 0

index: -3 -2 -1 ← last element always at index -1

`s[0]` → evaluates to "a"

`s[1]` → evaluates to "b"

`s[2]` → evaluates to "c"

`s[3]` → trying to index out of bounds, error

`s[-1]` → evaluates to "c"

`s[-2]` → evaluates to "b"

`s[-3]` → evaluates to "a"

STRINGS

- can **slice** strings using `[start:stop:step]`
- if give two numbers, `[start:stop]`, `step=1` by default
- 省略
  you can also omit numbers and leave just colons

If unsure what some command does, try it out in your console!

0 1 2 3 4 5 6 7

`s = "abcdefgh"`

-8 -7 -6 -5 -4 -3 -2 -1

sub string

`s[3:6]` → evaluates to `"def"`, same as `s[3:6:1]`

`s[3:6:2]` → evaluates to `"df"`

`s[::]` → evaluates to `"abcdefgh"`, same as `s[0:len(s):1]`

reverse

`s[::-1]` → evaluates to `"hgfedbca"`, same as `s[-1:-(len(s)+1):-1]`

`s[4:1:-2]` → evaluates to `"ec"`

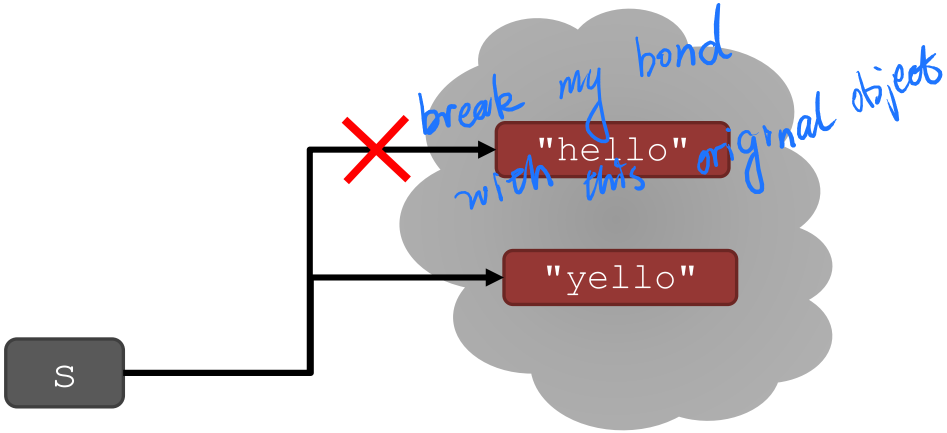
STRINGS

- strings are “**immutable**” – cannot be modified

不可变的

```
s = "hello"

s[0] = 'y'                → gives an error
s = 'y'+s[1:len(s)]       → is allowed,
                            s bound to new object
```



break my bond with this original object

概述 简要回顾

for LOOPS RECAP

- `for` loops have a **loop variable** that iterates over a set of values

loop variable

```
for var in range(4):
    <expressions>
```

→ `var` iterates over values 0,1,2,3

→ expressions inside loop executed with each value for `var`

```
for var in range(4,6):
    <expressions>
```

→ `var` iterates over values 4,5

- `range` is a way to iterate over numbers, but a for loop variable can **iterate over any set of values**, not just numbers!

STRINGS AND LOOPS

一小段

- these two code snippets do the same thing
- bottom one is more “pythonic”

```
s = "abcdefgh"
for index in range(len(s)):
    if s[index] == 'i' or s[index] == 'u':
        print("There is an i or u")

for char in s:
    if char == 'i' or char == 'u':
        print("There is an i or u")
```

CODE EXAMPLE:
ROBOT CHEERLEADERS

更直观
more intuitive

```
an_letters = "aefhilmnorsxAEFHILMNORSX"

word = input("I will cheer for you! Enter a word: ")
times = int(input("Enthusiasm level (1-10): "))
i = 0
while i < len(word):
    char = word[i]
    if char in an_letters:
        print("Give me an " + char + "! " + char)
    else:
        print("Give me a  " + char + "! " + char)
    i += 1
print("What does that spell?")
for i in range(times):
    print(word, "!!!")
```

```
for char in word:
```
✓

EXERCISE

```
s1 = "mit u rock"
s2 = "i rule mit"
if len(s1) == len(s2):
    for char1 in s1:
        for char2 in s2:
            if char1 == char2:
                print("common letter")
                break
```

猜测检验法

GUESS-AND-CHECK

完全枚举法.

- the process below also called **exhaustive enumeration**

- given a problem…
- you are able to **guess a value** for solution
- you are able to **check if the solution is correct**
- keep guessing until find solution or guessed all values

GUESS-AND-CHECK – cube root

```
cube = 8
for guess in range(cube+1):
    if guess**3 == cube:
        print("Cube root of", cube, "is", guess)
```

GUESS-AND-CHECK – cube root

```
cube = 8
                         绝对值  abs( )
for guess in range(abs(cube)+1):
    if guess**3 >= abs(cube):
        break // 先判断最小guess值
if guess**3 != abs(cube):
    print(cube, 'is not a perfect cube') // 排除错误情况
else:
    if cube < 0:
        guess = -guess // 检验负数情况
    print('Cube root of '+str(cube)+' is '+str(guess))
    打印结果
```

APPROXIMATE SOLUTIONS

- **good enough** solution
- start with a guess and increment by some **small value**
- keep guessing if |guess^3-cube| >= $\overset{\varepsilon}{\texttt{epsilon}}$ for some **small epsilon**

- decreasing increment size → slower program
- increasing epsilon → less accurate answer

极限思想

APPROXIMATE SOLUTION – cube root

guess
2
3
ε
2
3
ε

照样会越过ε.

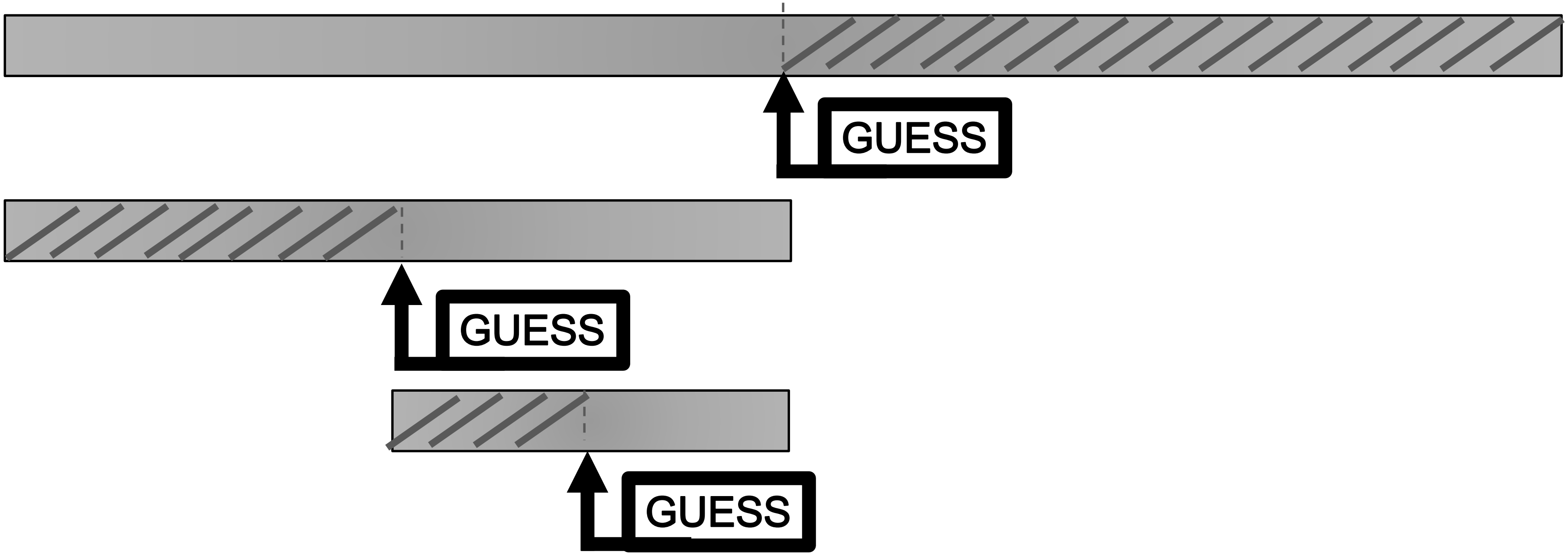
```
cube = 27
epsilon = 0.01
guess = 0.0
increment = 0.0001
num_guesses = 0 // 记数器，猜测的次数
while abs(guess**3 - cube) >= epsilon and guess <= cube :
    guess += increment
    num_guesses += 1
print('num_guesses =', num_guesses)
if abs(guess**3 - cube) >= epsilon: ?
    print('Failed on cube root of', cube)
else:
    print(guess, 'is close to the cube root of', cube)
```

1. 负数情况直接排除

2. 防止进入死循环

BISECTION SEARCH

- half interval each iteration
- new guess is halfway in between
- to illustrate, let's play a game!



二分搜索

BISECTION SEARCH – cube root

```
cube = 27
epsilon = 0.01
num_guesses = 0
low = 0
high = cube
guess = (high + low)/2.0
while abs(guess**3 - cube) >= epsilon:
    if guess**3 < cube :
        low = guess
    else:
        high = guess
    guess = (high + low)/2.0
    num_guesses += 1
print 'num_guesses =', num_guesses
print guess, 'is close to the cube root of', cube
```

initializing

```
if abs(cube) < 1:
    high = 1
    low = cube
```

小于1的情况

```
if cube < 0:
    guess = -guess
```

小于0的情况

BISECTION SEARCH CONVERGENCE

- 搜索空间
  search space N
  - first guess: $N/2$
  - second guess: $N/4$
  - kth guess: $N/2^k = 1$ 搜索空间里只有唯一的数了
- guess converges on the order of $\log_2 N$ steps
- bisection search works when value of function varies monotonically with input
  单调的
- code as shown only works for positive cubes > 1 – why?
- challenges → modify to work with negative cubes!
  → modify to work with x < 1!

x < 1

- if x < 1, search space is 0 to x but cube root is greater than x and less than 1
- modify the code to choose the search space depending on value of x