

We consider the following data set

$$\mathcal{D} = \{(x_i, y_i)\}_{i=1}^5 = \{(1, 0), (2, 1), (3, 0), (4, 1), (5, 1)\}$$

and compute the information gain associated with a split at $\xi = 3.5$. The entropy of the whole dataset is

$$H(\mathcal{D}) = -(p_0 \log_2 p_0 + p_1 \log_2 p_1), \quad p_0 = \frac{2}{5}, \quad p_1 = \frac{3}{5} \quad (1)$$

$$= -\frac{1}{5} (2 + 3 \log_2 3 - 5 \log_2 5) \quad (2)$$

The first set after the split is

$$\mathcal{D}_1 = \{(1, 0), (2, 1), (3, 0)\}$$

and its entropy is

$$H(\mathcal{D}_1) = -(p_0 \log_2 p_0 + p_1 \log_2 p_1), \quad p_0 = \frac{2}{3}, \quad p_1 = \frac{1}{3} \quad (3)$$

$$= -\frac{1}{3} (2 - 3 \log_2 3) \quad (4)$$

The second set after the split is

$$\mathcal{D}_2 = \{(4, 1), (5, 1)\}$$

and its entropy is

$$H(\mathcal{D}_2) = -(p_0 \log_2 p_0 + p_1 \log_2 p_1), \quad p_0 = 0, \quad p_1 = 1 \quad (5)$$

$$= 0 \quad (6)$$

The information gain is

$$\Delta H = 5H(\mathcal{D}) - (3H(\mathcal{D}_1) + 2H(\mathcal{D}_2)) \quad (7)$$

$$= -(2 + 3 \log_2 3 - 5 \log_2 5) + (2 - 3 \log_2 3) \quad (8)$$

$$= -(2 + 3 \log_2 3 - 5 \log_2 5 - 2 + 3 \log_2 3) \quad (9)$$

$$= -(6 \log_2 3 - 5 \log_2 5) \quad (10)$$

$$\approx -\left(6 \frac{16}{10} - 5 \frac{23}{10}\right) \quad (11)$$

$$= -\frac{96 - 115}{10} \quad (12)$$

$$= \frac{19}{10} = 1.9 \quad (13)$$