EC5321: Investment and Portfolio management
Lecture 1
Investment Environment and Fundamentals

Why do people invest?

- The inter-temporal consumer-savings model
  - One consumer, T periods
  - Income stream: Y_t, where; $t = 1, 2, \ldots, T$
  - consumer discounts future utility with a discount factor δ, where; $0 < \delta < 1$
  - Utility from each period: $U(.)$ where; $U' > 0$ & $U'' < 0$

    means slop is +ve → 1st derivative → 2nd derivative
  - consumer can borrow and lend freely at a rate $r > 0$
  - The consumer's problem is to choose consumption $C_1, C_2, \ldots, C_T$ in order to maximize lifetime utility.

- The consumer solves:

discount consumption — discounted income

$$\max_{(C_1, C_2, \ldots, C_T)} \sum_{t=0}^{T} \delta^t U(C_t) \quad \text{such that; } \sum_{t=0}^{T} \frac{1}{(1+r)^t} C_t \leq \sum_{t=0}^{T} \frac{1}{(1+r)^t} Y_t$$

- The Lagrangian:

$$L = \sum_{t=0}^{T} \delta^t U(C_t) - \lambda \left(\sum_{t=0}^{T} \frac{1}{(1+r)^t} (C_t - Y_t) \right)$$

- The FOC's for every t: $\delta^t U'(C_t) = \frac{\lambda}{(1+r)^t}$
- For any t and s: $\frac{U'(C_t)}{U'(C_s)} = [\delta(1+r)]^{s-t}$ — two periods s & t

where;

- T is period
- r is rate
- Y_t Income stream
- C_t consumption.
- δ discount factor
- U utility

* The discount factor $= \frac{1}{1 + \text{the discount rate}}$

* $U' > 0$ → means that the utility increases with consumption

* $U'' < 0$ → means utility function increases at a decreasing rate → so, the function is concave

- Conclusions

— the allocation of consumption over time does not depend on the allocation of income over time.

— In particular, when the market and individual discount factors are equal $\delta = \frac{1}{(1+r)}$

— perfect consumption smoothing occurs: $C_t = C_s$ for all s & t

— this model provides the main retionale for all investment: to smooth consumption over time.

— In other words, investment can be thought of as trading current for future consumption.

- Permanent income hypothesis

  — the hypothesis that current consumption depends only on one's estimate of the entire expected future income, rather than transitory shocks to income.

Investment

— the current commitment of money/wealth for a period of time to derive future payment.

- note that: the future payment will (in expectation) exceed the initial commitment in order to compensate the investor for:

— ~~thae~~ time the funds are committed (the real rate of return)

— the expected rate of inflation (inflation premium)

— uncertainty of future flow of funds (risk premium)

- the rate of return

— the "exchange rate" between current and future consumption.

- measuring the rate of return
  1) Gross return R
     - the ratio of final to initial value
  2) Net return r
     - also known as yield : $r = R - 1$
     - typically expressed in %

Historical rates of return

- Return over A Holding Period
  - Holding Period Return (HPR)

$$HPR = \frac{\text{Ending value of investment}}{\text{Beginning value of Investment}}$$

  - Holding Period Yield (HPY)

$$HPY = HPR - 1$$

  - Annual HPR and HPY

$$\text{Annual HPR} = HPR^{1/n}$$

$$\text{Annual HPY} = \text{Annual HPR} - 1 = HPR^{1/n} - 1$$

where;

n = number of years of the investment

Example: your investment of \$250 in stock A is worth \$350 in two years while the investment of \$100 in stock B is worth \$ 120 in six months. what are the annual HPRs and the HPYs on those two stocks?

- For stock A

$$\text{Annual HPR} = HPR^{1/n} = \left(\frac{\text{Ending value of investment}}{\text{Beginning value of investment}}\right)^{1/2} = \left(\frac{350}{250}\right)^{1/2} = 1.1832$$

$$\text{Annual HPY} = \text{Annual HPR} - 1 = 1.1832 - 1 = 18.32\%$$

- For stock B

$$\text{Annual HPR} = HPR^{1/n} = \left(\frac{120}{100}\right)^{1/0.5} = 1.2544$$

$$\text{Annual HPY} = \text{Annual HPR} - 1 = 1.2544 - 1 = 25.44\%$$

Compounding

- Why is annual HPY not equal to HPY/n ?
- Because of compounding

-Interest from each period gets re-invested

*suppose that annual yield is r & you invest principal, P

After one year

total investment value $= P(1+r)^1$

After n years,

It is $P(1+r)^n$

where, P = invest principal n=time, r=annual yield

→the entire holding period return is then

$HPR = (1+r)^n$

- The annual yield equivalent to this HPR is then

$r = HPR^{1/n} - 1$

Frequency of compounding

- for different investments, compounding may be more frequent than once per year
- there is then a difference between simple annual yield (SAY) and the aforementioned annualized yield (AHPY).

* $SAY = r_m m$

- when an investment yields r_m% m times per year.

Example:

suppose an investment yields 2% per quarter

$\therefore SAY = 2\% \times 4 = 8\%$

and, $P(1+0.02)^4 = 1.0824P$ -the value at the end of the year

and

$\therefore HPR(\text{one year}) = 1.0824$

- the holding return for one year.

& $AHPY = (1.0824)^{1/1} - 1 = 8.24\% > SAY$

- the annualized yield.

- compounding frequency & simple annual versus annualized returns: the generalized case

- suppose P_0 is invested for a holding period of n years at simple annual yield r, with interest compounded m times per year
  - The final value is $P_1 = P_0\left(1+\frac{r}{m}\right)^{mn}$

where;

P_0 = is investment

n = is period

r = is simple annual yield

m = is interest compounded.

- with continuous compounding, $m \to \infty$

$$\lim_{m\to\infty} P_1 = \lim_{m\to\infty} P_0\left(1+\frac{r}{m}\right)^{mn} = P_0 e^{nr}$$

Historical Rates of returns

- computing Mean Historical Returns

- suppose you have a set of annual rates of return (HPYs or HPRs) for an investment.

How do you measure the mean annual return?

1) Arithmetic Mean Return (AM)

$$AM = \frac{\Sigma HPY}{n}$$

where;

ΣHPY = is the sum of all the annual HPYs

n = is the number of years.

2) Geometric Mean Return (GM)

$$GM = [\pi HPY]^{1/n} - 1$$

where;

πHPR = is the product of all the annual HPRs

n = is the number of years.

Example

Suppose you invested \$100 three years ago and it is worth \$110.40 today.

Year	Beginning Value	Ending Value	HPR	HPY
1	100	115.0	1.15	0.15
2	115	138.0	1.20	0.20
3	138	110.4	0.80	-0.20

$$AM = \frac{\Sigma HPY}{n} = \frac{[(0.15)+(0.20)+(-0.20)]}{3} = \frac{0.15}{3} = 5\%$$

$$GM = [\pi\, HPY]^{1/n} - 1 = [(1.15)\times(1.20)\times(0.80)]^{1/3} - 1$$
$$= (1.104)^{1/3} - 1$$
$$= 1.03353 - 1$$
$$= 3.353\%$$

- Comparison of AM and GM

- when rates of return are the same for all years, the AM and the GM will be equal.
- when rates of return are not the same for all years, the AM will always be higher than the GM
- while the AM is best used as an "expected value" for an individual year, the GM is the best measure of an asset's long-term performance.

- A portfolio of Investments
  - Portfolio HPY

- the mean historical rate of return for a portfolio of investments is measured as the weighted average of the HPYs for the individual investments in the portfolio, or the overall change in the value of the original portfolio.

- the weights used in the computation are the relative beginning market values for each investment, which is often referred to as dollar-weighted or value-weighted mean rate of return.

Computation of Holding Period Yield for a Portfolio

Investment	No. of shares	Begining Price	Beginning Market Value	Ending Price	Ending Market Value	HPR	HPY	Market Weight	Weighted HPY
~~A~~ ~~B~~									
A	100,000	$10	$1,000,000	$12	$1,200,000				
B	200,000	20	4,000,000	21					
C	500,000	30	15,000,000	33					
Total			$20,000,000						

Expected rates of returns

- we discussed realized historical rates of return. In contrast, an investor would be more interested in the expected return on a future risky investment.
- Risk
  - refers to the uncertainty of the future outcomes of an investment

- computing Expected Rate of Return

$$E(R_i) = \sum_{i=1}^{n} (\text{Probability of Return}) \times (\text{Possible Return})$$

$$= [(P_1)(R_1) + (P_2)(R_2) + \dots + (P_n R_n)]$$

$$= \sum_{i=1}^{n} (P_i)(R_i)$$

where;

P_i = is Probability for possible return i

R_i = is Possible return i

- For most investments, the rate of return is a random variable.
  i.e., it is not known when the investment is made
- In general, we can therefore characterize the returns by their distribution and density functions (pdfs & cdfs)
- In many applications, just the mean and the variance (or the standard deviation) is sufficient
- The variance of returns in the investment context measures risk.

• Risk

– refers to the uncertainty of an investment; therefore the measure of risk should reflect the degree of the uncertainty.

• The risk of expected return reflect the degree of uncertainty that actual return will be different from the expect return.

• The common measures of risk
– are based on the variance of rates of return distribution of an investment.

Mean-Variance Preferences

• Higher means preferred to lower means
• Lower variances preferred to higher

$$\text{Variance } \sigma^2 = \sum_{i=1}^{n} \text{Prob} \times (\text{Possible Return} - \text{Expected Return})^2$$

$$= \sum_{i=1}^{n} P_i \times [R_i - E(R_i)]^2$$

$$\text{standard deviation } \sigma = \sqrt{\text{Variance}}$$

• Coefficient of Variation (CV):
– measures the risk per unit of expected return and is a relative measure of risk.

$$CV_i = \frac{\text{Standard of Deviation of Return}}{\text{Expected Return}} = \frac{\sigma_i}{E(R_i)}$$

• Given a series of historical returns measured by HPY, the risk is quantified as:

$$\sigma^2 = \frac{1}{n}\left[\sum_{i=1}^{n} [HPY_i - E(HPY)]^2\right]$$

• The risk is

$$\sigma^2 = \frac{1}{n}\left[\sum_{i=1}^{n}[HPY_i - E(HPY)]^2\right]$$

Where;
σ^2 = is the variance of the series
HPY_i = is the holding period yield during period i
$E(HPY)$ = is the expected value of the HPY equal to the arthemetic mean of the series (AM)
n = is the number of observations.

Determinants of required rates of return

• Three factors influence an investor's required rate of return
- real risk-free rate of return
- Expected rate of inflation during the period
- Risk.

• The Real Risk Free Rate (RRFR)
- assumes no inflation
- assumes no uncertainty about future cash flows
- influenced by time preference for consumption of income and investment opportunities in the economy.

• Nominal Risk-Free Rate (NRFR)
- conditions in the capital market
- expected rate of inflation

$$NRFR = (1 + RRFR) \times (1 + \text{Rate of Inflation}) - 1$$

The Fisher Equation

$$RRFR = [(1 + NRFR) / (1 + \text{Rate of Inflation})] - 1$$

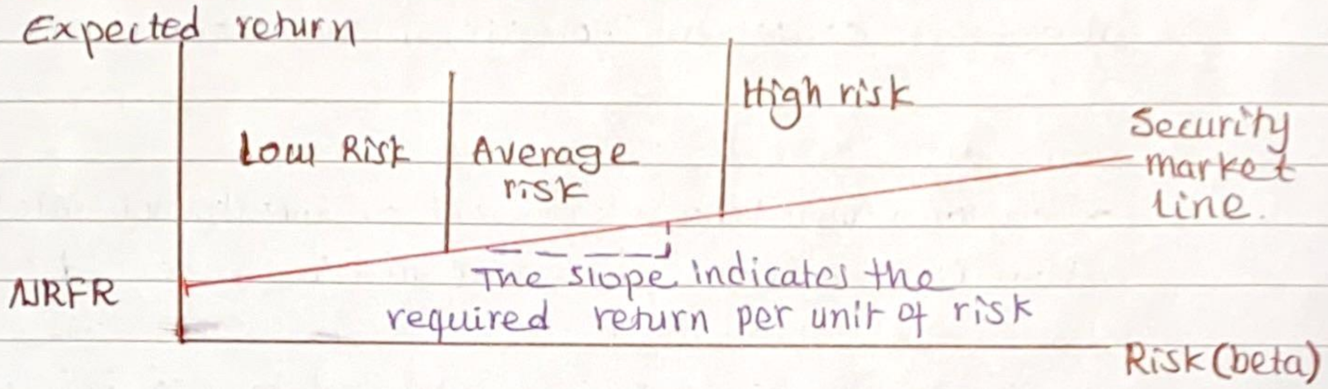
- Business Risk
  - uncertainty of income flows caused by the nature of a firm's business.
  - sales volatility and operating leverage determine the level of business risk.
- Financial Risk
  - uncertainty caused by the use of debt financing
  - borrowing requires fixed payments which must be paid ahead of payments to stockholders.
  - the use of debt increases uncertainty of stockholder income & causes an increase in the stock's risk premimum.

- Liquidity Risk
  - how long will take to convert an investment into cash?
  - how certain is the price that will be received?
- Exchange Rate Risk
  - uncertainty of return is introduced by acquiring securities denominated in a currency different from that of the investor.
  - change in exchange rates affect the investors return when converting an investment back into the "home" currency.
- Country Risk
  - political risk is the uncertainty of returns caused by the possibility of a major change in the political or economic environment in a country.
  - individuals who invest in countries that have unstable political-economic systems must include a country risk-premium when determining their required rate of return.

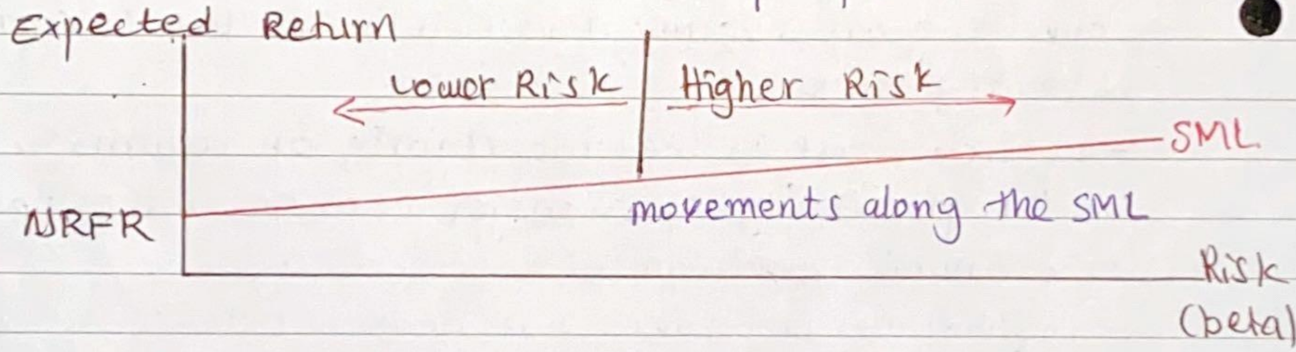
• Risk Premium and Portfolio Theory

- from a portfolio theory perspective, the relevant risk measure for an individual asset is its co-movement with the market portfolio.
- systematic risk relates the variance of the investment to the variance of the market
- Beta measures this systematic risk of an asset.
- According to portfolio theory, the risk premium depends on the systematic risk.

Risk-Return relationship: The SML



• NRFR = Nominal Risk-Free Rate
= (1 + RRFR) x (1 + Rate of Inflation) - 1



movements along the SML

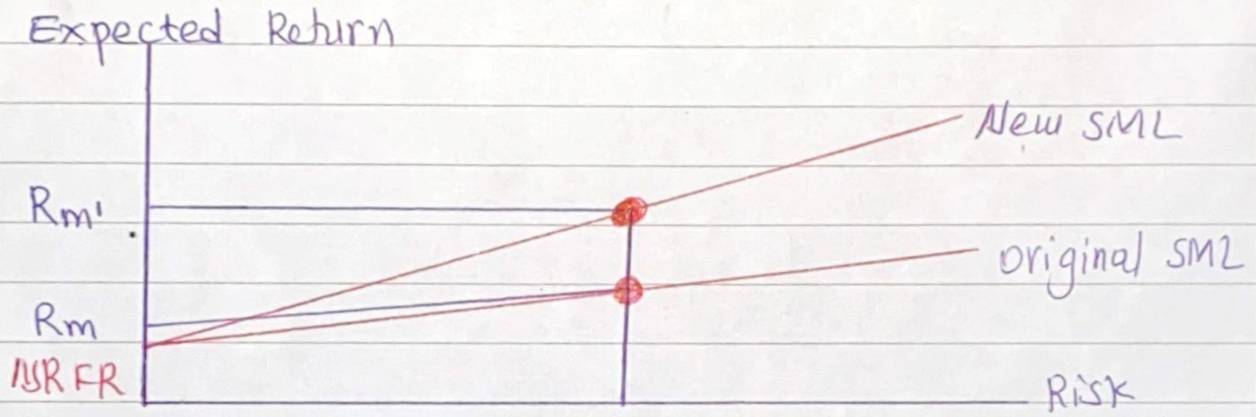
- when the risk changes, the expected return will also change,
  • moving along the SML
- Risk premium: $RP_i = E(R_i) - NRFR$

Changes in the slope of SML

→ the slope of the SML indicates the return per unit of risk required by all investors.

- The market risk premium
  - is the yield spread between the market portfolio & the risk free rate of return.
  - this changes over time, although the underlying reasons are not always clear.
  - However, a change in the market risk premium will affect the return required on all risky assets.

- when there is a change in the attitude of investors toward risk,
  - the slope of the SML will also change.
- If investor become more risk averse,
  - then the SML will have a steeper slope,
  - indicating a higher risk premium, RP, for the same risk level.



Changes in Market Condition or Inflation

- A change in the RRFR or the expected rate of inflation will
  - cause a parallel shift in the SML.
- when nominal risk-free rate increases,
  - the SML will shift up,
  - implying a higher rate of return (for all assets)
  - while still having the same risk premium.