

# IRT and CFA

Below, I will analyze the same dataset using both Item Response Theory (IRT) and Confirmatory Factor Analysis (CFA) to compare and contrast between the two.

## The Data

The 2015-2016 NHANES Mental Health - Depression Screener.

Originally a rating scale (0, Not at all; 1, several days; 2, more than half the days; 3, nearly everyday)

Q1: "Have little interest in doing things"

Q2: "Feeling down, depressed, or hopeless"

Q3: "Trouble sleeping or sleeping too much"

Q4: "Feeling tired or having little energy"

Q5: "Poor appetite or overeating"

Q6: "Feeling bad about yourself"

Q7: "Trouble concentrating on things"

Q8: "Moving or speaking slowly or too fast"

Q9: "Thought you would be better off dead"

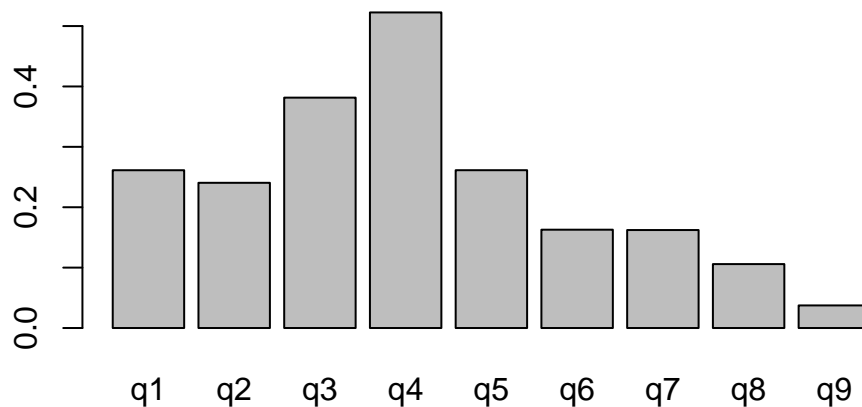
Q10: "Difficulty these problems have caused"

Looking at the questions, we clearly see that Q10 does not fit in with the rest. It violates the assumption of local independence (A participant's answer to Q10 depends on their answers to the other questions). To that end, I had Q10 dropped from the data set.

## Import Dataset

I opted to dichotomize the data by having any value above 0 changed to a 1.

```
# Mental Health - Depression Screener from  
# https://wwwn.cdc.gov/nchs/nhanes/search/datapage.aspx?Component=Questionnaire&CycleBeginn  
  
ds <- haven::read_xpt("https://wwwn.cdc.gov/Nchs/Nhanes/2015-2016/DPQ_I.XPT")  
ds <- ds[- 11]  
  
names(ds) <- c("id", paste0("q", 1:9))  
  
ds$id <- NULL  
  
ds <- ds %>%  
  mutate_at(vars(q1:q9), ~ifelse(. > 3, NA, .))  
  
ds_dich <- ds %>%  
  mutate_at(vars(q1:q9), ~ifelse(. > 0, 1, 0))  
  
qmeans <- apply(ds_dich, 2, mean, na.rm = TRUE)  
  
barplot(qmeans)
```

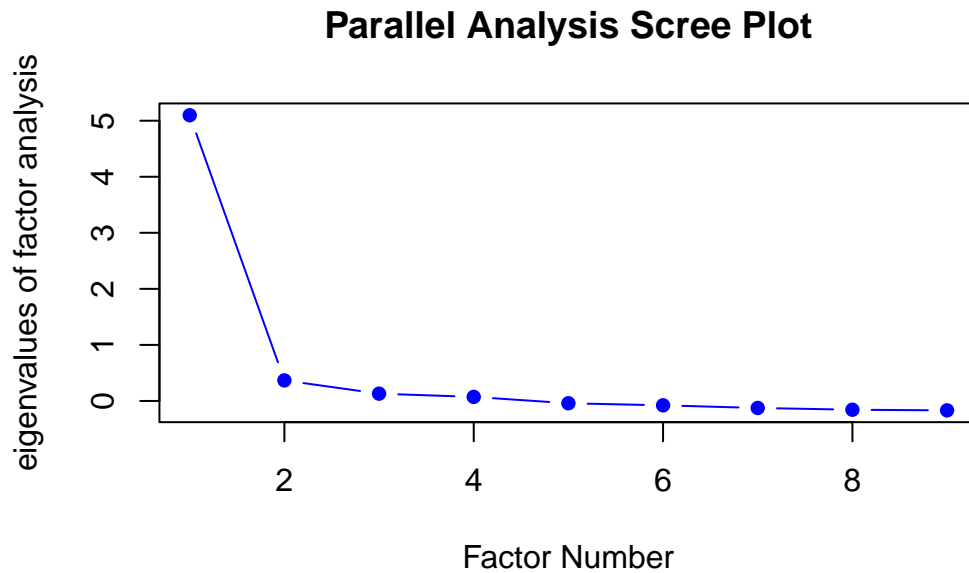


This graph shows mean number of endorsements an answer received. Here, we can see that q4 (“Feeling tired or having little energy”) had the most positive answers, while q9 (“Thought you would be better off dead”) had very few endorsements.

Before attempting to fit an IRT model, it would be good to examine whether the 9 items are all measuring the same latent variable. In our case, depression. This is done with a *scree test*, the components left of the “elbow” (the sharpest drop) of the graph should be retained. As we want to measure just 1 latent variable, we want there to be just 1 component before the elbow.

```
screeplot <- fa.parallel(ds_dich, cor = "tet")

x <- 1:length(screeplot$fa.values)
plot(x, screeplot$fa.values, type = "b", pch = 16, col = "blue",
     xlab = "Factor Number",
     ylab = "eigenvalues of factor analysis",
     main = "Parallel Analysis Scree Plot")
```



As shown, the scree plot suggests there is only 1 underlying factor. We can now proceed to fit the IRT models.

### Fit 1PL Model

Here, I fitted a 1PL (1-parameter logistic) model to estimate item difficulty based on how many people answered the items.

```
pl1 <- ltm::rasch(ds_dich)
kable(summary(pl1)$coefficients, digits = 2)
```

	value	std.err	z.vals
Dffclt.q1	0.84	0.03	32.27
Dffclt.q2	0.93	0.03	34.64
Dffclt.q3	0.40	0.02	16.86
Dffclt.q4	-0.07	0.02	-3.17
Dffclt.q5	0.84	0.03	32.27
Dffclt.q6	1.30	0.03	42.11
Dffclt.q7	1.30	0.03	42.17
Dffclt.q8	1.66	0.04	45.85
Dffclt.q9	2.40	0.05	45.52

	value	std.err	z.vals
Dscrmn	2.01	0.03	57.66

Q1 has a difficulty score of 0.84. This refers to the fact that an individual with the “depression level” of 0.84 would have a 50/50 chance of endorsing Q1. Every item was fixed to have a discrimination, which will be explained further on.

## Fit 2PL Model

Next, I fitted a 2PL model to estimate each item’s discrimination parameter. I will later graph this information, to better show what it means.

```
pl2 <- ltm(ds_dich ~ z1)

kable(summary(pl2)$coefficients, digits = 2)
```

	value	std.err	z.vals
Dffclt.q1	0.84	0.03	29.56
Dffclt.q2	0.81	0.02	34.27
Dffclt.q3	0.45	0.03	16.19
Dffclt.q4	-0.07	0.02	-2.93
Dffclt.q5	0.95	0.04	27.10
Dffclt.q6	1.17	0.03	39.27
Dffclt.q7	1.34	0.04	33.81
Dffclt.q8	1.69	0.05	33.75
Dffclt.q9	2.20	0.07	31.25
Dscrmn.q1	2.07	0.08	24.34
Dscrmn.q2	3.18	0.16	20.19
Dscrmn.q3	1.57	0.06	24.89
Dscrmn.q4	1.91	0.08	23.99
Dscrmn.q5	1.56	0.06	24.08
Dscrmn.q6	2.75	0.13	21.24
Dscrmn.q7	1.89	0.08	22.55
Dscrmn.q8	1.94	0.10	20.29
Dscrmn.q9	2.48	0.17	14.31

Here, I test to see if PL2 has a significantly better fit than PL1, by evaluating their model characteristics within an ANOVA.

```
anova(pl1, pl2)
```

Likelihood Ratio Table

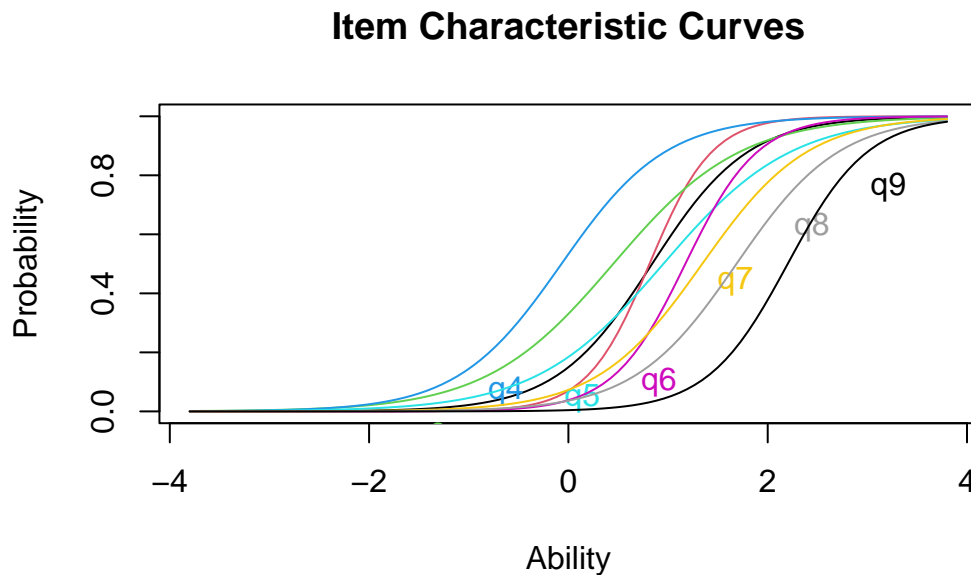
	AIC	BIC	log.Lik	LRT	df	p.value
pl1	38038.94	38105.49	-19009.47			
pl2	37849.45	37969.23	-18906.72	205.49	8	<0.001

The significant p-value in this chart tells us that the 2PL is a better fit to the data than the 1PL. The fit of the model has been improved by estimating the discrimination parameter of each item, instead of fixing it to one value.

### Item Characteristic Curves

Below, I plotted the item characteristic curves of the 10 items to better see the discriminability across items.

```
plot(pl2, type = c("ICC"))
```

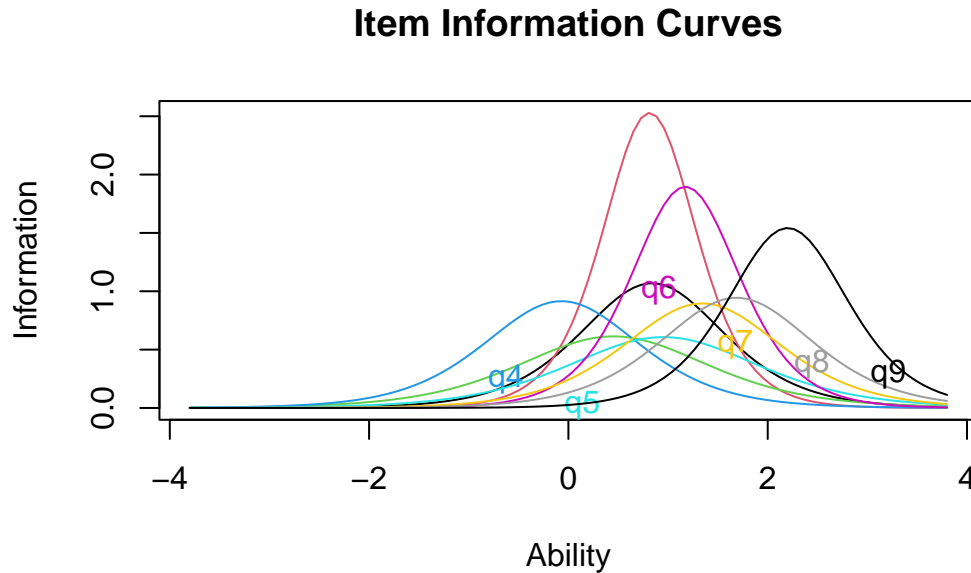


In the ICCs, we can better see the probability of endorsing an answer at varying ability levels. We see that q4 (“Feeling tired or having little energy”) has a range of abilities endorsing it,

while with q9 (“Thought you would be better off dead”), only individuals with the highest level of depression endorsed it. So, discrimination can be seen as the steepness of these curves.

### Item Information Curves

```
plot(pl2, type = c("IIC"))
```

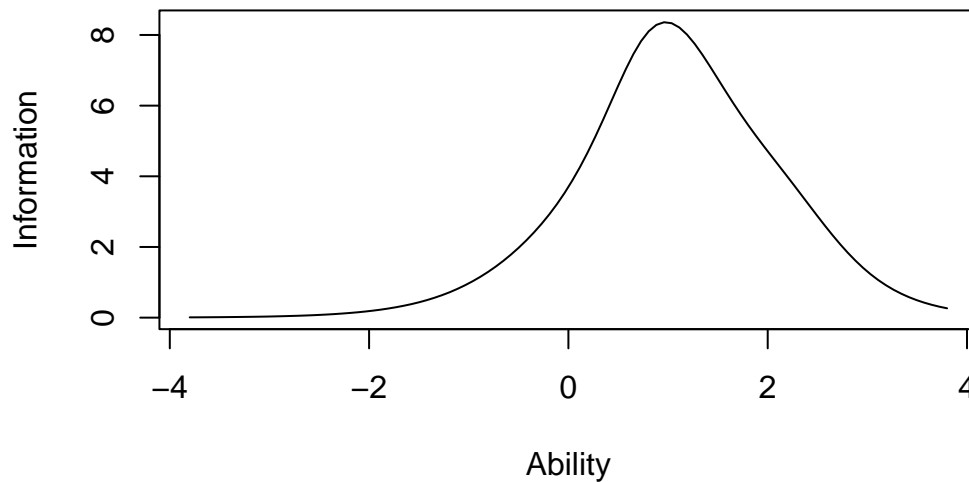


The IICs demonstrate that items range in how much information they provide about an individual's depression level for different ability levels. The red curve, q10, gives us the most information at moderate depression levels. In contrast, q4 (the blue curve), gives us very low information because of how a range of depression levels it covers.

### Sum of all IIC Curves

```
plot(pl2, type = c("IIC"), items = c(0))
```

## Test Information Function



The test information function shows that the items as a whole provide the most information about low-to-moderate depression levels, and less about extreme high or low depression levels. This is desirable, as it is not important to discriminate between those with very low or very high depression. It is important to discriminate between those of moderate depression levels, which is what the test information function tells us it does.

## Confirmatory Factor Analysis with Dichotomous Data

```
mod <-  
  "depression =~ q1 + q2 + q3 + q4 + q5 +  
    q6 + q7 + q8 + q9  
  
"  
  
cfafit <- cfa(mod, data = ds_dich,  
  ordered = c("q1", "q2", "q3", "q4", "q5",  
    "q6", "q7", "q8", "q9"),  
  estimator = "WLSMV",  
  check.gradient = TRUE  
)
```



```
summary(cfafit, fit.measures = TRUE, standardized = TRUE)
```

lavaan 0.6.15 ended normally after 17 iterations

Estimator	DWLS
Optimization method	NLMINB
Number of model parameters	18

	Used	Total
Number of observations	5134	5735

Model Test User Model:

	Standard	Scaled
Test Statistic	269.518	404.621
Degrees of freedom	27	27
P-value (Chi-square)	0.000	0.000
Scaling correction factor		0.669
Shift parameter		1.484
simple second-order correction		

Model Test Baseline Model:

Test statistic	28028.387	19894.317
Degrees of freedom	36	36
P-value	0.000	0.000
Scaling correction factor		1.410

User Model versus Baseline Model:

Comparative Fit Index (CFI)	0.991	0.981
Tucker-Lewis Index (TLI)	0.988	0.975
Robust Comparative Fit Index (CFI)		0.901
Robust Tucker-Lewis Index (TLI)		0.868

Root Mean Square Error of Approximation:

RMSEA	0.042	0.052
90 Percent confidence interval - lower	0.037	0.048
90 Percent confidence interval - upper	0.046	0.057
P-value H <sub>0</sub> : RMSEA ≤ 0.050	0.998	0.203

P-value H_0: RMSEA >= 0.080	0.000	0.000
Robust RMSEA		0.146
90 Percent confidence interval - lower		0.131
90 Percent confidence interval - upper		0.161
P-value H_0: Robust RMSEA <= 0.050		0.000
P-value H_0: Robust RMSEA >= 0.080		1.000

Standardized Root Mean Square Residual:

SRMR	0.053	0.053
------	-------	-------

Parameter Estimates:

Standard errors	Robust.sem
Information	Expected
Information saturated (h1) model	Unstructured

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )	Std.lv	Std.all
depression =~						
q1	1.000				0.765	0.765
q2	1.160	0.022	52.372	0.000	0.887	0.887
q3	0.890	0.023	39.502	0.000	0.681	0.681
q4	0.965	0.022	43.802	0.000	0.738	0.738
q5	0.877	0.023	38.143	0.000	0.671	0.671
q6	1.106	0.022	49.387	0.000	0.846	0.846
q7	0.956	0.024	39.331	0.000	0.731	0.731
q8	0.946	0.027	35.642	0.000	0.723	0.723
q9	1.020	0.033	31.139	0.000	0.780	0.780

Intercepts:

	Estimate	Std.Err	z-value	P(> z )	Std.lv	Std.all
.q1	0.000				0.000	0.000
.q2	0.000				0.000	0.000
.q3	0.000				0.000	0.000
.q4	0.000				0.000	0.000
.q5	0.000				0.000	0.000
.q6	0.000				0.000	0.000
.q7	0.000				0.000	0.000
.q8	0.000				0.000	0.000
.q9	0.000				0.000	0.000
depression	0.000				0.000	0.000

Thresholds:

	Estimate	Std.Err	z-value	P(> z )	Std.lv	Std.all
q1 t1	0.643	0.019	34.054	0.000	0.643	0.643
q2 t1	0.709	0.019	36.944	0.000	0.709	0.709
q3 t1	0.303	0.018	17.027	0.000	0.303	0.303
q4 t1	-0.056	0.018	-3.182	0.001	-0.056	-0.056
q5 t1	0.640	0.019	33.946	0.000	0.640	0.640
q6 t1	0.986	0.021	47.035	0.000	0.986	0.986
q7 t1	0.989	0.021	47.130	0.000	0.989	0.989
q8 t1	1.257	0.024	53.327	0.000	1.257	1.257
q9 t1	1.782	0.032	54.885	0.000	1.782	1.782

Variances:

	Estimate	Std.Err	z-value	P(> z )	Std.lv	Std.all
.q1	0.415				0.415	0.415
.q2	0.213				0.213	0.213
.q3	0.537				0.537	0.537
.q4	0.456				0.456	0.456
.q5	0.550				0.550	0.550
.q6	0.285				0.285	0.285
.q7	0.466				0.466	0.466
.q8	0.477				0.477	0.477
.q9	0.391				0.391	0.391
depression	0.585	0.019	30.385	0.000	1.000	1.000

Scales y\*:

	Estimate	Std.Err	z-value	P(> z )	Std.lv	Std.all
q1	1.000				1.000	1.000
q2	1.000				1.000	1.000
q3	1.000				1.000	1.000
q4	1.000				1.000	1.000
q5	1.000				1.000	1.000
q6	1.000				1.000	1.000
q7	1.000				1.000	1.000
q8	1.000				1.000	1.000
q9	1.000				1.000	1.000

The thresholds of the CFA have a similar pattern to that of the difficulty ability of each item. Similarly, the factor loadings of the CFA seem similar to the discrimination parameters from the IRT.

## Check Modification Indices

Next, I checked the modification indices for the assumption of *local independence* for any theoretical justification of adding covariances to the model.

```
modindices(cfafit) %>% dplyr::arrange(desc(mi)) %>% head()
```

	lhs	op	rhs	mi	epc	sepc.lv	sepc.all	sepc.nox
1	q3	~~	q4	77.087	0.172	0.172	0.348	0.348
2	q2	~~	q6	52.803	0.143	0.143	0.581	0.581
3	q7	~~	q8	29.487	0.138	0.138	0.292	0.292
4	q6	~~	q9	28.424	0.154	0.154	0.462	0.462
5	q2	~~	q3	28.022	-0.122	-0.122	-0.361	-0.361
6	q2	~~	q4	21.572	-0.105	-0.105	-0.337	-0.337

Some of the modification indices are high (e.g., q2~~q6), and the SEPC's are moderate.

For example:

q3: "Trouble sleeping or sleeping too much"

q4: "Feeling tired or having little energy"

These seem to be closely related, though because this would be different to the IRT models, I won't rerun the model with q3 and q4 added as covariances.

q2: "Feeling down, depressed, or hopeless"

q6: "Feeling bad about yourself"

Not closely related.

## Compare discrimination's with factor loadings

Below I take a better look at the difference between my CFA's factor loadings and 2PL's discrimination abilities. To do this, I use a formula to transform IRT discrimination parameter estimates to standardized factor loadings (Cho, 2023):

$$\lambda = \frac{\alpha/D}{\sqrt{1 + (\alpha/D)}}$$

Where alpha is the discrimination parameter, and  $D$  is a scaling constant which the author of the ltm package has chosen to be 1.702 (Rizopoulos, 2006).

```

model_loadings <- inspect(cfafit, what = "std")[["lambda"]]

discrims <- pl2$coefficients[, 2]

D <- 1.7

df_loadings <- cbind(loadings = model_loadings,
                     discrims_to_loadings = (discrims / D)
                     / (sqrt(1 + ((discrims / D)^2))))

df_loadings <- df_loadings %>%
  as.data.frame() %>%
  dplyr::rename(cfa_loadings = depression)

df_loadings %>% as.data.frame() %>%
  dplyr::mutate(dif = cfa_loadings-discrims_to_loadings, rat = cfa_loadings/discrims_to_lo

```

	cfa_loadings	discrims_to_loadings	dif	rat
q1	0.7647710	0.7725016	-0.007730544	0.9899928
q2	0.8868587	0.8819424	0.004916318	1.0055744
q3	0.6807793	0.6777242	0.003055145	1.0045079
q4	0.7377550	0.7475920	-0.009836942	0.9868418
q5	0.6709435	0.6758790	-0.004935506	0.9926976
q6	0.8455697	0.8509317	-0.005361990	0.9936987
q7	0.7309156	0.7440796	-0.013163991	0.9823084
q8	0.7233743	0.7528095	-0.029435287	0.9608994
q9	0.7802641	0.8252215	-0.044957360	0.9455209

Close! The values are close enough to have shown that for dichotomous data, CFA and 2PL IRT models are theoretically equivalent. The slight differences may be due to that this is only an approximation, and have slight rounding / computation errors; see Forero & Maydeu-Olivares (2009).

Everything done previously was on a dichotomized version of the data set. Next, I will compare two polytomous IRT models using the original data: the Graded Response Model (GRM) and Rating Scale Model (RSM).

## Graded Response Model

```
suppressWarnings({
  # Code generates "Warning: Nans produced" for every missing item
  grm1 <- ltm::grm(ds, IRT.param = TRUE)
})
grm1_coefs <- summary(grm1)$coefficients %>%
  as.data.frame() %>%
  t() %>%
  as.data.frame()
row.names(grm1_coefs) <- paste("Q", 1:nrow(grm1_coefs))

kable(grm1_coefs)
```

	Extrmt1	Extrmt2	Extrmt3	Dscrmn
Q 1	0.8601114	1.757451	2.283813	1.978272
Q 2	0.8207081	1.693291	2.193152	3.010173
Q 3	0.4323291	1.499742	2.049230	1.640595
Q 4	-0.0691675	1.260808	1.870432	1.905477
Q 5	0.9188324	1.918980	2.491899	1.636566
Q 6	1.1506598	1.880156	2.329262	2.802304
Q 7	1.2938260	2.061777	2.507890	1.997446
Q 8	1.6967355	2.441015	2.907672	1.895346
Q 9	2.2083221	2.803847	3.220459	2.458099

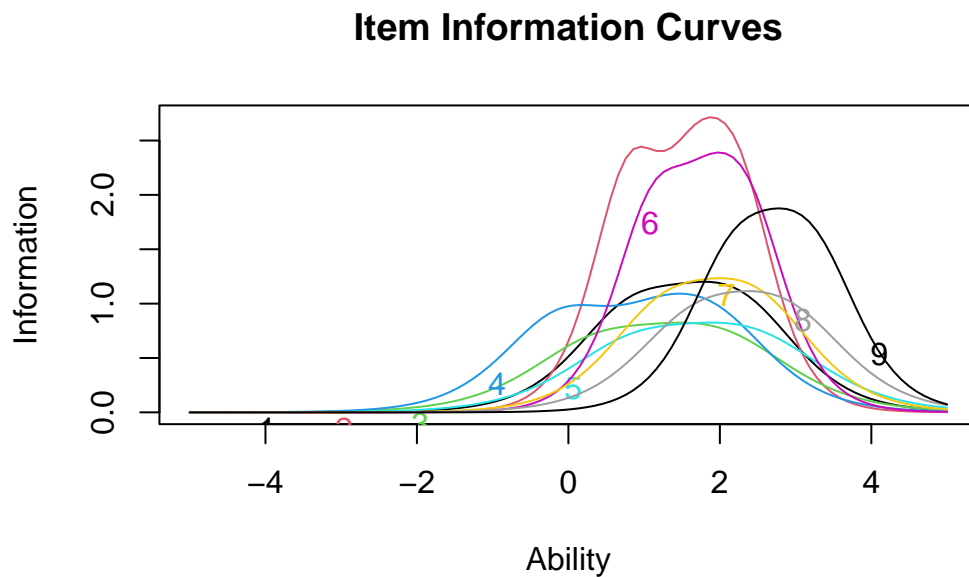
Examining the output, two things are immediately apparent. There are 3 extremity parameters; one for each of the 3 possible levels of endorsement of 1, 2, or 3). The *Extrmt1* values are roughly equivalent to the 2PL model's difficulty parameter values. This is expected, as the *Extrmt1* scores refer to the "depression level" an individual would need to have a 50/50 chance of selecting a 1 (In the dichotomous IRT, an endorsement) for that item. *Extrmt3* scores refer to the trait level an individual would need to have a 50/50 chance of selecting not only a 1, but also a 2 or 3. Polytomous IRT models give more information than dichotomous, where we discard these other probabilities. Because the GRM has more information, its discrimination parameters are slightly different to the discrimination parameters of the 2PL model.

## Item Characteristic and Item Information Curves

Much the same as with the 2PL Model, I plotted the ICC to get a better view of the discrimination and difficulty of each question. For this, I chose to plot only the ICC of items 2 and

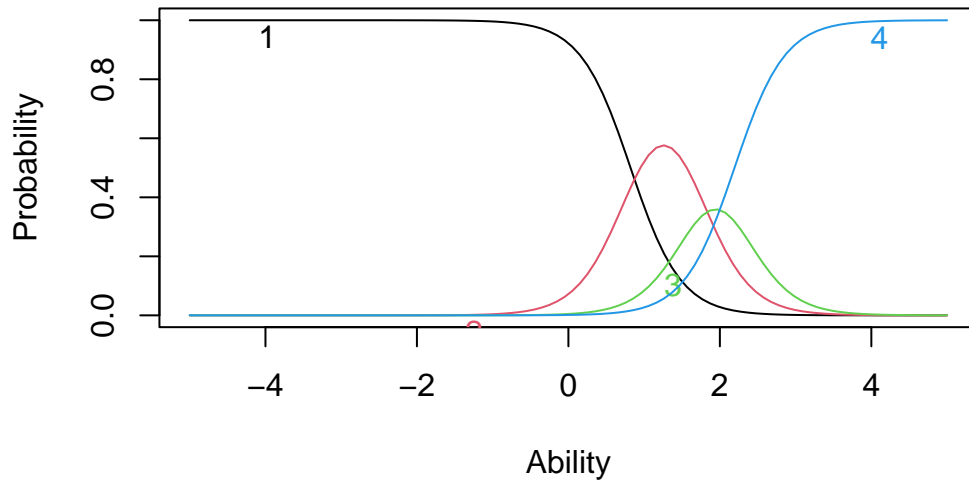
5, each having the largest difference in discriminations (3.010 for item 2, and 1.637 for item 5).

```
plot(grm1, type = "IIC", xrange = c(-5, 5))
```



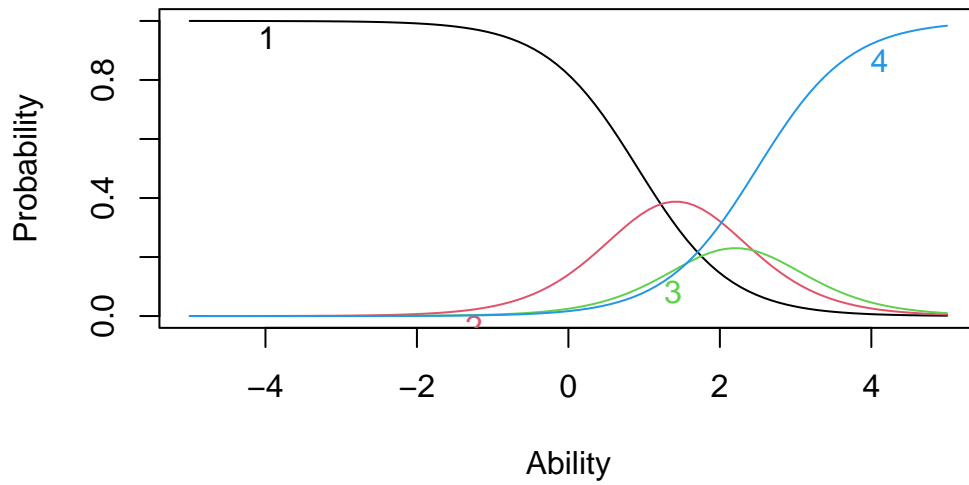
```
plot(grm1, type = "ICC", items = 2, xrange = c(-5, 5))
```

### Item Response Category Characteristic Curves – Item: q



```
plot(grm1, type = "ICC", items = 5, zrange = c(-5, 5))
```

### Item Response Category Characteristic Curves – Item: q





The discrimination parameter acts as the slope of the curves, so item 2 (with a very high discrimination) has a very steep slope. Item 5 (a low discrimination) has a very shallow slope.

For each item: As ability (level of depression) increases, the probability of selecting response category 1, “Not at all”, decreases. And for response category 4, “nearly everyday”, it’s vice versa. This is expected, as someone without depression would easily select “not at all” for many of the questions, it’s only if they have some level of depression they would start endorsing a higher category.

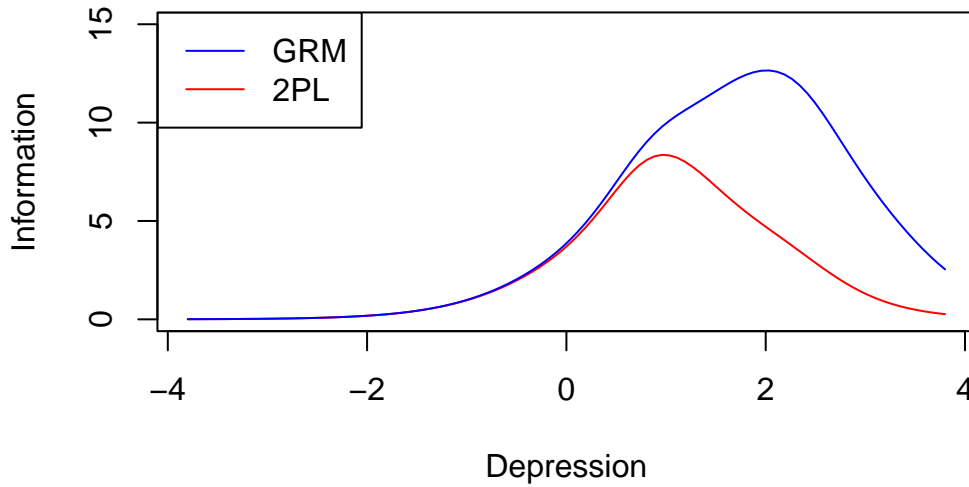
## 2PL and GRM Test Information Function

The Test Information Function best shows what information was lost when I dichotomized the data for the 2PL

```
p12_TIF <- plot(p12, type = c("IIC"),
               items = c(0),
               col = "red",
               ylim=c(0,15),
               xlab = "Depression")
grm_TIF <- plot(grm1, type = "IIC", items = 0, plot = FALSE)
lines(grm_TIF, col = "blue")

legend("topleft", legend = c("GRM", "2PL"), col = c("blue", "red"), lty = 1)
```

## Test Information Function



Up until a low-moderate depression level, both the dichotomous and polytomous IRT yielded the same information. GRM gives more information about the more severely depressed people, which you lose if you just ask, “Do you have any symptoms?” (essentially what the dichotomized data was).

## Rating Scale Model

```
rsm1 <-TAM::tam.mml(ds,irtmodel ="RSM", verbose = FALSE)

kable(rsm1$item)
```

	item	N	M	xsi.item	AXsi_.Ca	AXsi_.Ca	AXsi_.Ca	BCat1.Dim1	BCat2.Dim1	BCat3.Dim1
q1	q1	5151	0.403610	2.053358	1.607553	74.093673	6.160075	1	2	3
q2	q2	5161	0.342763	0.251587	1.805782	74.490131	6.754763	1	2	3
q3	q3	5161	0.612672	0.516058	1.070253	323.019072	4.548174	1	2	3
q4	q4	5161	0.782406	51.170635	0.724830	22.328226	3.511905	1	2	3
q5	q5	5158	0.400736	72.061523	1.615718	34.110002	6.184569	1	2	3
q6	q6	5159	0.244039	2.645112	2.199307	45.277180	7.935336	1	2	3
q7	q7	5159	0.255475	0.593513	2.147707	75.173981	7.780538	1	2	3
q8	q8	5158	0.163241	63.091736	2.645931	06.170427	9.275207	1	2	3

	item	N	M	xsi.item	AXsi_.Ca	AXsi_.Ca	AXsi_.Ca	B1Cat1.Dir	B1Cat2.Dir	B1Cat3.Dir	Dim1
q9	q9	5157	0.0562342	4.2110653	7.6526018	4.09085	12.633195	1	2	3	

The difficulties, from the “AXsi\_.Cat1, AXsi\_.Cat2, and AXsi\_.Cat3” columns, while scaled differently, show the expected values. Q9 has the highest difficulty, and Q4 has the lowest. There are no discrimination parameters, which lowers the complexity of the model quite a bit. Next, I will graph the Item Characteristic Curves to better see what this constrained discriminatory parameter looks like.

## RSM: Item Characteristic Curves

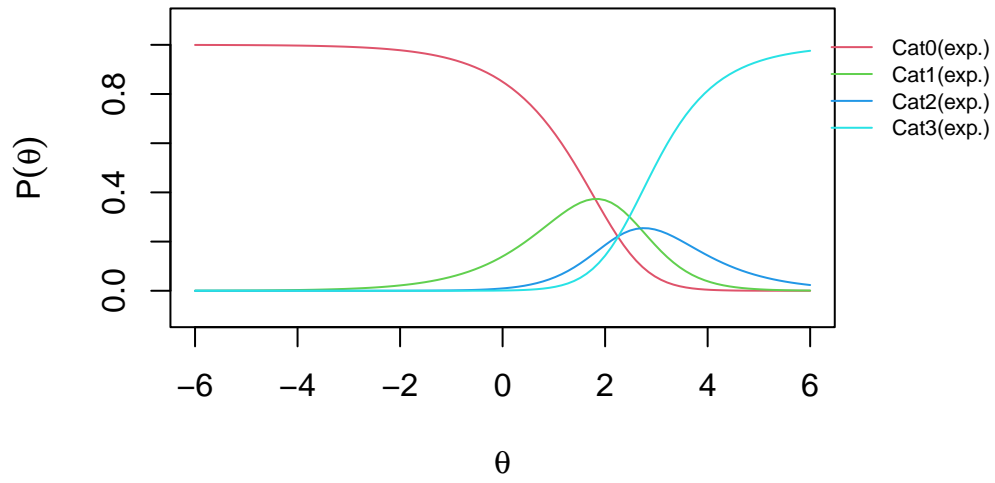
```
plot(rsm1,
     type = "items",
     export = FALSE,
     package = "graphics",
     observed = FALSE,
     low = -6,
     high = 6,
     items = c(2, 5))
```

```
Iteration in WLE/MLE estimation 1 | Maximal change 2.9707
Iteration in WLE/MLE estimation 2 | Maximal change 2.8242
Iteration in WLE/MLE estimation 3 | Maximal change 1.577
Iteration in WLE/MLE estimation 4 | Maximal change 0.4525
Iteration in WLE/MLE estimation 5 | Maximal change 0.1469
Iteration in WLE/MLE estimation 6 | Maximal change 0.0165
Iteration in WLE/MLE estimation 7 | Maximal change 6e-04
Iteration in WLE/MLE estimation 8 | Maximal change 0
```

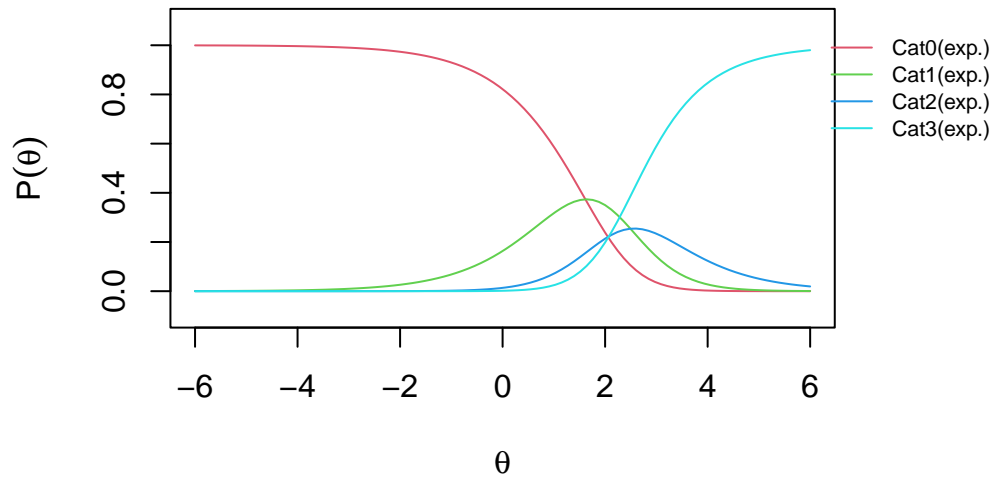
----

WLE Reliability= 0.412

**Item q2**



**Item q5**

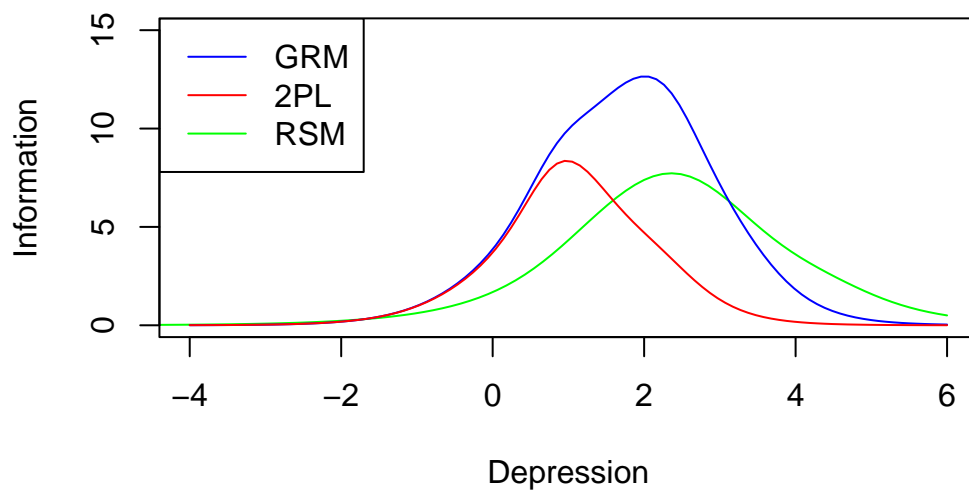


Interesting! The RSM would have us believe that Q2 and Q5 are extremely similar, even while they differ wildly in how discriminating they are. Next, I plot it's test information curve with

the 2PL and GRM models.

## RSM: Test Information Curve

```
rsm1_TIF <- plot(IRT.informationCurves(rsm1),  
  col = "green",  
  xlim = c(-4, 6),  
  ylim = c(0,15),  
  ylab = "Information",  
  xlab = "Depression")  
pl2_TIF <- plot(pl2, type = "IIC", items = 0, zrange = c(-4, 6), plot = FALSE)  
grm_TIF <- plot(grm1, type = "IIC", items = 0, zrange = c(-4, 6), plot = FALSE)  
  
lines(grm_TIF, col = "blue")  
lines(pl2_TIF, col = "red")  
  
legend("topleft",  
  legend = c("GRM", "2PL", "RSM"),  
  col = c("blue", "red", "green"),  
  lty = 1)
```



This may not be accurate, as I don't know how the TAM package creates the Test Information Curve. With that, it's noteworthy how much more information the GRM gives compared to the RSM. Again, this may be different if the discrimination parameters of the questions weren't so different.

## Log-Likelihood and AIC of GRM and RSM

Because GRM is the more complex model, I expect it to have a larger Log-likelihood. However, it should be a better fit due to the added discrimination abilities giving the model more information (as I've shown with the Test Information Function), which a lower AIC would signify.

```
comparison <- IRT.compareModels(grm1, rsm1)
```

Warning in rbind(dfr, dfr1): number of columns of result is not a multiple of vector length (arg 1)

```
summary(comparison, extended = FALSE)
```

Absolute and relative model fit

	Model	loglike	Deviance	Npars	Nobs	AIC	BIC	AIC3	GHP
1	grm1	-27895.93	55791.85	36	5735	55863.85	56103.41	55899.85	-27895.93
2	rsm1	-28752.26	57504.52	12	5164	57528.52	57607.11	57540.52	0.62

Likelihood ratio tests - model comparison

	Model1	Model2	Chi2	df	p
1	rsm1	grm1	1712.665	24	0

Comparing the AIC's of each model, the Graded Response Model is the better fit. If the discriminatory parameters of the GRM were all very similar, then the RSM may have been a better fit due to the lower complexity.

## Confirmatory Factor Analysis with Polytomous Data

Here, I fit a CFA with the original, polytomous data. This should be very close to the GRM, now that it is using the same data.

```

mod2 <-
  "depression =~ q1 + q2 + q3 + q4 + q5 +
    q6 + q7 + q8 + q9
"

cfafit2 <- cfa(mod2, data = ds,
  ordered = c("q1", "q2", "q3", "q4", "q5",
    "q6", "q7", "q8", "q9"),
  estimator = "WLSMV",
  check.gradient = TRUE
)

summary(cfafit2, fit.measures = TRUE, standardized = TRUE)

```

lavaan 0.6.15 ended normally after 17 iterations

Estimator	DWLS
Optimization method	NLMINB
Number of model parameters	36

	Used	Total
Number of observations	5134	5735

Model Test User Model:

	Standard	Scaled
Test Statistic	325.090	583.771
Degrees of freedom	27	27
P-value (Chi-square)	0.000	0.000
Scaling correction factor		0.559
Shift parameter		2.037
simple second-order correction		

Model Test Baseline Model:

Test statistic	41255.954	23189.999
Degrees of freedom	36	36
P-value	0.000	0.000
Scaling correction factor		1.780

User Model versus Baseline Model:

Comparative Fit Index (CFI)	0.993	0.976
Tucker-Lewis Index (TLI)	0.990	0.968

Robust Comparative Fit Index (CFI)	0.925
Robust Tucker-Lewis Index (TLI)	0.900

Root Mean Square Error of Approximation:

RMSEA	0.046	0.063
90 Percent confidence interval - lower	0.042	0.059
90 Percent confidence interval - upper	0.051	0.068
P-value H <sub>0</sub> : RMSEA ≤ 0.050	0.902	0.000
P-value H <sub>0</sub> : RMSEA ≥ 0.080	0.000	0.000

Robust RMSEA	0.124
90 Percent confidence interval - lower	0.113
90 Percent confidence interval - upper	0.136
P-value H <sub>0</sub> : Robust RMSEA ≤ 0.050	0.000
P-value H <sub>0</sub> : Robust RMSEA ≥ 0.080	1.000

Standardized Root Mean Square Residual:

SRMR	0.045	0.045
------	-------	-------

Parameter Estimates:

Standard errors	Robust.sem
Information	Expected
Information saturated (h1) model	Unstructured

Latent Variables:

	Estimate	Std.Err	z-value	P(> z )	Std.lv	Std.all
depression =~						
q1	1.000				0.738	0.738
q2	1.174	0.020	59.412	0.000	0.866	0.866
q3	0.940	0.020	45.940	0.000	0.693	0.693
q4	1.002	0.018	55.103	0.000	0.739	0.739
q5	0.912	0.021	44.119	0.000	0.673	0.673
q6	1.148	0.020	57.410	0.000	0.847	0.847
q7	1.009	0.022	46.399	0.000	0.744	0.744
q8	0.966	0.026	37.308	0.000	0.712	0.712
q9	1.091	0.030	36.901	0.000	0.804	0.804



## Intercepts:

	Estimate	Std.Err	z-value	P(> z )	Std.lv	Std.all
.q1	0.000				0.000	0.000
.q2	0.000				0.000	0.000
.q3	0.000				0.000	0.000
.q4	0.000				0.000	0.000
.q5	0.000				0.000	0.000
.q6	0.000				0.000	0.000
.q7	0.000				0.000	0.000
.q8	0.000				0.000	0.000
.q9	0.000				0.000	0.000
depression	0.000				0.000	0.000

## Thresholds:

	Estimate	Std.Err	z-value	P(> z )	Std.lv	Std.all
q1 t1	0.643	0.019	34.054	0.000	0.643	0.643
q1 t2	1.313	0.024	54.152	0.000	1.313	1.313
q1 t3	1.691	0.030	55.561	0.000	1.691	1.691
q2 t1	0.709	0.019	36.944	0.000	0.709	0.709
q2 t2	1.471	0.026	55.614	0.000	1.471	1.471
q2 t3	1.875	0.035	53.823	0.000	1.875	1.875
q3 t1	0.303	0.018	17.027	0.000	0.303	0.303
q3 t2	1.040	0.021	48.581	0.000	1.040	1.040
q3 t3	1.399	0.025	55.116	0.000	1.399	1.399
q4 t1	-0.056	0.018	-3.182	0.001	-0.056	-0.056
q4 t2	0.939	0.021	45.584	0.000	0.939	0.939
q4 t3	1.378	0.025	54.911	0.000	1.378	1.378
q5 t1	0.640	0.019	33.946	0.000	0.640	0.640
q5 t2	1.327	0.024	54.333	0.000	1.327	1.327
q5 t3	1.689	0.030	55.572	0.000	1.689	1.689
q6 t1	0.986	0.021	47.035	0.000	0.986	0.986
q6 t2	1.604	0.029	55.863	0.000	1.604	1.604
q6 t3	1.945	0.037	52.812	0.000	1.945	1.945
q7 t1	0.989	0.021	47.130	0.000	0.989	0.989
q7 t2	1.553	0.028	55.872	0.000	1.553	1.553
q7 t3	1.853	0.034	54.107	0.000	1.853	1.853
q8 t1	1.257	0.024	53.327	0.000	1.257	1.257
q8 t2	1.789	0.033	54.815	0.000	1.789	1.789
q8 t3	2.103	0.042	49.893	0.000	2.103	2.103
q9 t1	1.782	0.032	54.885	0.000	1.782	1.782
q9 t2	2.231	0.047	47.032	0.000	2.231	2.231
q9 t3	2.521	0.064	39.365	0.000	2.521	2.521

Variances:

	Estimate	Std.Err	z-value	P(> z )	Std.lv	Std.all
.q1	0.456				0.456	0.456
.q2	0.251				0.251	0.251
.q3	0.519				0.519	0.519
.q4	0.454				0.454	0.454
.q5	0.547				0.547	0.547
.q6	0.283				0.283	0.283
.q7	0.446				0.446	0.446
.q8	0.493				0.493	0.493
.q9	0.353				0.353	0.353
depression	0.544	0.017	31.727	0.000	1.000	1.000

Scales y\*:

	Estimate	Std.Err	z-value	P(> z )	Std.lv	Std.all
q1	1.000				1.000	1.000
q2	1.000				1.000	1.000
q3	1.000				1.000	1.000
q4	1.000				1.000	1.000
q5	1.000				1.000	1.000
q6	1.000				1.000	1.000
q7	1.000				1.000	1.000
q8	1.000				1.000	1.000
q9	1.000				1.000	1.000

## Compare Discrimination Parameter of GRM and Factor Loadings of Polytomous CFA

```

model_loadings_poly <- inspect(cfafit2, what = "std")["lambda"]

discrims_poly <- list()
for (i in 1:9) {
  discrims_poly[[paste0("q", i)]] <- summary(grm1)$coefficients[[paste0("q", i)]]["4"]
}

discrims_poly <- unlist(discrims_poly)
names(discrims_poly) <- names(discrims_poly)

D <- 1.7

df_loadings_poly <- cbind(loadings = model_loadings_poly,

```

```

discrims_to_loadings = (discrims_poly / D)
/
(sqrt(1 + ((discrims_poly / D)^2)))

df_loadings_poly <- df_loadings_poly %>%
  as.data.frame() %>%
  dplyr::rename(cfa_loadings = depression)

df_loadings_poly %>%
  as.data.frame() %>%
  dplyr::mutate(dif = cfa_loadings-discrims_to_loadings,
               rat = cfa_loadings/discrims_to_loadings)

```

	cfa_loadings	discrims_to_loadings	dif	rat
q1	0.7375195	0.7584342	-0.020914731	0.9724238
q2	0.8655098	0.8707365	-0.005226748	0.9939973
q3	0.6932849	0.6944226	-0.001137694	0.9983617
q4	0.7391335	0.7461941	-0.007060588	0.9905379
q5	0.6728642	0.6935380	-0.020673861	0.9701907
q6	0.8466470	0.8549768	-0.008329818	0.9902573
q7	0.7441284	0.7615307	-0.017402315	0.9771482
q8	0.7121545	0.7444280	-0.032273496	0.9566466
q9	0.8042705	0.8224679	-0.018197394	0.9778746

Almost equivalent. The values are close enough to have shown that for unidimensional polytomous data, CFA and the GRM are theoretically equivalent.

## Conclusion

The IRT analysis showed that the questions were all positive, that the discrimination parameters were all good, that the IIC suggests that the scale is overall reliable, but the reliability peaks on low-moderate depression. But in a screening tool, this is probably what we want.

The factor analysis gave extremely similar results to the IRT analysis, theoretically identical. However, the model fit was not perfect, suggesting that it was not uni-dimensional, which might be a problem. The modification indices suggested some additional covariances, but I didn't as it would be no longer equivalent to the IRT.

Also, the fact that I can add covariances to the CFA leads me to believe that CFA is the more flexible model. The IRT models did however yield more interpretable results, and in that way

seem specialized for uni-dimensional testing. CFA has a much wider application of uses (i.e., multivariate systems) and seems to focus on the relationship between variables.

I wanted to try running a CFA using Maximum Likelihood, however the model would not converge when the estimator was “MML”. This would have allowed me to use the `AIC()` function to compare my CFA models to the IRT models. Instead, I compared the factor loadings and discrimination parameters. This showed that the CFA is largely equivalent to an IRT model when using the same data.

Comparing the RSM and the GRM, the GRM was a better fit. This was due to the fact that the questions all had different discriminatory parameters, something that the RSM does not calculate (and for that reason, took much less time to run).

## References

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- Forero, C. G., & Maydeu-Olivares, A. (2009). Estimation of IRT graded response models: Limited versus full information methods. *Psychological Methods*, 14(3), 275–299. <https://doi.org/10.1037/a0015825>
- Rizopoulos, D. (2006). ltm: An R package for latent variable modeling and item response theory analyses. *Journal of Statistical Software*, 17(5). <https://doi.org/10.18637/jss.v017.i05>