IRT and CFA

This analysis was written in Quarto, and the source file can be found at

https://github.com/alexm123/CFAvsIRT

Below, I will analyze the same dataset using both Item Response Theory (IRT) and Confirmatory Factor Analysis (CFA) to compare and contrast between the two.

The Data

The 2015-2016 NHANES Mental Health - Depression Screener.

Originally a rating scale (0, Not at all; 1, several days; 2, more than half the days; 3, nearly everyday)

Q1: "Have little interest in doing things"

Q2: "Feeling down, depressed, or hopeless"

Q3: "Trouble sleeping or sleeping too much"

Q4: "Feeling tired or having little energy"

Q5: "Poor appetite or overeating"

Q6: "Feeling bad about yourself"

Q7: "Trouble concentrating on things"

Q8: "Moving or speaking slowly or too fast"

Q9: "Thought you would be better off dead"

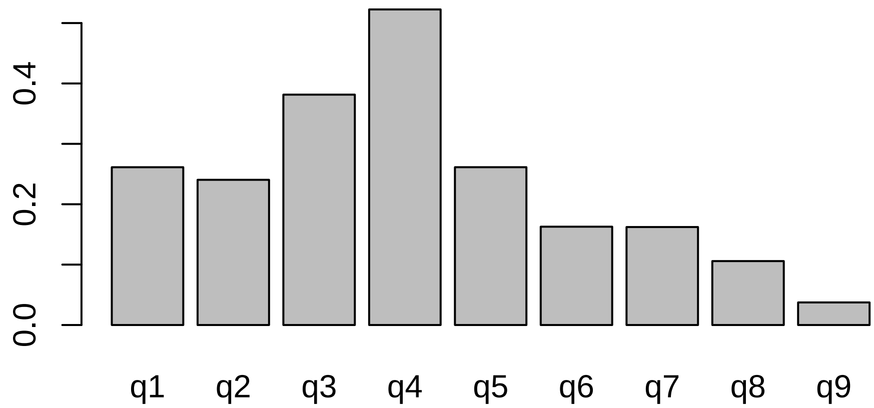
Q10: "Difficulty these problems have caused"

Looking at the questions, we clearly see that Q10 does not fit in with the rest. It violates the assumption of local independence (A participant's answer to Q10 depends on their answers to the other questions). To that end, I had Q10 dropped from the data set.

Import Dataset

For the initial analysis, I opted to dichotomize the data by having any value above 0 changed to a 1.

```
# Mental Health - Depression Screener from
# https://wwwn.cdc.gov/nchs/nhanes/search/datapage.aspx?Component=Questionnaire&CycleBegin

ds <- haven::read_xpt("https://wwwn.cdc.gov/Nchs/Nhanes/2015-2016/DPQ_I.XPT")
ds <- ds[- 11]


names(ds) <- c("id", paste0("q", 1:9))

ds$id <- NULL

ds <- ds %>%
  mutate_at(vars(q1:q9), ~ifelse(. > 3, NA, .))

ds_dich <- ds %>%
  mutate_at(vars(q1:q9), ~ifelse(. > 0, 1, 0))

qmeans <- apply(ds_dich, 2, mean, na.rm = TRUE)

barplot(qmeans)
```

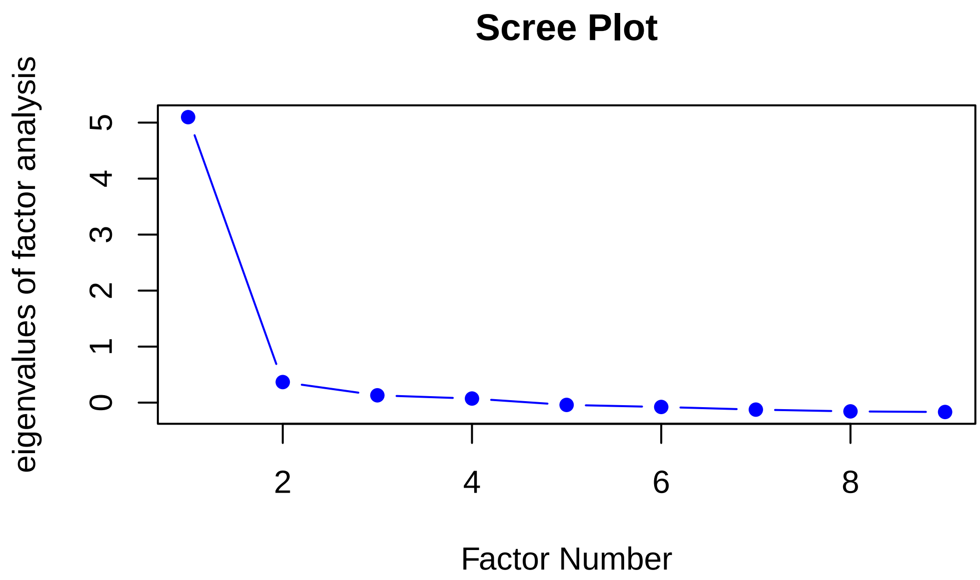


This graph shows the mean number of endorsements an answer received. Here, we can see that q4 ("Feeling tired or having little energy") had the most positive answers, while q9 ("Thought you would be better off dead") had very few endorsements.

Before attempting to fit an IRT model, it would be good to examine whether the 9 items are all measuring the same latent variable. In our case, depression. This is done with a *scree test*, the components left of the "elbow" (the sharpest drop) of the graph should be retained. As we want to measure just 1 latent variable, we want there to be just 1 component before the elbow.

```
screep <- fa.parallel(ds_dich, cor = "tet")
```

```
x <- 1:length(screep$fa.values)
plot(x, screep$fa.values, type = "b", pch = 16, col = "blue",
     xlab = "Factor Number",
     ylab = "eigenvalues of factor analysis",
     main = "Scree Plot")
```



As shown, the scree plot suggests there is only 1 underlying factor. We can now proceed to fit the IRT models.

Fit 1PL Model

Here, I fitted a 1PL (1-parameter logistic) model to estimate item difficulty based on how many people answered the items.

```
pl1 <- ltm::rasch(ds_dich)
kable(summary(pl1)$coefficients, digits = 2)
```

	value	std.err	z.vals
Dffclt.q1	0.84	0.03	32.27
Dffclt.q2	0.93	0.03	34.64
Dffclt.q3	0.40	0.02	16.86
Dffclt.q4	-0.07	0.02	-3.17
Dffclt.q5	0.84	0.03	32.27
Dffclt.q6	1.30	0.03	42.11
Dffclt.q7	1.30	0.03	42.17
Dffclt.q8	1.66	0.04	45.85
Dffclt.q9	2.40	0.05	45.52

	value	std.err	z.vals
Dscrmn	2.01	0.03	57.66

Q1 has a difficulty score of 0.84. This refers to the fact that an individual with the "depression level" of 0.84 would have a 0.5 probability of endorsing Q1. Every item was fixed to have the same discrimination, which will be explained further on.

Fit 2PL Model

Next, I fitted a 2PL model to estimate each item's discrimination parameter. I will later graph this information, to better show what it means.

```
pl2 <- ltm(ds_dich ~ z1)

kable(summary(pl2)$coefficients, digits = 2)
```

	value	std.err	z.vals
Dffclt.q1	0.84	0.03	29.56
Dffclt.q2	0.81	0.02	34.27
Dffclt.q3	0.45	0.03	16.19
Dffclt.q4	-0.07	0.02	-2.93
Dffclt.q5	0.95	0.04	27.10
Dffclt.q6	1.17	0.03	39.27
Dffclt.q7	1.34	0.04	33.81
Dffclt.q8	1.69	0.05	33.75
Dffclt.q9	2.20	0.07	31.25
Dscrmn.q1	2.07	0.08	24.34
Dscrmn.q2	3.18	0.16	20.19
Dscrmn.q3	1.57	0.06	24.89
Dscrmn.q4	1.91	0.08	23.99
Dscrmn.q5	1.56	0.06	24.08
Dscrmn.q6	2.75	0.13	21.24
Dscrmn.q7	1.89	0.08	22.55
Dscrmn.q8	1.94	0.10	20.29
Dscrmn.q9	2.48	0.17	14.31

Here, I test to see if 2PL model has a significantly better fit than 1PL model, by evaluating their model characteristics within an ANOVA. 2PL model has many more parameters, as seen by each question's discrimination parameter. The ANOVA will compare the AIC's of the two models, to see if the more complex model fits the data better.

```
anova(pl1, pl2)
```

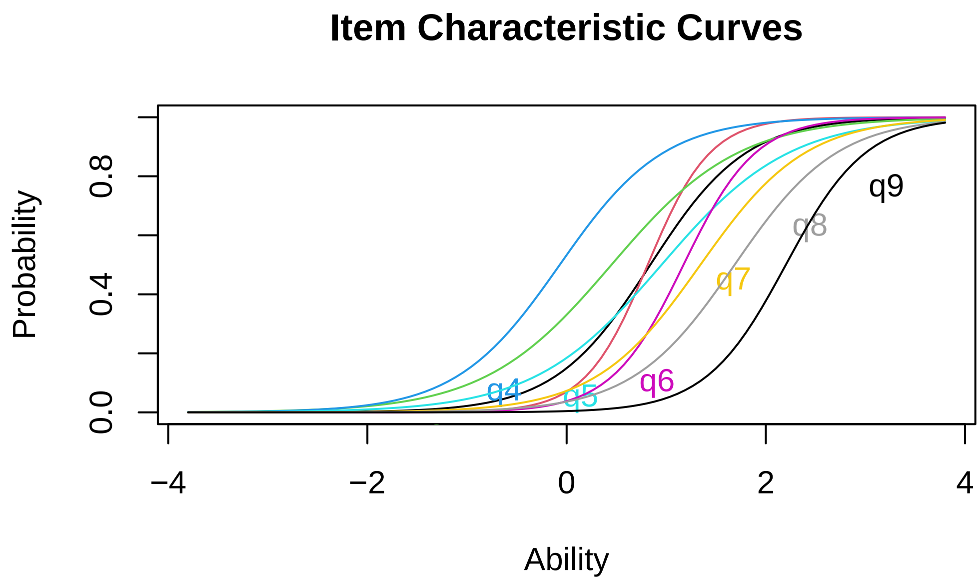
```
 Likelihood Ratio Table
         AIC      BIC   log.Lik    LRT df p.value
pl1 38038.94 38105.49 -19009.47
pl2 37849.45 37969.23 -18906.72 205.49  8  <0.001
```

The significant p-value in this chart tells us that the 2PL is a better fit to the data the 1PL. The fit of the model has been improved by estimating the discrimination parameter of each item, instead of fixing it to one value. If both models had fit the data similarly well, then the parsimony principle tells us that the simpler model should have been favored.

Item Characteristic Curves

Below, I plotted the item characteristic curves of the 10 items to better see the discriminability across items.

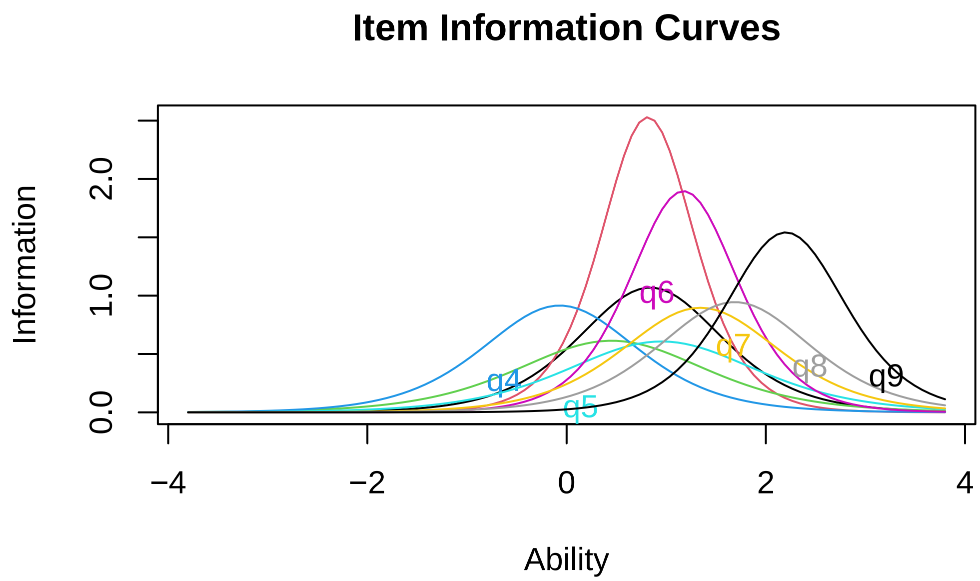
```
plot(pl2, type = c("ICC"))
```



In the ICCs, we can better see the probability of endorsing an answer at varying ability levels. We see that q4 (the blue curve on the left; "Feeling tired or having little energy") has a range of abilities endorsing it, while with q9 (the black curve on the right; "Thought you would be better off dead"), only individuals with the highest level of depression endorsed it. So, discrimination can be seen as the steepness of these curves.

Item Information Curves

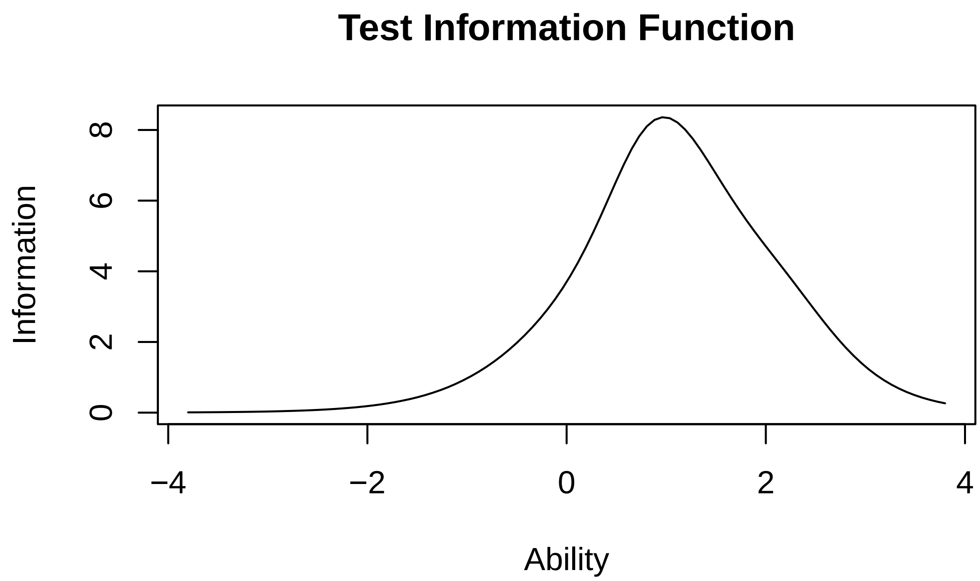
```
plot(pl2, type = c("IIC"))
```



The IICs demonstrate that items range in how much information they provide about an individuals depression level for different ability levels. Item q10 (the red curve) gives us a the most information at moderate depression levels. In contrast, item q4 (the blue curve), gives us very low information because of how wide a range of depression levels it covers.

Sum of all IIC Curves

```
plot(pl2, type = c("IIC"), items = c(0))
```



The test information function shows that the items as a whole provide the most information about low-to-moderate depression levels, and less about extreme high or low depression levels. This is desirable, as it is not important to discriminate between those with very low or very high depression. It is important to discriminate between those of moderate depression levels, which is what the test information function tells us it does.

Confirmatory Factor Analysis with Dichotomous Data

```
mod <-
  "depression =~ q1 + q2 + q3 + q4 + q5 +
                 q6 + q7 + q8 + q9

"


cfafit <- cfa(mod, data = ds_dich,
              ordered = c("q1", "q2", "q3", "q4", "q5",
                          "q6", "q7", "q8", "q9"),
              estimator = "WLSMV",
              check.gradient = TRUE
              )
```

```r
summary(cfafit, fit.measures = TRUE, standardized = TRUE)
```

```
lavaan 0.6.15 ended normally after 17 iterations

  Estimator                                       DWLS
  Optimization method                           NLMINB
  Number of model parameters                        18

                                                  Used       Total
  Number of observations                          5134        5735

Model Test User Model:
                                              Standard      Scaled
  Test Statistic                               269.518     404.621
  Degrees of freedom                                27          27
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  0.669
  Shift parameter                                            1.484
    simple second-order correction

Model Test Baseline Model:

  Test statistic                             28028.387   19894.317
  Degrees of freedom                                36          36
  P-value                                        0.000       0.000
  Scaling correction factor                                  1.410

User Model versus Baseline Model:

  Comparative Fit Index (CFI)                    0.991       0.981
  Tucker-Lewis Index (TLI)                       0.988       0.975

  Robust Comparative Fit Index (CFI)                         0.901
  Robust Tucker-Lewis Index (TLI)                            0.868

Root Mean Square Error of Approximation:

  RMSEA                                          0.042       0.052
  90 Percent confidence interval - lower         0.037       0.048
  90 Percent confidence interval - upper         0.046       0.057
  P-value H_0: RMSEA <= 0.050                    0.998       0.203
```

```
  P-value H_0: RMSEA >= 0.080                        0.000       0.000

  Robust RMSEA                                                   0.146
  90 Percent confidence interval - lower                         0.131
  90 Percent confidence interval - upper                         0.161
  P-value H_0: Robust RMSEA <= 0.050                             0.000
  P-value H_0: Robust RMSEA >= 0.080                             1.000

Standardized Root Mean Square Residual:

  SRMR                                               0.053       0.053

Parameter Estimates:

  Standard errors                               Robust.sem
  Information                                     Expected
  Information saturated (h1) model            Unstructured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  depression =~
    q1                1.000                                0.765    0.765
    q2                1.160    0.022   52.372    0.000    0.887    0.887
    q3                0.890    0.023   39.502    0.000    0.681    0.681
    q4                0.965    0.022   43.802    0.000    0.738    0.738
    q5                0.877    0.023   38.143    0.000    0.671    0.671
    q6                1.106    0.022   49.387    0.000    0.846    0.846
    q7                0.956    0.024   39.331    0.000    0.731    0.731
    q8                0.946    0.027   35.642    0.000    0.723    0.723
    q9                1.020    0.033   31.139    0.000    0.780    0.780

Intercepts:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .q1                0.000                                0.000    0.000
   .q2                0.000                                0.000    0.000
   .q3                0.000                                0.000    0.000
   .q4                0.000                                0.000    0.000
   .q5                0.000                                0.000    0.000
   .q6                0.000                                0.000    0.000
   .q7                0.000                                0.000    0.000
   .q8                0.000                                0.000    0.000
   .q9                0.000                                0.000    0.000
    depression        0.000                                0.000    0.000
```

```
Thresholds:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    q1|t1             0.643    0.019   34.054    0.000    0.643    0.643
    q2|t1             0.709    0.019   36.944    0.000    0.709    0.709
    q3|t1             0.303    0.018   17.027    0.000    0.303    0.303
    q4|t1            -0.056    0.018   -3.182    0.001   -0.056   -0.056
    q5|t1             0.640    0.019   33.946    0.000    0.640    0.640
    q6|t1             0.986    0.021   47.035    0.000    0.986    0.986
    q7|t1             0.989    0.021   47.130    0.000    0.989    0.989
    q8|t1             1.257    0.024   53.327    0.000    1.257    1.257
    q9|t1             1.782    0.032   54.885    0.000    1.782    1.782

Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .q1                0.415                               0.415    0.415
   .q2                0.213                               0.213    0.213
   .q3                0.537                               0.537    0.537
   .q4                0.456                               0.456    0.456
   .q5                0.550                               0.550    0.550
   .q6                0.285                               0.285    0.285
   .q7                0.466                               0.466    0.466
   .q8                0.477                               0.477    0.477
   .q9                0.391                               0.391    0.391
    depression        0.585    0.019   30.385    0.000    1.000    1.000

Scales y*:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    q1                1.000                               1.000    1.000
    q2                1.000                               1.000    1.000
    q3                1.000                               1.000    1.000
    q4                1.000                               1.000    1.000
    q5                1.000                               1.000    1.000
    q6                1.000                               1.000    1.000
    q7                1.000                               1.000    1.000
    q8                1.000                               1.000    1.000
    q9                1.000                               1.000    1.000
```

The thresholds of the CFA have a similar pattern to that of the difficulty ability of each item. Similarly, the factor loadings of the CFA seem similar to the discrimination parameters from the IRT.

Check Modification Indices

Next, I checked the modification indices for the assumption of *local independence* for any theoretical justification of adding covariances to the model.

```
modindices(cfafit) %>% dplyr::arrange(desc(mi)) %>% head()
```

```
  lhs op rhs     mi    epc sepc.lv sepc.all sepc.nox
1  q3 ~~  q4 77.087  0.172   0.172    0.348    0.348
2  q2 ~~  q6 52.803  0.143   0.143    0.581    0.581
3  q7 ~~  q8 29.487  0.138   0.138    0.292    0.292
4  q6 ~~  q9 28.424  0.154   0.154    0.462    0.462
5  q2 ~~  q3 28.022 -0.122  -0.122   -0.361   -0.361
6  q2 ~~  q4 21.572 -0.105  -0.105   -0.337   -0.337
```

Some of the modification indices are high (e.g., q2~~q6), and the SEPC's are moderate.

For example:

q3: "Trouble sleeping or sleeping too much"

q4: "Feeling tired or having little energy"

These seem to be closely related, though because this would be different to the IRT models, I won't rerun the model with q3 and q4 added as covariances.

q2: "Feeling down, depressed, or hopeless"

q6: "Feeling bad about yourself"

Not closely related.

Compare discrimination's with factor loadings

Below I take a better look at the difference between my CFA's factor loadings and 2PL's discrimination abilities. To do this, I use a formula to transform IRT discrimination parameter estimates to standardized factor loadings (Cho, 2023):

$$\lambda = \frac{\alpha/D}{\sqrt{1 + (\alpha/D)}}$$

Where alpha is the discrimination parameter, and D is a scaling constant which the author of the ltm package has chosen to be 1.702 (Rizopoulos, 2006).

```r
model_loadings <- inspect(cfafit, what = "std")[["lambda"]]

discrims <- pl2$coefficients[, 2]

D <- 1.7

df_loadings <- cbind(loadings = model_loadings,
                     discrims_to_loadings = (discrims / D)
                     / (sqrt(1 + ((discrims / D)^2))))


df_loadings <- df_loadings %>%
  as.data.frame() %>%
  dplyr::rename(cfa_loadings = depression)

df_loadings %>% as.data.frame() %>%
  dplyr::mutate(dif = cfa_loadings-discrims_to_loadings, rat = cfa_loadings/discrims_to_lo
```

```
   cfa_loadings discrims_to_loadings          dif       rat
q1    0.7647710            0.7725016 -0.007730544 0.9899928
q2    0.8868587            0.8819424  0.004916318 1.0055744
q3    0.6807793            0.6777242  0.003055145 1.0045079
q4    0.7377550            0.7475920 -0.009836942 0.9868418
q5    0.6709435            0.6758790 -0.004935506 0.9926976
q6    0.8455697            0.8509317 -0.005361990 0.9936987
q7    0.7309156            0.7440796 -0.013163991 0.9823084
q8    0.7233743            0.7528095 -0.029435287 0.9608994
q9    0.7802641            0.8252215 -0.044957360 0.9455209
```

Close! The values are close enough to have shown that for dichotomous data, CFA and 2PL IRT models are theoretically equivalent. The slight differences may be due to that this is only an approximation, and have slight rounding / computation errors; see Forero & Maydeu-Olivares (2009).

Everything done previously was on a dichotomized version of the data set. Next, I will compare two polytomous IRT models using the original data: the Graded Response Model (GRM) and Rating Scale Model (RSM).

Graded Response Model

```r
suppressWarnings({
  # Code generates "Warning: Nans produced" for every missing item
  grm1 <- ltm::grm(ds, IRT.param = TRUE)
})
grm1_coefs <- summary(grm1)$coefficients %>%
  as.data.frame() %>%
  t() %>%
  as.data.frame()
row.names(grm1_coefs) <- paste("Q", 1:nrow(grm1_coefs))

kable(grm1_coefs)
```

	Extrmt1	Extrmt2	Extrmt3	Dscrmn
Q 1	0.8601114	1.757451	2.283813	1.978272
Q 2	0.8207081	1.693291	2.193152	3.010173
Q 3	0.4323291	1.499742	2.049230	1.640595
Q 4	-0.0691675	1.260808	1.870432	1.905477
Q 5	0.9188324	1.918980	2.491899	1.636566
Q 6	1.1506598	1.880156	2.329262	2.802304
Q 7	1.2938260	2.061777	2.507890	1.997446
Q 8	1.6967355	2.441015	2.907672	1.895346
Q 9	2.2083221	2.803847	3.220459	2.458099

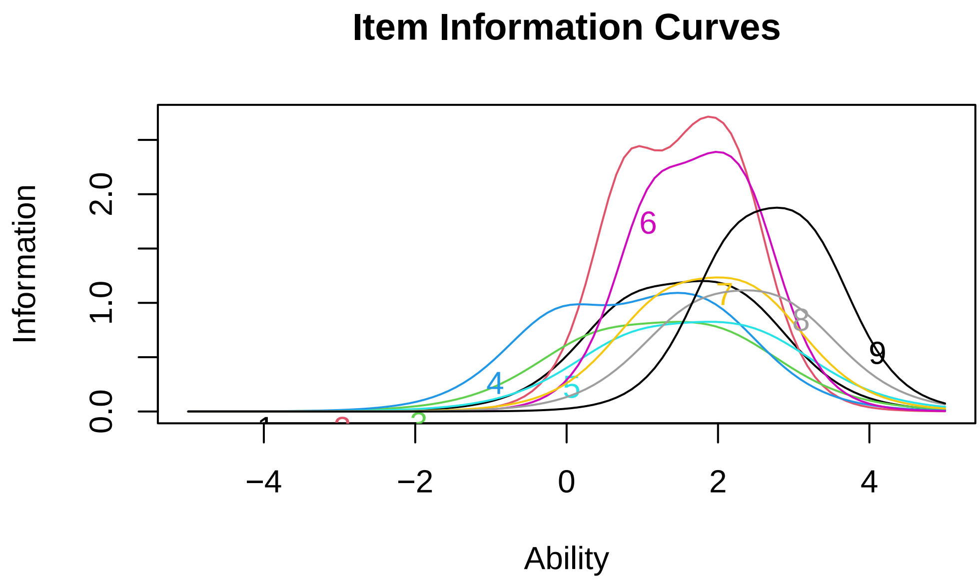
Examining the output, two things are immediately apparent. There are 3 extremity parameters; one for each of the 3 possible levels of endorsement of 1, 2, or 3). The *Extrmt1* values are roughly equivalent to the 2PL model's difficulty parameter values. This is expected, as the *Extrmt1* scores refer to the "depression level" an individual would need to have a 50/50 chance of selecting a 1 (In the dichotomous IRT, an endorsement) for that item. Extrt3 scores refer to the trait level an individual would need to have a 50/50 chance of selecting not only a 1, but also a 2 or 3. Polytomous IRT models give more information than dichotomous, where we discard these other probabilities. Because the GRM has more information, it's discrimination parameters are sightly different to the discrimination parameters of the 2PL model.

Item Characteristic and Item Information Curves

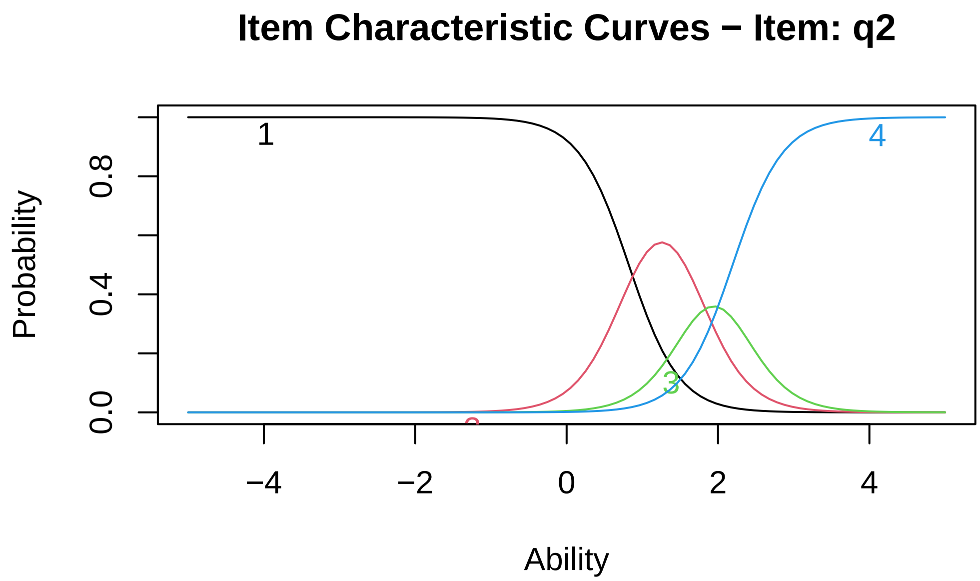
Much the same as with the 2PL Model, I plotted the ICCs to get a better view of the discrimination and difficulty of each question. For this, I chose to plot only the ICCs of items 2 and

5, each having the largest difference in discriminations (3.010 for item 2, and 1.637 for item 5).

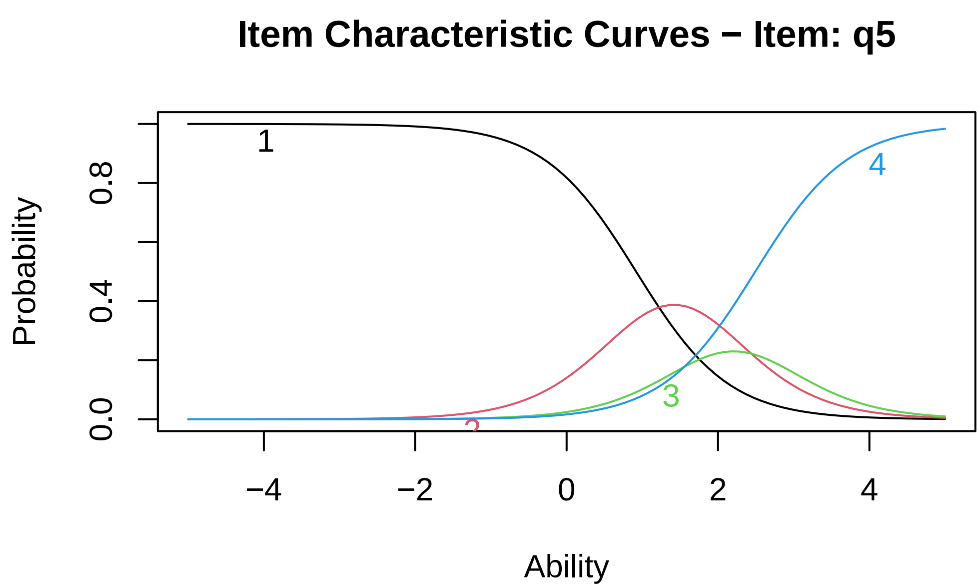
```
plot(grm1, type = "IIC", zrange = c(-5, 5))
```




```
plot(grm1, type = "ICC", items = 2, zrange = c(-5, 5),
     main = "Item Characteristic Curves - Item: q2")
```




```r
plot(grm1, type = "ICC", items = 5, zrange = c(-5, 5),
     main = "Item Characteristic Curves - Item: q5")
```



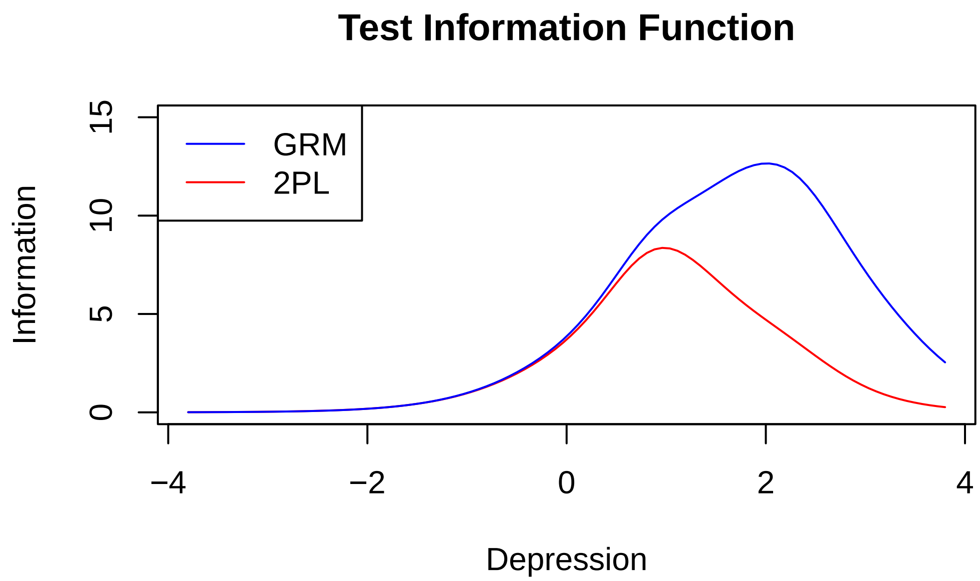
The discrimination parameter acts as the slope of the curves, so the response categories of item 2 (with a very high discrimination) all have a very steep slope. The response categories of item 5 (a low discrimination) have a very shallow slope.

For each item: As ability (level of depression) increases, the probability of selecting response category 1, "Not at all", decreases. And for response category 4, "nearly everyday", it's vice versa. This is expected, as someone without depression would easily select "not at all" for many of the questions, it's only if they have some level of depression they would start endorsing a higher category.

2PL and GRM Test Information Function

The Test Information Function best shows what information was lost when I dichotomized the data for the 2PL

```r
pl2_TIF <- plot(pl2, type = c("IIC"),
                items = c(0),
                col = "red",
                ylim=c(0,15),
                xlab = "Depression")
grm_TIF <- plot(grm1, type = "IIC", items = 0, plot = FALSE)
lines(grm_TIF, col = "blue")


legend("topleft", legend = c("GRM", "2PL"), col = c("blue", "red"), lty = 1)
```



Up until a low-moderate depression level, both the dichotomous and polytomous IRT yielded the same information. GRM gives more information about the more severely depressed people, which you lose if you just ask, "Do you have any symptoms?" (essentially what the dichotomized data was).

Rating Scale Model

```
rsm1 <-TAM::tam.mml(ds,irtmodel ="RSM", verbose = FALSE)

kable(rsm1$item[1:7])
```

	item	N	M	xsi.item	AXsi_.Cat1	AXsi_.Cat2	AXsi_.Cat3
q1	q1	5151	0.4036109	2.053358	1.6075537	4.093673	6.160075
q2	q2	5161	0.3427630	2.251587	1.8057827	4.490131	6.754763
q3	q3	5161	0.6126720	1.516058	1.0702532	3.019072	4.548174
q4	q4	5161	0.7824065	1.170635	0.7248302	2.328226	3.511905
q5	q5	5158	0.4007367	2.061523	1.6157183	4.110002	6.184569
q6	q6	5159	0.2440395	2.645112	2.1993074	5.277180	7.935336
q7	q7	5159	0.2554759	2.593513	2.1477077	5.173981	7.780538
q8	q8	5158	0.1632416	3.091736	2.6459310	6.170427	9.275207

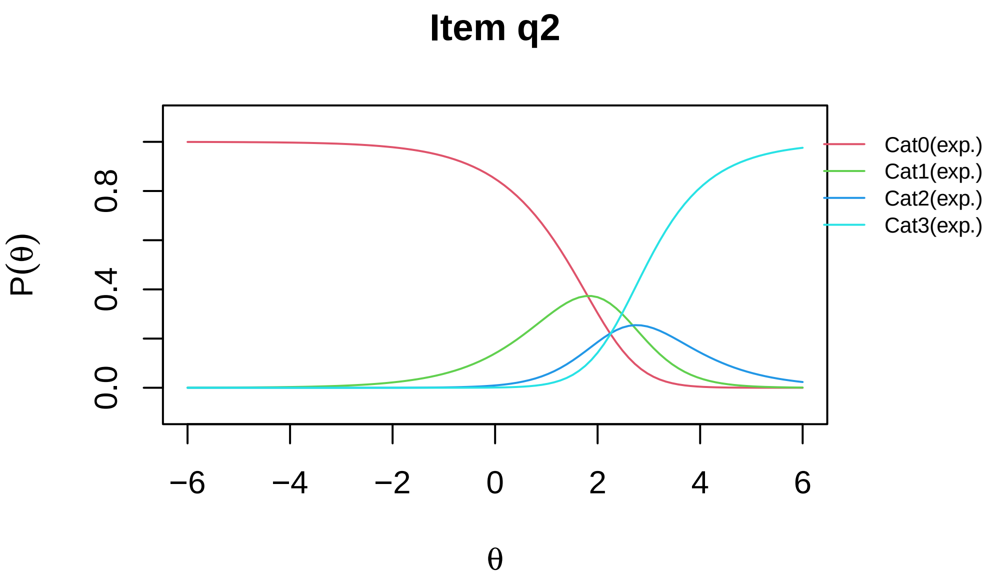
	item	N	M	xsi.item	AXsi_.Cat1	AXsi_.Cat2	AXsi_.Cat3
q9	q9	5157	0.0562342	4.211065	3.7652601	8.409085	12.633195

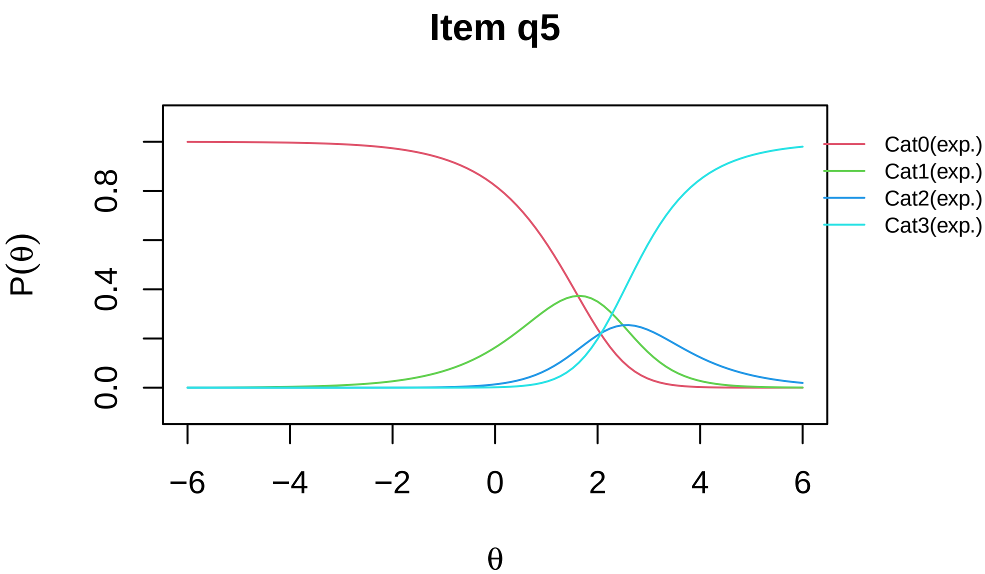
The difficulties, from the "AXsi_.Cat1, AXsi_.Cat2, and AXsi_.Cat3" columns, while seemingly scaled differently, show the expected values. Q9 has the highest difficulty, and Q4 has the lowest. There are no discrimination parameters, which lowers the complexity of the model quite a bit. Next, I will graph the Item Characteristic Curves to better see what this constrained discriminatory parameter looks like.

RSM: Item Characteristic Curves

```
plot(rsm1,
     type = "items",
     export = FALSE,
     package = "graphics",
     observed = FALSE,
     low = -6,
     high = 6,
     items = c(2, 5))
```

```
Iteration in WLE/MLE estimation  1   | Maximal change  2.9707
Iteration in WLE/MLE estimation  2   | Maximal change  2.8242
Iteration in WLE/MLE estimation  3   | Maximal change  1.577
Iteration in WLE/MLE estimation  4   | Maximal change  0.4525
Iteration in WLE/MLE estimation  5   | Maximal change  0.1469
Iteration in WLE/MLE estimation  6   | Maximal change  0.0165
Iteration in WLE/MLE estimation  7   | Maximal change  6e-04
Iteration in WLE/MLE estimation  8   | Maximal change  0
----
 WLE Reliability= 0.412
```

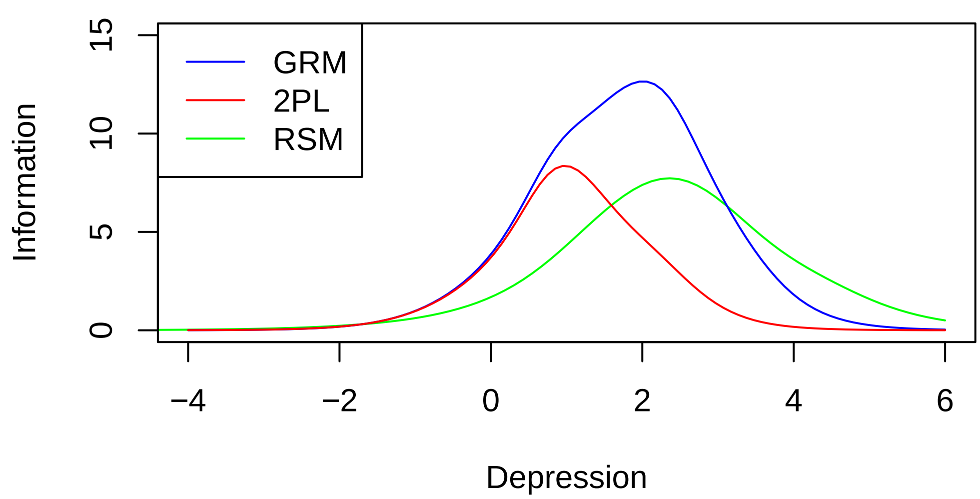




Interesting! The RSM would have us believe that Q2 and Q5 are extremely similar, even while they differ wildly in how discriminating they are. Next, I plot it's test information curve with the 2PL and GRM models.

RSM: Test Information Curve

```
rsm1_TIF <- plot(IRT.informationCurves(rsm1),
                col = "green",
                xlim = c(-4, 6),
                ylim = c(0,15),
                ylab = "Information",
                xlab = "Depression")
pl2_TIF <- plot(pl2,  type = "IIC", items = 0, zrange = c(-4, 6), plot = FALSE)
grm_TIF <- plot(grm1, type = "IIC", items = 0, zrange = c(-4, 6), plot = FALSE)


lines(grm_TIF, col = "blue")
lines(pl2_TIF, col = "red")

legend("topleft",
       legend = c("GRM", "2PL", "RSM"),
       col = c("blue", "red", "green"),
       lty = 1)
```



This may not be accurate, as I don't know how the TAM package creates the Test Information Curve. With that, it's noteworthy how much more information the GRM gives compared to the RSM. Again, this may be different if the discrimination parameters of the questions weren't so different.

Log-Likelihood and AIC of GRM and RSM

Because GRM is the more complex model, I expect it to have a larger Log-likelihood. However, it should be a better fit due to the added discrimination abilities giving the model more information (as I've shown with the Test Information Function), which a lower AIC would signify.

```
comparison <- IRT.compareModels(grm1, rsm1)
```

```
Warning in rbind(dfr, dfr1): number of columns of result is not a multiple of
vector length (arg 1)
```

```
summary(comparison, extended = FALSE)
```

```
Absolute and relative model fit

  Model    loglike Deviance Npars Nobs      AIC      BIC     AIC3       GHP
1  grm1 -27895.93 55791.85    36 5735 55863.85 56103.41 55899.85 -27895.93
2  rsm1 -28752.26 57504.52    12 5164 57528.52 57607.11 57540.52      0.62

Likelihood ratio tests - model comparison

  Model1 Model2     Chi2 df p
1   rsm1   grm1 1712.665 24 0
```

Comparing the AIC's of each model, the Graded Response Model is the better fit. If the discriminatory parameters of the GRM were all very similar, then the RSM may have been a better fit due to the lower complexity.

Confirmatory Factor Analysis with Polytomous Data

Here, I fit a CFA with the original, polytomous data. This should be very close to the GRM, now that it is using the same data.

```r
mod2 <-
  "depression =~ q1 + q2 + q3 + q4 + q5 +
                q6 + q7 + q8 + q9
"


cfafit2 <- cfa(mod2, data = ds,
              ordered = c("q1", "q2", "q3", "q4", "q5",
                          "q6", "q7", "q8", "q9"),
              estimator = "WLSMV",
              check.gradient = TRUE
              )

summary(cfafit2, fit.measures = TRUE, standardized = TRUE)
```

```
lavaan 0.6.15 ended normally after 17 iterations

  Estimator                                         DWLS
  Optimization method                             NLMINB
  Number of model parameters                          36

                                                    Used       Total
  Number of observations                            5134        5735

Model Test User Model:
                                                Standard      Scaled
  Test Statistic                                 325.090     583.771
  Degrees of freedom                                  27          27
  P-value (Chi-square)                             0.000       0.000
  Scaling correction factor                                    0.559
  Shift parameter                                              2.037
    simple second-order correction

Model Test Baseline Model:

  Test statistic                               41255.954   23189.999
  Degrees of freedom                                  36          36
  P-value                                          0.000       0.000
  Scaling correction factor                                    1.780

User Model versus Baseline Model:
```

```
  Comparative Fit Index (CFI)                    0.993       0.976
  Tucker-Lewis Index (TLI)                       0.990       0.968

  Robust Comparative Fit Index (CFI)                         0.925
  Robust Tucker-Lewis Index (TLI)                            0.900

Root Mean Square Error of Approximation:

  RMSEA                                          0.046       0.063
  90 Percent confidence interval - lower         0.042       0.059
  90 Percent confidence interval - upper         0.051       0.068
  P-value H_0: RMSEA <= 0.050                    0.902       0.000
  P-value H_0: RMSEA >= 0.080                    0.000       0.000

  Robust RMSEA                                               0.124
  90 Percent confidence interval - lower                     0.113
  90 Percent confidence interval - upper                     0.136
  P-value H_0: Robust RMSEA <= 0.050                         0.000
  P-value H_0: Robust RMSEA >= 0.080                         1.000

Standardized Root Mean Square Residual:

  SRMR                                           0.045       0.045

Parameter Estimates:

  Standard errors                           Robust.sem
  Information                                 Expected
  Information saturated (h1) model          Unstructured

Latent Variables:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
  depression =~
    q1                1.000                               0.738    0.738
    q2                1.174    0.020   59.412    0.000    0.866    0.866
    q3                0.940    0.020   45.940    0.000    0.693    0.693
    q4                1.002    0.018   55.103    0.000    0.739    0.739
    q5                0.912    0.021   44.119    0.000    0.673    0.673
    q6                1.148    0.020   57.410    0.000    0.847    0.847
    q7                1.009    0.022   46.399    0.000    0.744    0.744
    q8                0.966    0.026   37.308    0.000    0.712    0.712
    q9                1.091    0.030   36.901    0.000    0.804    0.804
```

Intercepts:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
.q1	0.000				0.000	0.000
.q2	0.000				0.000	0.000
.q3	0.000				0.000	0.000
.q4	0.000				0.000	0.000
.q5	0.000				0.000	0.000
.q6	0.000				0.000	0.000
.q7	0.000				0.000	0.000
.q8	0.000				0.000	0.000
.q9	0.000				0.000	0.000
depression	0.000				0.000	0.000

Thresholds:

	Estimate	Std.Err	z-value	P(>|z|)	Std.lv	Std.all
q1|t1	0.643	0.019	34.054	0.000	0.643	0.643
q1|t2	1.313	0.024	54.152	0.000	1.313	1.313
q1|t3	1.691	0.030	55.561	0.000	1.691	1.691
q2|t1	0.709	0.019	36.944	0.000	0.709	0.709
q2|t2	1.471	0.026	55.614	0.000	1.471	1.471
q2|t3	1.875	0.035	53.823	0.000	1.875	1.875
q3|t1	0.303	0.018	17.027	0.000	0.303	0.303
q3|t2	1.040	0.021	48.581	0.000	1.040	1.040
q3|t3	1.399	0.025	55.116	0.000	1.399	1.399
q4|t1	-0.056	0.018	-3.182	0.001	-0.056	-0.056
q4|t2	0.939	0.021	45.584	0.000	0.939	0.939
q4|t3	1.378	0.025	54.911	0.000	1.378	1.378
q5|t1	0.640	0.019	33.946	0.000	0.640	0.640
q5|t2	1.327	0.024	54.333	0.000	1.327	1.327
q5|t3	1.689	0.030	55.572	0.000	1.689	1.689
q6|t1	0.986	0.021	47.035	0.000	0.986	0.986
q6|t2	1.604	0.029	55.863	0.000	1.604	1.604
q6|t3	1.945	0.037	52.812	0.000	1.945	1.945
q7|t1	0.989	0.021	47.130	0.000	0.989	0.989
q7|t2	1.553	0.028	55.872	0.000	1.553	1.553
q7|t3	1.853	0.034	54.107	0.000	1.853	1.853
q8|t1	1.257	0.024	53.327	0.000	1.257	1.257
q8|t2	1.789	0.033	54.815	0.000	1.789	1.789
q8|t3	2.103	0.042	49.893	0.000	2.103	2.103
q9|t1	1.782	0.032	54.885	0.000	1.782	1.782
q9|t2	2.231	0.047	47.032	0.000	2.231	2.231
q9|t3	2.521	0.064	39.365	0.000	2.521	2.521

```
Variances:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
   .q1                0.456                               0.456    0.456
   .q2                0.251                               0.251    0.251
   .q3                0.519                               0.519    0.519
   .q4                0.454                               0.454    0.454
   .q5                0.547                               0.547    0.547
   .q6                0.283                               0.283    0.283
   .q7                0.446                               0.446    0.446
   .q8                0.493                               0.493    0.493
   .q9                0.353                               0.353    0.353
    depression        0.544    0.017   31.727    0.000    1.000    1.000

Scales y*:
                   Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
    q1                1.000                               1.000    1.000
    q2                1.000                               1.000    1.000
    q3                1.000                               1.000    1.000
    q4                1.000                               1.000    1.000
    q5                1.000                               1.000    1.000
    q6                1.000                               1.000    1.000
    q7                1.000                               1.000    1.000
    q8                1.000                               1.000    1.000
    q9                1.000                               1.000    1.000
```

Compare Discrimination Parameter of GRM and Factor Loadings of Polytomous CFA

```
model_loadings_poly <- inspect(cfafit2, what = "std")[["lambda"]]

discrims_poly <- list()
for (i in 1:9) {
  discrims_poly[[paste0("q", i)]] <- summary(grm1)$coefficients[[paste0("q", i)]][4]
}

discrims_poly <- unlist(discrims_poly)
names(discrims_poly) <- names(discrims_poly)

D <- 1.7

df_loadings_poly <- cbind(loadings = model_loadings_poly,
```

```
                        discrims_to_loadings = (discrims_poly / D)
                        /
                        (sqrt(1 + ((discrims_poly / D)^2))))

df_loadings_poly <- df_loadings_poly %>%
  as.data.frame() %>%
  dplyr::rename(cfa_loadings = depression)


df_loadings_poly %>%
  as.data.frame() %>%
  dplyr::mutate(dif = cfa_loadings-discrims_to_loadings,
                rat = cfa_loadings/discrims_to_loadings)
```

```
   cfa_loadings discrims_to_loadings          dif       rat
q1    0.7375195            0.7584342 -0.020914731 0.9724238
q2    0.8655098            0.8707365 -0.005226748 0.9939973
q3    0.6932849            0.6944226 -0.001137694 0.9983617
q4    0.7391335            0.7461941 -0.007060588 0.9905379
q5    0.6728642            0.6935380 -0.020673861 0.9701907
q6    0.8466470            0.8549768 -0.008329818 0.9902573
q7    0.7441284            0.7615307 -0.017402315 0.9771482
q8    0.7121545            0.7444280 -0.032273496 0.9566466
q9    0.8042705            0.8224679 -0.018197394 0.9778746
```

Almost equivalent. The values are close enough to have shown that for unidimensional polytomous data, CFA and the GRM are theoretically equivalent.

Conclusion

The IRT analysis showed that the questions were all positive, that the discrimination parameters were all good, that the IIC suggests that the scale is overall reliable, but the reliability peaks on low-moderate depression. But in a screening tool, this is probably what we want.

The factor analysis gave extremely similar results to the IRT analysis, theoretically identical. However, the model fit was not perfect, suggesting that it was not uni-dimensional, which might be a problem. The modification indices suggested some additional covariances, but I didn't as it would be no longer equivalent to the IRT.

Also, the fact that I can add covariances to the CFA leads me to believe that CFA is the more flexible model. The IRT models did however yield more interpretable results, and in that way seem specialized for uni-dimensional testing. CFA has a much wider application of uses (i.e., multivariate systems) and seems to focus on the relationship between variables.

I wanted to try running a CFA using Maximum Likelihood, however the model would not converge when the estimator was "MML". This would have allowed me to use the AIC() function to compare my CFA models to the IRT models. Instead, I compared the factor loadings and discrimination parameters. This showed that the CFA is largely equivalent to an IRT model when using the same data.

Comparing the RSM and the GRM, the GRM was a better fit. This was due to the fact that the questions all had different discriminatory parameters, something that the RSM does not calculate (and for that reason, took much less time to run).

References

Cho, E. (2023). Interchangeability between factor analysis, logistic IRT, and normal Ogive IRT. *Frontiers in Psychology*, *14*. https://doi.org/10.3389/fpsyg.2023.1267219

Forero, C. G., & Maydeu-Olivares, A. (2009). Estimation of IRT graded response models: Limited versus full information methods. *Psychological Methods*, *14*(3), 275–299. https://doi.org/10.1037/a0015825

Rizopoulos, D. (2006). ltm: An R package for latent variable modeling and item response theory analyses. *Journal of Statistical Software*, *17*(5). https://doi.org/10.18637/jss.v017.i05