Quantum Information: MAT4953/5983xPHY7973xQST6003. HW1, Fall 2025. Posted 09/04. Due 09/18, 11:59pm.

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Abstract

Problems to review concepts on: (1) an overview of quantum computing and information; (2) Classical Information Theory.

1. [Ex. 7.1 from Book] A Turing machine has the four tape symbols $\{\triangleright, 0, 1, \emptyset\}$ and the four processor states $\{S, I, II, F\}$. The instruction set, or program, is

$$
\begin{aligned}
(S, \triangleright) &\implies (I, \triangleright),\\
(I, 0) &\implies (I, 0),\\
(I, 1) &\implies (II, 0),\\
(II, 0) &\implies (I, 1),\\
(II, 1) &\implies (II, 1),\\
(II, \emptyset) &\implies (I, 1),\\
(I, \emptyset) &\implies (F, \emptyset),
\end{aligned}
$$

where, as in the text, each instruction is followed by moving the tape head one place to the right.

   (a) Calculate the effect of this program on tapes in the following two initial configurations:

$$\triangleright : 0 : 1 : \emptyset : \emptyset : \cdots$$

   and

$$\triangleright : 1 : 0 : 1 : \emptyset : \cdots$$

   (b) What simple mathematical operation does the program perform? (have a hunch)

2. A particle detector counts with an efficiency η. This means that each particle is detected with probability η and missed with probability $1 - \eta$. Let N be the number of particles present and n be the number detected.

(a) Show that the probability that n particles are detected given that there was a total of N is

$$P(n|N) = \frac{N!}{(N-n)!n!}\eta^n(1-\eta)^{N-n}$$

(b) In a real experiment we don't know the value of N. Indeed, we want to infer N given that we detected n particles. We therefore want to obtain $P(N|n)$, i.e. the probability that there were N particles given that we detected n and this can be obtained from $P(n|N)$ using Bayes theorem. Calculate $P(N|n)$ assuming that $P(N)$ follows a Poisson distribution, that is,

$$P(N) = \exp(-\overline{N})\frac{(\overline{N})^N}{N!}$$

since the Poisson distribution gives the probability that N random events occur if the average or expected number is $\overline{N}$. Hint: You will need to calculate $P(n)$ which obeys the identity $P(n) = \sum_{N=0}^{\infty} P(n|N)P(N)$. You can in fact simplify the resulting sum, i.e., re-sum the infinite series, which will make your answers look a lot neater.

(c) Calculate $P(N|n)$ for all $P(N)$ equally probable.

(d) [Bonus points: nonmandatory subexercise, because it's harder] Calculate $P(N|n)$ given only that the mean number of particles present is $\overline{N}$. Hint: If you are given very little information, a general principle in Physics is to maximize the entropy...