Aaron Weymouth Midterm 2: 5/2/25

(1.) Given two identical non-interacting particles with $m_1 = 0.98m$

(a) and $m_2 = 1.02m$. → 1D infinite well of width L.

- for 2 particles: $\hat{H} = \hat{H}_1 + \hat{H}_2 = \left(-\frac{\hbar}{2m_1}\frac{\partial^2}{\partial x_1^2}\right) + \left(-\frac{\hbar^2}{2m_2}\frac{\partial^2}{\partial x_2^2}\right)$

Total wavefunction is seperable:

$$\psi_{n,m}(x_1, x_2) = \psi_n^{(1)}(x_1) \cdot \psi_m^{(2)}(x_2)$$

and for each: $\psi_n^{(i)}(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$, $i = 1, 2$.

with coorsponding energies: $E_n^{(1)} = \frac{n^2\pi^2\hbar^2}{2m_1 L^2}$, $E_m^{(2)} = \frac{m^2\pi^2\hbar^2}{2m_2 L^2}$

Total Energy

$\therefore$

$$E_{n,m} = E_n^{(1)} + E_n^{(2)} = \frac{\pi^2\hbar^2}{2L^2}\left(\frac{n^2}{m_1} + \frac{m^2}{m_2}\right)$$

$$\boxed{E_{n,m} = \frac{\pi^2\hbar^2}{2L^2} \cdot \frac{m^2 m_1 + n^2 m_2}{m_1 m_2}}$$

which are allowed energy eigenvalues for all $n, m \in \mathbb{N}$ ~~real~~.

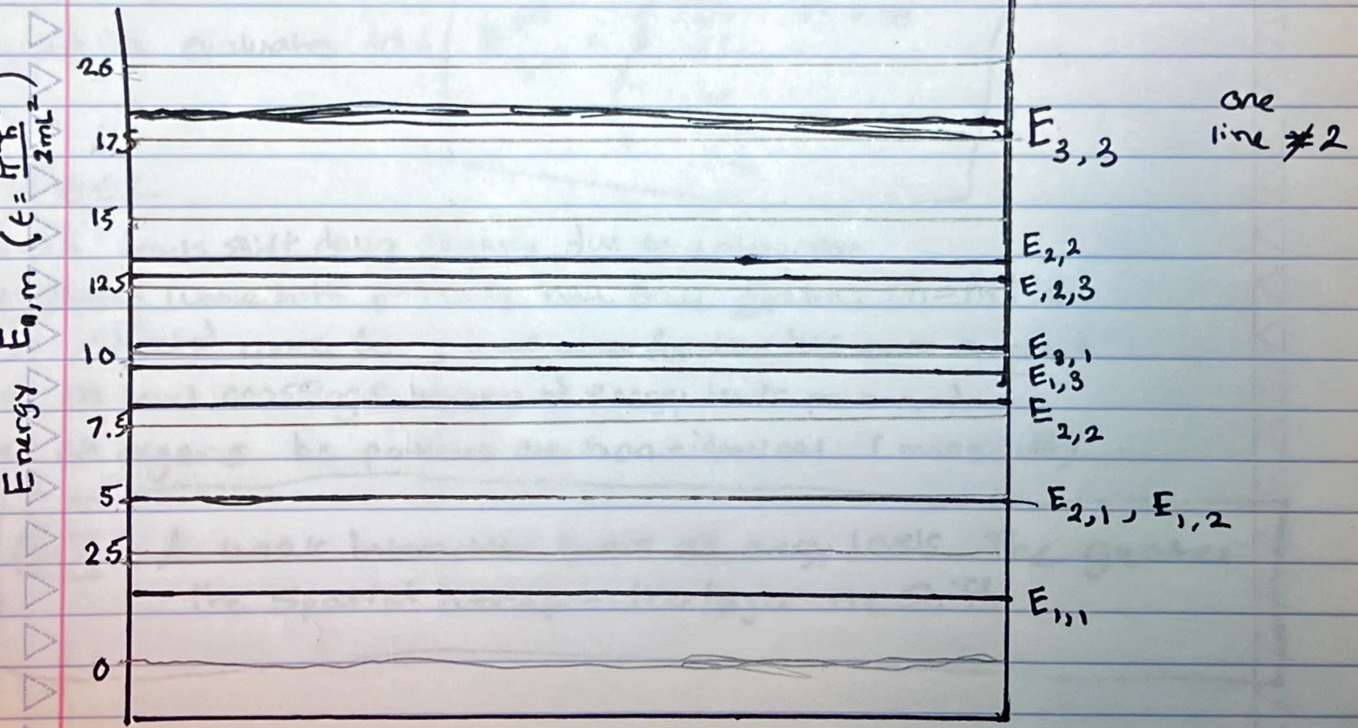
↳ plot on next pg + python.

(1a). cont let $m=1$, $\epsilon = \frac{\pi^2\hbar^2}{2mL^2}$; given $m_1 = 0.98m$
$m_2 = 1.02m$

State (m,n)	Energy (ϵ units)
$(1,1)$	~ 2.0008
$(1,2)$	~ 4.942
$(2,1)$	~ 5.062
$(2,2)$	~ 8.003

No degeneracy here due to mass difference $m_1 \neq m_2$

I am not good at graphing I will; do so here + provide python code and plot.



1b. Applying weak Attractive interaction.

When two nonidentical particles interact via weak attractive potential the short range interaction is perturbed: $\hat{V}_{init} = -V_0 \delta(x_1 - x_2)$ Since interaction is weak we can use first order non-degen perturbation theory.

$$\hat{H} = \hat{H}_1 + \hat{H}_2 + \hat{V}_{int}$$

and $E^{(1)}_{n,m} = \langle n, m | \hat{V}_{int,} | n, m \rangle < 0$ and we have for delta function

it becomes $E^{(1)}_{n,m} = -V_0 \int_0^L \psi_n(x)^2 \psi_m(x)^2 dx$

Which evaluates to: $\boxed{E^{(1)}_{n,m} = \begin{cases} \dfrac{-V_0}{2L} & n = m \\ \dfrac{-V_0}{4L} & n \neq m \end{cases}}$

Summary

- all levels shift down slightly due to interaction
- States where both particals have same spatial $(n=m)$ are effected more bc. spatial wavefunction has more overlap
- No level crossings happen → energy levels perserved.
- No degens bc particles are non-identical (mass diff)

SO A weak interaction lowers all energy levels. The greater the spatial overlap - the larger the shift