Aaron W: R+L PSet

Pr 3.1 : Pulsar:

(a) • Given a nuetron star w/ dipole $\vec{m}$ rotating at ω axis tilted by α; time varying $\vec{m}$ produces magnetic dipole rad.

• Dipole approx $R \ll \lambda$

for mag dipole moment let B_0 be field strength + radius R

$$B_{pole} = \frac{2m}{R^3} \rightarrow m = \frac{B_0 R^3}{2}$$

• only component of m perpendicular to rotation axis radiates.

↳ component along axis is static.

$m_\perp = m \sin\alpha$ and rotates in a plane w/ angular frequency ω

• Power radiated by magnetic dipole $\propto$ square of 2nd derv. m.

eq 3.19: $$P = \frac{2}{3c^3} |\ddot{m}|^2$$ and for a rotating

dipole w/ constant mag + direction

$$|\ddot{m}| = \cancel{\frac{2}{3c^3}} \omega^2 m_\perp$$

$$\therefore \; P = \frac{2}{3c^3} \omega^4 m^2 \sin^2\alpha$$

$$\therefore \; \boxed{P = \frac{B_0^2 R^6 \omega^4 \sin^2\alpha}{6c^3}}$$

In CGS matches mag dipole radiation analogous to larmor

3.1.b.

Spin down time:

b. Pulsars rotational ke: $E_{rot} = \frac{1}{2} I \omega^2$

let $\tau \equiv \frac{E_{rot}}{P}$ and using power derived in part a.

$$\tau = \frac{\frac{1}{2} \vec{I} \omega^2}{\left(\frac{B_0^2 \cdot R^6 \cdot \omega^4 \sin^2\alpha}{6c^3} \right)} = \frac{3 \cdot I c^3}{B_0^2 R^6 \omega^2 \sin^2\alpha}$$

from Goldstien: $I = \frac{2}{5} M R^2$ (moment of inertia uniform sphere)

$$\tau = \frac{3c^3 (2/5 \cdot M R^2)}{B_0^2 R^6 \omega^2 \sin^2\alpha} = \boxed{\frac{6}{5} \frac{M c^3}{B_0^2 R^4 \omega^2 \sin^2\alpha} = \tau}$$

Logic checks: higher mag field / faster spin shortens time and more inertia prolongs.

c. Ref python:

$M = 2 \times 10^{33}$ G

$R = 10^6$ cm

$B_0 = 10^{12}$ G

$\alpha = 90° \rightarrow \sin^2\left(\frac{\pi}{2}\right) = 1$

$c = 3 \times 10^{10}$ cm/s

Dimensional Analysis check:

for power: I derived $P = \dfrac{B_0^2 \cdot R^6 \omega^4 \sin^2\alpha}{6c^3}$

$[B_0] = \text{gauss} = \dfrac{g^{1/2}}{cm^{1/2} \cdot s}$

$[B_0^2] = g/cm \cdot s^2$

$[R^6] = cm^6$

$[\omega^4] = s^{-4}$

$[c^3] = \left(\dfrac{cm}{s}\right)^3 = \dfrac{cm^3}{s^3}$

$[P] = \dfrac{\left(\frac{g}{cm \cdot s^2}\right) \cdot cm^6 \cdot s^{-4}}{\frac{cm^3}{s^3}} = \dfrac{g \cdot cm^5}{s^6} \cdot \dfrac{s^3}{cm^3}$

$= \dfrac{g \cdot cm^2}{s^3} = \dfrac{erg}{s}$ ✓

Spin down $\tau = \dfrac{6}{5} \cdot \dfrac{M c^3}{B_0^2 R^4 \omega^2 \sin^2\alpha}$

$[M] = g$

$[c^3] = cm^3/s^3$

$[B_0^2] = g/cm \cdot s^2$

$[R^4] = cm^4$

$[\omega^2] = s^{-2}$

$[\tau] = \dfrac{g \cdot cm^3/s^3}{(g/(cm \cdot s^2)) \cdot cm^4 \cdot s^{-2}}$

$= \dfrac{g \cdot cm^3/s^2}{g \cdot cm^3/s^4} = s$ ✓

3.1.c

cont:

- $\omega = 10^4\ s^{-1}$

  ↳ $P \approx 6.2 \times 10^{43}$ ergs/s

  ↳ $\tau \approx 6.5 \times 10^8\ s \approx 20$ years.

- $\omega = 10^3\ s^{-1}$

  ↳ $P \approx 6.2 \times 10^{39}$ erg/s

  ↳ $\tau \approx 6.5 \times 10^{10}\ s \approx 2.1 \times 10^3$ years

- $\omega = 10^2\ s^{-1}$

  ↳ $P \approx 6.2 \times 10^{35}$ ergs/s

  ↳ $\tau = 6.5 \times 10^{12}\ s \approx 2.1 \times 10^5$ years

So: Newer, faster rotating stars (large ω) lose rotational energy rapidly; older slow ones spin down over long time scales.