SSY130
Hand in problem 1

October 29, 2018

1. A real valued continuos time signal $x(t)$ should be sampled and processed in a DSP. Assume the real valued signal is passed through an analog low-pass filter before sampling. The filter has the frequency function

$$H(\omega) = \frac{1}{1 + j\omega/\omega_0} \tag{1}$$

where $\omega_0 = 16\pi \times 10^3$ [rad/s]. The signal $x(t)$ has two additive components and we write

$$x(t) = x_s(t) + n(t) \tag{2}$$

where signal $x_s(t)$ is the signal of interest while $n(t)$ is a noise signal. The magnitude of the Fourier transform of the two signal components are

$$X_s(\omega) = \begin{cases} 1, & |\omega| < 12\pi \times 10^3 \, [\text{rad/s}] \\ 0, & |\omega| \geq 12\pi \times 10^3 \, [\text{rad/s}] \end{cases} \tag{3}$$

$$N(\omega) = \begin{cases} 0.1, & 16\pi \times 10^3 \leq |\omega| < 32\pi \times 10^3 \, [\text{rad/s}] \\ 0, & |\omega| < 16\pi \times 10^3 \, [\text{rad/s}] \\ 0, & |\omega| \geq 32\pi \times 10^3 \, [\text{rad/s}] \end{cases} \tag{4}$$

Determine the minimum sample rate which guarantees that

(a) the continuos time signal $x_s(t)$ can be perfectly reconstructed from the sampled signal if the noise signal is not present.

(b) the magnitude of the DTFT of the filtered and sampled noise signal for $|\omega| < 12\pi \times 10^3$ is at least 20 times lower than the magnitude of the DTFT of the sampled and filtered desired signal at $\omega = 0$. (2pt)

2. Consider a discrete time sinusoidal signal

$$x_d(n) = \sin(2\pi n f_0 / f_s) \tag{5}$$

where $f_0 = 3$ kHz and the sample rate is $f_s = 10$ kHz. A continuous signal $x(t)$ is constructed using the zero-order-hold method

$$x(t) \triangleq x_d(n), \quad n\Delta t \leq t < (n+1)\Delta t. \tag{6}$$

Since the reconstruction is not ideal $x(t)$ will also contain sinusoidal components with frequencies higher than the fundamental component at 3 kHz, often called harmonics. Determine the magnitude of the fundamental component and the frequencies and magnitudes of the 3 first harmonic components (the 3 harmonics closest to the fundamental frequency) in the ZOH reconstructed signal. (2pt)