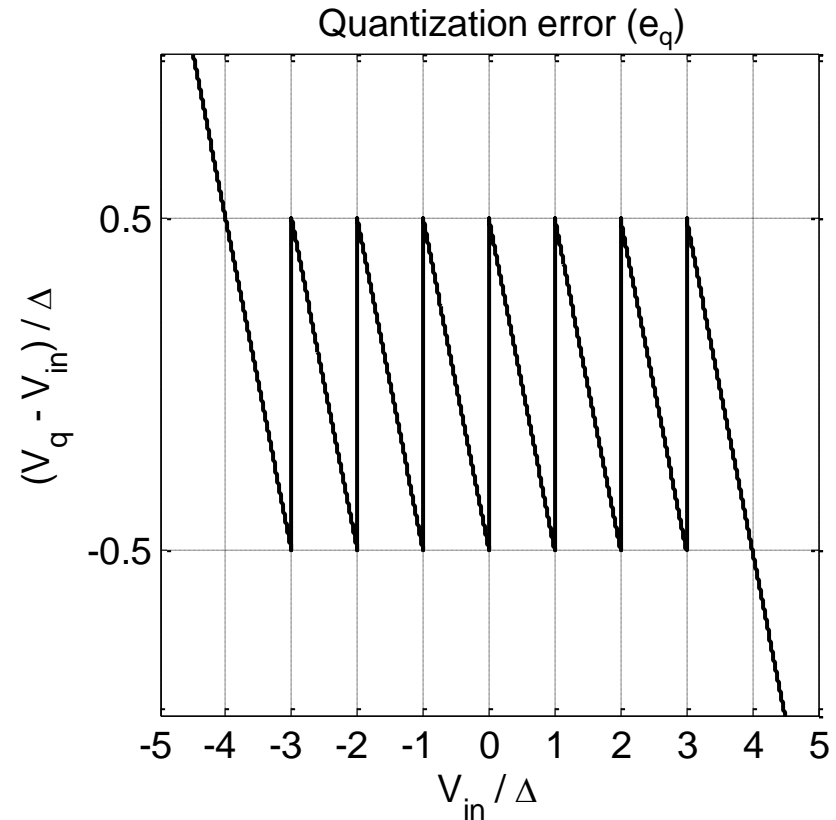
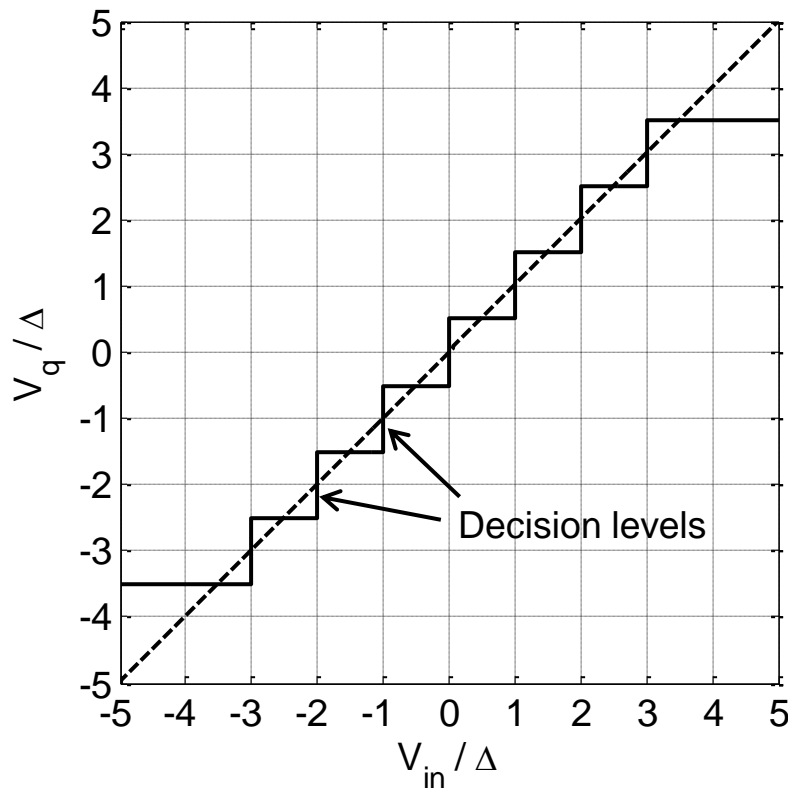


Incremental Delta-Sigma Modulators

Boris Murmann

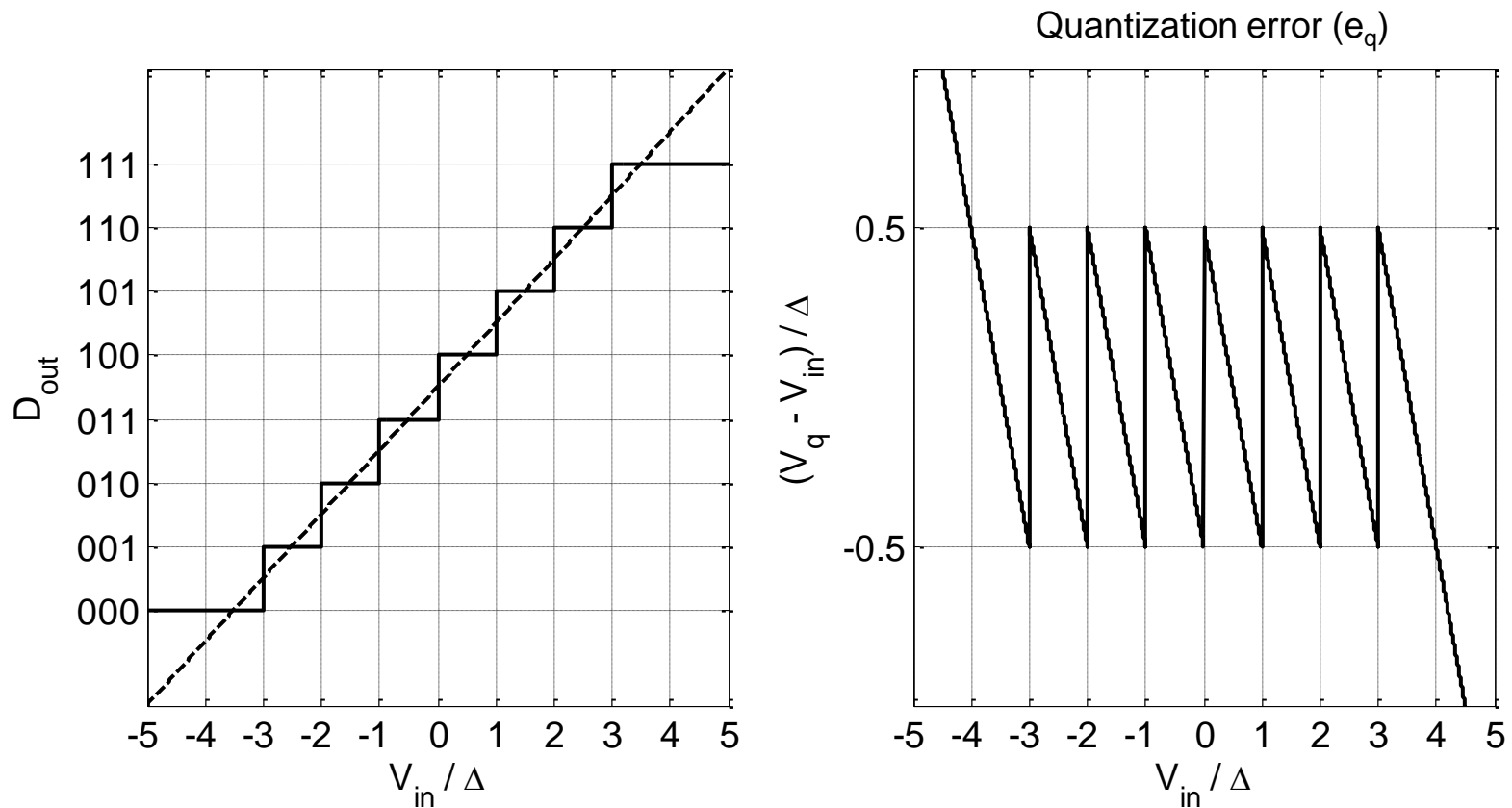
bmurmann@hawaii.edu

Basics of Quantization



- Example of a 3-bit quantizer (2^3 output levels)
- Quantization step size (or “LSB size”) is Δ
- Full-scale range defined by region with quantization error magnitude $< \Delta/2$

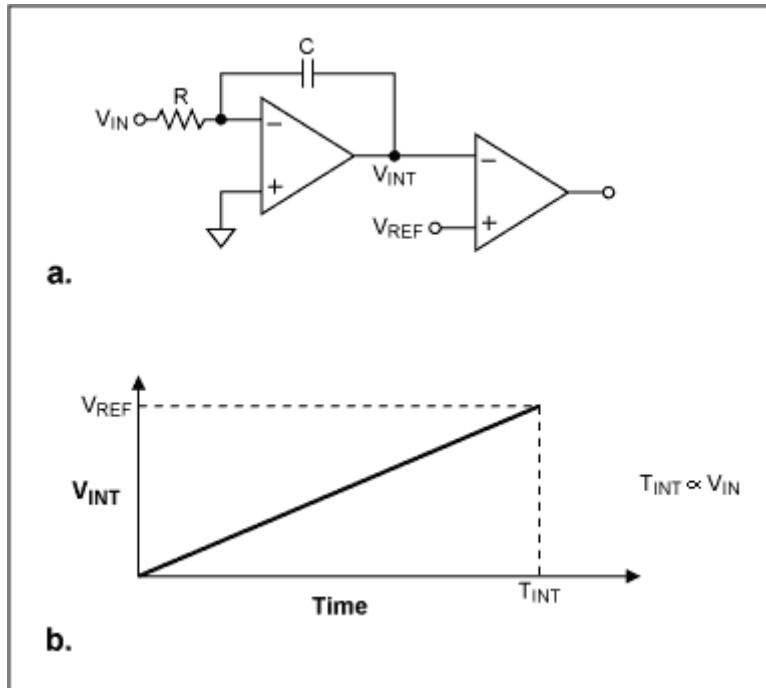
Output Encoding



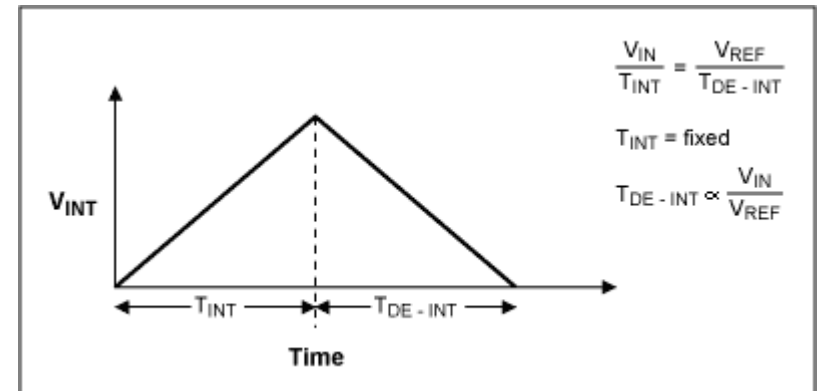
- The output of a real quantizer is mapped into a digital code
 - Many options exist, shown is a basic binary encoding

Single- and Dual-Slope ADCs

Single slope

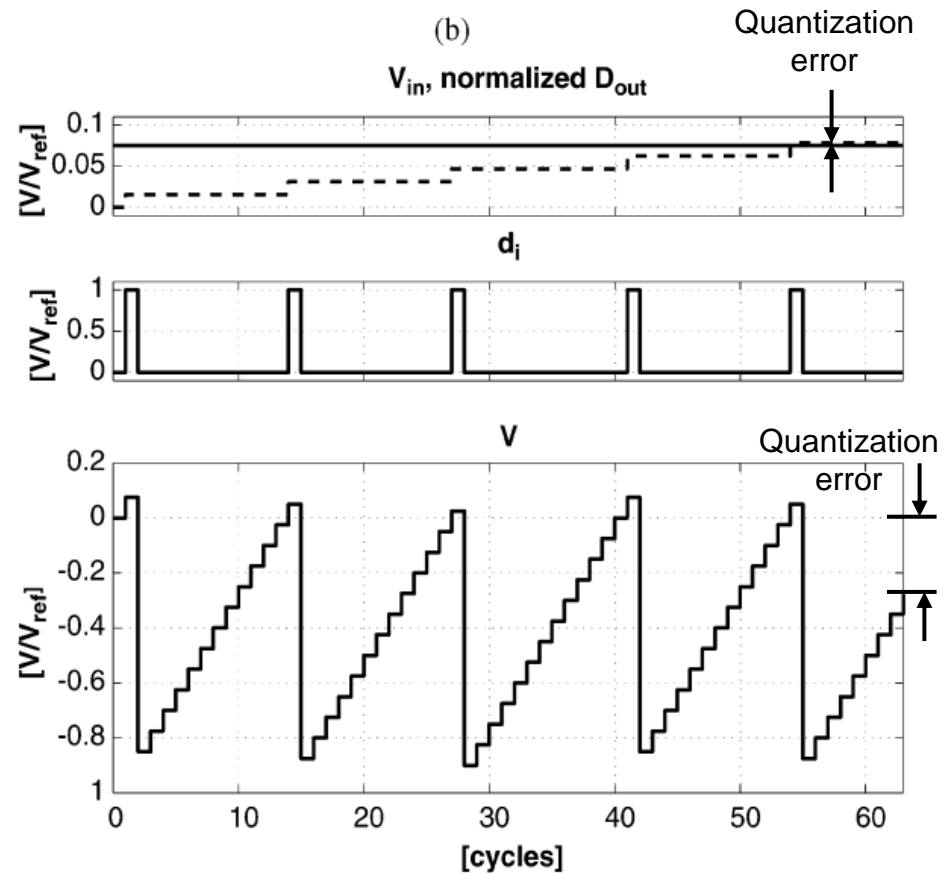
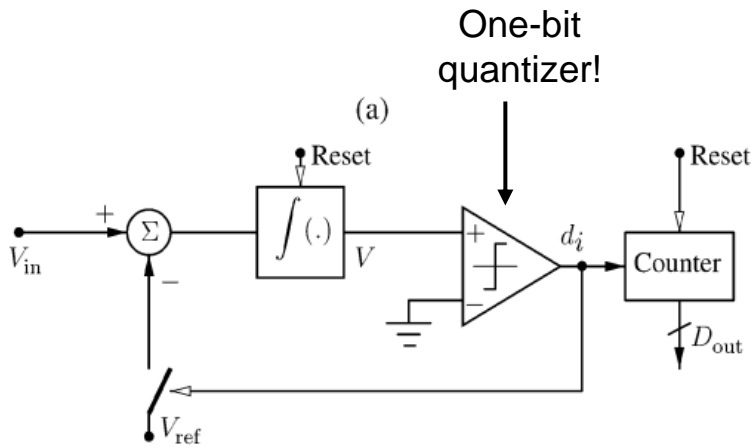


Dual slope



Quantization error comes from discretization of steps or threshold crossing

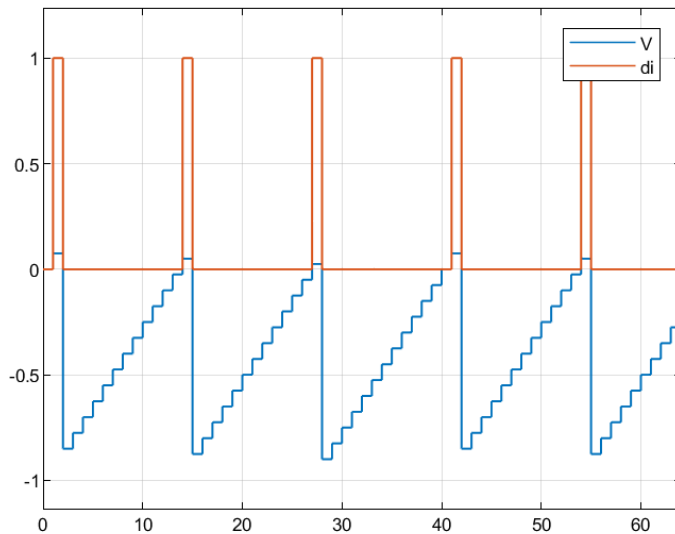
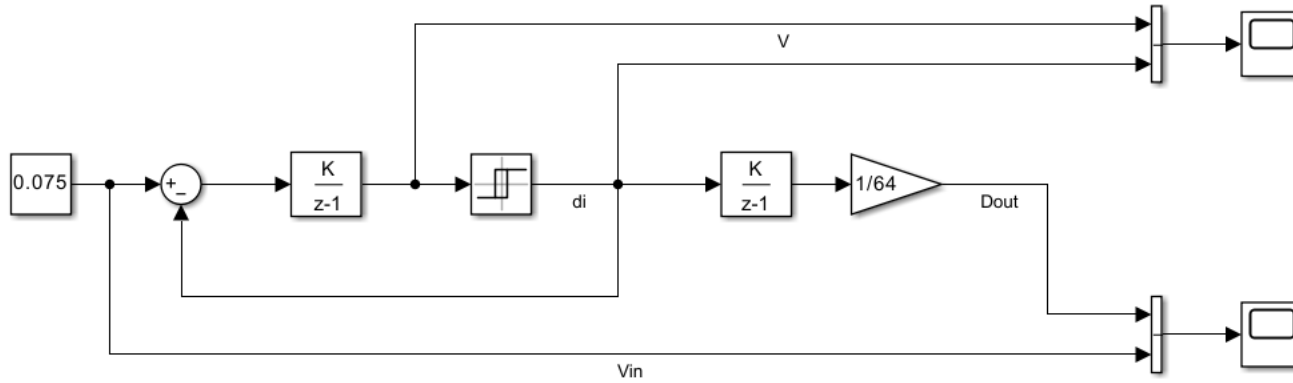
First Order Incremental Delta Sigma Converter



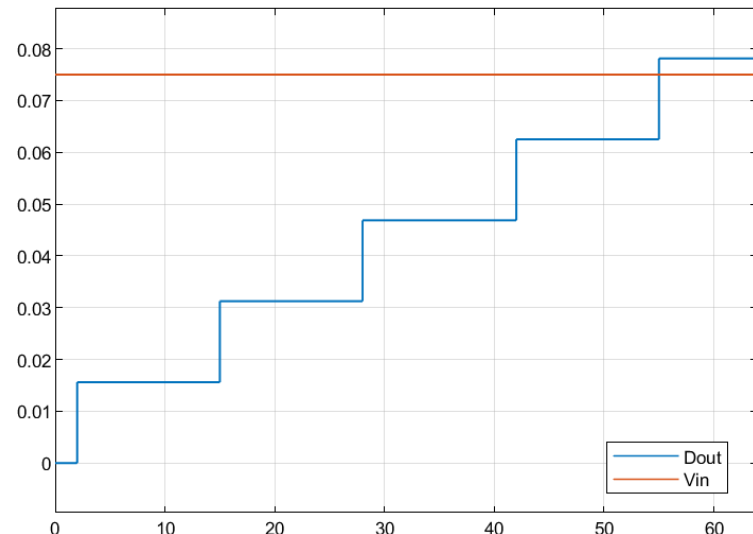
- Similar to dual-slope, but V_{in} integration and V_{ref} de-integration occur together
- Number of de-integration pulses (d_i) is proportional to $V_{in} \rightarrow$ use a counter
- Ideally V should be zero at the end of conversion, but it can be anywhere between $-V_{ref}$ and $V_{in} \rightarrow$ quantization error

J. Markus, J. Silva and G. C. Temes, "Theory and applications of incremental Delta Sigma converters," in IEEE Transactions on Circuits and Systems I, April 2004. <https://ieeexplore.ieee.org/document/1284742>

Simulink Model



Time step



Time step

Analysis

$$V = \sum (V_{in} - d_i V_{ref})$$

$$\sum d_i = D_{out}$$

(counter)

$$V = nV_{in} - D_{out}V_{ref} = \epsilon$$

$$\frac{D_{out}}{n} = \frac{V_{in}}{V_{ref}} - \frac{1}{n} \frac{\epsilon}{V_{ref}}$$

- Error is divided by the number of clock cycles per conversion (n)
- Can we do better, e.g., n^2 ?

Second-Order Approach

$$V = \sum \sum (V_{in} - d_i V_{ref})$$

$$\sum \sum d_i = D_{out}$$

(cascaded counters)

$$V = \frac{n(n+1)}{2} V_{in} - D_{out} V_{ref} = \epsilon$$

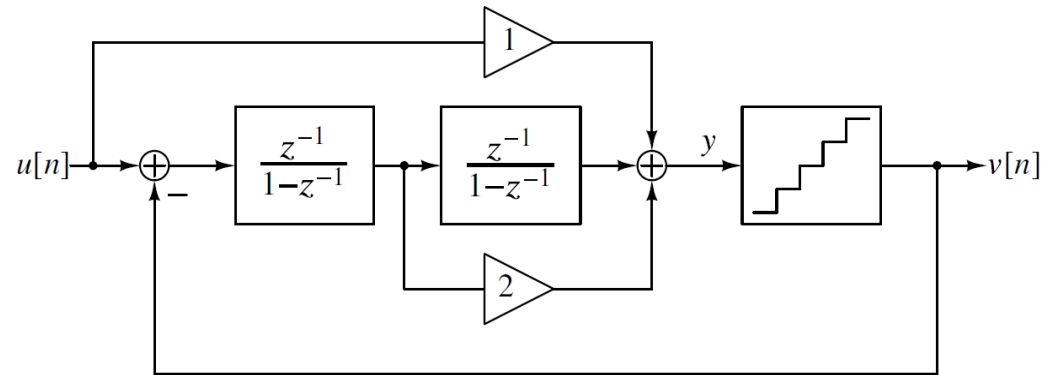
$$\sum_i^n i = \frac{n(n+1)}{2}$$

$$\frac{D_{out}}{\frac{n(n+1)}{2}} \approx \frac{V_{in}}{V_{ref}} - \frac{2}{n^2} \frac{\epsilon}{V_{ref}}$$

- Much better! Need fewer clock cycles to get accurate reading

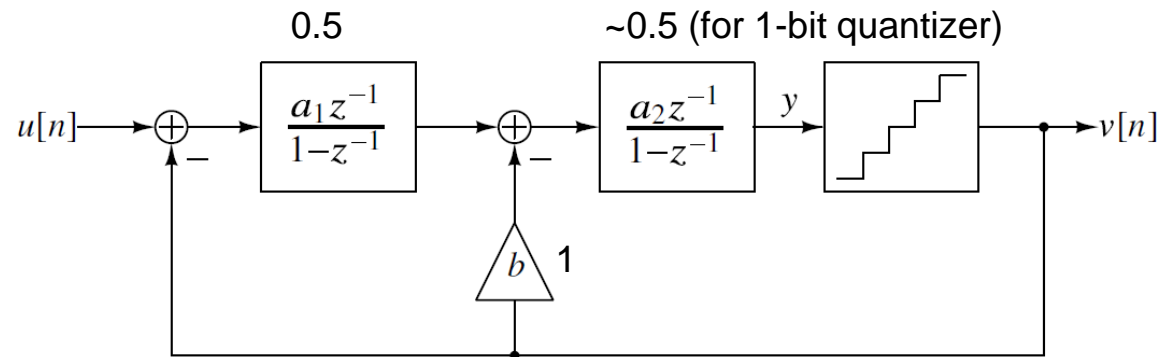
How to Implement?

CIFF architecture



Silva–Steenstgaard modulator

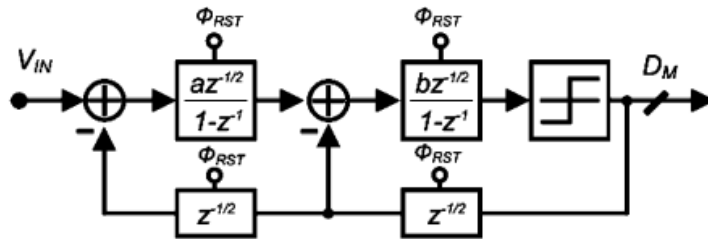
CIFB architecture



Boser-Wooley modulator

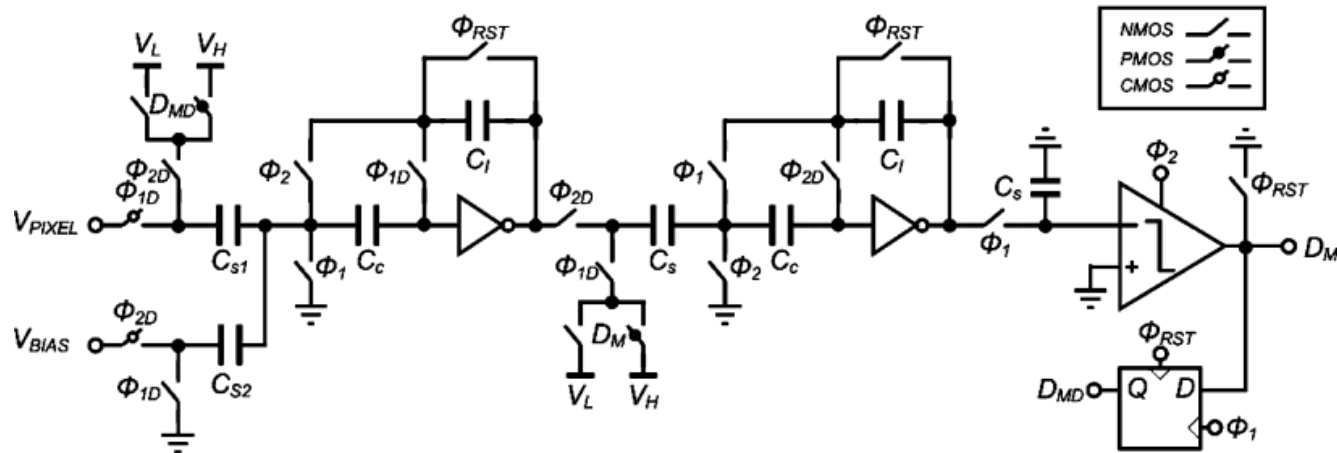
- Cannot just use two plain integrators in a loop (stability)

Template Project

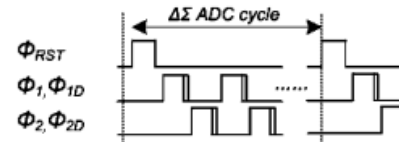


CIFB

(a)

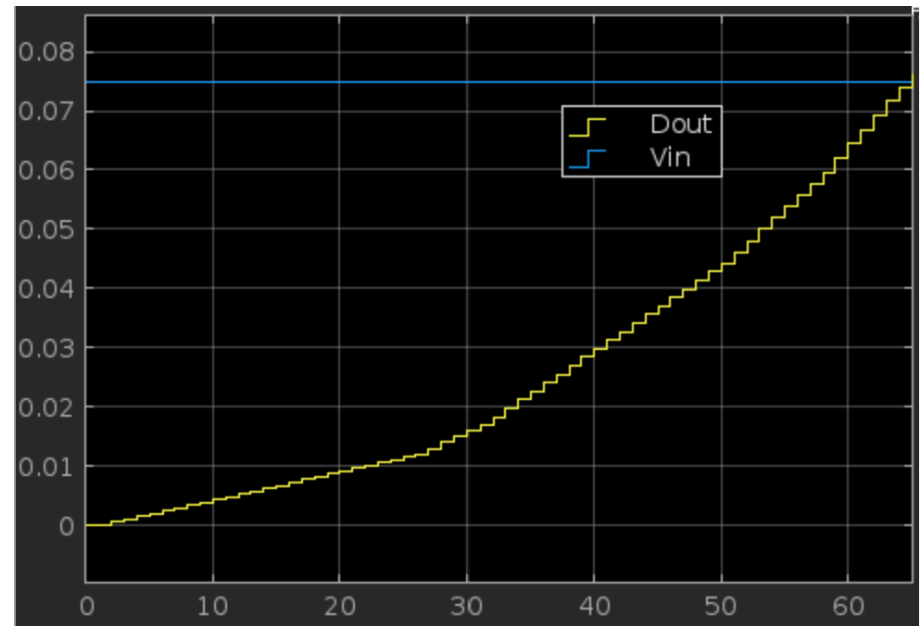
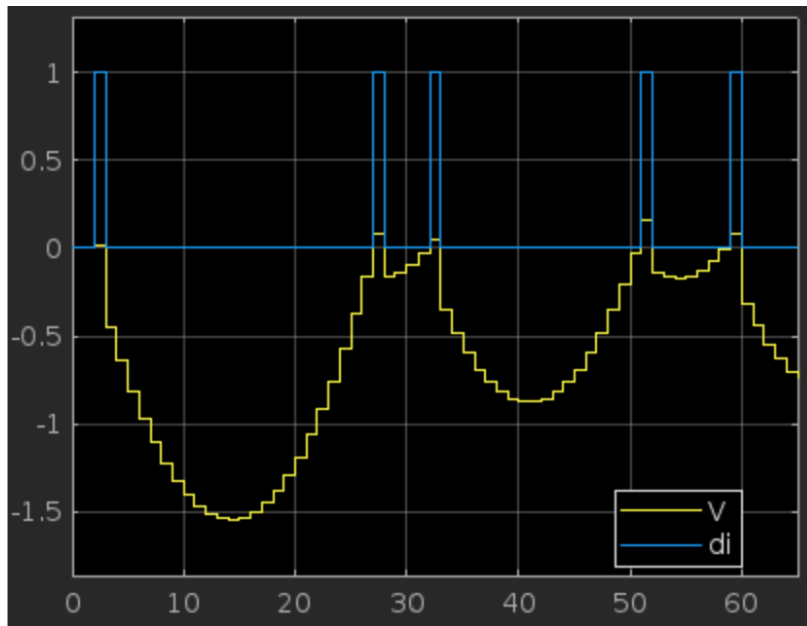
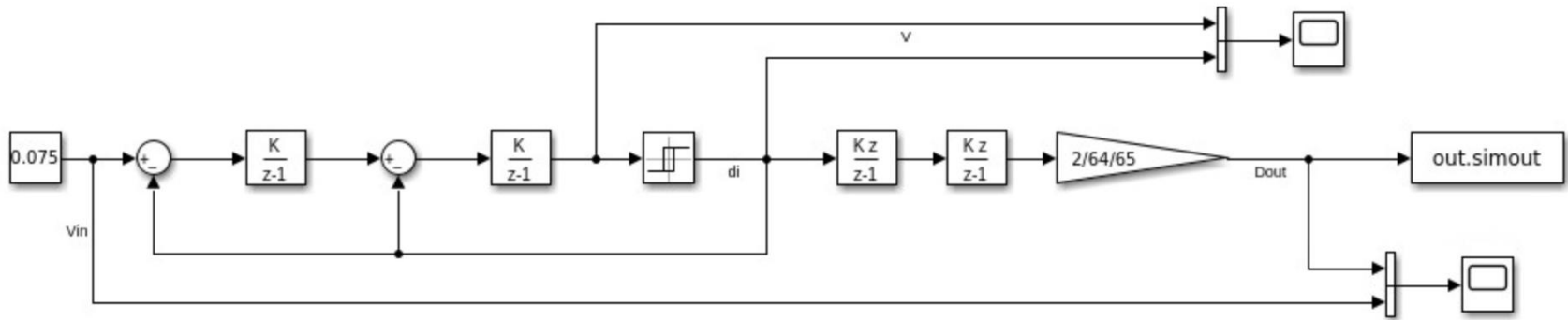


(b)

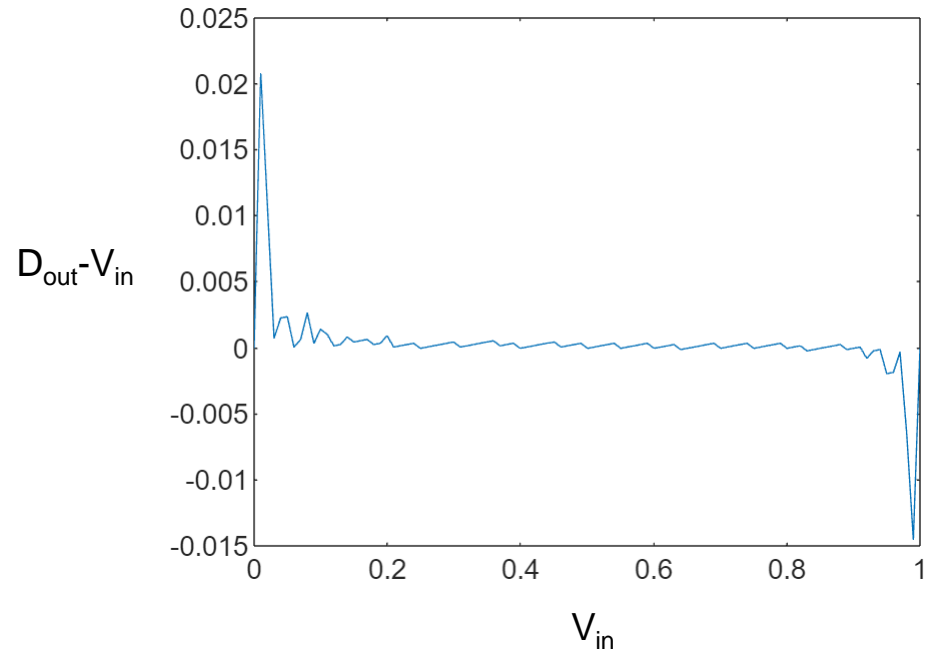
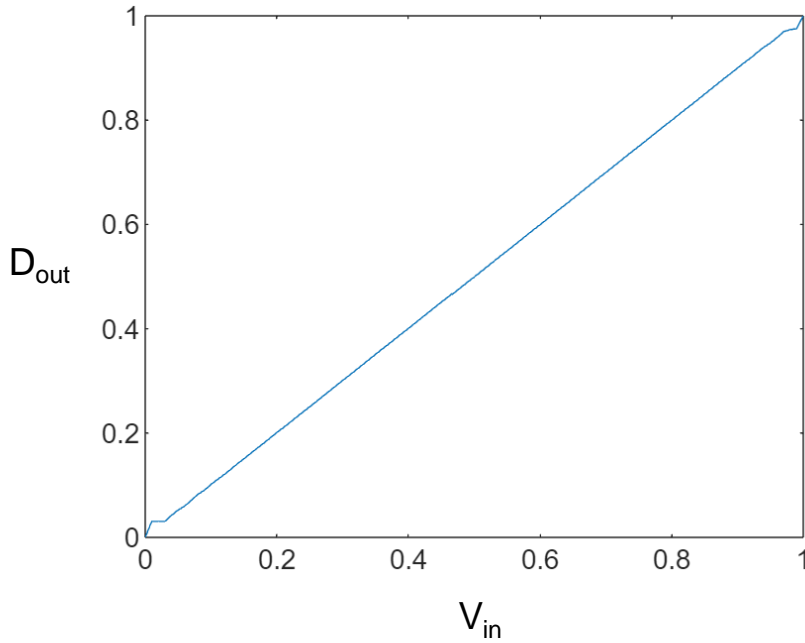


Y. Chae et al., "A 2.1 M Pixels, 120 Frame/s CMOS Image Sensor With Column-Parallel ADC Architecture," in IEEE Journal of Solid-State Circuits, Jan. 2011. <https://ieeexplore.ieee.org/document/5641589>

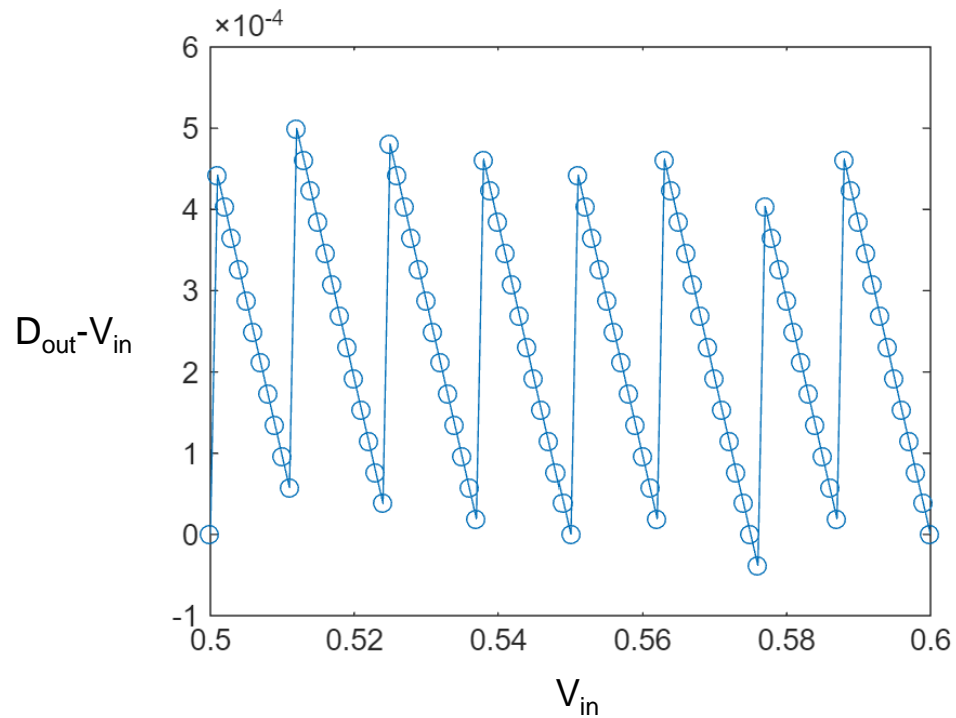
Simulink Model



Transfer Function and Quantization Error

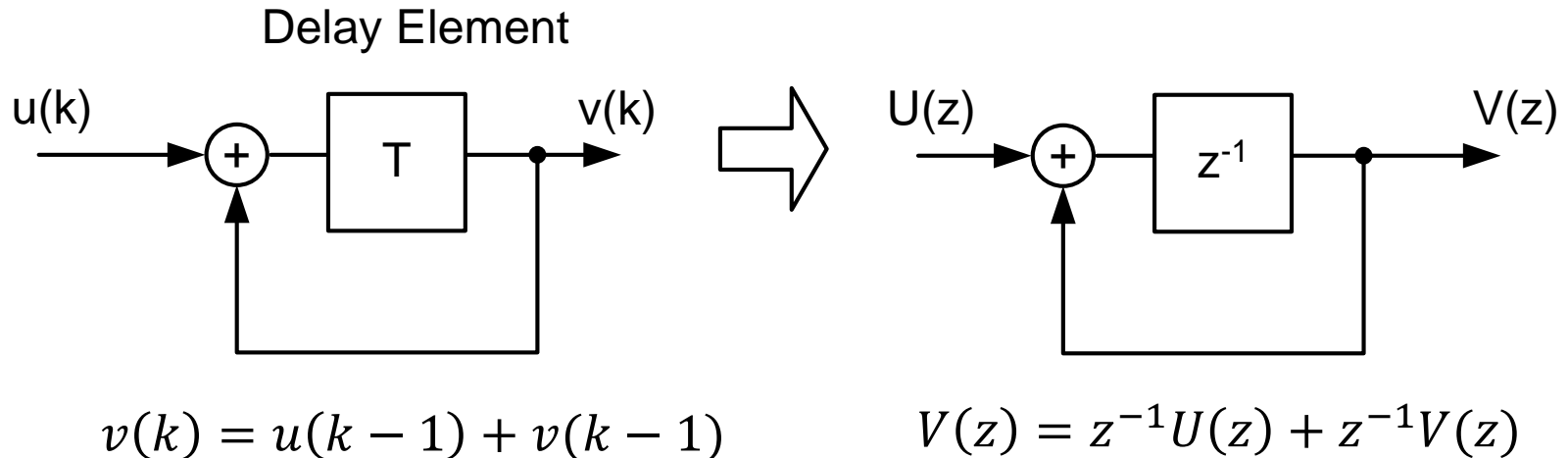


Zooming In



Expected bound $\sim 2/n^2 \sim 4.9 \cdot 10^{-4} \rightarrow$ pretty close!

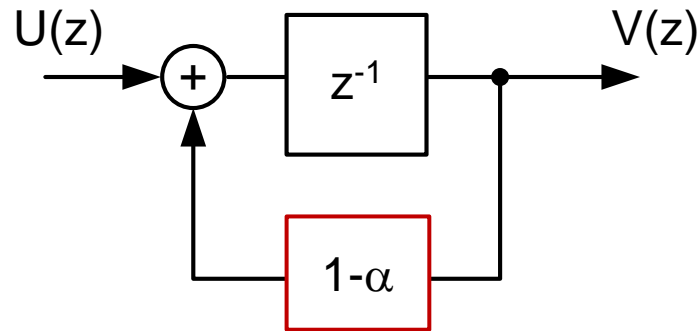
Ideal Discrete Time Integrator



$$\frac{V(z)}{U(z)} = \frac{z^{-1}}{1 - z^{-1}} = \frac{1}{z - 1} \quad z = e^{j\omega T}$$

- Infinite gain at DC ($\omega=0$, $z=1$)
- Finite gain in the analog circuit realization will be a problem...

Leaky Discrete Time Integrator



$$V(z) = z^{-1}U(z) + (1 - \alpha)z^{-1}V(z)$$

$$\frac{V(z)}{U(z)} = \frac{z^{-1}}{1 - (1 - \alpha)z^{-1}}$$

- DC gain is $1/\alpha$
- Detailed analysis shows that we need $1/\alpha > n$ (number of cycles)
 - Can try this out using our simulation model!
 - Replace ideal integrator with above model