Efficient Design and Simulation of Distributed Microwave Circuits for Quantum Devices

Workshop Slides

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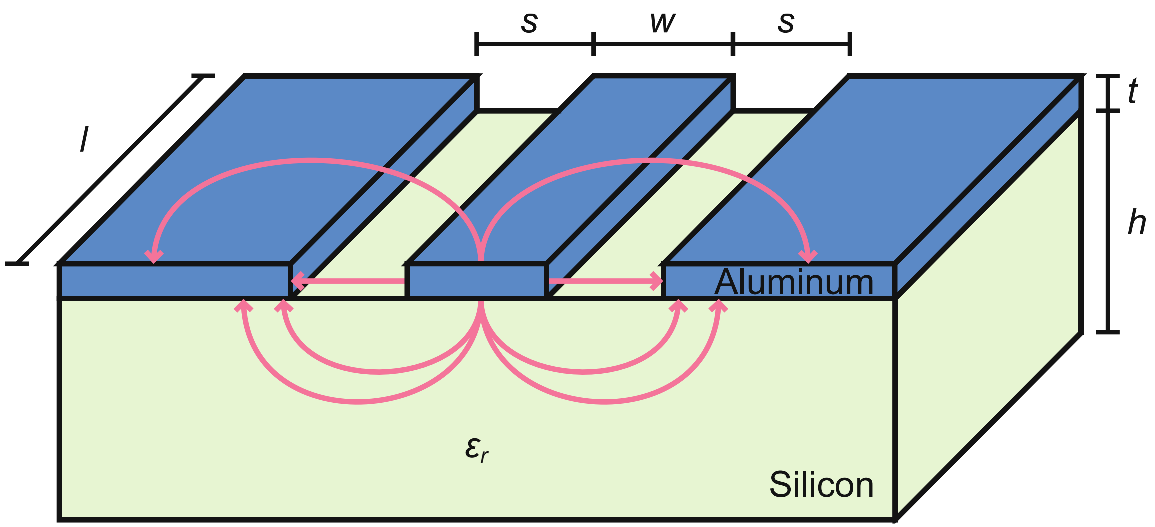
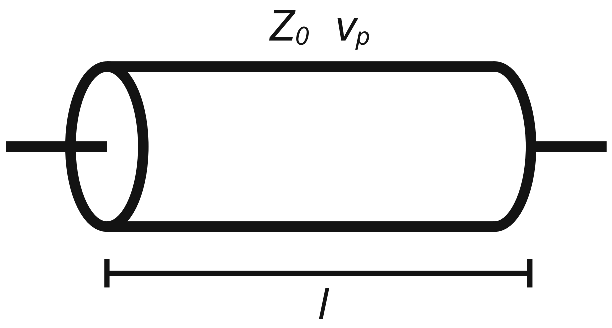
ABCD Formalism





$$v_p = \frac{c_0}{\sqrt{\varepsilon_{\mathrm{eff}}}}$$

$$v_p = \frac{1}{\sqrt{L'C'}} \qquad Z_0 = \sqrt{\frac{L'}{C'}}$$

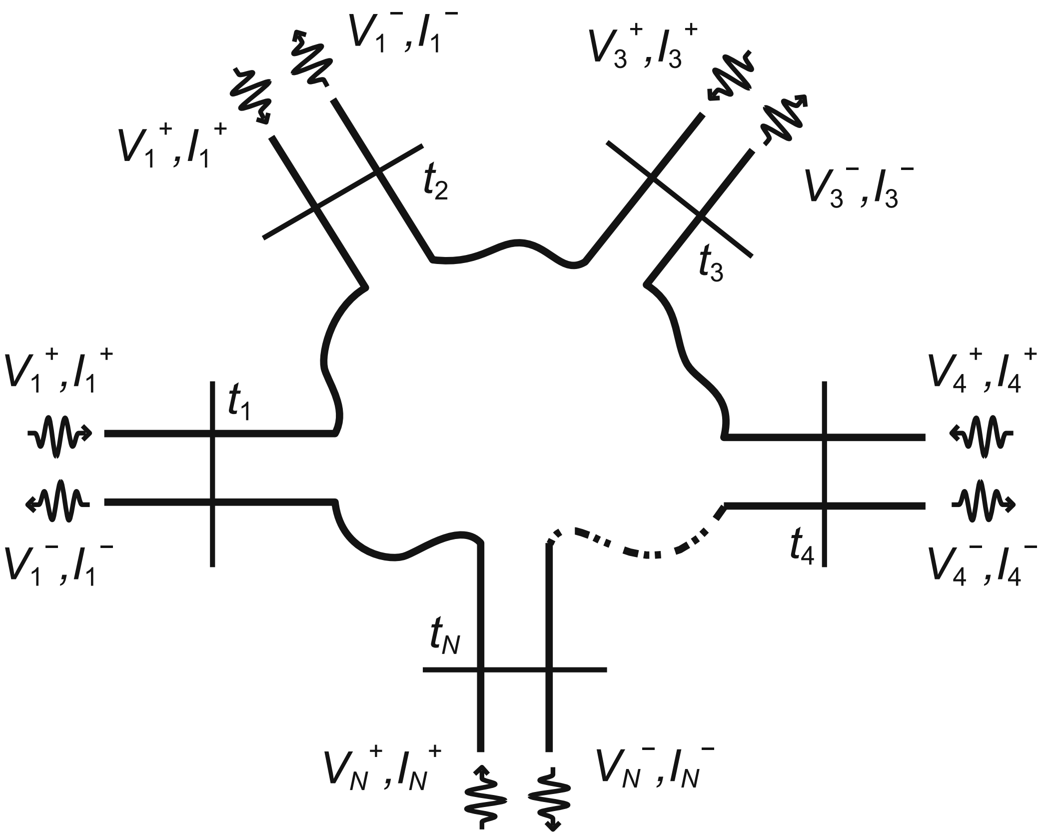
c_0: Speed of light

$\varepsilon_{\mathrm{eff}}$: Effective dielectric constant

L': Inductance per length

C': Capacitance per length

ABCD Formalism

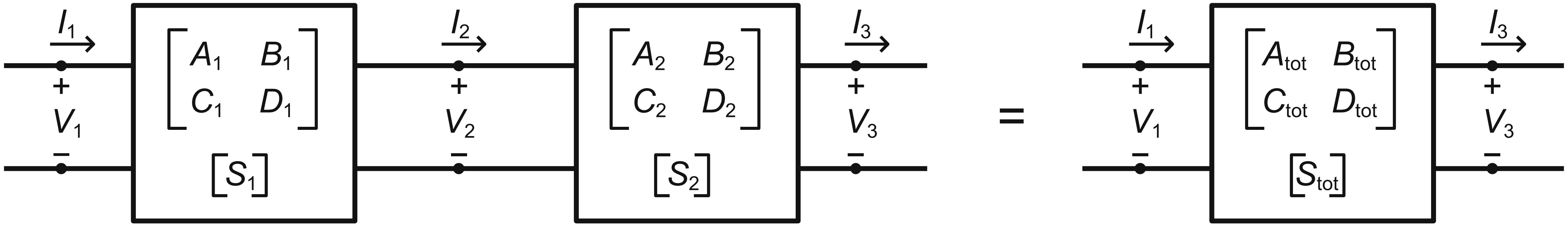


$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & & \vdots \\ \vdots & & \ddots & \\ S_{N1} & \cdots & & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

- Treat any EM-environment as **N-port** network
- S-parameters **fully characterize** the network

[1] Pozar, David M. (2012). Microwave engineering. Wiley

ABCD Formalism



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_1 \end{bmatrix}$$

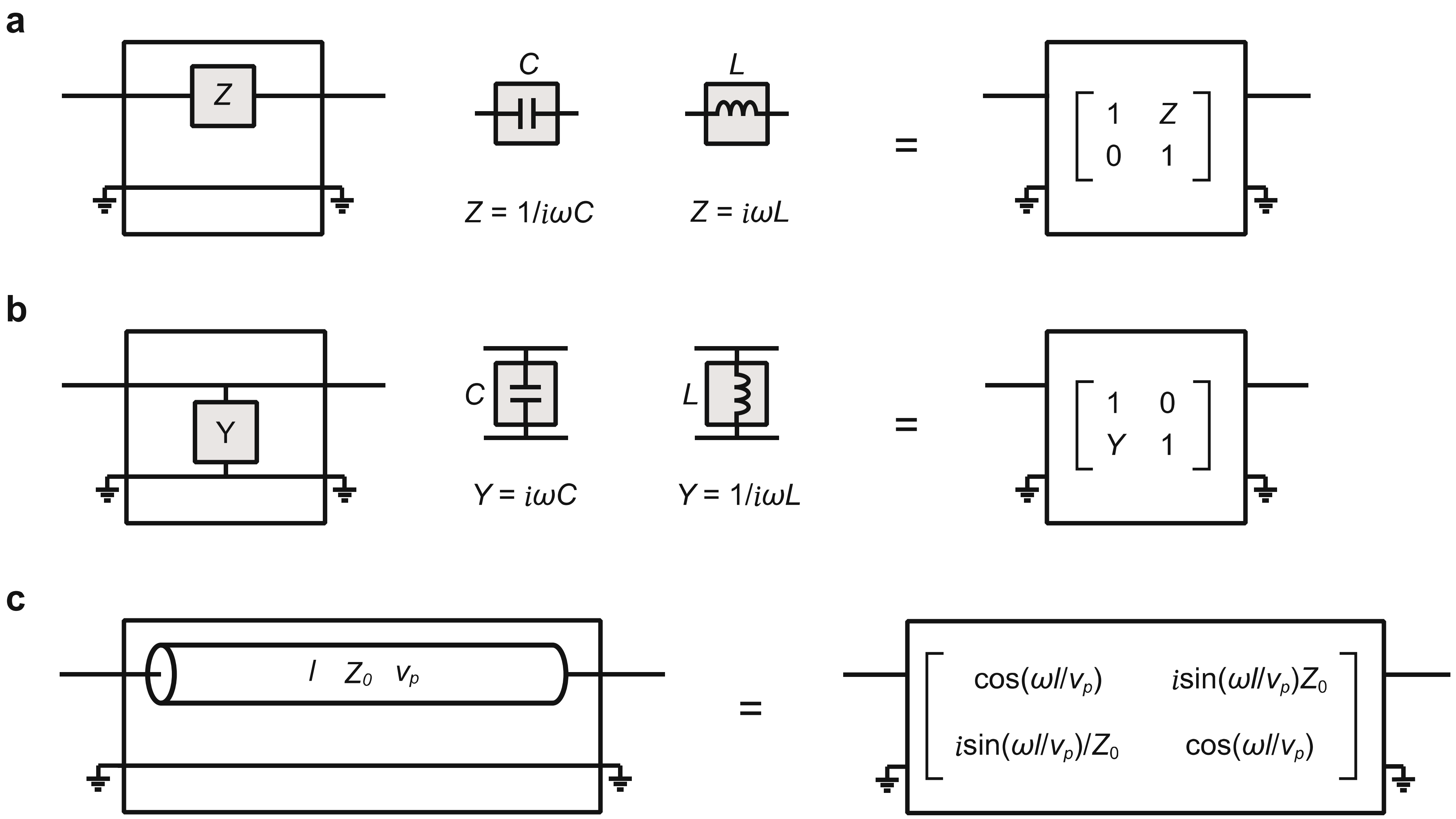
$$\begin{bmatrix} A_{\mathrm{tot}} & B_{\mathrm{tot}} \\ C_{\mathrm{tot}} & D_{\mathrm{tot}} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

$$S_{21} = \frac{2}{A + B/Z_{\mathrm{port}} + C Z_{\mathrm{port}} + D}$$

- Extensible and computationally **efficient**
- **Assumption**: Sub-circuits have no crosstalk

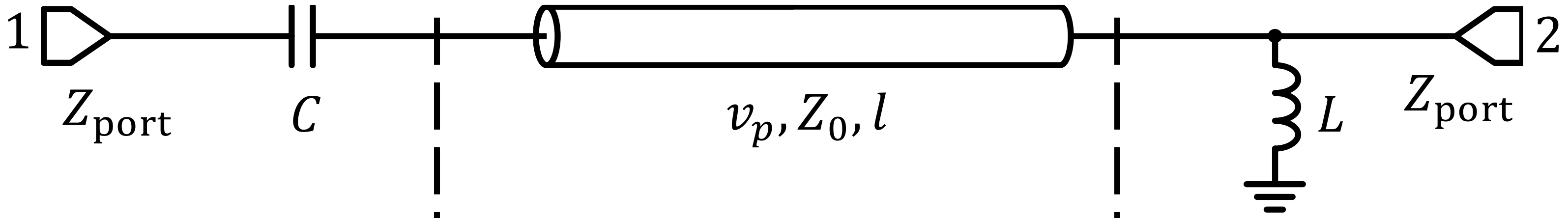
[1] Pozar, David M. (2012). Microwave engineering. Wiley

ABCD Formalism



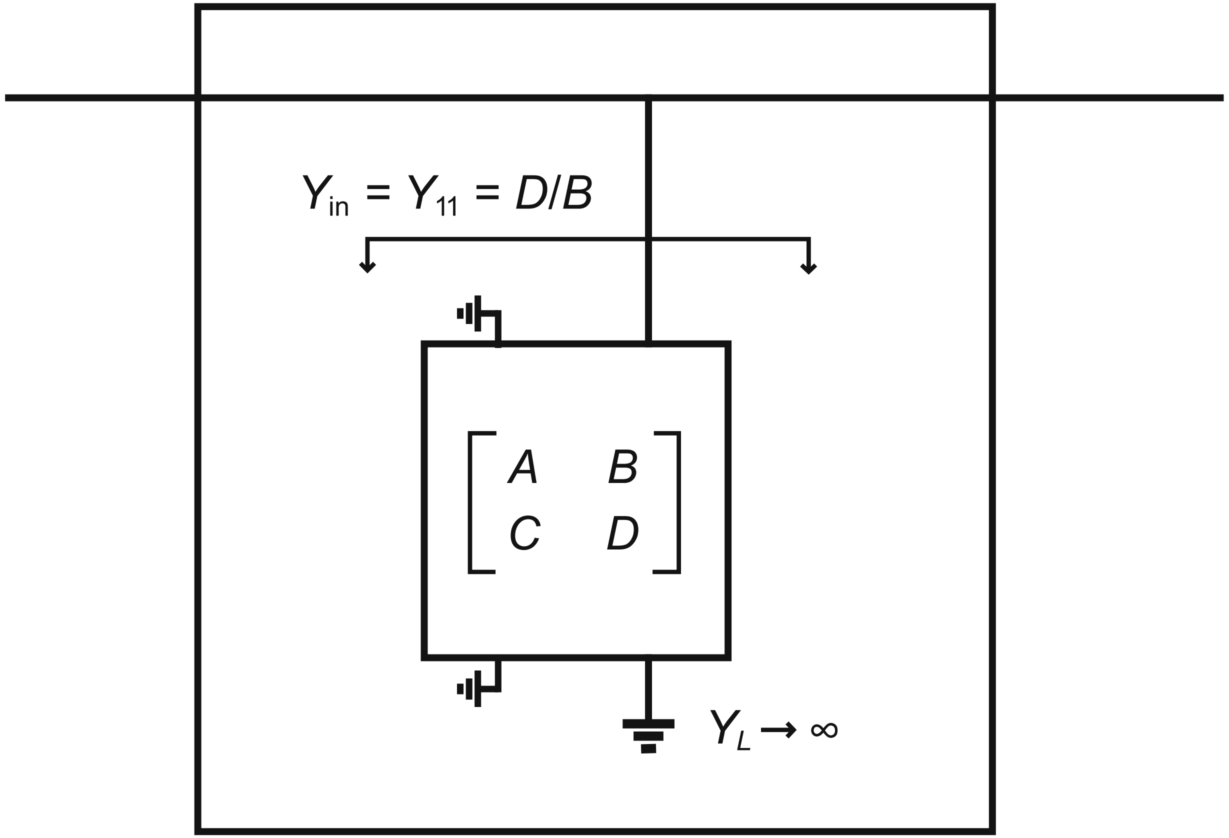
[1] Pozar, David M. (2012). Microwave engineering. Wiley

ABCD Transfer Formalism [3]: Example



$$\begin{bmatrix} 1 & 1/j\omega C \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\frac{\omega}{v_p}l & jZ_0\sin\frac{\omega}{v_p}l \\ j\sin\frac{\omega}{v_p}l\,/Z_0 & \cos\frac{\omega}{v_p}l \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 1/j\omega L & 1 \end{bmatrix} = \begin{bmatrix} A_{\text{tot}} & B_{\text{tot}} \\ C_{\text{tot}} & D_{\text{tot}} \end{bmatrix}$$

ABCD Formalism



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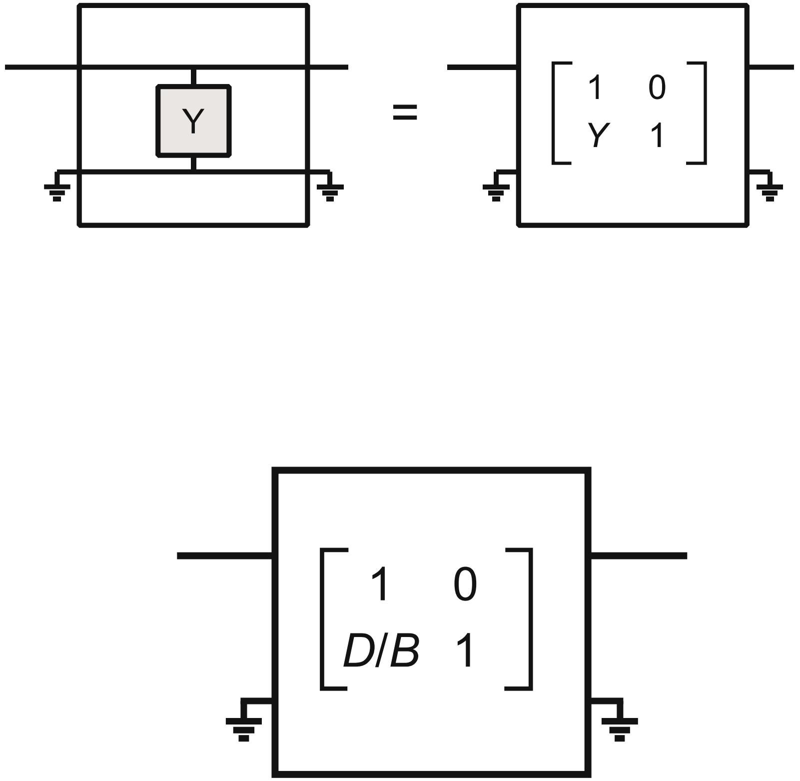
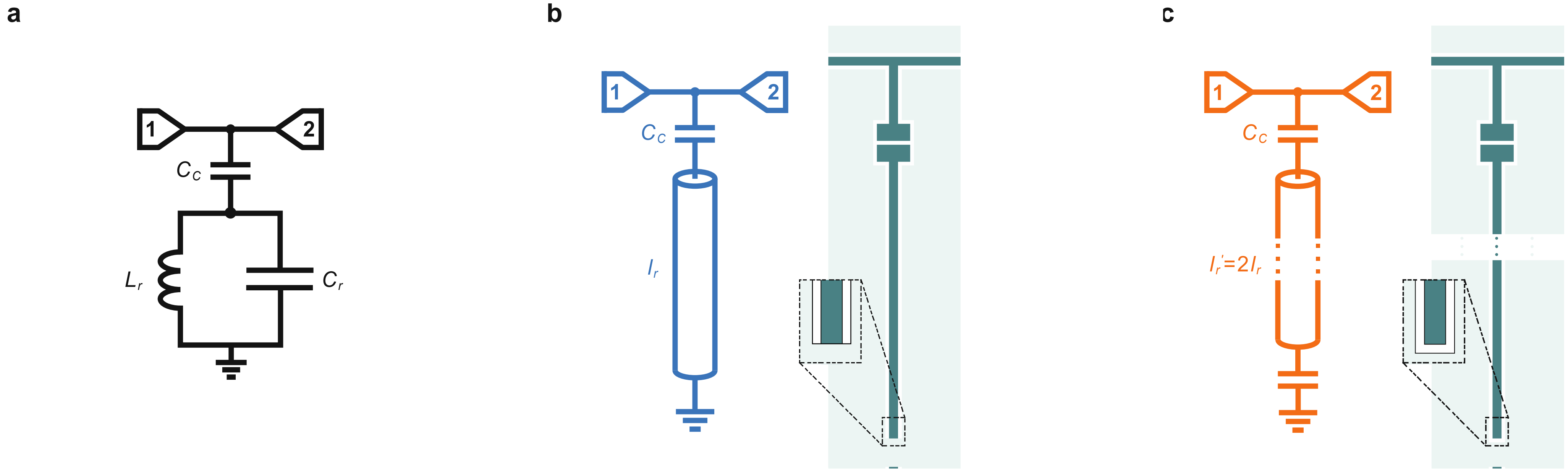


Distributed resonators



$$f_r = \frac{1}{2\pi\sqrt{L_r C_r}}$$

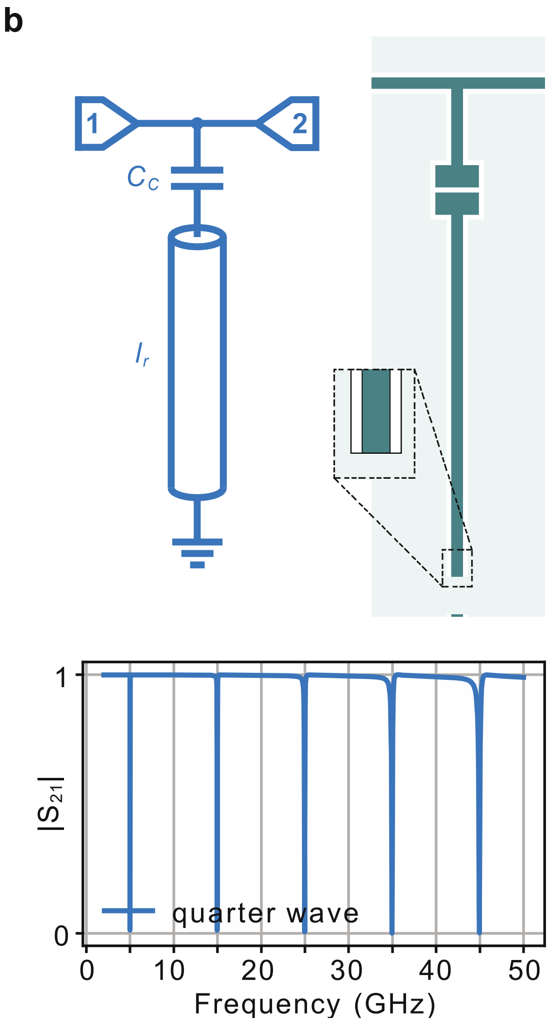
$$f_n = \frac{v_p}{4l_r} \cdot (2n-1)$$

$$f_n = \frac{v_p}{2l_r'} \cdot n$$

$$v_p = \lambda \cdot f_0$$

λ: Wavelength

f_0: Resonance frequency



Quarter-wave resonator

We can identify the resistance of the equivalent circuit as

$$R = \frac{Z_0}{\alpha \ell} \tag{6.30a}$$

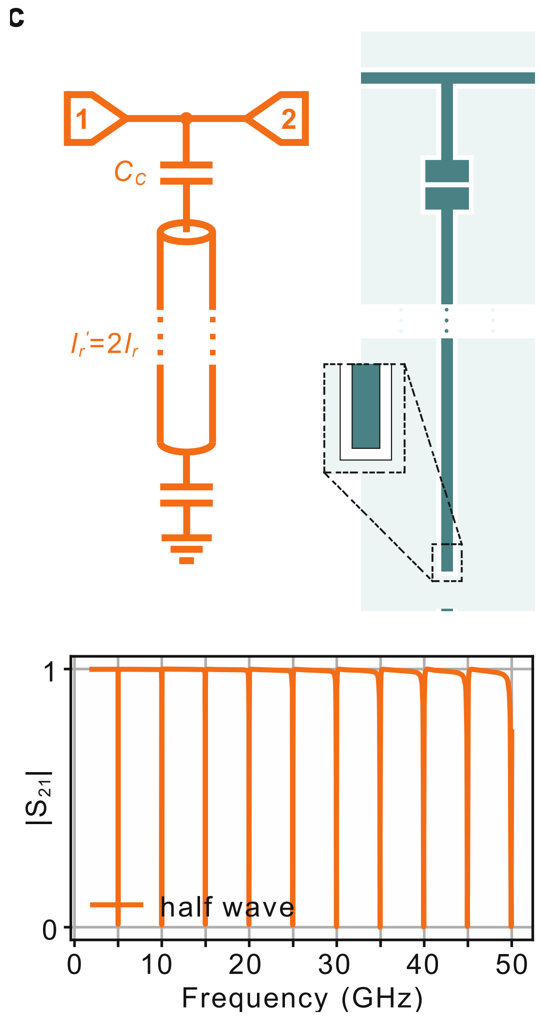
and the capacitance of the equivalent circuit as

$$C = \frac{\pi}{4\omega_0 Z_0}. \tag{6.30b}$$

The inductance of the equivalent circuit can be found as

$$L = \frac{1}{\omega_0^2 C}. \tag{6.30c}$$

[1] Pozar, David M. (2012). Microwave engineering. Wiley



Half-wave resonator

Comparison with the input impedance of a parallel resonant circuit, as given by (6.19), suggests that the resistance of the equivalent *RLC* circuit is

$$R = \frac{Z_0}{\alpha\ell}, \tag{6.34a}$$

and the capacitance of the equivalent circuit is

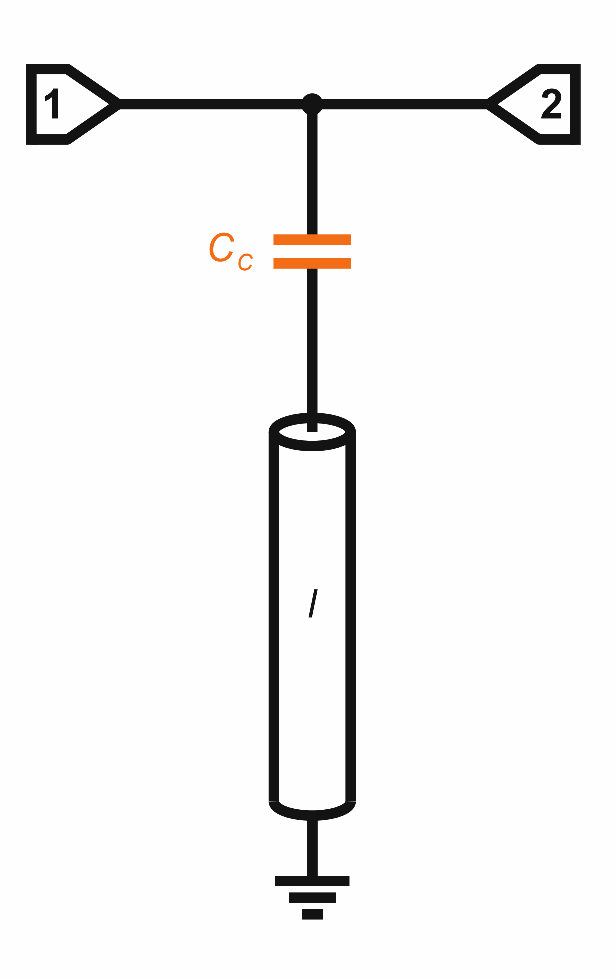
$$C = \frac{\pi}{2\omega_0 Z_0}. \tag{6.34b}$$

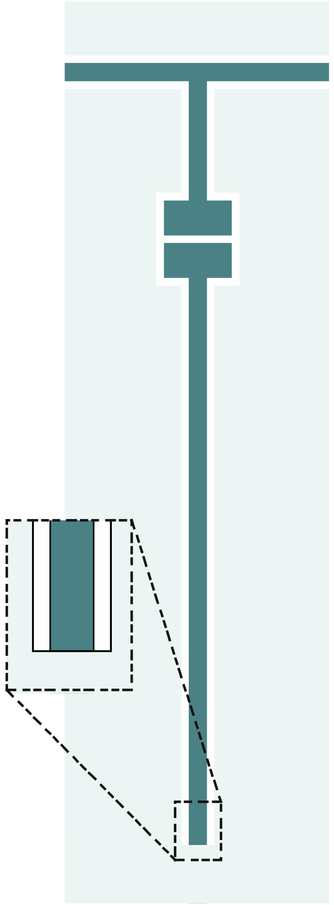
The inductance of the equivalent circuit is

$$L = \frac{1}{\omega_0^2 C}. \tag{6.34c}$$

[1] Pozar, David M. (2012). Microwave engineering. Wiley

Capacitance to ground





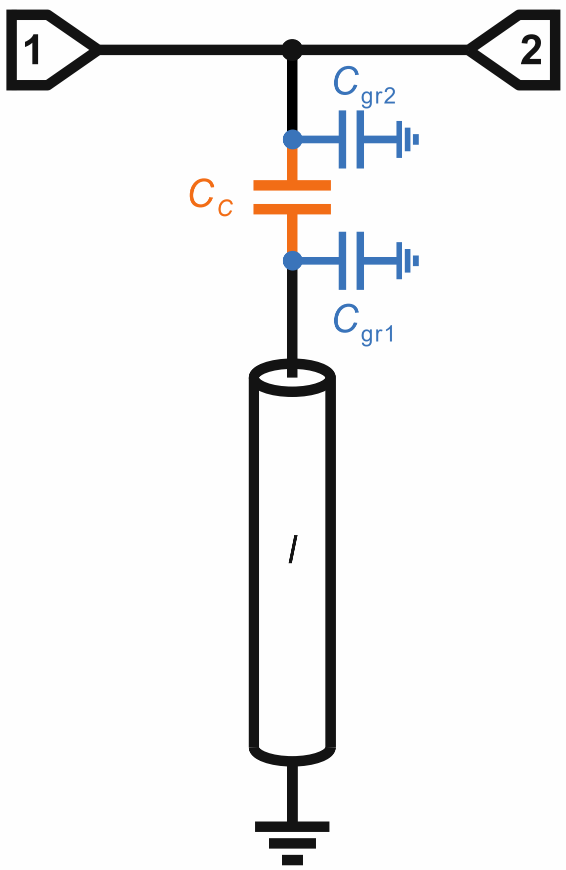
$$f_0 = \frac{v_p}{4 l_r}$$

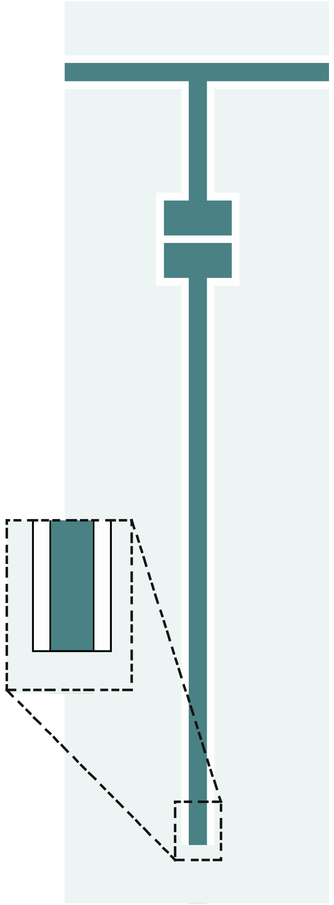
$$\frac{1}{f_0} = \frac{4}{v_p}\underbrace{\left(l + v_p Z_0 \cdot C_c\right)}_{l_{\text{eff}}}$$

Coupling capacitance:

- Increases effective length

Capacitance to ground





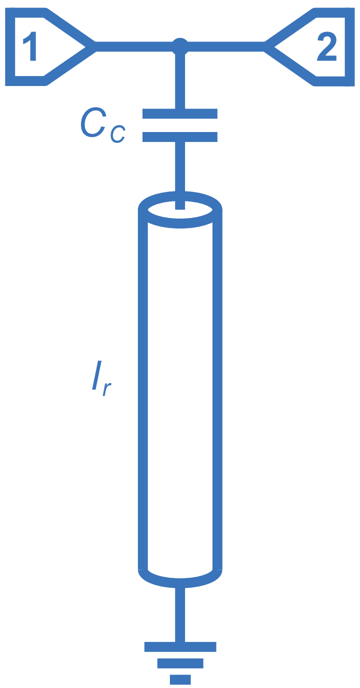
$$f_0 = \frac{v_p}{4l_r}$$

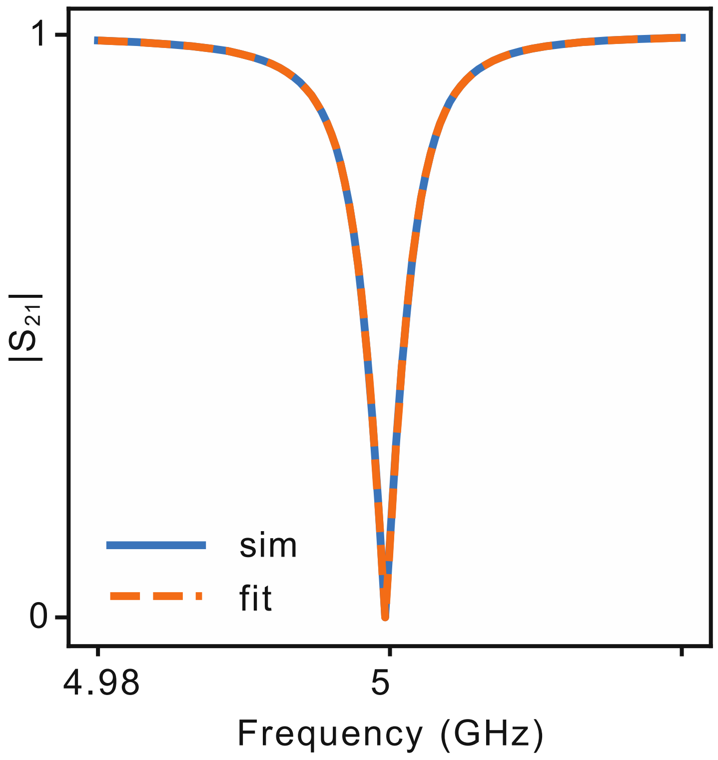
$$\frac{1}{f_0} = \frac{4}{v_p}\underbrace{\left(l + v_p Z_0 \cdot \left(C_c + C_{gr1}\right)\right)}_{l_{\text{eff}}}$$

Coupling capacitance:

- Increases effective length
- Changes the capacitance to ground

Semi-infinite Waveguide





Generate S_{21} trace using:

$$S_{21} = \frac{2}{A + B/Z_{\mathrm{port}} + CZ_{\mathrm{port}} + D}$$

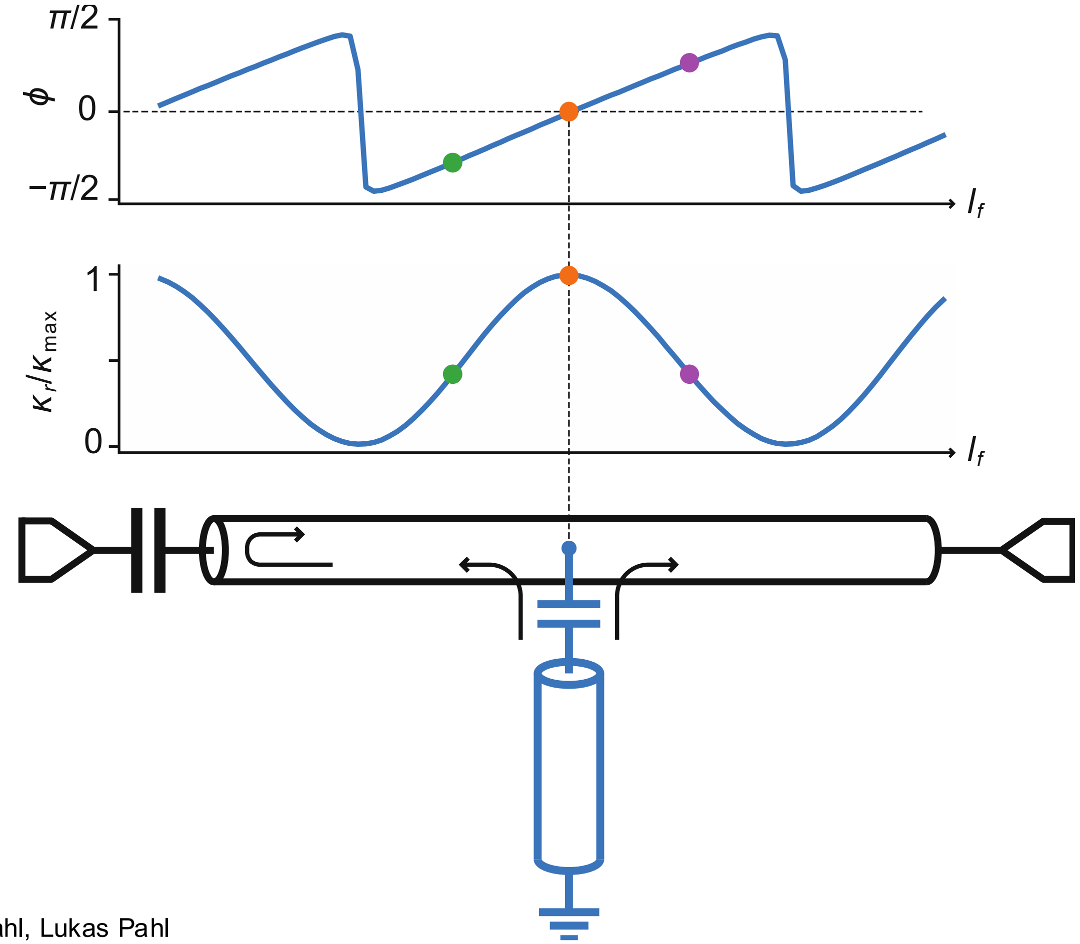
Fit S_{21} trace using:

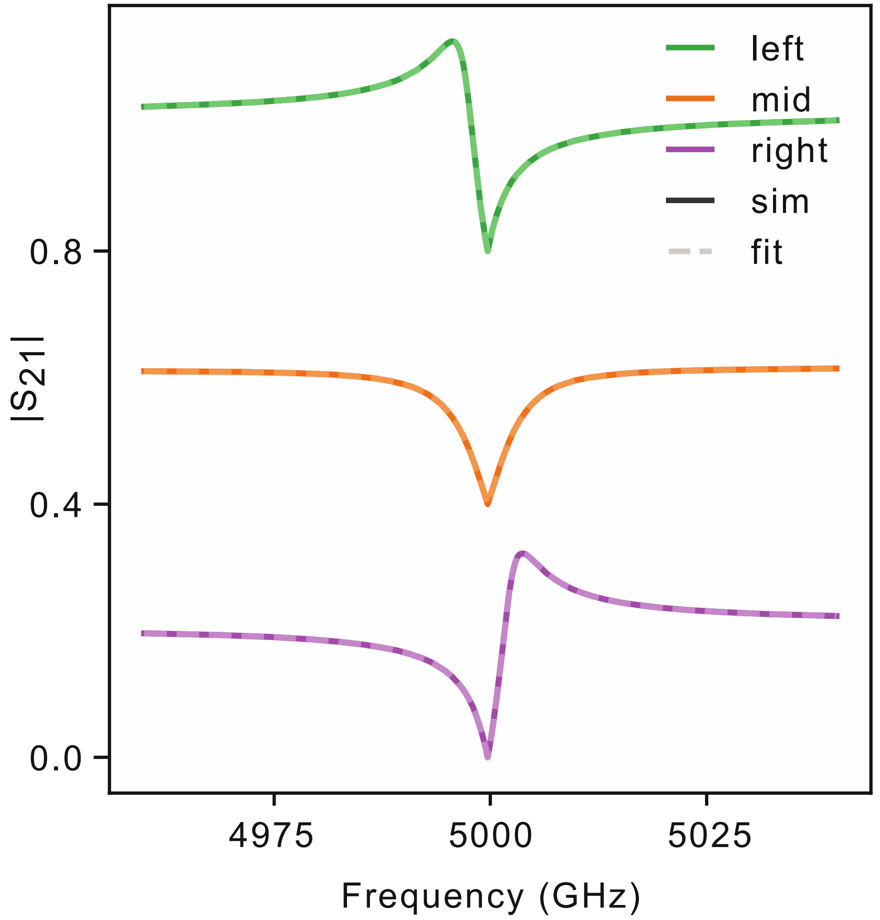
$$S_{21} = 1 - \frac{\kappa}{\kappa + 2i(\omega_r - \omega)}$$

➔ Extract ω_r and κ

Semi-infinite Waveguide

$$S_{21} = 1 - \frac{\kappa}{\kappa + 2i(\omega_r - \omega)} \quad \rightarrow \quad S_{21} \propto \cos(\phi) - e^{i\phi} \frac{\kappa}{\kappa + 2i(\omega_r - \omega)}$$





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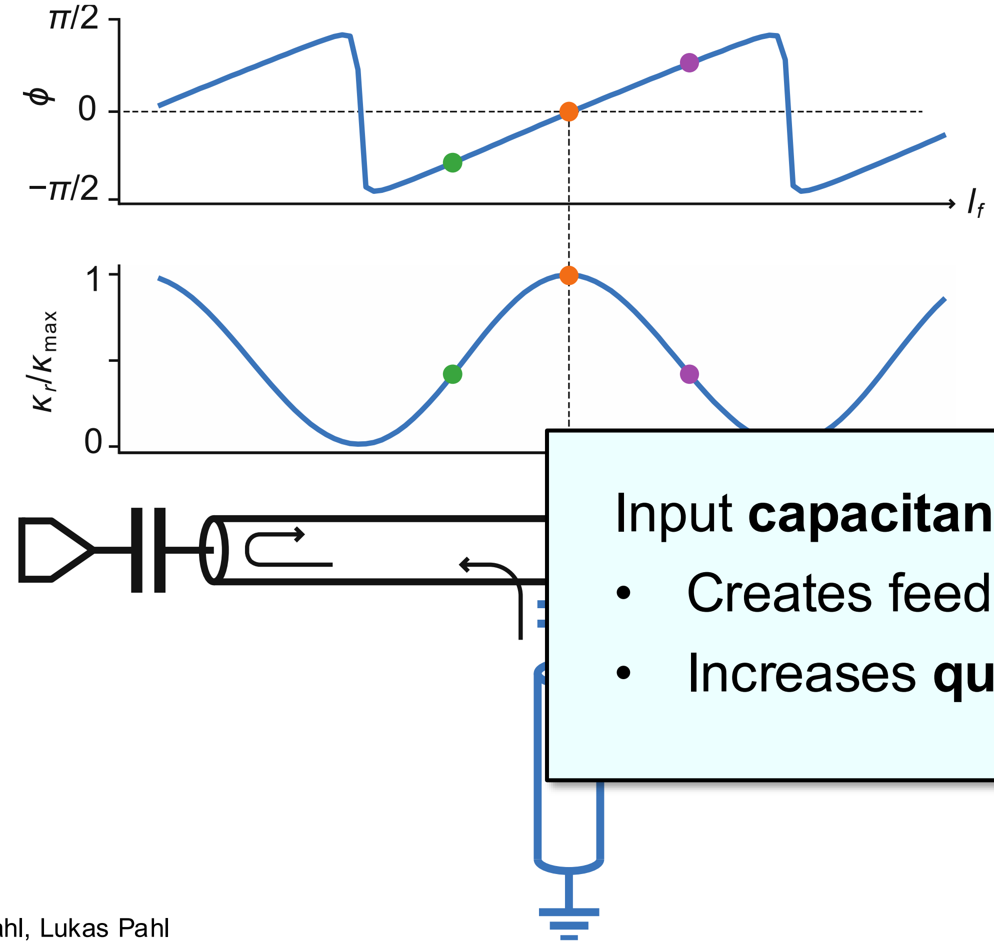
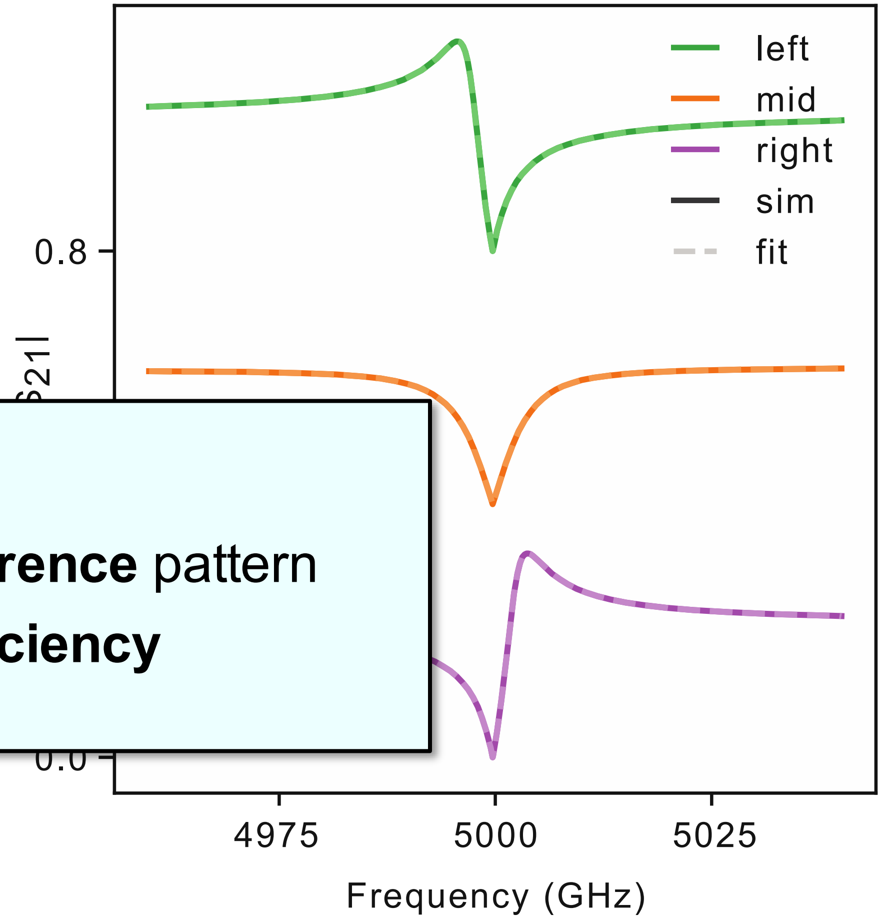
Semi-infinite Waveguide

$$S_{21} = 1 - \frac{\kappa}{\kappa + 2i(\omega_r - \omega)} \quad \Rightarrow \quad S_{21} \propto \cos(\phi) - e^{i\phi}\frac{\kappa}{\kappa + 2i(\omega_r - \omega)}$$

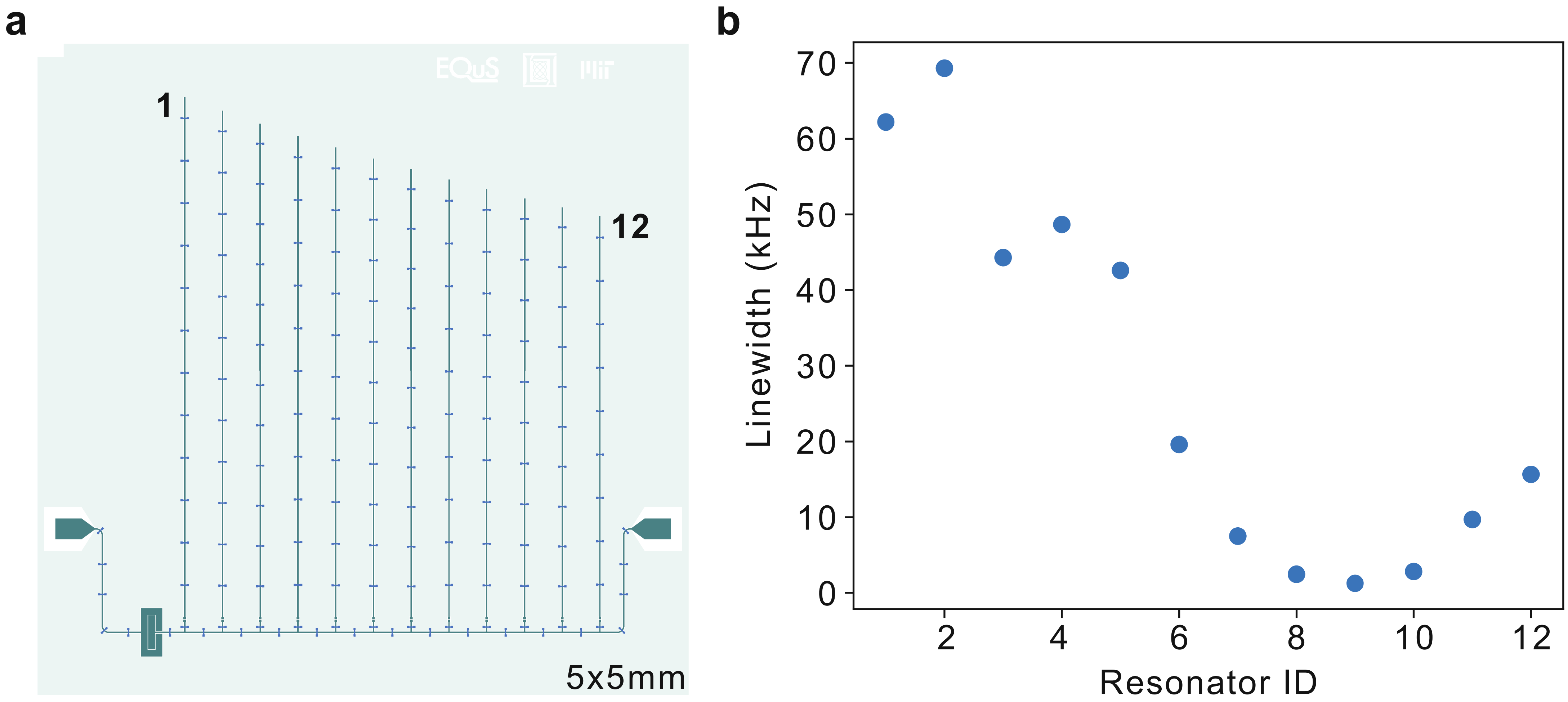




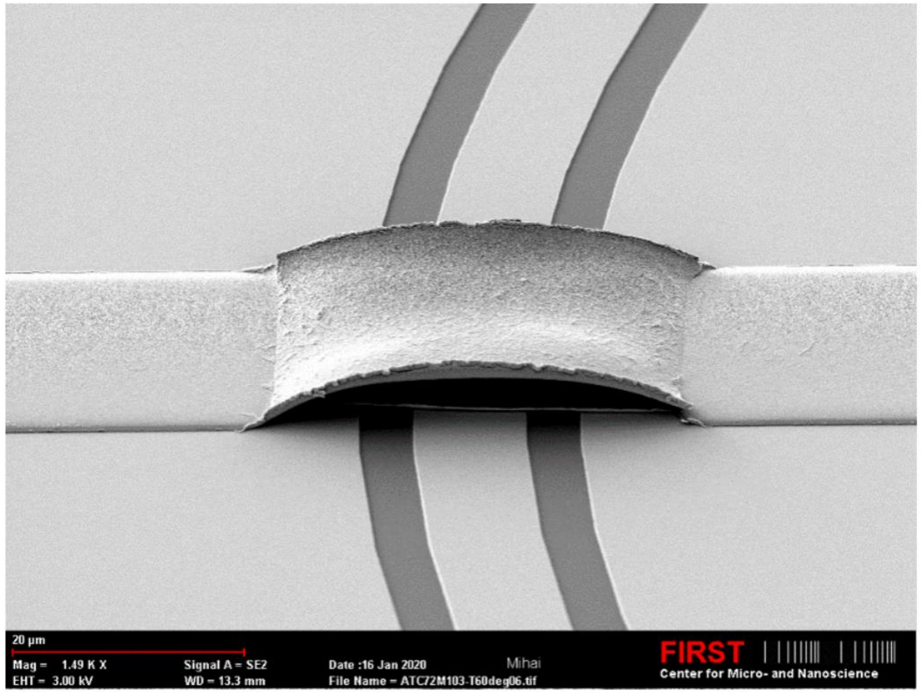
Input **capacitance**:

- Creates feedline **interference** pattern
- Increases **quantum efficiency**

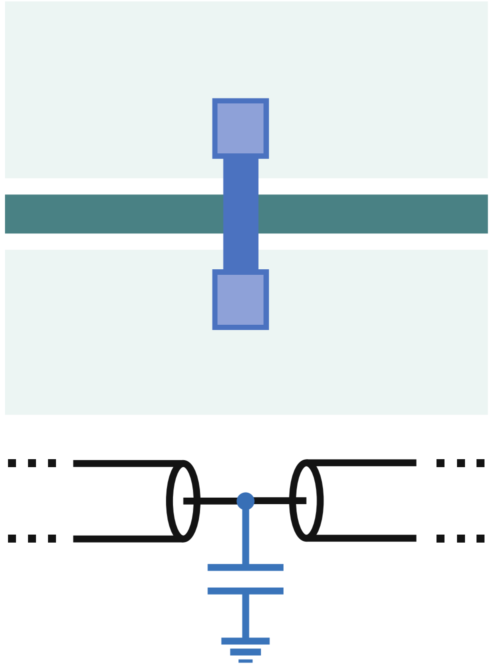
Semi-infinite Waveguide



Airbridges



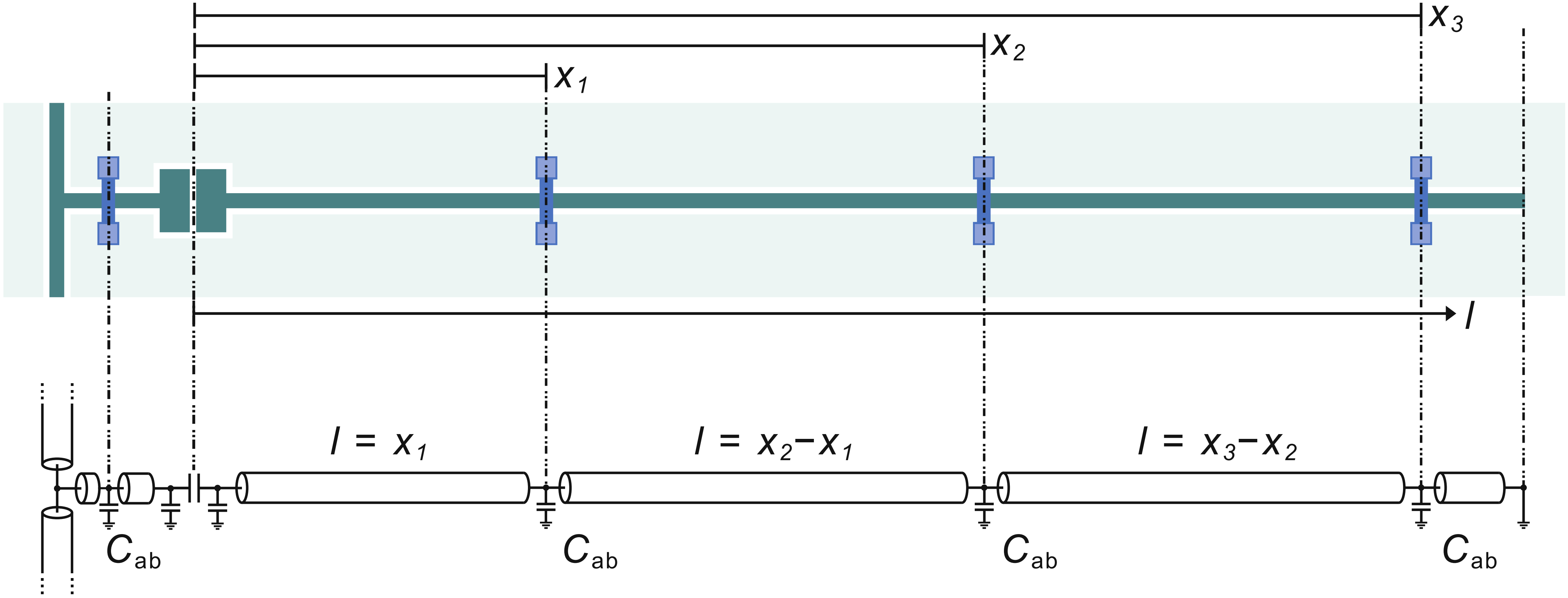
[2]



Simulated airbridge capacitance: $C_{\mathrm{ab}} \approx 1.0\ \mathrm{fF}$
(Depends on airbridge geometry)

[2] ETH Zurich, 2021

Airbridges



Closed loop simulation

Need to simulate **full feedline**