CSE300_Assignment1
Introduction to LaTeX
Fourier Analysis

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Contents

1 Fourier Representation of Signals and Systems **3**
1.1 Introduction to Signals 3
1.2 The Fourier Transform 3
1.2.1 Definition 3
1.2.2 Dirichlets conditions 4
1.3 Properties of the Fourier Transform 4

2 Fourier Series **5**
2.1 Fourier Series Expansion 5
2.2 Non Periodic function 5

3 Some Applications of Fourier Analysis **6**

1 Fourier Representation of Signals and Systems

1.1 Introduction to Signals

In mathematical terms, a signal is ordinarily described as a *function of time*, which is how we usually see the signal when its waveform is displayed on an oscilloscope.However,from the perspective of a communication system it is important that we know the *frequency content* of the signal in question. The mathematical tool that relates the frequency-domain description of the signal to its time-domain description is the Fourier transform. There are in fact several versions of the Fourier transform available. In this article we will discuss about two specific versions:

- The *continuous Fourier transform*, or the **Fourier Transform(FT)** for short, which works with continuous functions in both the time and frequency domains.
- The *discrete Fourier transform*, or DFT for short, which works with discrete data in both the time and frequency domains.

1.2 The Fourier Transform

1.2.1 Definition

Let $g(t)$ denote a *nonperiodic deterministic signal*, expressed as some function of time t. By definition, the Fourier transform of the signal $g(t)$ is given by the integral

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt \tag{1}$$

where $j = \sqrt{-1}$, and the variable f denotes frequency; the exponential function $e^{-j2\pi ft}$is referred to as the *kernel* of the formula defining the Fourier transform. Given the Fourier transform $G(f)$, the original signal $g(t)$ is recovered exactly using the formula for the *inverse* Fourier transform:

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} dt \tag{2}$$

where the exponential $e^{j2\pi ft}$ is the *kernel* of the formula defining the inverse Fourier transform. The two kernels of Eqs. 1 and 2 are therefore the complex conjugate of each other. We refer to Eq. 1 as the ***analysis*** equation. Given the time-domain behavior of a system, we are enabled to analyze the frequency-domain behavior of a system. The basic advantage of transforming the time-domain behavior into the frequency domain is that *resolution into eternal sinusoids presents the behavior as the superposition of steady-state* effects. For systems whose time-domain behavior is described by linear differential equations, the separate steady-state solutions are usually simple to understand in theoretical as well as experimental terms.

Conversely, we refer to Eq. 2 as the ***synthesis*** equation. Given the superposition of steady-state effects in the frequency-domain, we can *reconstruct the original time-domain behavior of the system without any loss of information.* The analysis and synthesis equations, working side by side as depicted in Fig. 1, enrich the representation of signals and systems by making it possible to view the representation in two interactive domains: the time domain and the frequency domain.

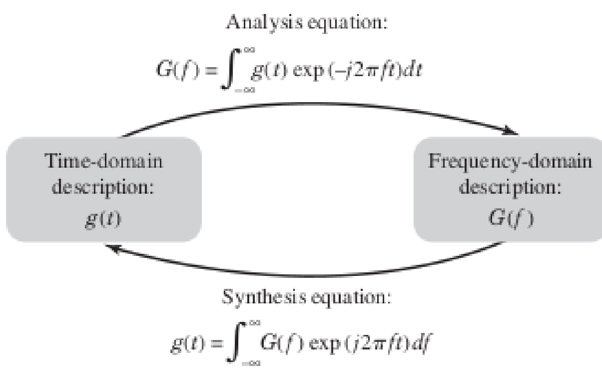

Figure 1: Sketch of the interplay between the synthesis and analysis equations embodied in Fourier transformation.

1.2.2 Dirichlets conditions

For the Fourier Transformof a signal $g(t)$ to exist, it is sufficient, but not necessary, that $g(t)$ satisfies three conditions known collectively as *Dirichlets conditions*:

1. The function $g(t)$ is single-valued, with a finite number of maxima and minima in any finite time interval.

2. The function $g(t)$ has a finite number of discontinuities in any finite time interval.

3. The function $g(t)$ is absolutely integrablethat is,

$$\int_{-\infty}^{\infty} |g(t)|dt < \infty$$

1.3 Properties of the Fourier Transform

It is useful to have insight into the relationship between a time function $g(t)$ and its Fourier transform $G(f)$, and also into the effects that various operations on the function $g(t)$ have on the transform $G(f)$. This may be achieved by examining certain properties of the Fourier transform. There are four properties of **Fourier Transform** Due to shortage of time we will only be able to discuss one property in this section.

PROPERTY 1 **: Linearity (Superposition)**

Let $g_1(t) \Longleftrightarrow G_1(f)\ and\ g_2(t) \Longleftrightarrow G_2(f)$. Then for all constants $c_1$and c_2 we have

$$c_1 g_1(t) + c_2 g_2(t) \Longleftrightarrow c_1 G_1(f) + c_2 G_2(f)$$

Property 1 permits us to find the Fourier transform $G(f)$ of a function $g(t)$ that is a linear combination of two other functions $g_1(t)$ and $g_2(t)$ whose Fourier transforms $G_1(f)$ and $G_2(f)$ are known.

2 Fourier Series

In the previous section 1.2, we studied the **Fourier Transform**. Now, we will review the formulation of the **Fourier series** and develop the Fourier transform as a generalization of the Fourier series.

2.1 Fourier Series Expansion

Let $gT_0(t)$ denote a *periodic* signal with period T_0 . By using a Fourier series expansion of this signal, we are able to resolve it into an infinite sum of sine and cosine terms. The expansion may be expressed in the trigonometric form:

$$gT_0(t) = a_0 + 2\sum_{n=1}^{\infty}[a_n cos(2\pi n f_0 t) + b_n sin(2\pi n f_0 t)] \tag{3}$$

where f_0 is the *fundamental frequency*:

$$f_0 = \frac{1}{T_0} \tag{4}$$

2.2 Non Periodic function

We can develop a representation for a signal $g(t)$ that is non-periodic in terms of complex exponential signals. In order to do this, we first construct a periodic function $gT_0(t)$ of period T_0 in such a way that $g(t)$ defines one cycle of this periodic function, as illustrated in Fig. 2 In the limit, we let the period T_0 become infinitely large, so that we may write

$$g(t) = \lim_{T_0 \to \infty} gT_0(t) \tag{5}$$

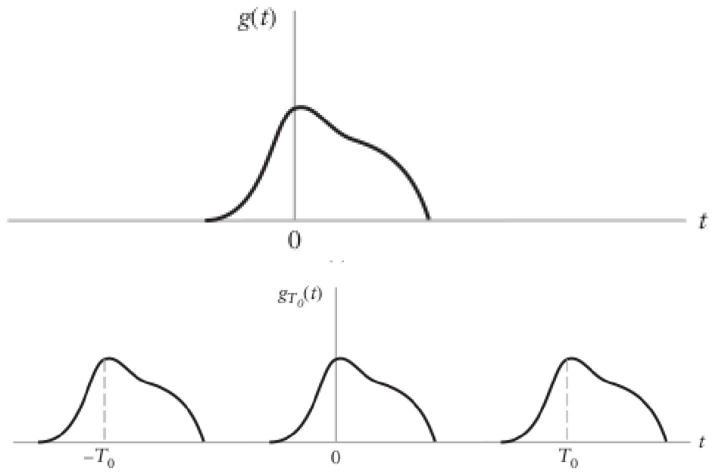

Figure 2: Illustration of the use of an arbitrarily defined function of time to construct a periodic waveform.

3 Some Applications of Fourier Analysis

Fourier synthesis The operation of rebuilding the function from these pieces is known as **Fourier synthesis**. For example, determining what component frequencies are present in a musical note would involve computing the Fourier Transformof a sampled musical note. One could then re-synthesize the same sound by including the frequency components as revealed in the Fourier analysis. In mathematics, the term Fourier analysis often refers to the study of both operations.

Fourier transformation The Fourier transform (FT) decomposes a function of time (a signal) into the frequencies that make it up, in a way similar to how a musical chord can be expressed as the frequencies (or pitches) of its constituent notes.