Introduction to Satellite Geodesy

Basics of Global Navigation Satellite Systems

Lesson (Part 2 of 2)

The presentation script will be updated soon.

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GPS Signal structure: CA code

GPS navigation signal (electromagnetic wave carrying navigation signals and messages), continuously propagating from the orbiting satellites towards the Earth's surface can be mathematically described by the following equation:

$$s(t)=\sqrt{(2A)}\,C(t)\,D(t)\cos(2\pi f_{ca}t + \phi_{ca}) \qquad \textbf{(1.20)}$$

where:

- A power of the signal
- f_{ca} the carrier frequency
- φ_{ca} phase at t_0
- P(t) pseudo random number code (PRN) code – precise+encrypted =Y
- C(t) pseudo random number code – Coarse Acquisition
- D(t) navigation data messages (Broadcast Ephemeris)

RF carrier waves

The radio--frequency continue waves are carrying:

A) C/A-Code on the L1 frequency
The system generates 1023 bits per millisecond with the chipping rate $1.023 * 10^6$ bps It corresponds to the **wave length** = 293.05 m

B) P-Code on L1 and L2
The onboard system generates **204600 chips per week, which** corresponds to the **wave length** = 29.305 m

C) Navigation message

The P-Code is an ncrypted code called W-Code.
Y-Code = P-Code · W-Code
The Y-Code protect direct using of L2-signal only by authorized users.
New: C/A code also on L5, in future on L2 also

GPS RF Carrier waves

The radio--frequency continue waves:
L1: 1575.42 MHz (19.05 cm)
L2: 1227.60 MHz (24.45 cm)
L5: 1176.45 MHz (25.5 cm) (new!)

are carrying:
- C/A-Code on the L1 frequency (Wavelength = 293.1m, period = 1 Millisecond)
New: C/A code also on L5, in future on L2 also

P-Code on L1 and L2
(Wavelength 29.31 m and period of 266.4 days)

The W-Code (encrypted code, when Anti-Spoofing active).
Y-Code = P-Code · W-Code
The Y-Code protect direct using of L2-signal only by authorized users.

Navigation signal transmitted by GPS satellites

$$S_{L1}(t) = A_P P(t) D(t) \sin(\omega_1 t + \phi_1) + A_C C(t) D(t) \cos(\omega_1 t + \phi_1) \qquad (2c.11)$$

I - term Q - term

The first NAVSTA-GPS signals ware transmitted on two frequencies *(see Lesson 4)*, *Later on three frequences (see Lesson*

More about GNSS signal generation will be given in the lectures:
"Signal Processing and Data Communication"

However at the L2 carrier the signal contains only the P-code.

$$S_{L2} = A_P P(t) D(t) \sin(\omega_2 t + \phi_2) \qquad (2c.12)$$

Summary: The four main types of the primary observables

The four main types of the GPS/GNSS primary observables are:

Doppler observable	(range-rate)	D
Code (phase) observable		P
Carrier phase observable		L
Signal/Noise Ratio		S/N (SN)

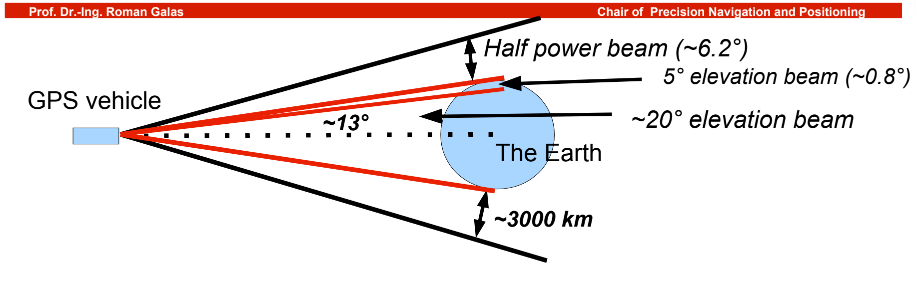
$$P_R^S = [(X^S - x_R)^2 + (Y^S - y_R)^2 + (Z^S - z_R)^2]^{1/2} + (u_R - u^S) \cdot c - (\beta_R + \beta^S) \qquad (2c.13)$$

Instrumental noise of the code-phase (code) observations is of order of
0.3 meters (P-Code) – 3 meters (C/A code).

$$L_R^S = [(X^S - x_R)^2 + (Y^S - y_R)^2 + (Z^S - z_R)^2]^{1/2} + (u_R - u^S) \cdot c + \lambda N_R^S - (\alpha_R + \alpha^S) \qquad (2c.14)$$

Instrumental noise of the carrier-phase observations is of order of
2 mm

GPS (GNSS) Signal transmissions



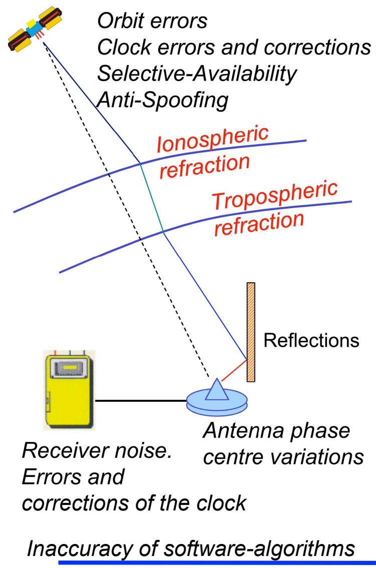
The signals are broadcast at two (in the future three) frequencies

The satellite transmission antennas point to the Earth's centre

The beam covers about 27.8°

The main beam is equal to:
L1 band: 21.4
L2 band: 23.4 dB

GPS biases and error sources – a SUMMARY



Satellites
- Accuracy of available Satellite orbits
- Accuracy and Stability of satellite clocks
- SA and AS

Signal propagation
- Ionospheric refraction
- Tropospheric refraction and accuracy of models
- Multipath effects (reflections)
- Radio-Interference

GPS Antenna
- Phase centre variations (PCV)

GPS Receiver
- Receiver clocks
- Observational/Instrumental noise
- Channel dependent delays

Data processing
- Cycle-Slips
- Ambiguities

and so on ....

Satellite-Receiver geometry

Biases (systematic errors) and random errors

Inaccuracy of broadcast satellite orbits	±3 -	±5 m
Inaccuracy of Precise IGS Orbits	±3 -	±10 cm
Inaccuracy of broadcast satellite clocks corrections	±5 -	±20 cm
Ionosphere delay/advance (if corr. not available)	1 -	100 m
Troposphere (standard atmosphere applied)	5 -	40 cm
Antenna PCV (if no correction)	1 -	10 cm
Multipath (code-phase)	1 -	10 m
Multipath (carrier-phase)	1 -	5 cm
Measurement noise (code-phase)	±10 -	±100 cm
Measurement noise (carrier-phase)	±0.2 -	±5 mm

The total influences of the random errors onto satellite-receiver range is called UERE (User Equivalent Range Error).
Accuracy of the GPS positioning depends:
- *from UERE or standard deviation of the used observations* σ_r *and*
- *from geometrical configuration of the satellites used*

Observational models of the primary observables: L & P

For (more) precise application the simplified models, (previous lesson, Page 18, equations 1.19)

$$L^S_R = [(X^S - x_R)^2 + (Y^S - y_R)^2 + (Z^S - z_R)^2]^{1/2} + (u_R - u^S)\,c + \lambda N^S_R + (\alpha_R + \alpha^S) + B_{carrier}$$
$$P^S_R = [(X^S - x_R)^2 + (Y^s - y_R)^2 + (Z^S - z_R)^2]^{1/2} + (u_R - u^S)\,c + (\beta_R + \beta^S) + B_{code}$$

should be completed with additional parameters as:
- satellite clock biases
- signal propagation delays
  - ionosphere refraction
  - troposphere refraction
- multipath
- instrumental delays
- relativistic correction
- measurement error

and some more.

(Extended) nonlinear observational models

$$P_i^s = \rho_i^s + c\,u_i - c\,u^s + I_i^s + T_i^s + \epsilon_{Ri}^s + \beta_i - \beta^s + \dots + \varepsilon_R \qquad (2.1)$$

$$L_i^s = \rho_i^s + c\,u_i - c\,u^s - I_i^s + T_i^s + \epsilon_{\Phi i}^s + \lambda(\alpha_i - \alpha^s - N_i^s) + \dots + \varepsilon_\Phi \qquad (2.2)$$

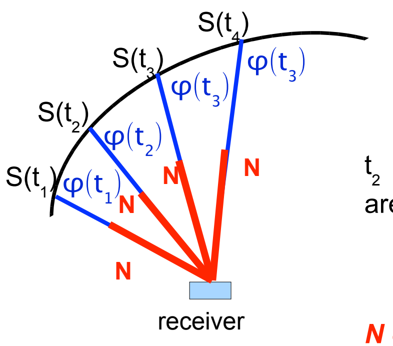
$$B \equiv \lambda(\alpha_i - \alpha^s - N_i^s) \qquad (2.3)$$

The system of observational equations (2.1) and (2.2) can be used for positioning with GPS/GNSS.

However achieved accuracy is
influanced by various biases and random errors
and inaccuracies of the applied models parameters.

Carrier phase ambiguity

Only the fractional phase (modulo 360° / 1 cycle) of the first measurement during the signal acquisition is recorded. There is no way to measure the total phase (between the satellite's and the receivers's antennas) of the incoming satellite signal.



t_1 - Satellite locked. Receiver started to track.
Reading of the phase value = $\varphi(t_1)$
Integer number of waves corresponding to the distance is ambiguious, because N is unknown.

t_2 - Satellite locked. Receiver tracks. Full waves are
continuously counted (measured id only the floating part).
Reading of the phase value = $\varphi(t_2)$

N can be calculated during processing of observations

Carrier phase ambiguity

GNSS field observation approaches

There are two main field **observation** approaches *(data recording)*:
"Static" and
"Kinematic"

The recorded primary observables will **processed** for PVT solutions in:
„**real-time**", or
post-processing mode (in an office).

Various GNSS **processing algorithms** allow to estimate coordinates, clock corrections and velocity in two main methods:
Absolute, or
Relative
Positioning.
In addition the two above (abolute, relative), various
Differential corrections
methods have been also developped.

GNSS observation- and processing approaches

In addition to the previous:

Static GNSS positioning:
- Permanent
- Long static (simply called "static")
- Rapid static (known also "short static")

Kinematic GNSS positioning:
- "STOP and GO"
  - off-line post-processing: "Stop and go"
  - Processing in real-time: RTK (Real Time Kinematic)
- Continuous kinematic approach (simply „kinematic")

Main methods of GPS/GNSS data processing

Positioning, surveying

Standalone/Absolute

Post & RT Kin. / Stat.
PPP

Relative

Post-processed ***Real-Time***

Static
Permanent Static
(Long) Static
Rapid static (*also known as "short static"* or *"pseudo-kinematic"*)

Kinematic
(Pure) Kinematic
Semi-kinematic
called also "stop-and-go"

Processing of the carrier-phase obs. (CDGPS/PDGPS)

RTK
(Real-Time Kin.)

Navigation

Navigation is always kinematic (or "stop-and-go") and in a "real-time"

Differential corrections

Absolute *(Standalone)*

Single Point Pos.:
- code obs.,
- smoothed code

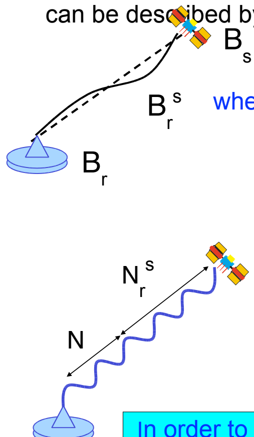
- code & carr. ph.

Code-phase differential (DGPS)

Mod. of Langlay's "RTK GPS", GPS World, Sept. 1998. R. Galas, TUB,2008

Very general model of GPS ranging observables

Let us (temporary) assume that GPS-measurements at a particular observation epoch can be described by the following simplified equation:



$$O_r^s = \rho_r^s + B^s + B_r + B_r^s + (b_r^s) + \varepsilon_r^s \qquad (2.4)$$

where
- ρ_r^s - geometrical distance between satellite s and receiver pair
- B^s - satellite dependent Bias-Parameters
- B_r - receiver dependent Bias-Parameters
- B_r^s - electromagnetic wave transmission dependent bias
- ε_r^s - measurement noise
- b_r^s - bias specific to a receiver/satellite pair (in case of positioning with ambiguous ranges)

Subscripts refer to a particular receiver and superscripts refer to a particular satellite.

In order to reduce (or to eliminate) influence of GPS biases, "new" observations can be derived from the original ones. Those "new" obs. are created from the "raw" ones ***by forming various types of differences and/or linear combinations of the primary observables.***

1. Absolute methods

1.1 Autonomous/**absolute single point positioning with pseudo-ranges** derived from code phases

1.2 Autonomous/**absolute single point positioning with code-smoothed by carrier phases**

1.3 **Precise Point Positioning** approach (PPP)
1.4 Position, Velocity and Time with Doppler observables

Selection of the method of observation and processing depends on:
- required accuracy (not allways the highest one is needed)
- availability of external products (satellite clock corrections, orbit parameters, model parameters, . . .).

Let's introduce

a simple absolute point positioning approach

for

(autonomous) standard point positioning (SPP)

with: Broadcast Ephemeris (BCE) and
code measurements, or
code measurements smoothed by carrier phase measurements

It is sometimes referred as navigation solution.

Procedure:
Linearisation of the system of the non-linear equations

$$P_r^s = \rho_r^s + c\,u_r - c\,u^s + I_r^s + Z_r^s + \beta_r - \beta^s + d\rho_{mul,r} + \ldots + \varepsilon_R \quad \textbf{(2.5)}$$

$$\rho_r^s = [\,(X^s - x_R)^2 + (Y^s - y_R)^2 + (Z^s - z_R)^2\,]^{1/2}$$

The biases must be calculated or cancelled, as far as it possible, or modeled.
Instrumental noise of the code-measurements is 3m (C/A) or 30cm (P)

Absolute point positioning with code observations (1)

Calculation of the observer's position from code observations of a single receiver was already very briefly discussed in previous lessons.

Absolute positioning – Single Point Positioning:
Done: Observations
Orbits
Satellite clocks
Estimation of some correction terms
Calculated: Station coordinates and receiver clock correction (PNT)
Procedure:
Linearisation of the system of non-linear equations
Solution by:
Least Squares Adjustment, or
Kalman filter
Post-processing (off-line) or „Real-Time“ (on-line) processing

Accuracy of absolute posiitoning with the code-measurements of order of 10 to 30m is possible

Absolute Single Point Positioning with code- smoothed by carrier phase observables

PRN Code: Nominal instrumental accuracy: ca. 30 cm – 3 m. (1% of code length)

RF Carrier: L1: 1575.42 MHz (19.05 cm) , L2: 1227.60 MHz (24.45 cm)
Nominal instrumental accuracy: ca. 2 mm. (1% of wave length)

Why smoothing code- with carrier phases?
- to reduce noise
- omit challenge of carrier phase ambiguity (see later viewgraphs below)
- and achieve better accuracy in the navigation software module of GPS receiver
- enhance in a simple way accuracy of autonomous single receiver navigation (real-time)

Usually ionosphere free combinations of code- and carrier phases are used (requirement: multi-frequency receiver).

Cycle slips must be detected (but they must not be corrected → re-initialization)

Precise Point Positioning approach (PPP)
Static and kinematic

Multi frequency receiver absolutely required → ***Ionospheric free combination of code- and carrier phases used***

Unknown parameters: coordinates and time
Biases must be modelled (see equations 2.1 - 2.3):
- ***Ionosphere** – higher order terms needed*,
- troposphere,
- satellite clocks,
- antenna phase centre offset, antenna PCV,
- phase wind-up, relativity, … .

Required:
- Precise orbits and satellite clocks (delay of order of 10 days)
- Perfect Ionosphere models
- Ambiguity estimation issues (among them calibration of the hardware delays, ...)

PPP (cont.)

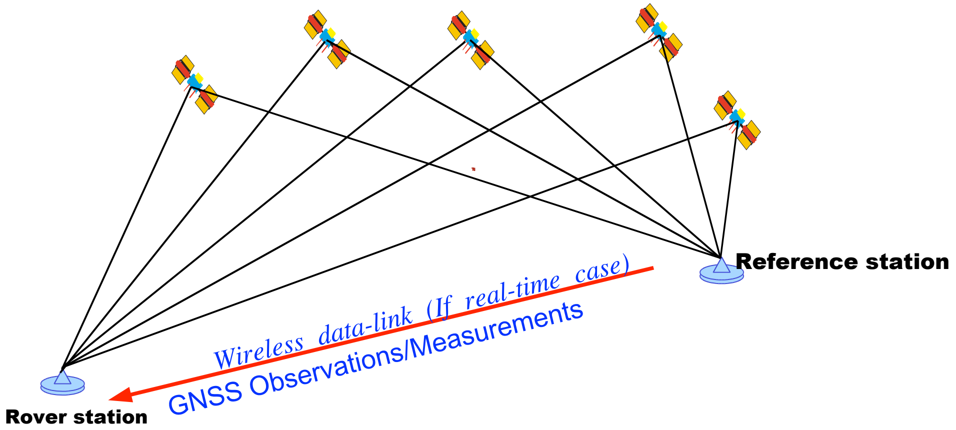
Parameter estimation: Kalman filter (forwards and backwards) is broadly used for kinematic and static applications.

Unknowns:
- station coordinates
- receiver's clock correction
- tropospheric bias
- carrier-phase ambiguities

Derived coordinates should be corrected (depends on application) for:
- motion of tectonic plates
- Earth tides,
- ocean and atmosphere loadings
- polar motion, ...

2. Relative positioning methods: static & kinematic, post-processing & real-time

Calculation of coordinate differences between two stations from the synchronous Observations of the same satellites at the both stations. Coordinates of one of them (reference station)must be known.



Relative positioning: static / kinematic

At least two receivers must be operated simultaneously on both sites, ***„reference“*** receiver and a “***slave***” one.

Coordinates of the reference station must be given in a well defined reference frame (for example WGS 84, ITRF, …)

If $\underline{\mathbf{x}}_R$ - ***known*** state vector of the reference station $[x_R, y_R, z_R]^T$

$\underline{\mathbf{x}}_S$ - ***unknown*** state vector of the user's (slave) receiver $[x_S, y_S, z_S]^T$

$\underline{\mathbf{d}}_{RS}$ - vector between the both, which components are to be determined with GNSS, then

$$\underline{\mathbf{x}}_S = \underline{\mathbf{x}}_R + \underline{\mathbf{d}}_{RS} \qquad (2.6)$$

Components of the vector “reference-slave” (baseline)

$$\underline{\mathbf{d}}_{RS} \ ([\Delta x_{RS}, \Delta x_{RS}, \Delta x_{RS}]^T)$$

can be derived by processing of.
code-, or carrier-,
or Doppler or combined observations (e.g. code & carrier)

Non-linear model for relative positioning with DD carrier phases (see lesson)

Double differenced observations (second differences), can be then written
As differences between then the first ones $\nabla\Delta L^{jk}_{AB}(t) = \Delta L^{k}_{BA}(t) - \Delta L^{j}_{BA}(t)$

$$= \underbrace{\rho^{k}_{B}(t) - \rho^{k}_{A}(t) - \rho^{j}_{B}(t) + \rho^{j}_{A}(t)}_{\nabla\Delta\rho^{jk}_{AB}(t)} + \underbrace{\lambda N^{k}_{B} - \lambda N^{k}_{A} - \lambda N^{j}_{B} + \lambda N^{j}_{A}}_{\lambda\nabla\Delta N^{jk}_{AB}} + \dots$$

$$\boxed{\nabla\Delta L^{ji}_{BA}(t_k) = \nabla\Delta\rho^{ji}_{BA}(t_k) + \lambda\nabla\Delta N^{ji}_{BA} + \dots} \qquad (2.9)$$

$$\nabla\Delta L^{ji}_{BA}(t_k) = \nabla\Delta\rho^{ji}_{BA}(t_k) + \lambda\nabla\Delta N^{ji}_{BA} - \nabla\Delta I^{ji}_{BA}(t_k) + \nabla\Delta Tr^{ji}_{BA}(t_k) + \varepsilon_{\Phi}$$

$$\nabla\Delta P^{ji}_{BA}(t_k) = \nabla\Delta\rho^{ji}_{BA}(t_k) + \nabla\Delta I^{ji}_{BA}(t_k) + \nabla\Delta Tr^{ji}_{BA}(t_k) + \varepsilon_{P} \qquad (2.10)$$

Reachable accuracies in *relative positioning* with code & carrier phases. Ambiguity fixed solutions

Method	obs. period	comments	accuracy
Static	>24 hours (days)	network solutions	3/6 mm
Static	several hours	network solutions	~2 cm
Rapid static	few minutes	baseline solution (baseline < 12 km)	2-3 cm
Stop and Go	few minutes	short distance to reference station	3-5 cm
Kinematic	few minutes	short baselines (d < 12 km)	10 cm

"Stop-and-go" or "Semi-kinematic positioning"

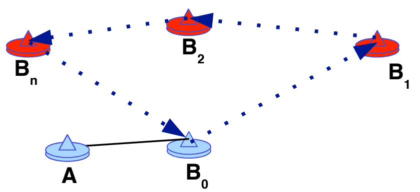
Stop and Go (similar as Rapid Static) but the signal must be continuously traced while the slave receiver is roving through all stations

1) Reference receiver on site A continuously tracking

2) Rover receiver occupies B_0 ca. 30-40 minutes and initial carrier phase ambiguity will be fixed. The length of this „first" baseline should be short

3) Slave (rover) receiver moves (kinematic way) to B_1 and records GPS data. If no loss-of-lock, the fixed ambiguities remain unchanged and a short (very few minutes) static observation period is enough to determine static coordinates of B_1

4) Slave receiver visits all stations B_i collecting data (static)



The receiver is tracking the satellite signal continuously and without loss off lock, otherwise longer re-occupation is needed

"Stop and Go" or „semi kinematic"

The method is no more used. It „converted" to the "RTK"

Data ware stored in the field, and

in the office downloaded and post-processed.

Accuracy depends on DOP, number of satellites tracked, receiver used (single/dual frequency) … .

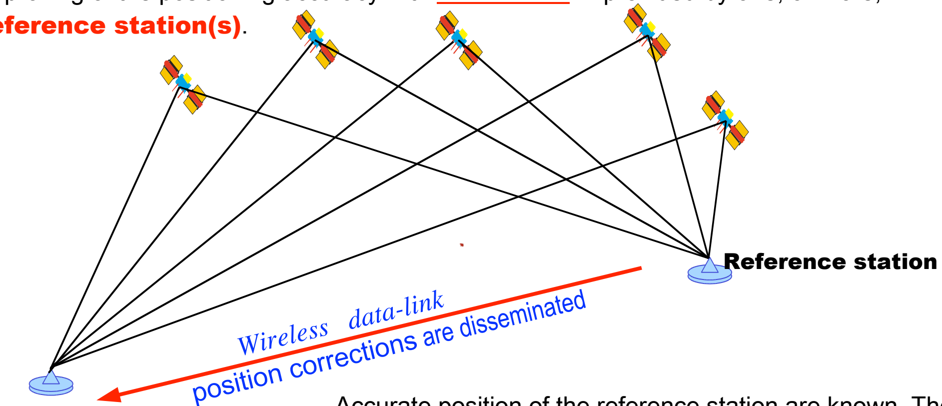
Mathematical model is exactly the same as that given in one of the previous lessons. i. e. :

$$\boxed{L^{jk}_{AB}(t) = \rho^{jk}_{AB}(t) + \lambda N^{jk}_{AB}} \qquad (2.11)$$

The approach is used for a local surveying in an area of very few kilometres (short baselines!)

Differential GNSS with a reference station (DGNSS)

Improving of the positioning accuracy with ***corrections*** provided by one, or more, **reference station(s)**.



Accurate position of the reference station are known. The "current" position is continuously calculated.

DGNSS with: a) station coordinates, b) code-phases, c) carrier-phases

RTK – Real Time Kinematic

It is, as the previous one,
carrier-phase-based GNSS relative positioning,
but it is real-time technique.

and it is also called
carrier-phase differential GPS
or (another name)
Real Time Kinematic (RTK)

However it is a „Semi-Kinematic“ method!

It requires at least two receivers, base (reference) station and a rover station.
tracking simultaneously
the same satellites *(only common satellites are processed)*.
The receivers are recording observables with 1Hz rate. The observations from the reference station and the reference coordinates are transmitted to the rover receiver(s).

Wide- & Local Area Augmentation services

Corrections in the state space domain: WADGPS

In an area there are operating continuously reference stations.

From the recorded data a set of "space correlated" corrections is generated and transmitted to the authorized users.

The corrections are valid for the working area of course.

This approach allows for application of the DGPs over a large areas. The systems are called Local and Wide Area Differential GPS

LADGPS, WADGPS
Space Based Augmentation Systems
Ground Based Augmentation Systems

The concept will be discussed in lectures "Methodology of the GNSS".

Wide Area Augmentation System

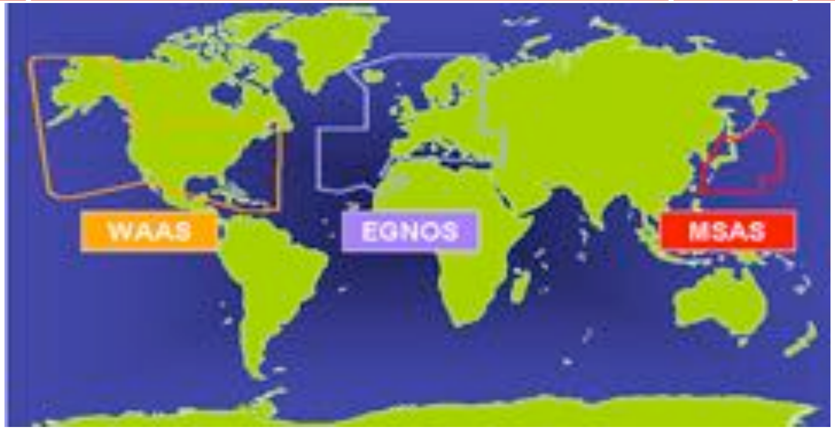
The system is composed of:
- GPS ground tracking stations
- Reference stations
- Two geostationary Inmarsat satellites (Broadcasting of corrections)

The SBAS corrections are calculated on the base of all measurements from all reference stations. The values of corrections are issued for each node of a grid covering the service area.
Each particular user can calculate its individual corrections and can correct own coordinates.

WAAS signals are valid only on the territory of US.

The WAAS corrections can be applied only there

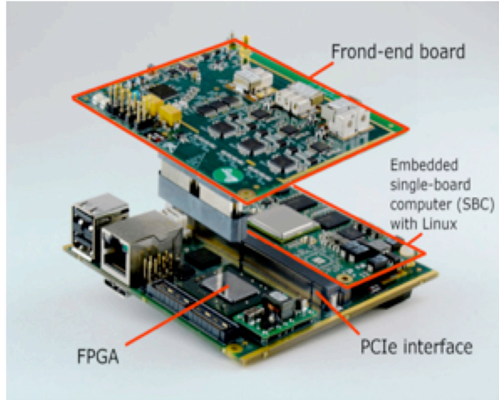
SBAS -Space Based Augmentation Systems



Availability of the SBAS corrections depends on:
- location of the RIMS stations, and
- in which geographical area SBAS signals from the geostationary satellites can be received (foot-plots).

Selected themes in research and development

- Fundamentals of GNSS
- GNSS algorithms for post- and real-time processing
- GNSS augmentation systems (e.g. local-area augmentation
- GPS/Galileo digital signal processing (for software receivers)
- Real-time data communication
- GPS/INS integration:
  loosely & tightly coupled systems,
  ultra-tight & deep integration
- Outdoor-to-indor navigation
- Analysis of time series of
  measurements in time-domain
  and in spectral-domain



Selected themes in research and development *(cont.)*

Development of the software **„TUB-NavSolutions“** for research and learning.
The software package includes various modules for positioning & navigation.
Among others: attitude control of UAV and ships.

My lectures:

1) **Methodology of positioning and navigation with GNSS**
   2nd semester (Summer), 3 CP, portfolio examination

2) **GNSS (real-time) signal processing and data communication**
   3rd semester (Winter), 3 CP, portfolio examination

3) **Space Geodesy and Navigation Seminar** *(shared with other Profs)*

4) **Space Geodetic techniques** *(Prof. H. Schuh, Dr. B. Männel, Prof. R. Galas)*