

[30] **Homework 3. Programming Assignment**

The goal of this assignment is to find good approximations of

$$H_n^{(2)} = \sum_{k=1}^n \frac{1}{k^2}$$

and

$$n! = 1 \cdot 2 \cdot 3 \cdots n$$

for large  $n$  ( $n \rightarrow \infty$ ).

[15] **Evaluation of (Zeta Function)  $H_n^{(2)}$ :**

Tabulate and plot  $H_n^{(2)}$  for a range of  $n$  (e.g.,  $1 \leq n \leq 1000$ ) using your favorite programming language (MAPLE or MATHEMATICA are fine). Based on this numerical computation find a *good* approximation of  $H_n^{(2)}$  for large  $n$  up to a *constant* term. That is, find a simple function  $f(n)$  (e.g.,  $f(n) = \log n$ ) such that  $H_n^{(2)} \approx f(n) + \text{constant}$  (and this approximation you should check on your plots and/or tables). For example, your computations may indicate that  $H_n^{(2)} \approx n\sqrt{n} + 3.2n + 4$  (this is **not** the correct answer!).

[15] **Stirling's Approximation:**

Tabulate and plot

$$\sum_{k=1}^n \log k$$

for a range of  $n$  (e.g.,  $1 \leq n \leq 1000$ ). Based on this numerical computation find a *good* approximation of

$$\log n! = \sum_{k=1}^n \log k$$

for large  $n$  up to a  $\log n$  term, if possible. That is, your answer may look like this

$$\log n! \approx? n^2 \log n + n + 3 \log n.$$

**Include your program in the homework write-up.**