Solution of Homework 3

**** Note that the approximate functions shown below are *not* unique and are provided for the sake of illustration not perfection.**

Prob.1

$$H_n^{(2)} = \sum_{k=1}^{n} \frac{1}{k^2} \approx \frac{3}{2} n^{0.02}$$

<u>Algorithm</u>

H(1) =1
for k = 2 to n
  H(k) = H(k-1) + 1/k^2

n	exact	approximation
1	1	1.5
50	1.625132734	1.622073985
100	1.6349839	1.644717294
150	1.638289573	1.658109029
200	1.639946546	1.667676692
250	1.640942056	1.675135951
300	1.641606283	1.681255369
350	1.642081002	1.6864467
400	1.642437189	1.690956591
450	1.642714312	1.694944606
500	1.642936066	1.698519977
550	1.643117537	1.70176079
600	1.643268788	1.704724819
650	1.643396788	1.70745602
700	1.643506515	1.709988619
750	1.643601622	1.712349788
800	1.643684848	1.714561465
850	1.643758288	1.716641619
900	1.643823573	1.718605151
950	1.643881989	1.720464561
1000	1.643934567	1.722230432

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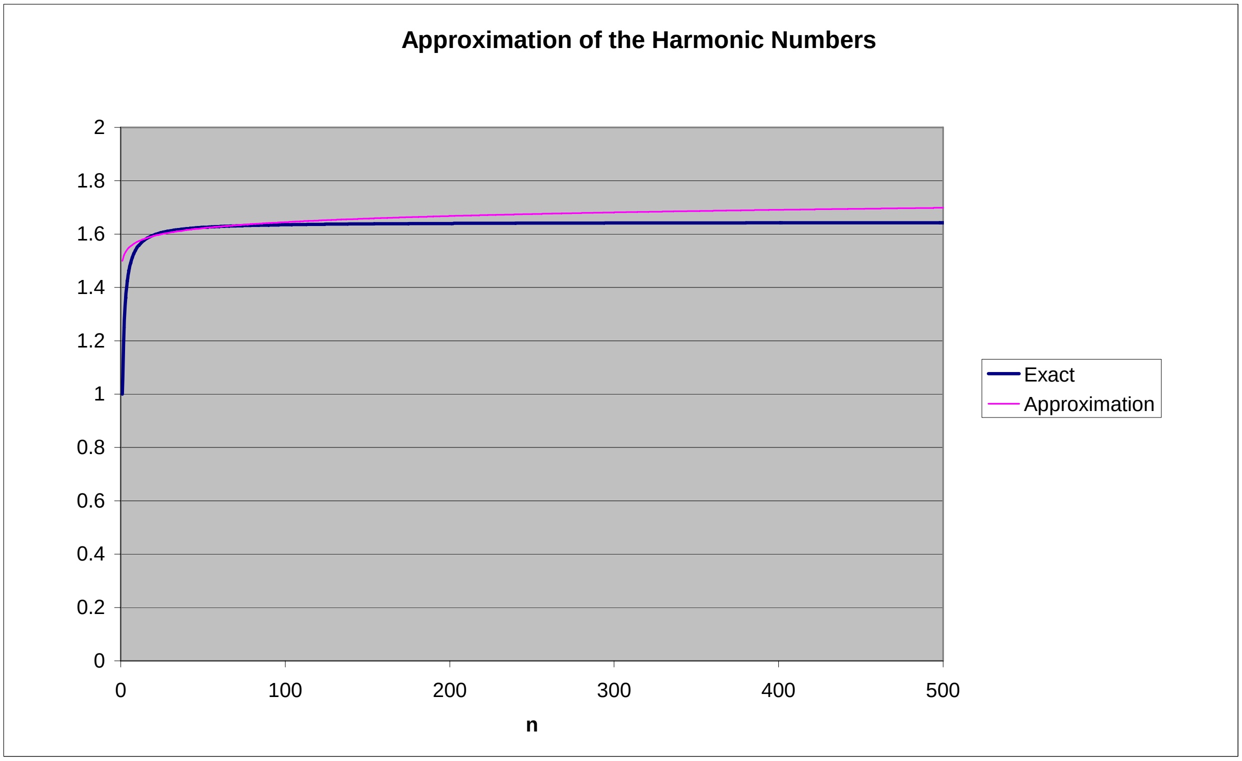

Solution of Homework 3

Prob. 2 Using Stirling's formula:

$$n! \approx \left(\frac{n}{e}\right)^n \sqrt{2n\pi} \Rightarrow$$

$$\sum_{k=1}^{n} \log k = \log(1 \times 2 \times \ldots \times n)$$

$$= \log n! \approx n(\log n - \log e) + 0.5 \log(2n\pi)$$

<u>Algorithm</u>

S(1) = 0
for k = 2 to n
  S(k) = S(k-1) + ln(k)

n	exact	approximation
1	0	-0.035117164
50	64.48307487	64.48243844
100	157.9700037	157.9697291
150	262.7568934	262.7567395
200	374.8968886	374.8967951
250	492.5095864	492.509529
300	614.485803	614.4857698
350	740.0919742	740.0919582
400	868.8064142	868.8064111
450	1000.238891	1000.238898
500	1134.086409	1134.086424
550	1270.106851	1270.106873
600	1408.102287	1408.102314
650	1547.907871	1547.907903
700	1689.384181	1689.384217
750	1832.411755	1832.411794
800	1976.887084	1976.887126
850	2122.719619	2122.719664
900	2269.829476	2269.829523
950	2418.145657	2418.145706
1000	2567.604644	2567.604695

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