Question 1. (10 points). A gambler (call him Bob) is playing at a casino the following game: Bob can bet any amount he wishes, then a fair coin is tossed, and if the outcome of the toss is "heads" he wins an amount equal to his bet, otherwise he loses the amount he bet. Bob can play as many rounds of this game as he wishes. Bob decides to follow the following strategy: He bets \$100 in round 1, and if he loses round 1 then he doubles the bet for the next round, and he keeps doubling the bet from one round to the next until he wins; he stops as soon as he wins a round, collects his money, and walks out of the casino.

1. What is the probability that Bob loses all of the first k rounds?
2. If Bob has lost the first k rounds and won round $k+1$, what is his net gain out of playing these $k+1$ rounds?

Question 2. (10 points). Solve the recurrence $a_n = 3a_{n-1}$, with boundary condition $a_0 = 1$.

Question 3. (15 points). Solve the recurrence $a_n = 4a_{n-1} - 4a_{n-2}$, with boundary conditions $a_0 = 1$ and $a_1 = 4$.

Question 4. (15 points). Let W be a 26-letter word that contains all of the 26 letters of the English alphabet, so that each letter occurs in W exactly once (obviously W is a strange word not found in the English dictionary).

You are given a sheet of paper on which W was written with a pencil. You are also given an eraser, and you are asked to erase as few letters as possible from W such that the surviving letters are either increasing or decreasing when they are read in left-to-right order (either is fine). For example, it is valid if the surviving word (after erasing) is *bempwx* or *xwpmeb*, but not *bmepwx*. Use the pigeonhole principle to prove that, for every such word W, you never need to erase more than 20 letters from W (which is equivalent to proving that there is always a valid surviving word of length 6 or more). Note that this question is about an existence proof of a long enough valid surviving word, not about an algorithm for determining which letters of W to erase for obtaining it (which would be a much harder question).

Date due: April 12, 2011 at the beginning of class.