Solutions of HW#1: *Basic Logic*

Q.1 Make truth tables for the following statement:

- $(p \to q) \to (q \to p)$

Answer

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

- $(p \to q) \wedge (\neg p \to q)$

Answer

p	q	$p \to q$	$\neg p \to q$	$(p \to q) \wedge (\neg p \to q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Q.2 Using *logical equivalences* discussed in class prove that

$$(p \wedge q) \to (p \vee q)$$

is a tautology, that is, prove that

$$(p \wedge q) \to (p \vee q) \equiv T.$$

Answer

$$\begin{aligned}
(p \wedge q) \to (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) \\
&\equiv (\neg p \vee \neg q) \vee (p \vee q) \\
&\equiv (\neg p \vee p) \vee (\neg q \vee q) \\
&\equiv T \vee T \\
&\equiv T
\end{aligned}$$

Note: Another way to solve this question is by constructing the truth table for the given logical expression and showing that it always yields T for all values of p and q.

Q.3 Let Determine the truth value of the following statements when x and y are real numbers:

1. $\exists_x \forall_y \ (x = y^2)$,
2. $\exists_x \forall_y \ (xy = 0)$,
3. $\forall_{x \neq 0} \exists_y \ xy = 1$,
4. $\exists_x \exists_y \ (x + y \neq y + x)$.

Answer

1. $\exists_x \forall_y \ (x = y^2)$: (False)
   Suppose $x = a$ satisfies the above statement. This means that $a = y^2$ for all real values of y. This is impossible since y would then be equal at most 2 real values ($\pm\sqrt{a}$).
2. $\exists_x \forall_y \ (xy = 0)$: (True)
   At $x = 0$, $xy = 0$ for all real values of y.
3. $\forall_{x \neq 0} \exists_y \ xy = 1$: (True)
   If we pick an arbitrary value for $x \neq 0$, say a, then there exists a value for y that satisfies the given statement, $y = \frac{1}{a}$.
4. $\exists_x \exists_y \ (x + y \neq y + x)$: (False)
   The addition of real numbers is always commutative operation.