[50] Homework 4. *Proof Techniques*

Each problem is worth 10 points

[10] Show that $\sqrt[5]{5}$ is irrational.

[10] The *harmonic number* H_n is defined as for $n \geq 1$

$$H_n = \sum_{k=1}^{n} \frac{1}{k}.$$

Prove by induction that

$$H_{2^n} \geq 1 + \frac{n}{2}$$

whenever n is a nonnegative natural number.

[10] Let A be a set of cardinality n. Let $P(A)$ be the power set, that is, the set of *all* subsets of A. Prove by induction that cardinality of $P(A)$ is 2^n, that is,

$$|P(A)| = 2^n.$$

[10] Prove using induction that for any natural n

$$\sum_{i=1}^{n} \frac{1}{i^2} \leq 2 - \frac{1}{n}$$

[10] Derive an explicit formula for the following recurrence for $n \geq 1$

$$a_n = \frac{n}{2} a_{n-1}$$

with $a_0 = 1$.