[50] Homework 2. *Language of Mathematics*

Each problem is worth 10 points

[10] Prove that for any sets A and B

$$A = (A - B) \cup (A \cap B).$$

[10] Let x and y be *integers*. Determine whether the following relations are reflexive, symmetric, antisymmetric, or transitive:

- $x \equiv y \mod 7$;
- $xy \geq 1$;
- $x = y^2$.

Justify your statements.

Finally, determine which of the above relations are equivalence and partial order relations. For equivalence relations, construct the equivalence classes.

[10] Determine whether the following function is bijection from $\mathbf{R}$ to $f(\mathbf{R})$:

- $f(x) = x^3$
- $f(x) = \sin^2(x)$
- $f(x) = \frac{x+1}{x+2}$.

[10] Let $g(x) = \lfloor x \rfloor$. Find

- $g^{-1}(\{0\})$;
- $g^{-1}(\{x : \ 0 < x < 1\})$

[10] What are the values of the following:

$$\sum_{i=1}^{500} 5^i,$$

$$\sum_{i=1}^{2} \sum_{j=1}^{3} (i + j)$$

$$\sum_{j=0}^{8} (3^j - 2^j).$$