[30] Homework 1. *Basic Logic*

Each problem is worth 10 points

[10] Make truth tables for the following statements:

1. $(p \to q) \to (q \to p)$
2. $(p \to q) \wedge (\neg p \to q)$

[10] Using *logical equivalences* discussed in class prove that

$$(p \wedge q) \to (p \vee q)$$

is a tautology, that is, prove that

$$(p \wedge q) \to (p \vee q) \equiv T.$$

(**Hint**: Observe first that $p \to q \equiv \neg p \vee q$.

[10] Determine the truth value of the following statements when x and y are real numbers:

- $\exists x \forall y \ \ (x = y^2)$;
- $\exists x \forall y \ (xy = 0)$;
- $\forall \ x \neq 0 \exists y \ \ xy = 1$;
- $\exists x \ \exists y \ \ (x + y \neq y + x.$