Problem 1: Fourier-Transform of Synthetic Signals

(a) Generation of clean synthetic signal: Consider a sinusoidal signal x_1 with fundamental frequency at $F_1 = 1Hz$ and a sinusoidal signal x_2 with fundamental frequency at $F_2 = 0.2Hz$, both sampled at $F_s = 10Hz$. Plot signals x_1, x_2, and their sum $x_s = x_1 + x_2$.

(b) Computation of Fourier transform: Compute the Fourier transform of signals x_1, x_2, x_s with $N = 100$ points. Plot the spectrum, i.e., magnitude of the Fourier transform over the frequency space $[0, \frac{F_s}{2}]$. What do you observe?

(c) Fourier transform of noisy signal: Add Gaussian noise to the signal of zero mean and 0.1 variance. Plot the noisy signal x_n. Compute the Fourier transform of the noisy signal with 100 points and plot it over the frequency space. What do you observe?

(d) Low-pass filtering of noisy signal: Create a low-pass filter with cutoff frequency at $1Hz$. Plot the spectrum of the filter. Apply the filter to the noisy signal x_n. Plot the resulting signal in the time and frequency domain. What do you observe?

Problem 2: Fourier-Transform of ECG Signals

In this problem we are going to work with two ECG signals from the MIT-BIH Arrhythmia Database, which contains ECG signals from healthy people and patients with cardiac diseases. The sampling frequency of these signals is $F_s = 360Hz$. More information can be found in this link: `https://www.physionet.org/physiobank/database/mitdb/`

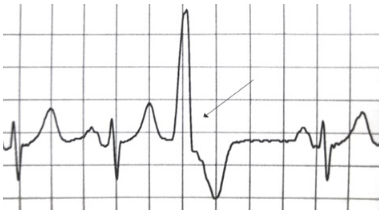
(a) Visualization of normal ECG signal: Load signal `123.txt`, which corresponds to a healthy patient. Using the corresponding annotation labels `123.lab` and what we have said in class, find an ECG beat and isolate the P, QRS, and T segments. Plot these three segments.

(b) Fourier computation of ECG components: Compute the 70-point Fourier transform of the above P, QRS, T segments and plot their spectrums. What do you observe?

(c) QRS detection: Take the first 10 seconds of the ECG signal. Use the Pan-Tomkins Algorithm to detect the QRS, i.e., bandpass filtering, differentiation, squaring, moving average filtering, peak detection.

(d) Visualization and Fourier computation of PVC beats: Load signal `208.txt`, which corresponds to a patient with frequent premature ventricular contractions, denoted with "V" in the annotation file `208.lab`. More information about how the premature ventricular beat is generated can be found here: `https://en.wikipedia.org/wiki/Premature_ventricular_contraction`. Based on the annotation file, isolate a premature ventricular beat (of length around 170-200 samples) and compute its Fourier transform.

Example of premature ventricular contraction (PVC) beat



(d) Comparison of the frequency spectrum between a normal and a PVC beat: Isolate the QRS complex of a normal beat from the healthy subject of the same length as the premature ventricular beat and compute its Fourier transform. Plot the Fourier magnitudes of the normal and the premature ventricular beat over the frequency space at the same graph. What do you observe?