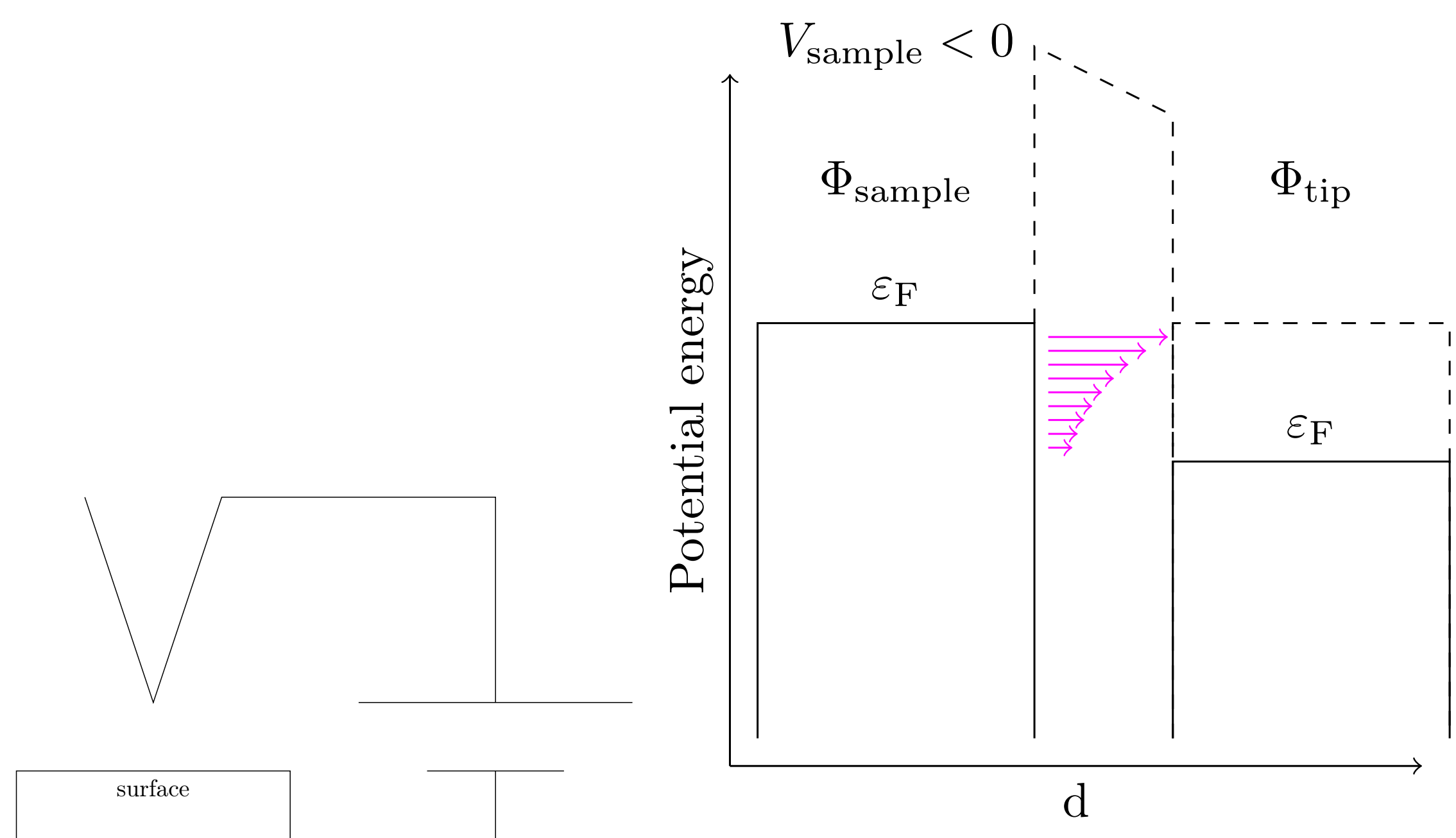


# SCANNING TUNNELLING MICROSCOPY AND ATOMIC FORCE MICROSCOPY

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## Scanning Tunnelling Microscopy

### principle



### Tunneling math!!!

$$P_{s \rightarrow t} = \left( \frac{2m}{\hbar^2} \right) \int |\Psi_t(r) H' \Psi_s(r)|^2 d\mathbf{r} \partial(E_t - E_s)$$
$$I_{t \rightarrow s} = \frac{2\pi e}{\hbar} \int |M_{ts}|^2 \rho_t(E - eV) \rho_s(E) f_t(E - eV) [1 - f_s(E)] dE$$
$$I_{s \rightarrow t} = \frac{2\pi e}{\hbar} \int |M_{ts}|^2 \rho_t(E - eV) \rho_s(E) [1 - f_t(E - eV)] f_s(E) dE$$

$\rho_t$  is the density of states in the tip.  $\rho_s$  is the density of states in the surface.  $eV$  is the shift due to bias.  $f_t$  is the Fermi—Dirac distribution for the tip.  $f_s$  is the Fermi—Dirac distribution for the surface.  $|M_{ts}|^2$  tunneling matrix element.

$$I_{t \rightarrow s} - I_{s \rightarrow t} = \frac{2\pi e}{\hbar} \int |M_{ts}|^2 \rho_t(E - eV) \rho_s(E) [f_s(E) - f_t(E - eV)] dE$$
$$I = I_{t \rightarrow s} - I_{s \rightarrow t} \approx \frac{2\pi e}{\hbar} \int_{E_f}^{E_f + eV} |M_{ts}|^2 \rho_t(E - eV) \rho_s(E) dE$$

Note version:

$$I \approx \int_{E_f}^{E_f + eV} |M_{ts}|^2 \rho_t(E - eV) \rho_s(E) dE = \int_0^{eV_A} T(E, eV_A) \rho_a(E) \rho_B(E - eV) dE$$

$$I \approx \int_0^{eV} T(E, eV) \rho_s(E) \rho_t(E - eV) dE$$

let  $\rho_t$  be constant and:

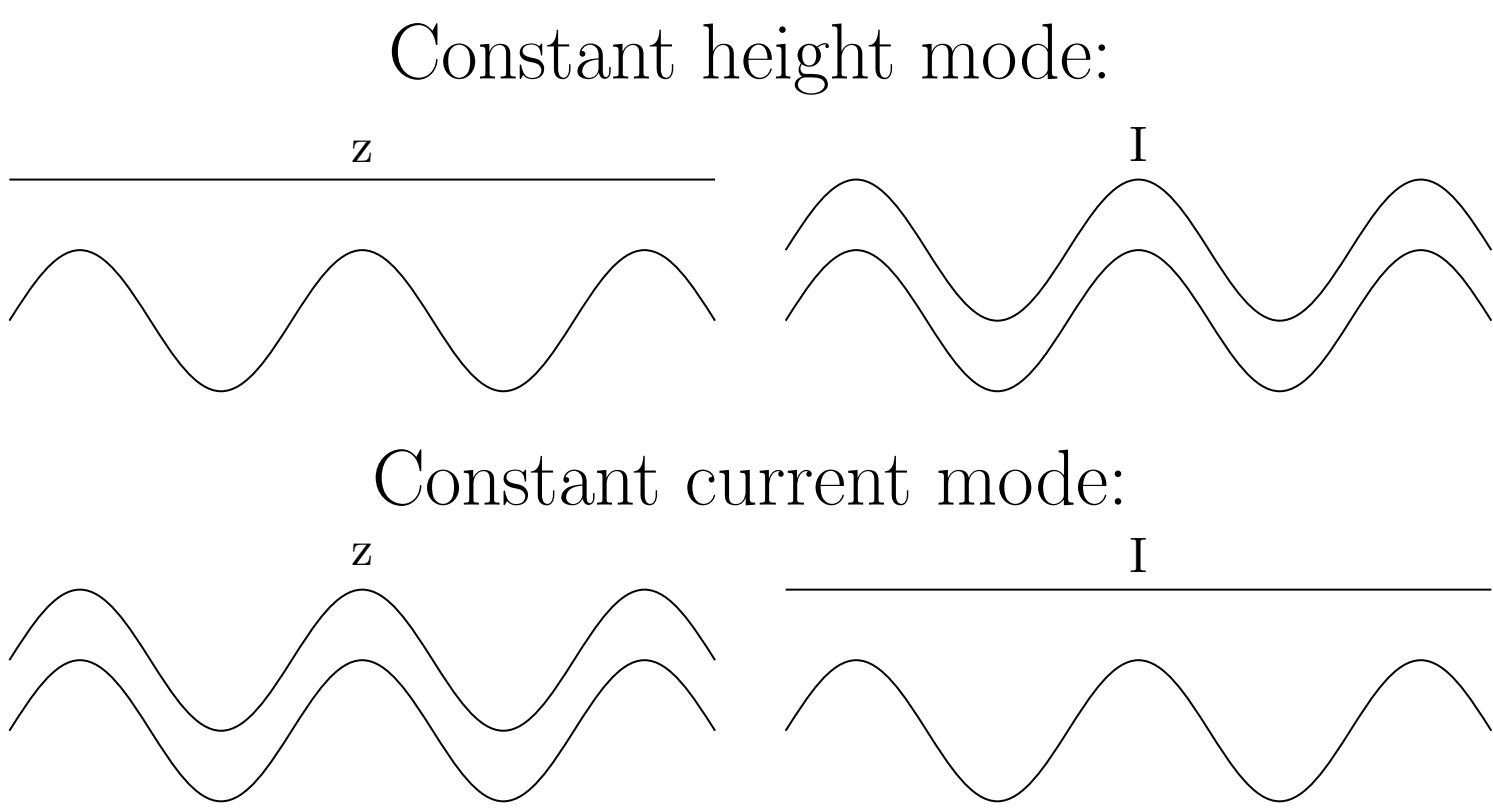
$$I \propto \int_0^{eV} T(E, eV) \rho_s(E) dE$$

$$T(E, eV) \approx e^{-2z \sqrt{\frac{2m}{\hbar^2} (\bar{\Phi} + \frac{eV}{2} - E)}}$$

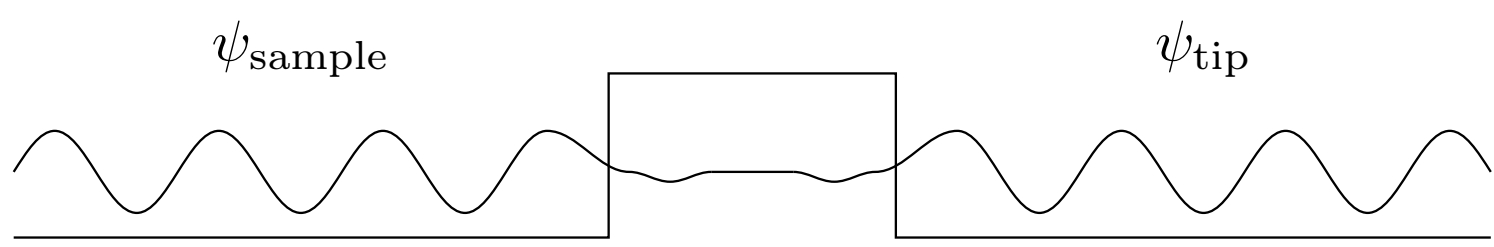
Is the tunneling probability,  $T$  is the barrier,  $z$  is the width of the barrier,  $\Phi$  is the avg work function and  $V$  is the applied bias. Next assume a low bias, to take the tunneling out of the integral:

$$I \propto \int_0^{eV} T(E, eV) \rho_s(E) dE$$
$$\approx T(E_f) \int_0^{eV} \rho_s dE$$
$$\approx e^{-2z \sqrt{\frac{2m}{\hbar^2} (\bar{\Phi})}} \int_0^{eV} \rho_s(E) dE$$
$$\approx e^{-2z \sqrt{\frac{2m}{\hbar^2} (\bar{\Phi})}} \rho_s(E_f) \cdot eV = \text{const} \cdot \rho_s(E_f, z) V$$

### modes

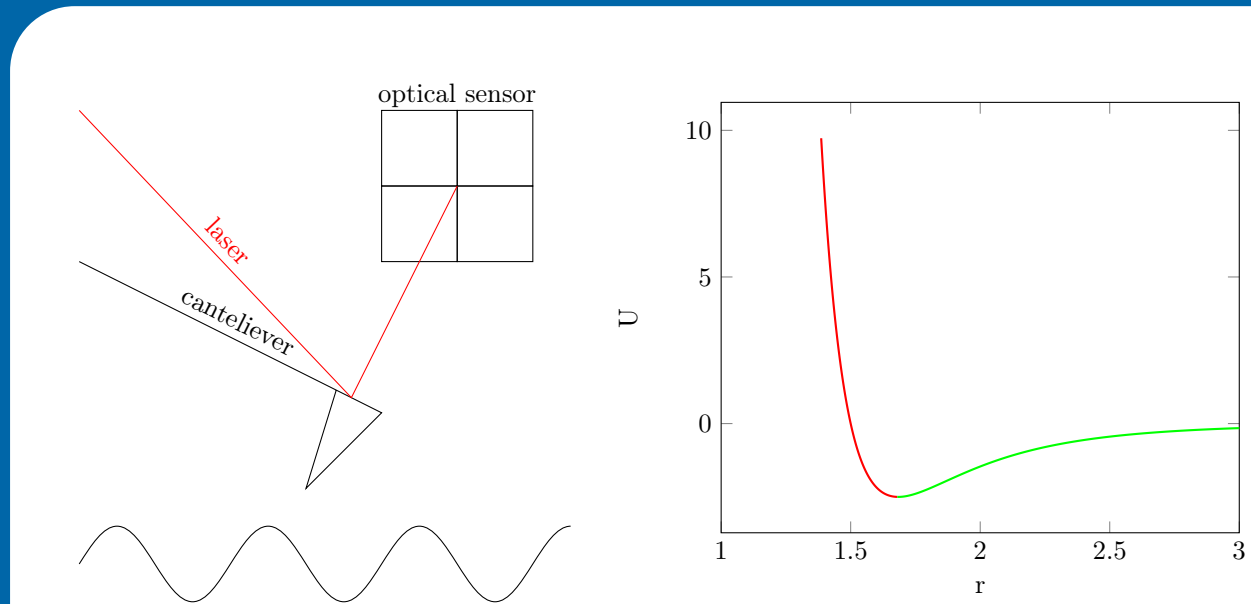


### Tunneling



## Atomic Force Microscopy

### setup



### Modes

There's contact mode and non contact/dynamic mode and tapping mode between the two. In contact mode the tip scrapes along the surface, in tapping mode the tip oscillates and make contact with the tip each time it reaches it's lowest point, this mode doesn't require as much calibration as non contact, but also doesn't damage the system it's probing as much as contact, so it can be used for studying biological environments. In non contact mode the cantilever vibrates above the surface, and is driven at some frequency, thinking of the surface—tip interactions as a spring between the surface and tip, makes the tip a “ball” suspended between two springs, this problem is known, see to the right

### forces

$$F_{tot} = \underbrace{F_{chem}}_{\text{Bonding between tip and sample}} + \underbrace{F_{mag}}_{\text{magnetically sensitive tips}} + \underbrace{F_{el}}_{-\frac{1}{2} \frac{\partial C}{\partial z} V^2} + \underbrace{F_{vdW}}_{-\frac{\hbar R}{6r^2}}$$

### Cantilever mechanics

$$m \frac{d^2 z}{dt^2} + \frac{m \omega_0}{Q} \frac{dz}{dt} + k_0 z = F_{\text{ext}} \cos \omega t$$
$$z(t) = A(\omega) \cos(\omega t - \phi)$$
$$A(\omega) = \frac{\frac{F_{\text{ext}}}{m}}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega \omega_0}{Q}\right)^2}}$$
$$\phi(\omega) = \arctan \left( \frac{\gamma \omega}{Q(\omega^2 - \omega_0^2)} \right)$$
$$m \frac{d^2 z}{dt^2} + \frac{m \omega_0}{Q} \frac{dz}{dt} + k_0 z = F_{\text{ext}} \cos \omega t + F_{\text{ts}}(z)$$
$$F_{\text{ts}}(x) \approx \frac{\partial F_{\text{ts}}}{\partial z} z = k_{\text{ts}} z$$
$$\Downarrow$$
$$m \frac{d^2 z}{dt^2} + \frac{m \omega_0}{Q} \frac{dz}{dt} + (k_0 - k_{\text{ts}}) z = F_{\text{ext}} \cos(\omega t)$$
$$k' = k_0 - k_{\text{ts}} \quad \text{Is the new spring constant}$$
$$\omega' = \sqrt{\frac{k'}{m}} \quad \text{Is the new resonance frequency}$$
$$\omega'(k_0 + k_{\text{ts}}) \approx \omega_0 + \frac{d\omega}{dk_{\text{ts}}} k_{\text{ts}}$$
$$= \omega_0 - \frac{1}{2} \frac{1}{\sqrt{m k_0}}$$
$$= \omega_0 \left( 1 - \frac{1}{2} \frac{k_{\text{ts}}}{k_0} \right)$$
$$\Downarrow$$
$$f' = f_0 + \underbrace{\Delta f}_{\text{detuning}}$$