The pn junction under forward and reverse bias

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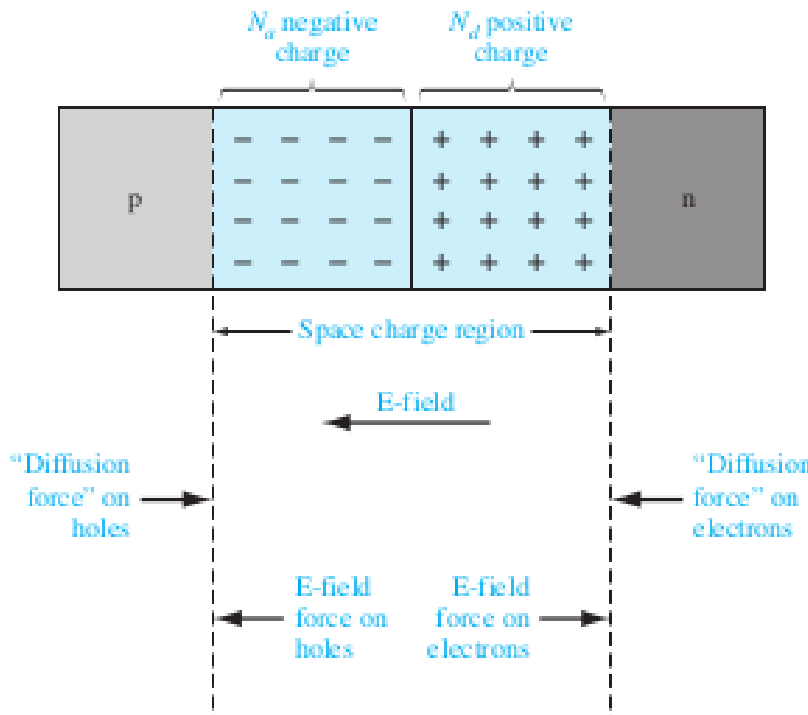
Part I

The pn junction

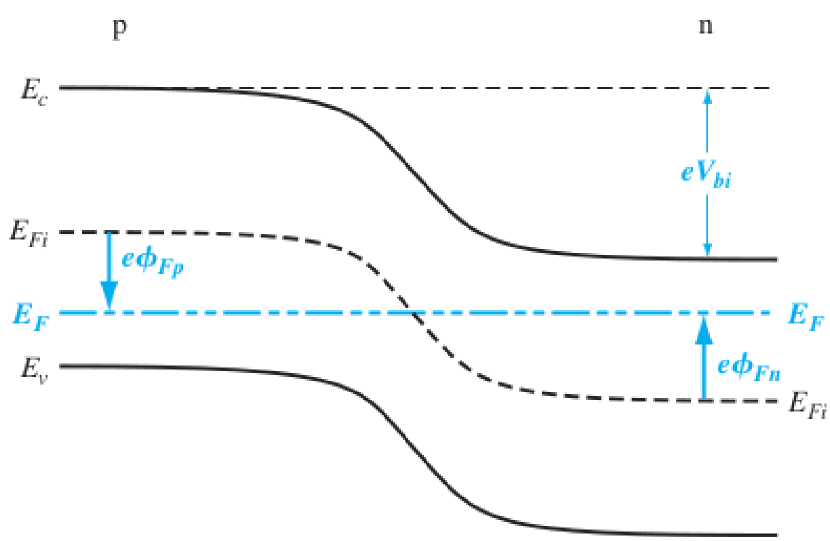
1 Basic structure of the pn junction

A space charge region appears between the n and p junctions:



Note the space charge region and how the electrons from the n-type move to the holes in the p-type

2 Zero applied Bias



$$V_{bi} = |\phi_{Fn}| + |\phi_{Fp}|$$

V_{bi} is the build in potential barrier, the ϕ values are the differences in from the Fermi level to the intrinsic Fermi level, in the n region ($N_d \approx n_0$)

$$\begin{aligned}
n_0 &= N_c \exp\left[-\frac{E_c - E_F}{kT}\right] = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \\
e\phi_{Fn} &= E_{Fi} - E_F \\
n_0 &= N_c \exp\left[-\frac{e\phi_{Fn}}{kT}\right]
\end{aligned}$$

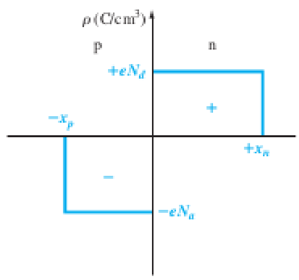
For the p type

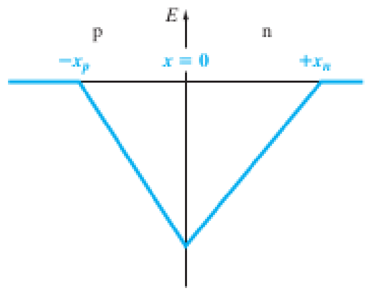
$$p_0 = N_a = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right] = n_i \exp\left[\frac{e\phi_{Fp}}{kT}\right]$$

$$\phi_{Fn} = -\frac{kT}{e} \ln\left(\frac{N_d}{n_i}\right) \qquad \phi_{Fp} = +\frac{kT}{e} \ln\left(\frac{N_a}{n_i}\right)$$

Thus:

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$





Here it's assumed that the space charge region is strictly within $-x_p$–x_n.

$$\frac{d^2\phi(x)}{dx^2} = \frac{-\rho(x)}{\epsilon_s} = -\frac{dE(x)}{dx}$$

$$\rho(x) = \begin{cases} -eN_a & \forall x \in [-x_p; 0] \\ eN_d & \forall x \in [0; x_n] \end{cases}$$

$$E = \int \frac{\rho(x)}{\epsilon_s}\, dx = -\int \frac{eN_a}{\epsilon_s}\, dx = -\frac{eN_a}{\epsilon_s} x + C_1$$

$$E = -\int \frac{eN_d}{\epsilon_s}\, dx = \frac{eN_d}{\epsilon_s} x + C_1$$

C_1 and C_2 is from integration, now let $E = 0$ at $x = -x_p$ and $x = x_n$:

$$E = \begin{cases} -\frac{eN_a}{\epsilon_s}(x + x_p) & \forall x \in [-x_p; 0] \\ \frac{eN_d}{\epsilon_s}(x - x_n) & \forall x \in [0; x_n] \end{cases}$$

$$N_a x_p = N_d x_n$$

$$\phi(x) = -\int E(x)\, dx$$

$$= \int \frac{eN_a}{\epsilon_s}(x + x_p)\, dx$$

$$= \frac{eN_a}{\epsilon_s}(\frac{x^2}{2} + x \cdot x_p) + C_1'\, dx$$

$$C_1' = \frac{eN_a}{2\epsilon_s} x_p^2 \qquad \text{because of the zero location}$$

$$\phi(x) = \frac{eN_a}{2\epsilon_s}(x + x_p)^2 \qquad \forall x \in [-x_p; 0]$$

now for the n region:

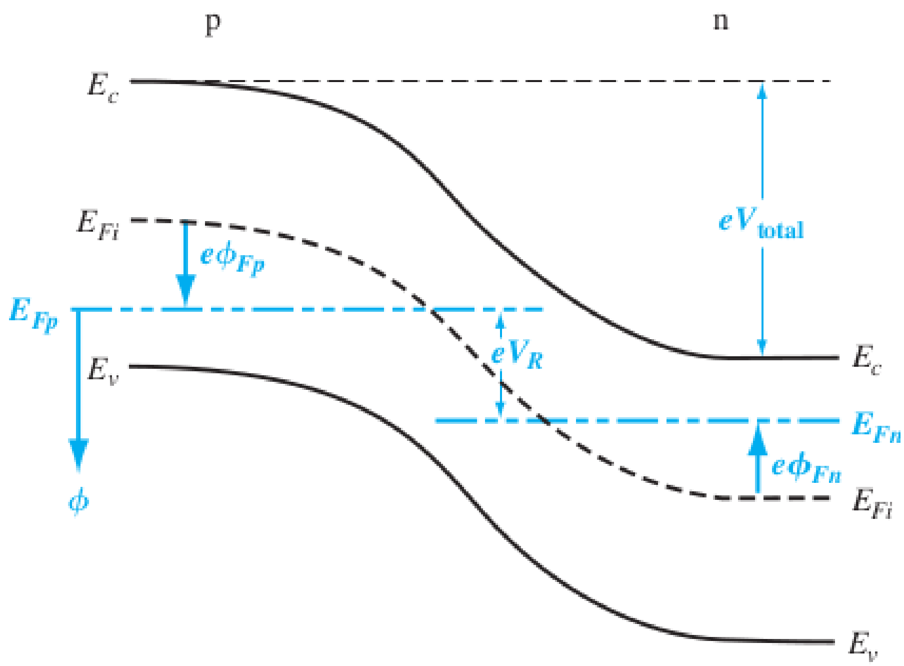
$$
\begin{aligned}
\phi(x) &= \int \frac{eN_d}{\epsilon_s}(x - x_n)\, dx \\
&= \frac{eN_d}{\epsilon_s}(x \cdot x_n - \frac{x^2}{2}) + C_2'\, dx \\
C_2' &= C_1' \\
\phi(x) &= \frac{eN_d}{\epsilon_s}\left(x \cdot x_n - \frac{x^2}{2}\right)^2 + \frac{eN_a}{2\epsilon_s}x_p^2 \qquad \forall x \in [0; x_n] \\
V_{bi} &= \frac{e}{2\epsilon_s}\left(N_d x_n^2 + N_a x_p^2\right) = |\phi(x = x_n)|
\end{aligned}
$$

Space charge width:

$$
\begin{aligned}
x_p &= \frac{N_d}{N_a}x_n \\
x_n &= \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a + N_d}\right]\right\}^{\frac{1}{2}} \\
x_p &= \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_d}{N_n}\right]\left[\frac{1}{N_a + N_d}\right]\right\}^{\frac{1}{2}} \\
W = x_n + x_p &= \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{\frac{1}{2}}
\end{aligned}
$$

3 Reverse applied bias

Reverse bias: Positive on the n side

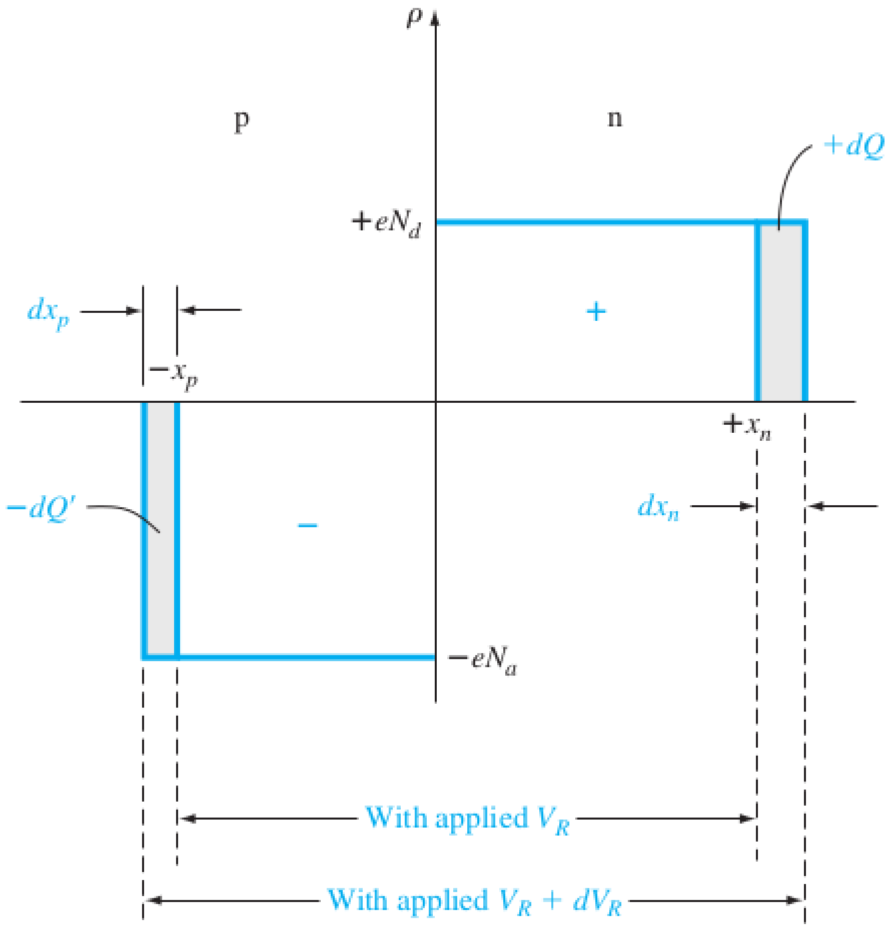


$$\begin{aligned} V_{\text{total}} &= |\phi_{Fn}| + |\phi_{Fp}| + V_R \\ &= V_{bi} + V_R \end{aligned}$$

New width:

$$\begin{aligned} W = x_n + x_p &= \left\{ \frac{2\epsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{\frac{1}{2}} \\ E_{\max} &= -\frac{eN_d x_n}{\epsilon_s} = -\frac{eN_a x_p}{\epsilon_s} \\ &= -\sqrt{\frac{2e\left(V_{bi} + V_R\right)}{\epsilon_s} \left(\frac{N_a N_d}{N_a + N_d} \right)} \\ &= -\frac{2\left(V_{bi} + V_R\right)}{W} \end{aligned}$$

Junction capacitance:



$$dQ' = eN_d dx_n = eN_a dx_p$$

$$C' = \frac{dQ'}{dV_R} = eN_d \frac{dx_n}{dV_R}$$

$$= \sqrt{\frac{e\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}} = \frac{\epsilon_s}{W}$$

One sided junction ($N_a \gg N_d$, known as p^+n junction, opposite the other way)

$$W \approx \sqrt{\frac{2\epsilon_s(V_{bi}+V_R)}{eN_d}}$$

$$x_p \ll x_n$$

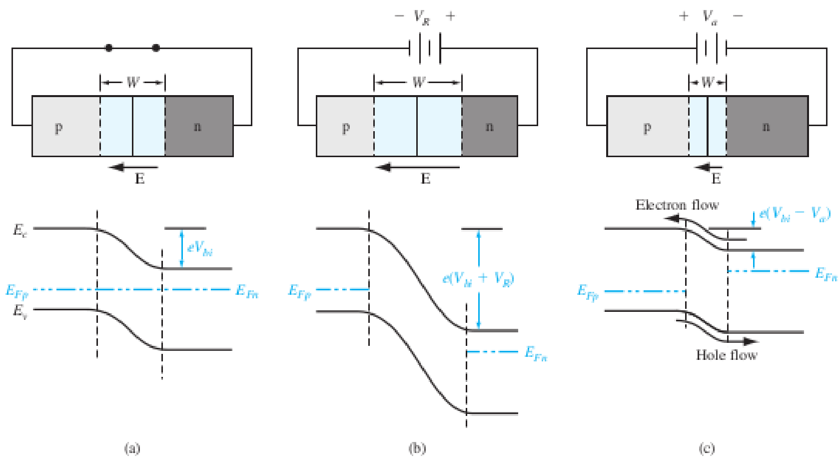
$$W \approx x_n$$

$$C' \approx \sqrt{\frac{e\epsilon_s N_d}{2(V_{bi}+V_R)}}$$

Part II

The pn junction diode

4 pn junction current



- The abrupt depletion layer approximation applies. The space charge regions have abrupt boundaries, and the semiconductor is neutral outside of the depletion region.
- The Maxwell–Boltzmann approximation applies to carrier statistics.
- The concepts of low injection and complete ionization apply.
-
  - The total current is a constant throughout the entire pn structure.
  - The individual electron and hole currents are continuous functions through the pn structure.
  - The individual electron and hole currents are constant throughout the depletion region.

$$
\begin{aligned}
V_{bi} &= V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \\
\frac{n_i^2}{N_a N_d} &= \exp\left(-\frac{eV_{bi}}{kT}\right) \\
n_{n0} &\approx N_d \\
n_{p0} &\approx \frac{n_i^2}{N_a} \\
n_{p0} &= n_{n0} \exp\left(-\frac{eV_{bi}}{kT}\right)
\end{aligned}
$$

n_n is the majority carrier electrons, n_p is the concentration of minority carrier electrons.

$$n_p = n_{p0} \exp\left(\frac{eV_a}{kT}\right)$$
$$p_n = p_{n0} \exp\left(\frac{eV_a}{kT}\right)$$

Minority carrier distribution

$$\frac{\partial(\delta p_n)}{\partial t} = D_p \frac{\partial^2(\delta p_n)}{\partial x^2} - \mu_p E \frac{\partial \delta p_n}{\partial x} + g' - \frac{\partial p_n}{\tau_{p0}}$$
$$\frac{d^2 \delta p_n}{dx^2} - \frac{\delta p_n}{L_p^2} = 0 \qquad (x > x_n), L_p^2 = D_p \tau_{p0}$$
$$\frac{d^2 \delta n_p}{dx^2} - \frac{\delta n_p}{L_n^2} = 0 \qquad (x > x_n), L_n^2 = D_n \tau_{n0}$$
$$p_n(x_n) = p_{n0} \exp\left(\frac{eV_a}{kT}\right) \xrightarrow[x \to \infty]{} p_{n0}$$
$$n_p(-x_p) = n_{p0} \exp\left(\frac{eV_a}{kT}\right) \xrightarrow[x \to -\infty]{} n_{p0}$$
$$\begin{aligned}\delta p_n(x) &= p_n(x) - p_{n0} = Ae^{\frac{x}{L_p}} + Be^{-\frac{x}{L_p}} \\ &= p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \exp\left(\frac{x_n - x}{L_p}\right)\end{aligned}$$
$$\begin{aligned}\delta n_p(x) &= n_p(x) - n_{p0} = Ce^{\frac{x}{L_n}} + De^{-\frac{x}{L_n}} \\ &= n_{p0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \exp\left(\frac{x_p - x}{L_n}\right)\end{aligned}$$
$$p = p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$
$$n = n_0 + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

At the space charge edge:

$$n_0 p_n(x_n) = n_0 p_{n0} \exp\left(\frac{V_a}{V_t}\right) = n_i^2 \exp\left(\frac{V_a}{V_t}\right)$$
$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{F_p}}{kT}\right)$$

Ideal pn junction current

$$J_p(x_n) = -eD_p \frac{dp_n(x)}{dx}\bigg|_{x=x_n}$$

$$J_p(x_n) = -eD_p \frac{d\delta p_n(x)}{dx}\bigg|_{x=x_n}$$

$$= \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$J_n(-x_p) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$J = \underbrace{\left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]}_{J_s}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

5 Generation-recombination currents and high-injection levels

$$R = \frac{C_n C_p N_t (np - n_i^2}{C_n(n+n') + C_p(p+p')}$$

Reverse bias:

$$R = \frac{-C_n C_p N_t n_i^2}{C_n n' + C_p p'}$$

$$= \frac{-n_i}{\frac{1}{N_t C_p} + \frac{1}{N_t C_n}}$$

$$= \frac{-n_i}{2\tau_0} \quad \tau_0 = \frac{\tau_{p0} + \tau_{n0}}{2}$$

$$J_{gen} = \int_0^w eG\,dx = \frac{en_i W}{2\tau_0}$$

$$J_R = J_s + J_{gen}$$

Forward bias:

$$R = \frac{np - n_i^2}{\tau_{p0}(n+n') + \tau_{n0}(p+p')}$$

$$n = n_i \exp\left[\frac{E_{Fn} - E_{Fi}}{kT}\right]$$

$$p = n_i \exp\left[\frac{E_{Fi} - E_{Fp}}{kT}\right]$$

figure 8.13

$$eV_a = (E_{Fn} - E_{Fi}) + (E_{Fi} - E_{Fp})$$

At the space charge region center:

$$\frac{eV_a}{2} = E_{Fn} - E_{Fi} = E_{Fi} - E_{Fp}$$
$$n = n_i \exp\left[\frac{eV_a}{2kT}\right]$$
$$p = n_i \exp\left[\frac{eV_a}{2kT}\right]$$
$$R_{max} = \frac{n_i}{2\tau_0}\frac{\exp\left(\frac{eV_a}{kT}\right) - 1}{\exp\left(\frac{eV_a}{kT} + 1\right)}$$

ignoring the ones ($V_a \gg kT/e$)

$$= \frac{n_i}{2\tau_0}\exp\left(\frac{eV_a}{2kT}\right)$$
$$J_{rec} = \int_0^{"} eR\,dx = ex'\frac{n_i}{2\tau_0}\exp\left(\frac{eV_a}{2kT}\right)$$
$$= \underbrace{\frac{eWn_i}{2\tau_0}}_{J_{r0}}\exp\left(\frac{eV_a}{2kT}\right)$$
$$J = J_{rec} + J_D$$
$$J_D = J_s \exp\left(\frac{eV_a}{kT}\right)$$
$$\ln J_{rec} = \ln J_{r0} + \frac{V_a}{2V_t}$$
$$\ln J_D = \ln J_s + \frac{V_a}{2V_t}$$
$$I = I_s\left[\exp\left(\frac{V_a}{nV_t}\right) - 1\right]$$

This n is not n but the ideality factor, now for high level injection:

$$np = n_i^2 \exp\left(\frac{V_a}{V_t}\right)$$

$$(n_0 + \delta n)(p_0 + \delta p) = n_i^2 \exp\left(\frac{V_a}{V_t}\right)$$

$$\delta n \delta p \approx n_i^2 \exp\left(\frac{V_a}{V_t}\right)$$

$$\delta n = \delta p \approx n_i^2 \exp\left(\frac{V_a}{2V_t}\right)$$

$$I \propto \exp\left(\frac{V_a}{2V_t}\right)$$