Semiconductors in equilibrium

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1 Equilibrium distribution of electrons and holes

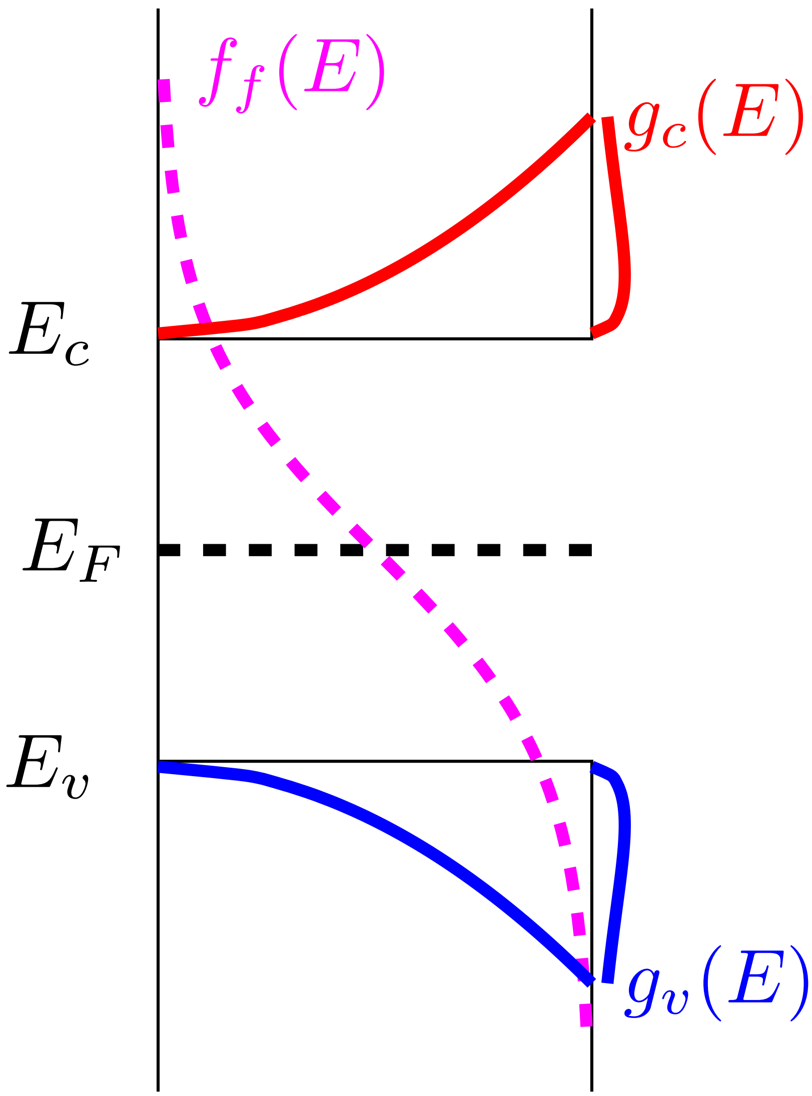
The equilibrium distributions are:

$$n(E) = g_c(E)f_F(E)$$
$$p(E) = g_v(E)(1 - f_F(E))$$

with f_F being a Fermi–Dirac distribution:

n is the electron distribution in the valence band and g_c the density of states (DOS) in the conducting band, p the hole distribution in the conducting band and g_v the DOS in the valence band.

An intrinsic semiconductor is a perfect and perfectly pure semiconductor crystal. At $T = 0k$ all states in the valence band is filled.



2 Thermal equilibrium electron distribution

assumptions: $E > E_c$, $E_c - E_F \gg kT$ then $E - E_F \gg kT$

$$n_0 = \int_{E_c}^{E_{\max}} g_c(E) f_F(E) dE$$

$E_{\max}$ is the maximum possible energy, it can be assumed equal to ∞ (due to $f_F \xrightarrow[E\to\infty]{} 0$)

$$
\begin{aligned}
f_F(E) &= \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} \\
&\approx e^{-\frac{E-E_F}{kT}}
\end{aligned}
$$

$$
\begin{aligned}
n_0 &= \int_{E_c}^{\infty} \frac{4\pi(2m_a^*)^{\frac{3}{2}}}{h^3}\sqrt{E-E_c}\exp\left(-\frac{E-E_F}{kT}\right)dE \qquad \text{Let:} \quad \eta = \frac{E-E_c}{kT} \\
&= \frac{4\pi\,(2m_n^*kT)^{\frac{3}{2}}}{h^3}\exp\left(-\frac{E_c-E_F}{kT}\right)\underbrace{\int_0^{\infty}\eta^{\frac{1}{2}}e^{-\eta}\,d\eta}_{=\frac{\sqrt{\pi}}{2}} \\
&= \underbrace{2\left(\frac{2\pi m_n^*kT}{h^2}\right)^{\frac{3}{2}}}_{N_c} e^{-\frac{E_c-E_F}{kT}} \\
&= N_c e^{-\frac{E_c-E_F}{kT}}
\end{aligned}
$$

The same thing can be shown for holes, just swap m_n^*(effective mass of electron) with m_p^*(effective mass of hole) and $E-E_F$ with E_F-E and E_c-E_F with E_F-E_v.

N_c is the *effective density of states function in the conduction band*

For holes the constant is $N_v = 2\left(\frac{2\pi m_p^*kT}{h^2}\right)^{\frac{3}{2}}$

3 Intrinsic carrier concentration

For the intrinsic system E_{Fi} is the intrinsic Fermi energy.

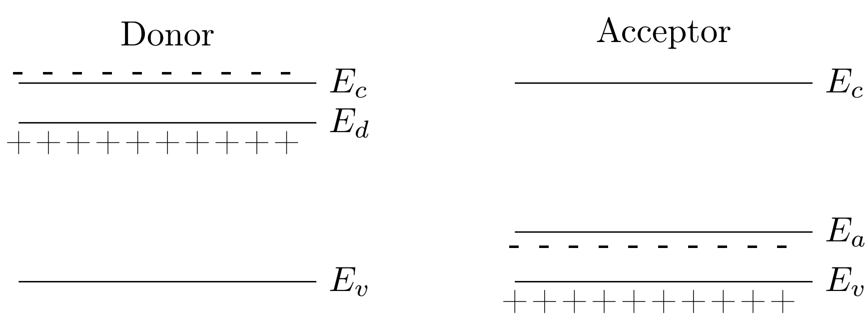
$$
\begin{aligned}
n_0 = p_0 = n_i = p_i &= N_c e^{-\frac{E_c-E_{Fi}}{kT}} \\
n_i^2 &= N_c N_v e^{-\frac{E_c-E_{Fi}}{kT}} e^{-\frac{E_{Fi}-E_v}{kT}} \\
&= N_c N_v e^{-\frac{E_c-E_v}{Kt}} \\
&= N_c N_v e^{-\frac{E_g}{Kt}}
\end{aligned}
$$

E_g is the band gap.

4 Intrinsic Fermi level

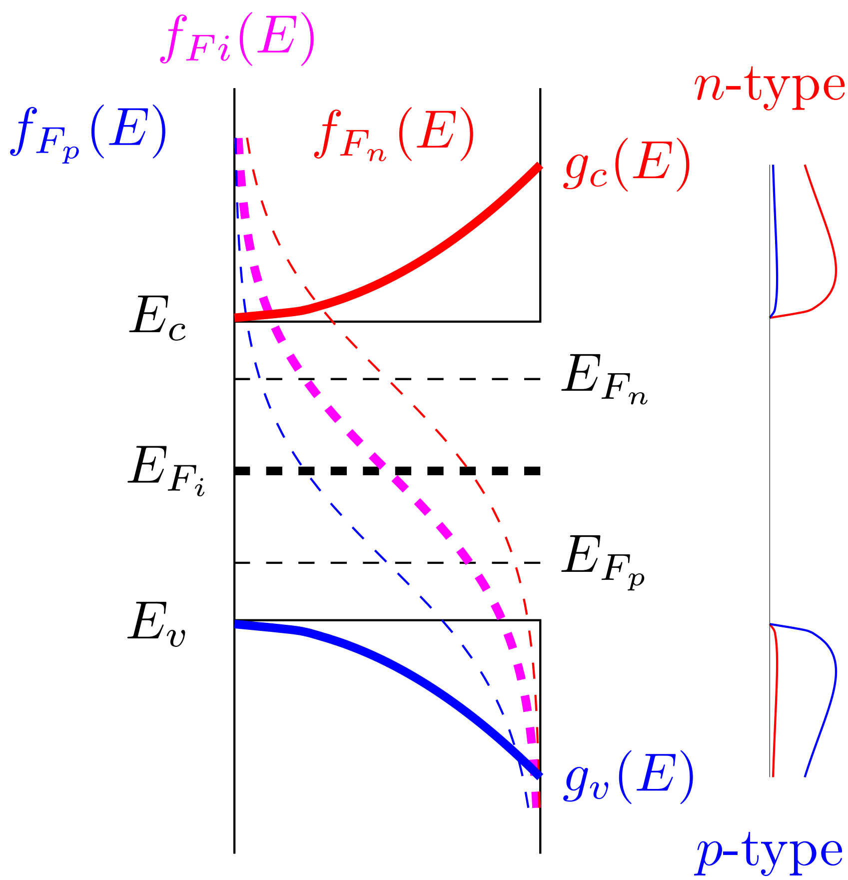
$$
\begin{aligned}
n_i &= N_c e^{-\frac{E_c - E_{Fi}}{kT}} \\
&= N_v e^{-\frac{E_{Fi} - E_v}{kT}} \\
E_{Fi} &= \frac{1}{2}\left(E_c + E_v\right) + \frac{1}{2} kT \ln\left(\frac{N_v}{N_c}\right) \\
&= \frac{1}{2}\left(E_c + E_v\right) + \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)
\end{aligned}
$$

5 Dopant atoms



Donors and acceptors change the properties, it also makes the semiconductor extrinsic instead of intrinsic.

6 The extrinsic semiconductor



The donors/acceptors create a new equilibrium. This is gotten from adding and subtracting the intrinsic Fermi energy to the exponential term:

$$\begin{aligned} n_0 &= N_c e^{-\frac{(E_c - E_{Fi}) + (E_F - E_{Fi})}{kT}} \\ &= \underbrace{N_c e^{-\frac{(E_c - E_{Fi})}{kT}}}_{n_i} e^{\frac{E_F - E_{Fi}}{kT}} \\ &= n_i e^{\frac{E_F - E_{Fi}}{kT}} \end{aligned}$$

This leads to a new product at thermal equilibrium:

$$\begin{aligned} n_0 p_0 &= n_i e^{\frac{E_F - E_{Fi}}{kT}} n_i e^{\frac{E_{Fi} - E_f}{kT}} \\ &= N_c N_v e^{-\frac{E_g}{kT}} \\ &= n_i^2 \end{aligned}$$

7 Probability function

$$n_d = \frac{N_d}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

n_d is the density of electrons occupying a donor level, N_d the concentration of donors and E_d the donor energy level. $\frac{1}{2}$ is from spin (also degeneracy factor)

$$= N_d - N_d^+$$

N_d^+ is the concentration of ionized donors, now assume $(E_d - E_F) \gg kT$:

$$\begin{aligned}
n_d &= \frac{N_d}{\frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)} \\
&= 2N_d \exp\left(-\frac{E_d - E_F}{kT}\right) \\
n_0 &= N_c \exp\left(-\frac{E_c - E_F}{kT}\right) \\
\frac{n_d}{n_d + n_0} &= \frac{2N_d \exp\left(-\frac{E_d - E_F}{kT}\right)}{2N_d \exp\left(-\frac{E_d - E_F}{kT}\right) + N_c \exp\left(-\frac{E_c - E_F}{kT}\right)} \\
&= \frac{1}{1 + \frac{N_c}{2N_d}\exp\left(-\frac{E_c - E_d}{kT}\right)}
\end{aligned}$$

8 Thermal equilibrium electron concentration

A compensated semiconductor has both acceptor and donor impurities

$$\begin{aligned}
n_0 + N_a^- &= p_0 + N_d^+ \\
n_0 + (N_a - p_a) &= p_0 + (N_d - n_d)
\end{aligned}$$

n_d is the concentration of electrons in the donor states, N_d^+ the concentration of positively charged donor states, p_a is the concentration of holes in the acceptor states, N_a^- the concentration of negatively charged acceptor states

$$\begin{aligned}
n_0 + N_a &= p_0 + N_d \\
&= \frac{n_i^2}{n_0} + N_d \\
0 &= n_0^2 - (N_d - N_a)\, n_0 - n_i^2 \\
n_0 &= \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}^2 + n_i^2\right)}
\end{aligned}$$

9 Fermi level position

$$\begin{aligned} E_c - E_F &= kT \ln\left(\frac{N_c}{n_0}\right) \\ &= kT \ln\left(\frac{N_c}{N_d}\right) \end{aligned}$$

since

$$\begin{aligned} N_d &\gg n_i \\ n_0 &\simeq N_d \end{aligned}$$

for an n type semiconductor. To get the intrinsic Fermi level:

$$\begin{gathered} n_0 = n_i e^{\frac{E_F - E_{Fi}}{kT}} \\ \Downarrow \\ E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right) \end{gathered}$$