Semiconductors in equilibrium

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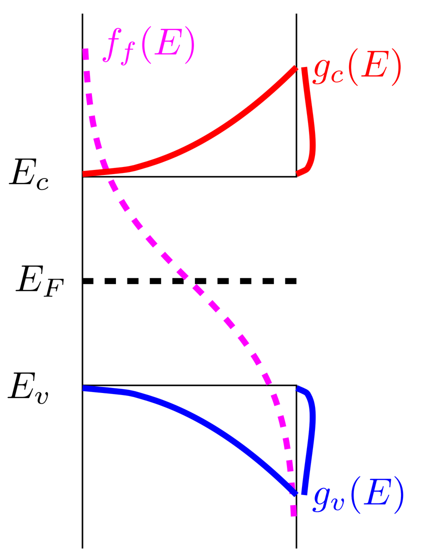
Intrinsic semiconductors

Equilibrium distributions

The equilibrium distributions are:

$$n(E) = g_c(E) f_F(E)$$
$$p(E) = g_v(E)(1 - f_F(E))$$



with f_F being a Fermi–Dirac distribution n is the electron distribution in the valence band and g_c the density of states (DOS) in the conducting band, p the hole distribution in the conducting band and g_v the DOS in the valence band.
An intrinsic semiconductor is a perfect and perfectly pure semiconductor crystal. At $T = 0k$ all states in the valence band is filled.

Intrinsic carrier concentration

For the intrinsic system E_{Fi} is the intrinsic Fermi energy.

$$n_0 = p_0 = n_i = p_i = N_c e^{-\frac{E_c - E_{Fi}}{kT}}$$
$$n_i^2 = N_c N_v e^{-\frac{E_c - E_{Fi}}{kT}} e^{-\frac{E_{Fi} - E_v}{kT}}$$
$$= N_c N_v e^{-\frac{E_c - E_v}{Kt}}$$
$$= N_c N_v e^{-\frac{E_g}{Kt}}$$

E_g is the band gap.

Intrinsic Fermi level

$$n_i = N_c e^{-\frac{E_c - E_{Fi}}{kT}}$$
$$= N_v e^{-\frac{E_{Fi} - E_v}{kT}}$$
$$E_{Fi} = \frac{1}{2}(E_c + E_v) + \frac{1}{2} kT \ln\left(\frac{N_v}{N_c}\right)$$
$$= \frac{1}{2}(E_c + E_v) + \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

Thermal equilibrium electron distribution

assumptions: $E > E_c$, $E_c - E_F \gg kT$ then $E - E_F \gg kT$

$$n_0 = \int_{E_c}^{E_{\max}} g_c(E) f_F(E) dE$$

$E_{\max}$ is the maximum possible energy, it can be assumed equal to ∞ (due to $f_F \xrightarrow[E\to\infty]{} 0$)

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$
$$\approx e^{-\frac{E - E_F}{kT}}$$
$$n_0 = \int_{E_c}^{\infty} \frac{4\pi (2m_a^*)^{\frac{3}{2}}}{h^3} \sqrt{E - E_c} \exp\left(-\frac{E - E_F}{kT}\right) dE \qquad \text{Let:} \quad \eta = \frac{E - E_c}{kT}$$
$$= \frac{4\pi (2m_n^* kT)^{\frac{3}{2}}}{h^3} \exp\left(-\frac{E_c - E_F}{kT}\right) \underbrace{\int_0^{\infty} \eta^{\frac{1}{2}} e^{-\eta} d\eta}_{=\frac{\sqrt{\pi}}{2}}$$
$$= \underbrace{2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{\frac{3}{2}}}_{N_c} e^{-\frac{E_c - E_F}{kT}}$$
$$= N_c e^{-\frac{E_c - E_F}{kT}}$$

The same thing can be shown for holes, just swap m_n^*(effective mass of electron) with m_p^*(effective mass of hole) and $E - E_F$ with $E_F - E$ and $E_c - E_F$ with $E_F - E_v$.
N_c is the *effective density of states function in the conduction band*
For holes the constant is $N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{\frac{3}{2}}$

Extrinsic semiconductors

Probability function

$$n_d = \frac{N_d}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

n_d is the density of electrons occupying a donor level, N_d the concentration of donors and E_d the donor energy level. $\frac{1}{2}$ is from spin (also degeneracy factor)

$$= N_d - N_d^+$$

N_d^+ is the concentration of ionized donors, now assume $(E_d - E_F) \gg kT$:

$$n_d = \frac{N_d}{\frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$
$$= 2N_d \exp\left(-\frac{E_d - E_F}{kT}\right)$$
$$n_0 = N_c \exp\left(-\frac{E_c - E_F}{kT}\right)$$
$$\frac{n_d}{n_d + n_0} = \frac{2N_d \exp\left(-\frac{E_d - E_F}{kT}\right)}{2N_d \exp\left(-\frac{E_d - E_F}{kT}\right) + N_c \exp\left(-\frac{E_c - E_F}{kT}\right)}$$
$$= \frac{1}{1 + \frac{N_c}{2N_d}\exp\left(-\frac{E_c - E_d}{kT}\right)}$$

Fermi level position

$$E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right)$$
$$= kT \ln\left(\frac{N_c}{N_d}\right)$$

since

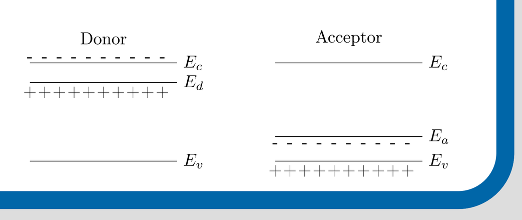
$$N_d \gg n_i$$
$$n_0 \simeq N_d$$

for an n type semiconductor. To get the intrinsic Fermi level:

$$n_0 = n_i e^{\frac{E_F - E_{Fi}}{kT}}$$
$$\Downarrow$$
$$E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right)$$

Dopant atoms

Donors and acceptors change the properties, it also makes the semiconductor extrinsic instead of intrinsic.



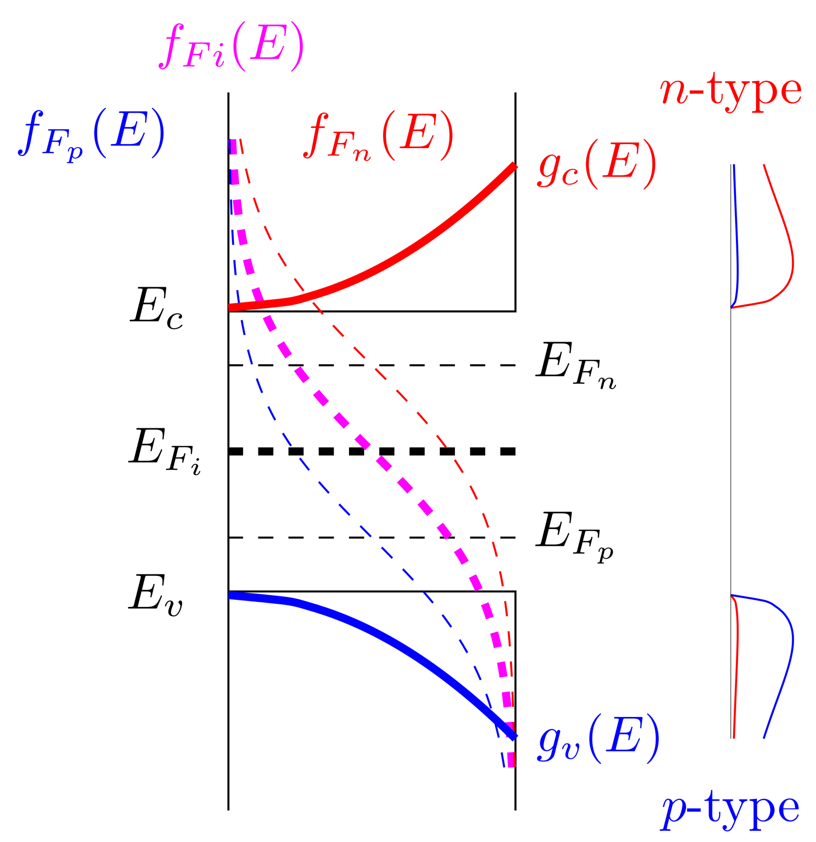
Extrinsic semiconductor

The donors/acceptors create a new equilibrium. This is gotten from adding and subtracting the intrinsic Fermi energy to the exponential term:

$$n_0 = N_c e^{-\frac{(E_c - E_{Fi}) + (E_F - E_{Fi})}{kT}}$$
$$= \underbrace{N_c e^{-\frac{(E_c - E_{Fi})}{kT}}}_{n_i} e^{\frac{E_F - E_{Fi}}{kT}}$$
$$= n_i e^{\frac{E_F - E_{Fi}}{kT}}$$

This leads to a new product at thermal equilibrium:

$$n_0 p_0 = n_i e^{\frac{E_F - E_{Fi}}{kT}} n_i e^{\frac{E_{Fi} - E_f}{kT}}$$
$$= N_c N_v e^{-\frac{E_g}{kT}}$$
$$= n_i^2$$



Thermal equilibrium electron concentration

A compensated semiconductor has both acceptor and donor impurities

$$n_0 + N_a^- = p_0 + N_d^+ = n_0 + (N_a - p_a) = p_0 + (N_d - n_d)$$

n_d is the concentration of electrons in the donor states, N_d^+ the concentration of positively charged donor states, p_a is the concentration of holes in the acceptor states, N_a^- the concentration of negatively charged acceptor states

$$n_0 + N_a = p_0 + N_d$$
$$= \frac{n_i^2}{n_0} + N_d$$
$$0 = n_0^2 - (N_d - N_a) n_0 - n_i^2$$
$$n_0 = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}^2 + n_i^2\right)}$$