Scanning Tunnelling Microscopy and Atomic Force Microscopy

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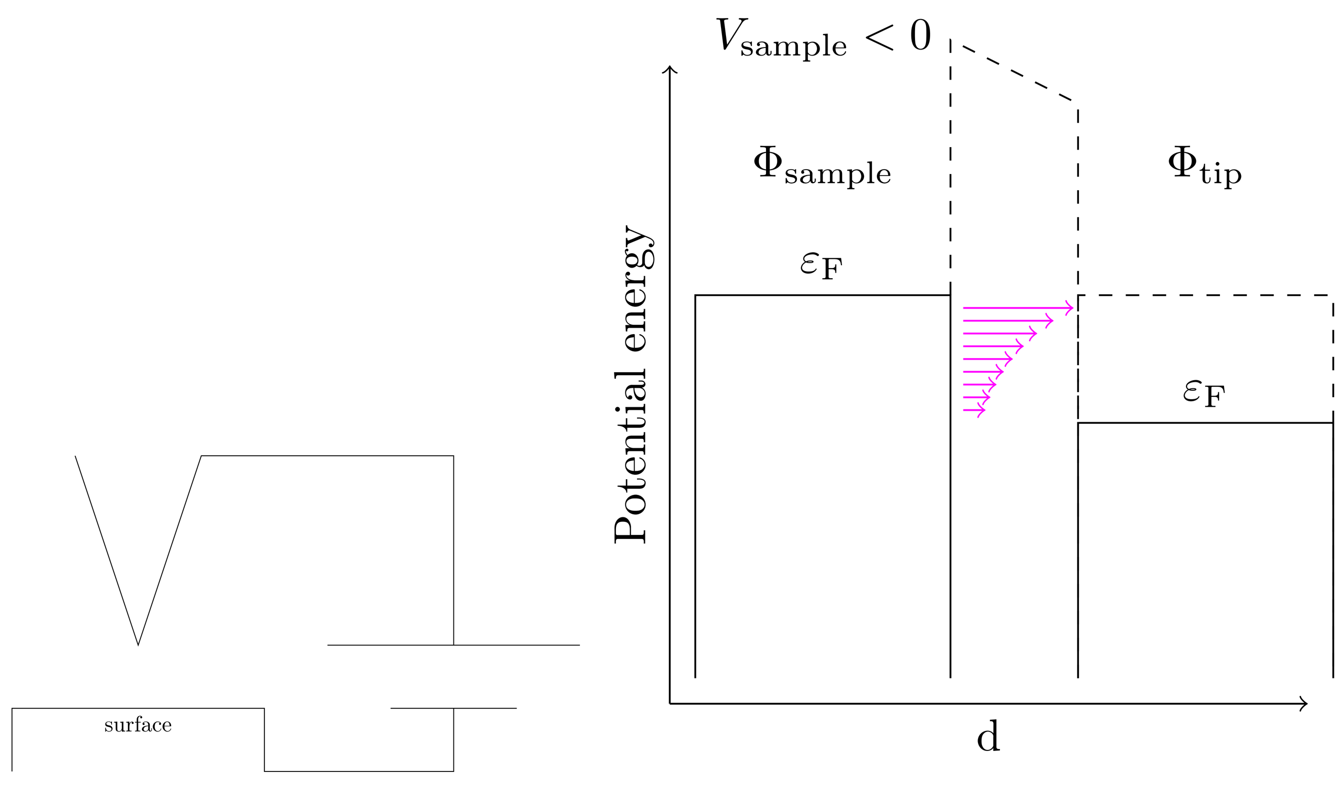
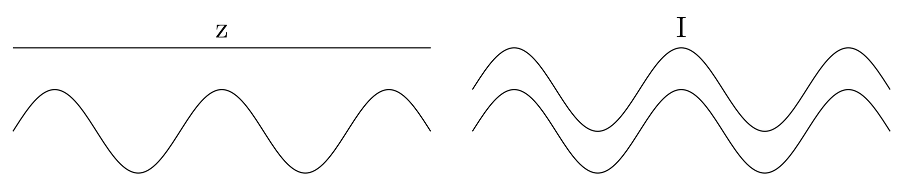
Scanning Tunnelling Microscopy

principle

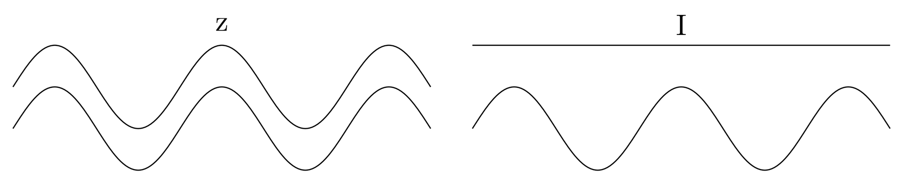


modes

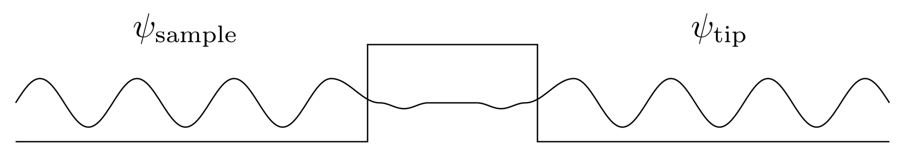
Constant height mode:



Constant current mode:



Tunneling



Tunneling math!!!

$$P_{s\to t} = \left(\frac{2m}{\hbar^2}\right) \int |\Psi_t(r) H' \Psi_s(r)|^2 \, d\mathbf{r} \partial(E_t - E_s)$$
$$I_{t\to s} = \frac{2\pi e}{\hbar} \int |M_{ts}|^2 \rho_t(E - eV) \rho_s(E) f_t(E - eV)[1 - f_s(E)] \, dE$$
$$I_{s\to t} = \frac{2\pi e}{\hbar} \int |M_{ts}|^2 \rho_t(E - eV) \rho_s(E)[1 - f_t(E - eV)] f_s(E) \, dE$$

ρ_t is the density of states in the tip.ρ_s is the density of states in the surface.eV is the shift due to bias.f_t is the Fermi—Dirac distribution for the tip.f_s is the Fermi—Dirac distribution for the surface.$|M_{ts}|^2$ tunneling matrix element.

$$I_{t\to s} - I_{s\to t} = \frac{2\pi e}{\hbar} \int |M_{ts}|^2 \rho_t(E - eV) \rho_s(E)[f_s(E) - f_t(E - eV)] \, dE$$
$$I = I_{t\to s} - I_{s\to t} \approx \frac{2\pi e}{\hbar} \int_{E_f}^{E_{f+eV}} |M_{ts}|^2 \rho_t(E - eV) \rho_s(E) \, dE$$

Note version:

$$I \approx \int_{E_f}^{E_{f+eV}} |M_{ts}|^2 \rho_t(E - eV) \rho_s(E) \, dE = \int_0^{eV_A} T(E, eV_A) \rho_a(E) \rho_B(E - eV) \, dE$$

$$I \approx \int_0^{eV} T(E, eV) \rho_s(E) \rho_t(E - eV) \, dE$$

let ρ_t be constant and:

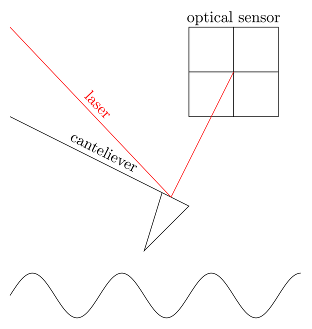
$$I \propto \int_0^{eV} T(E, eV) \rho_s(E) \, dE$$
$$T(E, eV) \approx e^{-2z\sqrt{\frac{2m}{\hbar^2}\left(\bar{\Phi} + \frac{eV}{2} - E\right)}}$$

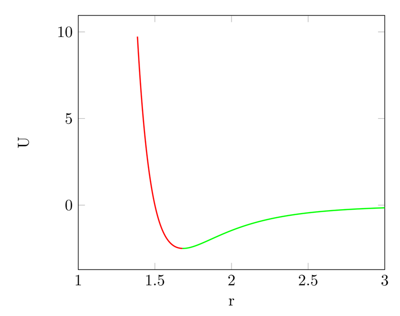
Is the tunneling probability, T is the barrier, z is the width of the barrier, Φ is the avg work function and V is the applied bias. Next assume a low bias, to take the tunneling out of the integral:

$$\begin{aligned} I &\propto \int_0^{eV} T(E, eV) \rho_s(E) \, dE \\ &\approx T(E_f) \int_0^{eV} \rho_s \, dE \\ &\approx e^{-2z\sqrt{\frac{2m}{\hbar^2}(\Phi)}} \int_0^{eV} \rho_s(E) \, dE \\ &\approx e^{-2z\sqrt{\frac{2m}{\hbar^2}(\Phi)}} \rho_s(E_f) \cdot eV = const \cdot \rho_s(E_f, z) V \end{aligned}$$

Atomic Force Microscopy

setup





Modes

There's contact mode and non contact/dynamic mode and tapping mode between the two. In contact mode the tip scrapes along the surface, in tapping mode the tip oscillates and make contact with the tip each time it reaches it's lowest point, this mode doesn't require as much calibration as non contact, but also doesn't damage the system it's probing as much as contact, so it can be used for studying biological environments. In non contact mode the cantilever vibrates above the surface, and is driven at some frequency, thinking of the surface–tip interactions as a spring between the surface and tip, makes the tip a "ball" suspended between two springs, this problem is known, see to the right

forces

$$F_{tot} = \underbrace{F_{chem}}_{\text{Bonding between tip and sample}} + \underbrace{F_{mag}}_{\text{magnitically sensitive tips}} + \underbrace{F_{el}}_{-\frac{1}{2}\frac{\partial C}{\partial z}V^2} + \underbrace{F_{vdW}}_{-\frac{HR}{6r^2}}$$

Cantilever mechanics

$$m\frac{d^2z}{dt^2} + \frac{m\omega_0}{Q}\frac{dz}{dt} + k_0 z = F_{\text{ext}} \cos \omega t$$
$$z(t) = A(\omega) \cos(\omega t - \phi)$$
$$A(\omega) = \frac{\frac{F_{\text{ext}}}{m}}{\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2}}$$
$$\phi(\omega) = \arctan\left(\frac{\gamma\omega}{Q(\omega^2 - \omega_0^2)}\right)$$
$$m\frac{d^2z}{dt^2} + \frac{m\omega_0}{Q}\frac{dz}{dt} + k_0 z = F_{\text{ext}} \cos \omega t + F_{\text{ts}}(z)$$
$$F_{\text{ts}}(x) \approx \frac{\partial F_{\text{ts}}}{\partial z} z = k_{\text{ts}} z$$
$$\Downarrow$$
$$m\frac{d^2}{dt^2} + \frac{m\omega_0}{Q}\frac{dz}{dt} + (k_0 - k_{\text{ts}}) z = F_{\text{ext}} \cos(\omega t)$$
$$k' = k_0 - k_{\text{ts}} \qquad \text{Is the new spring constant}$$
$$\omega' = \sqrt{\frac{k'}{m}} \qquad \text{Is the new resonance frequency}$$
$$\begin{aligned} \omega'(k_0 + k_{\text{ts}}) &\approx \omega_0 + \frac{d\omega}{dk_{\text{ts}}} k_{\text{ts}} \\ &= \omega_0 - \frac{1}{2}\frac{1}{\sqrt{mk_0}} \\ &= \omega_0 \big(1 - \frac{1}{2}\frac{k_{\text{ts}}}{k_0}\big) \end{aligned}$$
$$\Downarrow$$
$$f' = f_0 + \underbrace{\Delta f}_{\text{detuning}}$$

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