# The pn-junction under forward and reverse bias

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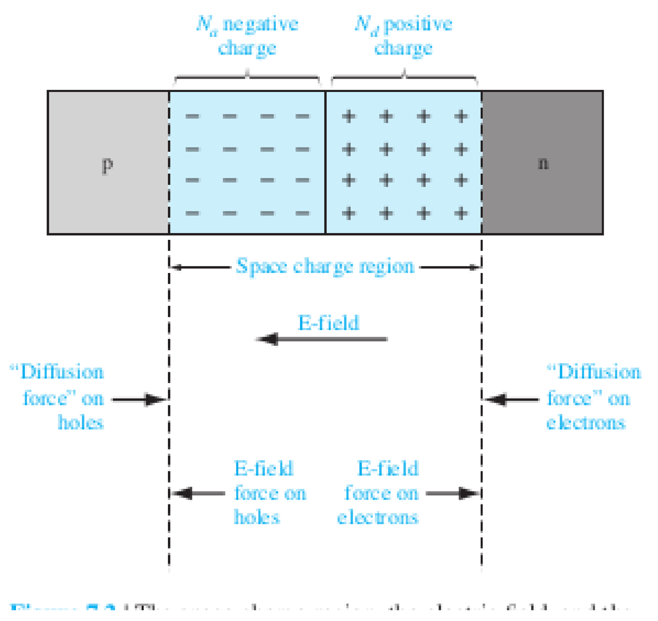
## The pn-junction

### Basic structure of the pn junction

A space charge region appears between the n and p junctions:




Note the space charge region and how the electrons from the n-type move to the holes in the p-type

### Zero applied Bias

$$V_{bi} = |\phi_{Fn}| + |\phi_{Fp}|$$

$V_{bi}$ is the build in potential barrier, the $\phi$ values are the differences in from the Fermi level to the intrinsic Fermi level, in the n region ($N_d \approx n_0$)

$$n_0 = N_c \exp\left[-\frac{E_c - E_F}{kT}\right] = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

$$e\phi_{Fn} = E_{Fi} - E_F$$

$$n_0 = N_c \exp\left[-\frac{e\phi_{Fn}}{kT}\right]$$

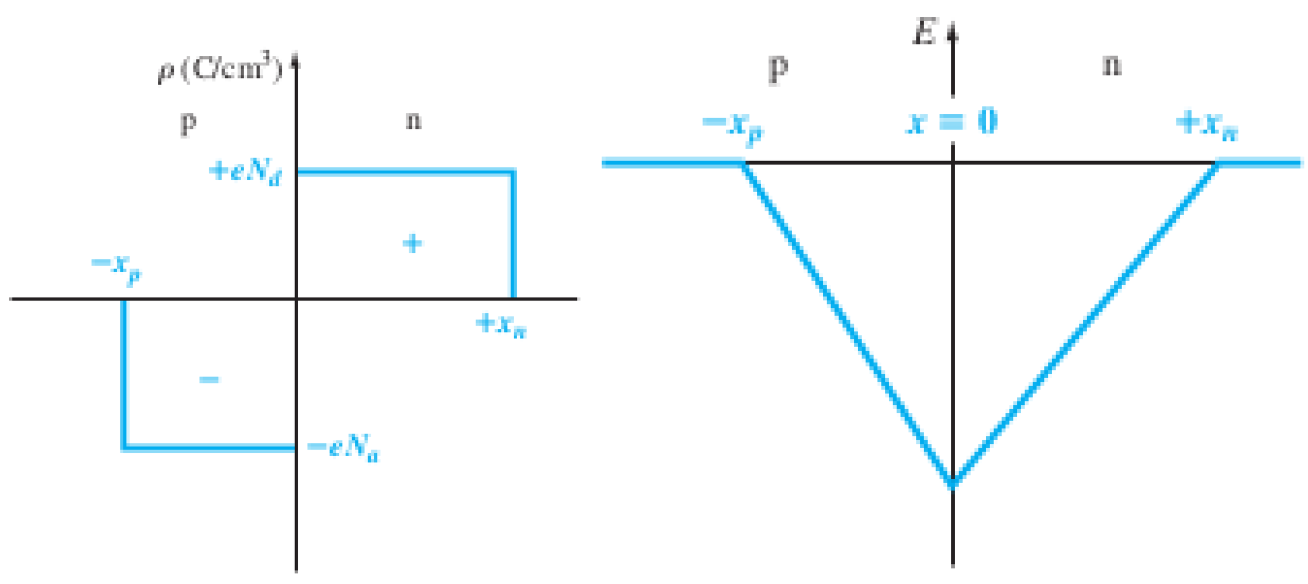
For the p type

$$p_0 = N_a = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right] = n_i \exp\left[\frac{e\phi_{Fp}}{kT}\right]$$

$$\phi_{Fn} = -\frac{kT}{e}\ln\left(\frac{N_d}{n_i}\right) \qquad \phi_{Fp} = +\frac{kT}{e}\ln\left(\frac{N_a}{n_i}\right)$$

Thus:

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$




Here it's assumed that the space charge region is strictly within $-x_p$–$x_n$.

$$\frac{d^2\phi(x)}{dx^2} = \frac{-\rho(x)}{\epsilon_s} = -\frac{dE(x)}{dx}$$

$$\rho(x) = \begin{cases} -eN_a & \forall x \in [-x_p; 0] \\ eN_d & \forall x \in [0; x_n] \end{cases}$$

$$E = \int \frac{\rho(x)}{\epsilon_s}\,dx = -\int \frac{eN_a}{\epsilon_s}\,dx = -\frac{eN_a}{\epsilon_s}x + C_1$$

$$E = -\int \frac{eN_d}{\epsilon_s}\,dx = \frac{eN_d}{\epsilon_s}x + C_1$$

$C_1$ and $C_2$ is from integration, now let $E = 0$ at $x = -x_p$ and $x = x_n$:

$$E = \begin{cases} -\frac{eN_a}{\epsilon_s}(x + x_p) & \forall x \in [-x_p; 0] \\ \frac{eN_d}{\epsilon_s}(x - x_n) & \forall x \in [0; x_n] \end{cases}$$

$$N_a x_p = N_d x_n$$

$$\begin{aligned} \phi(x) &= -\int E(x)\,dx \\ &= \int \frac{eN_a}{\epsilon_s}(x + x_p)\,dx \\ &= \frac{eN_a}{\epsilon_s}\left(\frac{x^2}{2} + x \cdot x_p\right) + C_1'\,dx \end{aligned}$$

$$C_1' = \frac{eN_a}{2\epsilon_s}x_p^2 \qquad \text{because of the zero location}$$

$$\phi(x) = \frac{eN_a}{2\epsilon_s}(x + x_p)^2 \qquad \forall x \in [-x_p; 0]$$

now for the n region:

$$\begin{aligned} \phi(x) &= \int \frac{eN_d}{\epsilon_s}(x - x_n)\,dx \\ &= \frac{eN_d}{\epsilon_s}\left(x \cdot x_n - \frac{x^2}{2}\right) + C_2'\,dx \end{aligned}$$

$$C_2' = C_1'$$

$$\phi(x) = \frac{eN_d}{\epsilon_s}\left(x \cdot x_n - \frac{x^2}{2}\right)^2 + \frac{eN_a}{2\epsilon_s}x_p^2 \qquad \forall x \in [0; x_n]$$

$$V_{bi} = \frac{e}{2\epsilon_s}\left(N_d x_n^2 + N_a x_p^2\right) = |\phi(x = x_n)|$$

Space charge width:

$$x_p = \frac{N_d}{N_a}x_n$$

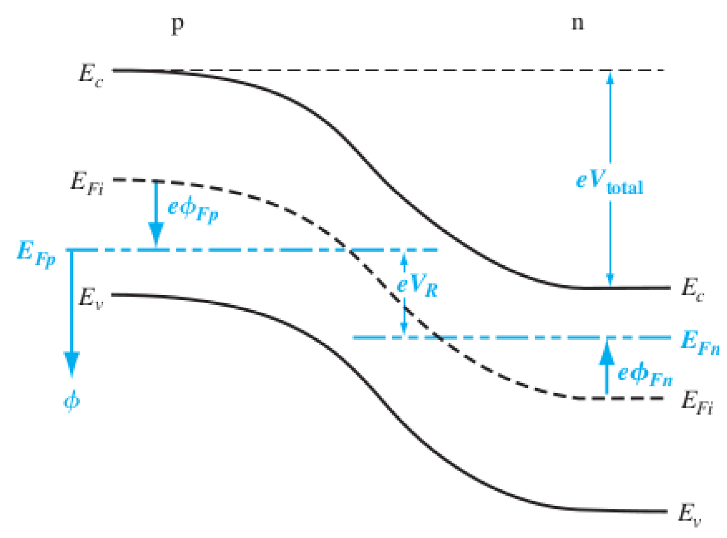
$$x_n = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a + N_d}\right]\right\}^{\frac{1}{2}}$$

$$x_p = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_d}{N_n}\right]\left[\frac{1}{N_a + N_d}\right]\right\}^{\frac{1}{2}}$$

$$W = x_n + x_p = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{\frac{1}{2}}$$

### Reverse applied bias

Reverse bias: Positive on the n side



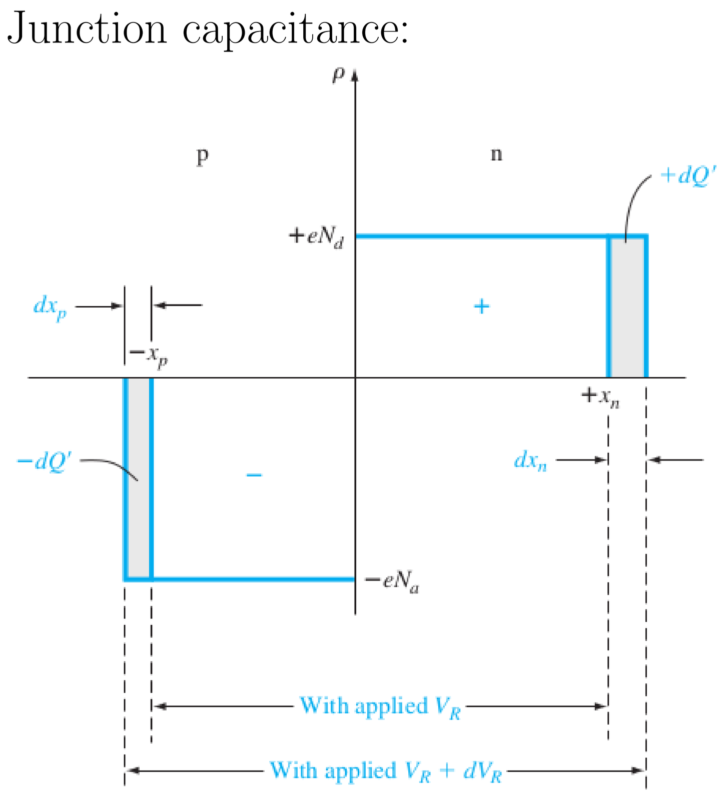

$$\begin{aligned} V_{\text{total}} &= |\phi_{Fn}| + |\phi_{Fp}| + V_R \\ &= V_{bi} + V_R \end{aligned}$$

New width:

$$W = x_n + x_p = \left\{\frac{2\epsilon_s(V_{bi} + V_R)}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{\frac{1}{2}}$$

$$\begin{aligned} E_{\max} &= -\frac{eN_d x_n}{\epsilon_s} = -\frac{eN_a x_p}{\epsilon_s} \\ &= -\sqrt{\frac{2e\,(V_{bi} + V_R)}{\epsilon_s}\left(\frac{N_a N_d}{N_a + N_d}\right)} \\ &= -\frac{2\,(V_{bi} + V_R)}{W} \end{aligned}$$

Junction capacitance:




$$dQ' = eN_d dx_n = eN_a dx_p$$

$$\begin{aligned} C' &= \frac{dQ'}{dV_R} = eN_d\frac{dx_n}{dV_R} \\ &= \sqrt{\frac{e\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}} = \frac{\epsilon_s}{W} \end{aligned}$$

One sided junction ($N_a \gg N_d$, known as $p^+n$ junction, opposite the other way)

$$W \approx \sqrt{\frac{2\epsilon_s(V_{bi} + V_R)}{eN_d}}$$

$$x_p \ll x_n$$

$$W \approx x_n$$

$$C' \approx \sqrt{\frac{e\epsilon_s N_d}{2(V_{bi} + V_R)}}$$