Transport and non-equilibrium behaviour in semiconductors

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Part I

Carrier transport phenomena

1 Carrier drift

$$J_{\mathrm{drf}} = \rho v_d$$

ρ is the charge density, v_d the drift current

$$J_{\mathrm{drf}} \quad \left[\frac{\mathrm{A}}{\mathrm{cm}^2}\right]$$

$$J_{p|\mathrm{drf}} = (ep)v_{dp}$$

v_p is the average drift velocity of holes, and $J_{p|\mathrm{drf}}$ the drift current density due to holes.

$$F = m^*_{cp} a = eE$$

is the equation of motion e is the magnitude of the electron charge E the electric field and m^*_{cp} the effective mass a the acceleration

$$v_{dp} = \mu_p$$

μ_p is the hole mobility

$$J_{p|\mathrm{drf}} = e\mu_p pE$$

$$J_{n|\mathrm{drf}} = e\mu_n nE$$

In total:

$$J_{\mathrm{drf}} = e(\mu_n n + \mu_p p)E$$

$$F = m^*_{cp}\frac{d_v}{dt} = eE$$

v is the velocity due to the electric field:

$$v = \frac{eEt}{m^*_{cp}}$$

Now let the time between collisions be τ_{cp}

$$v_{d|\mathrm{peak}} = \frac{eE\tau_{cp}}{m^*_{cp}}$$

$$\langle v_d \rangle = \frac{1}{2}\frac{eE\tau_{cp}}{m^*_{cp}}$$

$$\mu_p = \frac{v_{dp}}{E} = \frac{e\tau_{cp}}{m^*_{cp}}$$

$$\mu_n = \frac{e\tau_{cn}}{m^*_{cn}}$$

There are two types of scattering, lattice and ionized:

$$\mu_L \propto T^{-3/2}$$

$$\mu_L \propto \frac{T^{3/2}}{N_I}$$

$$\frac{dt}{\tau} = \frac{dt}{\tau_I} + \frac{dt}{\tau_L}$$

$$\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}$$

$$J_{\text{drf}} = e(\mu_n n + \mu_p p)E = \sigma E$$

$$\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)}$$

$$J = \frac{I}{A}$$

$$E = \frac{V}{L}$$

$$\frac{I}{A} = \sigma \frac{V}{L}$$

$$V = \frac{L}{\sigma A} I = \frac{\rho L}{A} I = IR$$

ohms law. now think p-tpe such that $N_a \gg n_i$

$$\sigma = e(\mu_n n + \mu_p p) \approx e\mu_p p \approx \frac{1}{\rho}$$

For an intrinsic material

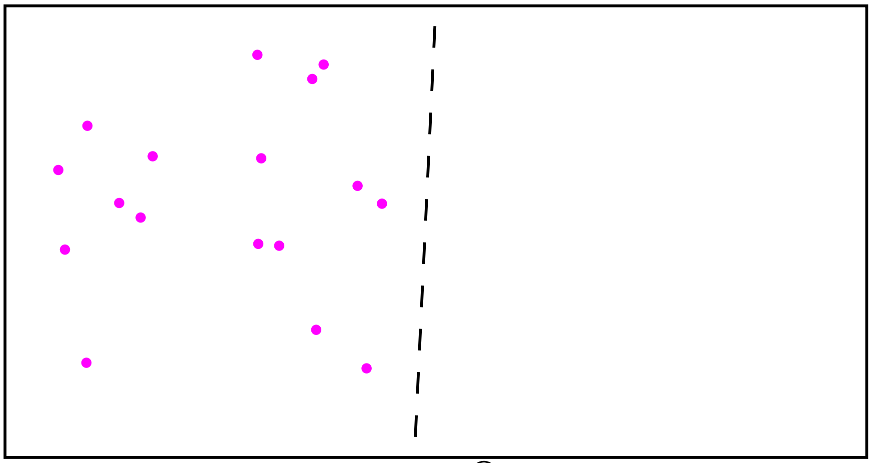
$$\sigma = e(\mu_n + \mu_p)n_i$$

$$v_n = \frac{v_s}{\sqrt{1 + \left(\frac{E_{on}}{E}\right)^2}}$$

$$v_p = \frac{v_s}{\sqrt{1 + \left(\frac{E_{op}}{E}\right)^2}}$$

$$v_n \approx \left(\frac{E}{E_{on}}\right) v_s$$

2 Carrier diffusion



$$F_n = \frac{1}{2}n(-l)v_{th} - \frac{1}{2}n(+l)v_{th} = \frac{1}{2}v_{th}\left[n(-l) - n(+l)\right]$$

Taylor expansion

$$\begin{aligned}
F_n &= \frac{1}{2}v_{th}\left(\left[n(0) - \frac{dn}{dx}\right] - \left[n(0) + l\frac{dn}{dx}\right]\right) \\
&= -v_{th}l\frac{dn}{dx} \\
J &= -eF_n = +ev_{th}l\frac{dn}{dx} \\
J_{nx|\text{dif}} &= eD_n\frac{dn}{dx} \\
J_{px|\text{dif}} &= -eD_p\frac{dp}{dx}
\end{aligned}$$

The total current is then

$$\begin{aligned}
J &= en\mu_n E_x + ep\mu_p E_x + eD_n\frac{dn}{dx} - eD_p\frac{dp}{dx} \\
J &= en\mu_n E + ep\mu_p E + eD_n\nabla n - eD_p\nabla p
\end{aligned}$$

3 Graded impurity distribution

$$\phi = \frac{1}{e}\left(E_F - E_{Fi}\right)$$
$$E_x = -\frac{d\phi}{dx} = \frac{1}{e}\frac{dE_{Fi}}{dx}$$

assuming quasi neutrality

$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \approx N_d(x)$$
$$E_F - E_{Fi} = kT \ln\left(\frac{N_d(x)}{n_i}\right)$$
$$-\frac{dE_{Fi}}{dx} = \frac{kT}{N_d(x)}\frac{dN_d(x)}{dx}$$
$$E_x = -\frac{kT}{e}\frac{1}{N_d(x)}\frac{dN_d(x)}{dx}$$

no electrical connection and thermal equilibrium:

$$J_n = 0 = en\mu_n E_x + eD_n\frac{dn}{dx}$$

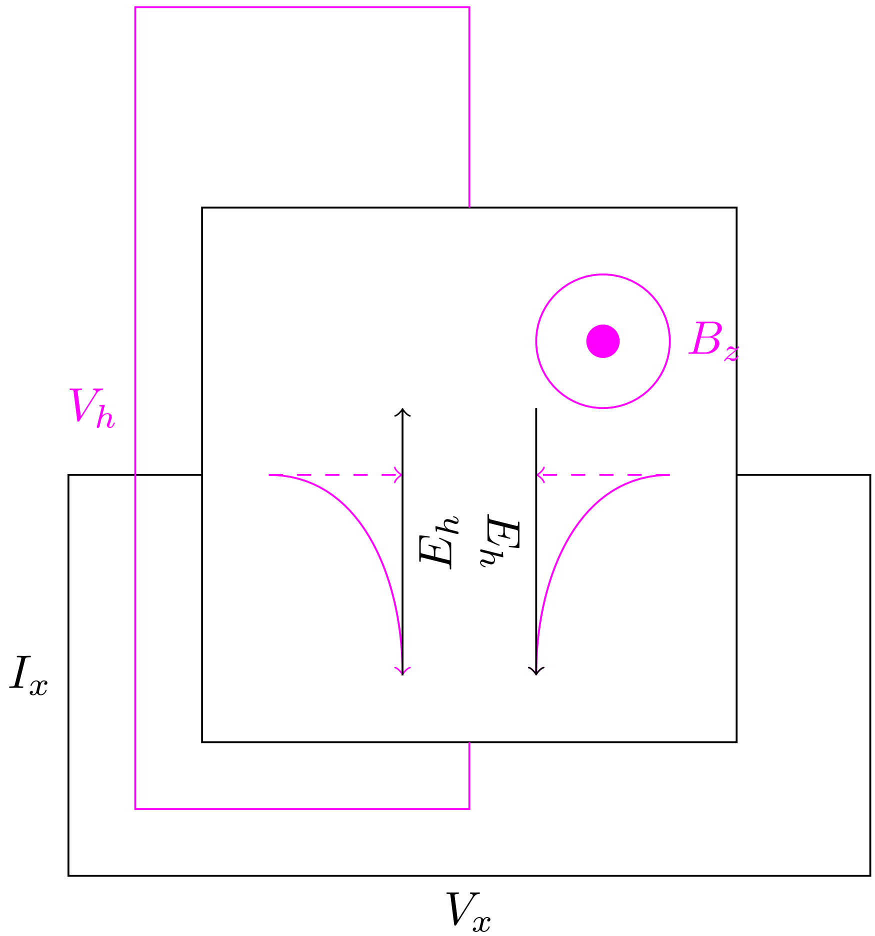
assume quasi-neutrality ($n \approx N_d(x)$)

$$= eN_d(x)\mu_n E_x + eD_n\frac{dN_d(x)}{dx} \qquad = -e\mu_n N_d(x)\frac{kT}{e}\frac{1}{N_d(x)}\frac{dN_d(x)}{dx} + eD_n\frac{dN_d(x)}{dx}$$

this needs the conditions (known as the Einstein relations

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$$

4 The Hall effect



$F = qv \times B$

As the holes and electrons move in the same direction the important thing is wheter or not there's an excess of one or the other. that's the case in an p or n type semiconducter. An electric field will then be made to compensate for this field:

$$
\begin{aligned}
F &= q\left[E + v \times B\right] = 0 \\
qE_y &= qv_x B_z \\
v_H &= v_x W B_z \\
v_{dx} &= \frac{J_x}{ep} = \frac{I_x}{epWd} \\
v_H &= \frac{I_x B_z}{epd} \\
p &= \frac{I_x B_z}{edV_H} \\
v_H &= \frac{I_x B_z}{end} \\
n &= -\frac{I_x B_z}{edV_H} \\
J_x &= ep\mu_p E_x \\
\frac{I_x}{Wd} &= \frac{epp\mu_p V_x}{L} \\
\mu_p &= \frac{I_x L}{epV_x Wd} \\
\mu_n &= \frac{I_x L}{enV_x Wd}
\end{aligned}
$$

In the above L is the length of the semiconductor in the direction of the current, W is the length of the semiconductor in the direction of neither the current nor the magnetic field, d is the length of the semiconductor in the direction of the magnetic field.

Part II

Nonequilibrium excess carriers in semiconductors

5 Carrier generation and recombination

$$G_{n0} = G_{p0} = R_{n0} = R_{p0}$$

Are the generation and recombination rates for holes and electrons respectively, when leaving equilibrium excess carriers will be generated and recombined at rates:

$$\begin{aligned} g'_n &= g'_p \\ R'_n &= R'_p \end{aligned}$$

The new concentrations are now:

$$\begin{aligned} n &= n_0 + \delta n \\ p &= p_0 + \delta p \end{aligned}$$

Note!

$$\begin{aligned} np \neq n_0 p_0 = n_i^2 \\ \frac{dn(t)}{dt} &= \alpha_r \left[n_i^2 - n(t)p(t) \right] \\ n(t) &= n_0 + \delta n(t) \\ p(t) &= p_0 + \delta p(t) \end{aligned}$$

The first term is in thermal equilibrium, $\delta n(t) = \delta p(t)$

$$\begin{aligned} \frac{d(\delta n(t))}{dt} &= \alpha_r \left[n_i^2 - (n_0 + \delta n(t))(p_0 + \delta p(t)) \right] \\ &= -\alpha_r \delta n(t) \left[(n_0 + p_0) + \delta n(t) \right] \end{aligned}$$

Here one must use the low-level injection condition: the excess carrier concentration is much less than the thermal-equilibrium majority carrier concentration

$$\begin{aligned} &= -\alpha_r p_0 \delta n(t) \\ \delta n(t) &= \delta n(0) e^{-\alpha_r p_0 t} \\ &= \delta n(0) e^{-\frac{t}{\tau_{n0}}} \end{aligned}$$

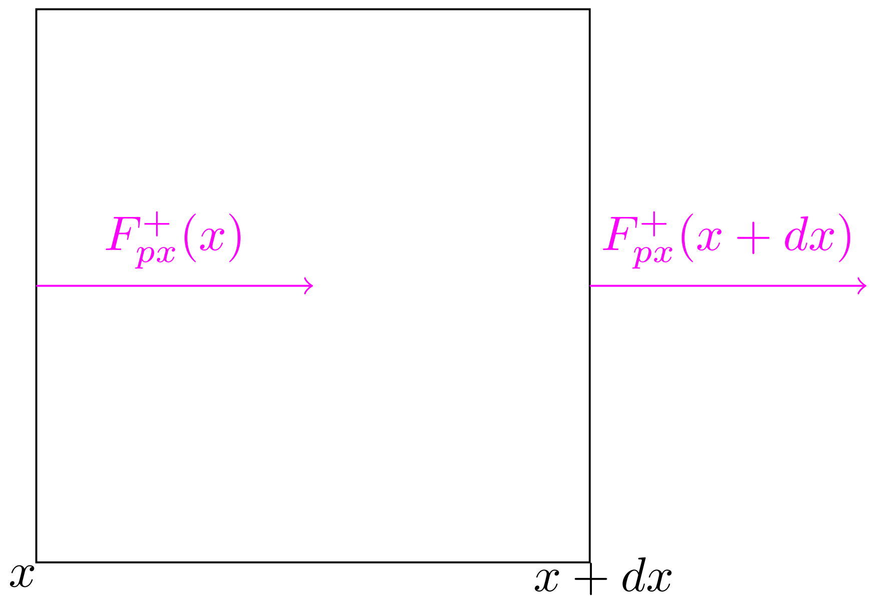
τ_{n0} is a constant for low-level injection, often called excess minority carrier lifetime, unrelated to collisions

$$\begin{aligned} R'_n &= \frac{-d(\delta n(t))}{dt} \\ &= +\alpha_r p_0 \delta n(t) = \frac{\delta n(t)}{\tau_{n0}} \end{aligned}$$

The recombining rates for majority and minority holes are the same:

$$R'_n = R'_p = \frac{\delta n(t)}{\tau_{n0}} \qquad \text{for majority}$$
$$R'_n = R'_p = \frac{\delta n(t)}{\tau_{p0}} \qquad \text{for minority}$$

6 Characteristic of excess carriers



$$F^+_{px}(x+dx) = F^+_{px}(x) + \frac{\partial F^+_{px}}{\partial x} \cdot dx$$
$$\frac{\partial p}{\partial t} dxdydz = \left[F^+_{px}(x+dx)\right] dydz = -\frac{\partial F^+_{px}}{\partial x} dxdyxz$$
$$\frac{\partial p}{\partial t} dxdydz = -\frac{\partial F^+_{px}}{\partial x} dxdyxz + g_p dxdydz - \frac{p}{\tau_{pt}} dxdydz$$

p is the density of holes, τ_{pt} thermal-equilibrium and excess carriar lifetimes

$$\begin{aligned}
\frac{\partial p}{\partial t} &= -\frac{\partial F_p^+}{\partial x} + g_p - \frac{p}{\tau_{pt}} \\
\frac{\partial n}{\partial t} &= -\frac{\partial F_n^-}{\partial x} + g_n - \frac{n}{\tau_{nt}} \\
J_p &= e\mu_p pE - eD_p \frac{\partial p}{\partial x} \\
J_n &= e\mu_n nE - eD_n \frac{\partial n}{\partial x} \\
\frac{J_p}{+e} &= F_p^+ = \mu_p pE - D_p \frac{\partial p}{\partial x} \\
\frac{J_n}{-e} &= F_n^+ = \mu_n nE - D_n \frac{\partial n}{\partial x} \\
\frac{\partial p}{\partial t} &= -\mu_p \frac{\partial (pE)}{\partial x} + D_p \frac{\partial^2 p}{\partial x^2} + g_p - \frac{p}{\tau_{pt}} \\
\frac{\partial n}{\partial t} &= -\mu_n \frac{\partial (nE)}{\partial x} + D_n \frac{\partial^2 n}{\partial x^2} + g_n - \frac{n}{\tau_{nt}}
\end{aligned}$$

With one dimensionality in mind

$$\frac{\partial (pE)}{\partial x} = E\frac{\partial p}{\partial x} + p\frac{\partial E}{\partial x}$$

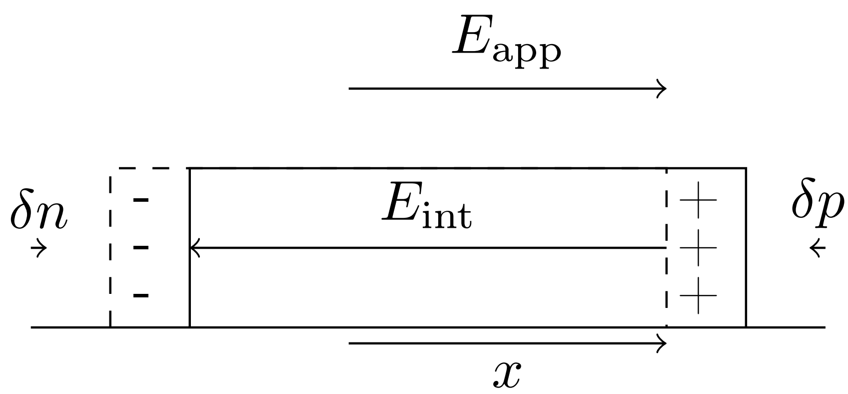
In the 3d case:

$$\begin{aligned}
D_p \frac{\partial^2 p}{\partial x^2} - \mu_p \left(E\frac{\partial p}{\partial x} + p\frac{\partial E}{\partial x} \right) + g_p - \frac{p}{\tau_{pt}} &= \frac{\partial p}{\partial t} \\
D_n \frac{\partial^2 n}{\partial x^2} - \mu_n \left(E\frac{\partial n}{\partial x} + n\frac{\partial E}{\partial x} \right) + g_n - \frac{n}{\tau_{nt}} &= \frac{\partial n}{\partial t}
\end{aligned}$$

For a homogeneous semiconductor:

$$\begin{aligned}
D_p \frac{\partial^2 \delta p}{\partial x^2} - \mu_p \left(E\frac{\partial \delta p}{\partial x} + p\frac{\partial E}{\partial x} \right) + g_p - \frac{p}{\tau_{pt}} &= \frac{\partial \delta p}{\partial t} \\
D_n \frac{\partial^2 \delta n}{\partial x^2} - \mu_n \left(E\frac{\partial \delta n}{\partial x} + n\frac{\partial E}{\partial x} \right) + g_n - \frac{n}{\tau_{nt}} &= \frac{\partial \delta n}{\partial t}
\end{aligned}$$

7 Ambipolar transport



$$\nabla \cdot E_{\text{int}} = \frac{e(\delta p - \delta n}{\epsilon_s} = \frac{\partial E_{\text{int}}}{\partial x}$$

ϵ_s is the permittivity of the semi conductor, form here on we assume $|E_{\text{int}}| \ll |E_{\text{app}}|$, and charge neutrality, the excess hole concentration will be balanced by an equal excesss hole conentration. now define:

$$\begin{aligned}
g_n &= g_p \equiv g \\
R_n &= \frac{n}{\tau_{nt}} = R_p = \frac{p}{\tau_{pt}} = R \\
\frac{\partial \delta p}{\partial t} &= D_p \frac{\partial^2 \delta p}{\partial x^2} - \mu_p \left(E \frac{\partial \delta p}{\partial x} + p \frac{\partial E}{\partial x} \right) + g_p - R \\
\frac{\partial \delta n}{\partial t} &= D_n \frac{\partial^2 \delta n}{\partial x^2} - \mu_n \left(E \frac{\partial \delta n}{\partial x} + n \frac{\partial E}{\partial x} \right) + g_n - R \\
(\mu_n n + \mu_p p) \frac{\partial \delta n}{\partial t} &= (\mu_n n D_p + \mu_p p D_n) \frac{\partial^2 \delta n}{\partial x^2} + (\mu_n \mu_p)(p - n) E \frac{\partial \delta n}{\partial x} + (\mu_n n + \mu_p p)(g - R) \\
D' \frac{\partial^2 \delta n}{\partial x} + \mu' E \frac{\partial \delta n}{\partial x} + g - R &= \frac{\partial \delta n}{\partial t} \\
D' &= \frac{\mu_n n D_p + \mu_p p D_n}{\mu_n n + \mu_p p} \\
\mu' &= \frac{\mu_n \mu_p (p - n)}{\mu_n n + \mu_p p}
\end{aligned}$$

The three above equations are the "ambipolar transport equation", "ambipolar diffusion coefficient" and the "ambipolar mobility", The einstein releation holds:

$$\frac{\mu_n}{D_n} = \frac{\mu_p}{D_p} = \frac{e}{kT}$$

$$D' = \frac{D_n D_p (n+p)}{D_n n + D_p p}$$

now with low-level injection:

$$= \frac{D_n D_p (n_0 + \delta n + p_0 + \delta n)}{D_n (n_0 + deltan + D_p (p_0 + \delta n)}$$

$$D' = D_n$$

$$\mu' = \mu_n$$

Thus the minority carrier becomes the most important for ambipolar transport under low level injection. Now fr generation/recombination

$$R_n = R_p = \frac{n}{\tau_{nt}} = \frac{p}{\tau_{pt}} = R$$

For the minority carrier $\tau_{it} = \tau_t$ where i is either n or p

$$g - R = g_n - R = (G_{n0} + g'_n) - (R_{n0} + R'_n)$$

$$G_{n0} = R_{n0}$$

$$g - R = g_n - R = g'_n - R'_n \qquad = g$$

same for hole generation

$$D_n \frac{\partial^2 \delta n}{\partial x^2} - \mu_n E \frac{\partial \delta n}{\partial x} + g' - \frac{n}{\tau_{n0}} = \frac{\partial \delta n}{\partial t}$$

$$D_p \frac{\partial^2 \delta p}{\partial x^2} - \mu_p E \frac{\partial \delta p}{\partial x} + g' - \frac{p}{\tau_{p0}} = \frac{\partial \delta p}{\partial t}$$

The dielectric relaxation time constant

$$\nabla \cdot E = \frac{\rho}{\epsilon}$$

$$J = \sigma E$$

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

$$= \sigma \nabla \cdot E = \frac{\sigma \rho}{\epsilon}$$

$$= -\frac{\partial \rho}{\partial t} = -\frac{d\rho}{dt}$$

$$\frac{d\rho}{dt} + \frac{\sigma}{\epsilon}\rho = 0$$

$$\rho(t) = \rho(0) e^{-\frac{t}{\tau_d}}$$

$$\tau_d = \frac{\epsilon}{\sigma}$$

Haynes–Shockley: A field of v_1 is applied to a semiconductor, a pulse is sent through the semiconductor and travels a distance d inside it:

$$x - \mu_p E_0 t = 0 \qquad x = d$$

$$\mu_p = \frac{d}{E_0 t_0}$$

$$(d - \mu_p E_0 t)^2 = 4 D_p t$$

$$D_p = \frac{(\mu_p E_0 \Delta t)^2}{16 t_0}$$

$$\Delta t = t_2 - t_1$$

$$S = K \exp\left(-\frac{t_0}{\tau_{p0}}\right) = K \exp\left(-\frac{d}{\mu_p E_0 \tau_{pd}}\right)$$

is the area under the curve.

8 Quasi Fermi energy levels

When excess carriers are present there's no thermal equilibrium, so the Fermi level is not defined, but one can define quasi Fermi levels.

$$n_0 = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$$

$$p_0 = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$

$$n_0 + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

9 Excess carrier lifetime

It's important the speed at which generation and recombination occurs a state in the energy within the bandgap is called a trap, and can act as a recombination center. 4 processes can hapen therere 1: Electrons jump from conducting band to the trap

$$R_{cn} = C_n N_t \left[1 - f_F(E_t)\right] n$$

R_{cn} is the capture rate, C_n ta constant proportional to the electron-capture cross section, N_t the concentration of traps, n the electron concentration in the conducting band and $f_F(E_t)$ the fermi function at the trap energy:

$$f_F(E_t) = \frac{1}{1 + \exp\left(\frac{E_t - E_F}{kT}\right)}$$

Process 2 describes electrons escaping from a negatively charged hole:

$$R_{en} = E_n N_t f_F(E_t)$$

R_{en} is the emmision rate, E_n a constant and $f_{F(E_t)}$ the probability that the trap is occupied. At thermal equilibrium:

$$\begin{aligned} R_{en} &= R_{cn} \\ E_n N_t f_{F0(E_t)} &= C_n N_t \left[1 - f_{F0}(E_t)\right] n_0 \\ E_n &= n' C_n \\ n' &= N_c \exp\left[-\frac{E_c - E_t}{kT}\right] \end{aligned}$$

n' corrosponds to the electron concentration in the conducting band if $E_t = E_F$, outside equilibrium:

$$\begin{aligned} R_n &= R_{cn} - R_{en} \\ &= \left[C_n N_t (1 - f_F(E_t)) n\right] - \left[E_n N_t f_F(E_t)\right] \\ &= C_n N_t \left[n(1 - f_F(E_t)) - n' f_F(E_t)\right] \end{aligned}$$

The same two processes exists for the holes:

$$\begin{aligned} R_p &= C_p N_t \left[p(1 - f_F(E_t)) - p'(1 - f_F(E_t))\right] \\ p' &= N_c \exp\left[-\frac{E_t - E_v}{kT}\right] \\ f_F(E_t) &= \frac{C_n n + C_p p'}{C_n(n + n') + C_p(p + p')} \qquad \text{By } R_p = R_n \\ R_n = R_p \equiv R &= \frac{C_n C_p N_t (np - n_i^2)}{C_n(n + n') + C_p(p + p')} = \\ frac\delta n\tau \end{aligned}$$

Now apllying the conditions of extrinsic doping and low injection, for an n type:

$$\begin{aligned} n_0 \gg p_0, \quad n_0 \gg \delta p, \quad n_0 \gg n', n_0 \gg p' \\ R &= C_p N_t \delta p \\ \frac{\delta n}{\tau} &= C_p N_t \delta p \equiv \frac{\delta p}{\tau_{p0}} \\ \tau_{p0} &= \frac{1}{C_p N_t} \end{aligned}$$

The fewer the number of excess carriers the longer the lifetime.