自然语言处理导论
#L7
句法分析

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The path so far

- 最初，将语言视为由词构成的序列
  - n-gram（语言模型）
- 接着，引入词的句法属性
  - part-of-speech tagging（词性标注）
- 现在，考察词之间的句法关系
  - Syntactic parsing（句法分析）

语法和语义

- 大部分情况下，一个不合乎语法的句子也可以被理解
  - The boy quickly in the house the ball found
  - 看清楚路先兄弟
- 合乎语法的句子也可能无法理解
  - Are gyre and gimble in the wabe? (non-sense words)
  - 不会做饭的裁缝不是一个好司机
- 但句法规则可以传递如下信息：
  - 句子的语法
  - 词的顺序
  - 短语成分
  - 句子的层次结构
  - 句法关系，例如主语、宾语
  - ……

句法分析的应用

- 句法分析被广泛且成功地应用到NLP的各个方面：
  - Meaning Representation [Jeffrey Flanigan, et al., ACL 2014]
  - High precision question answering [Pasca and Harabagiu, SIGIR 2011]
  - Source sentence analysis for machine translation [Xu et al., 2009]
  - Syntactically based sentence compression [Lin and Wilbur, 2007]
  - Extracting opinions about products [Bloom et al., NAACL 2007]
  - Relation extraction systems [Fundel et al., *Bioinformatics* 2006]
  - Improved interaction in computer games [Gorniak and Roy, 2005]
  - Helping linguists find data [Resnik et al., BLS 2005]
  - Improving biological named entity finding [Finkel et al., JNLPBA 2004]

Parsing on ACL 2016

- *29 papers:*
  - *Neural Greedy Constituent Parsing with Dynamic Oracles*
  - *Active Learning for Dependency Parsing with Partial Annotation*
  - *Sentence Rewriting for Semantic Parsing*
  - *A Fast Unified Model for Parsing and Sentence Understanding*
  - *……*

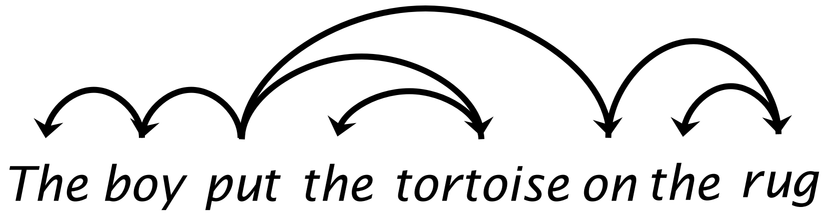
一个简单的句子

- I like the interesting lecture
- PRO VB DET JJ NN

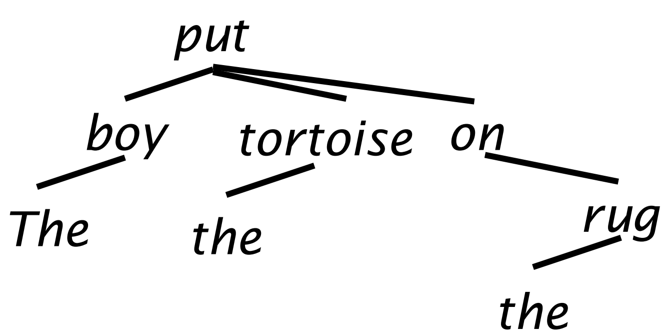
- 除了以上的词性标注，往往还关心：
  - 动词 *like*的主语是代词 *I*， 说明 who is doing the liking
  - 动词 *like*的宾语是名词*lecture*，说明 what is being liked
  - 定冠词*the* 指出说明名词*lecture*
  - 形容词*interesting* 给出更多关于名词*lecture*的信息

两种不同的句法结构

- 依存结构（Dependency structure）：
  - 说明词和其它词之间的依赖关系（从属关系、支配关系等）

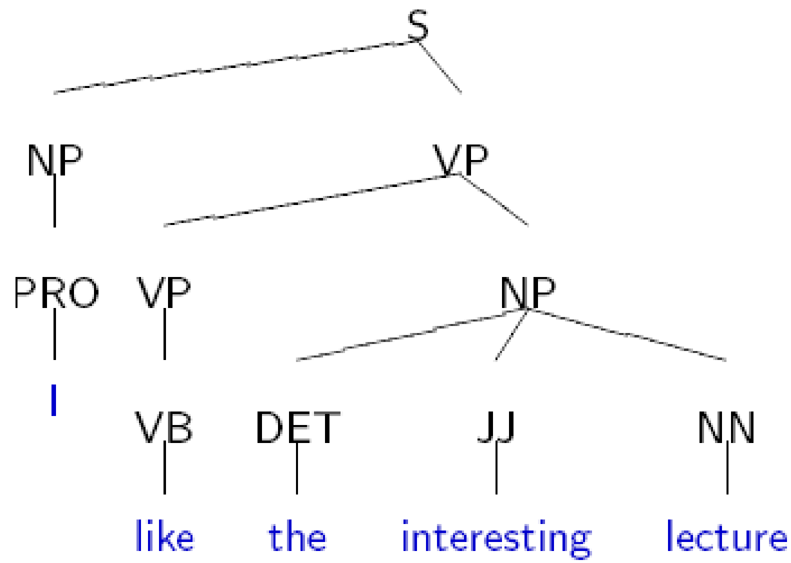


  - 可以表示为一个依存树（dependency tree）：



两种不同的句法结构

- 短语结构（Phrase structure）：
  - 将句子表示成嵌套的短语成分
- 父节点将子节点组合成较大的短语单元
  - 例如将DET JJ NN组合成NP



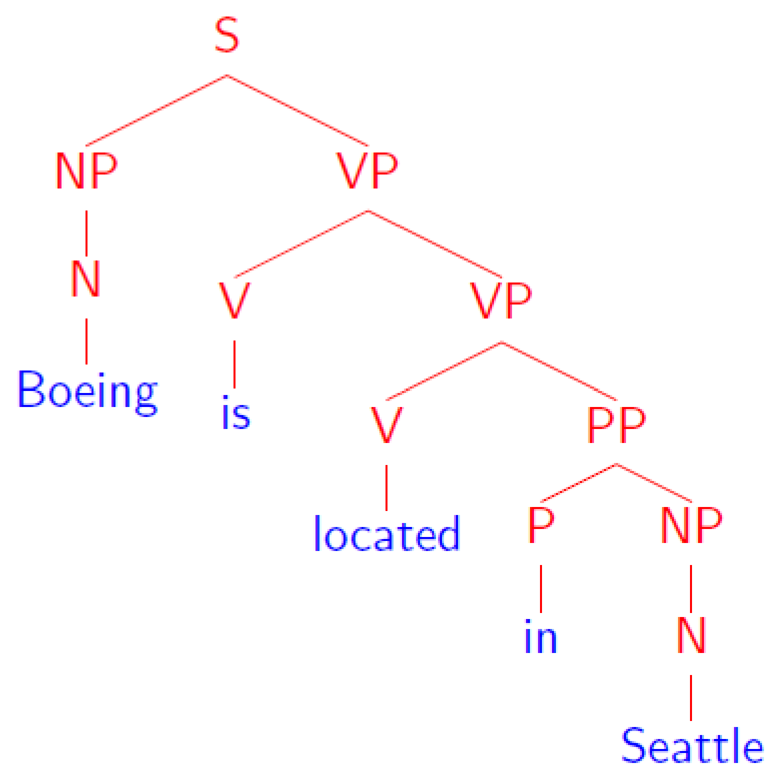
句法分析

- 这里的句法分析（Parsing）：专指短语结构分析

INPUT: 句子

Boeing is located in Seattle

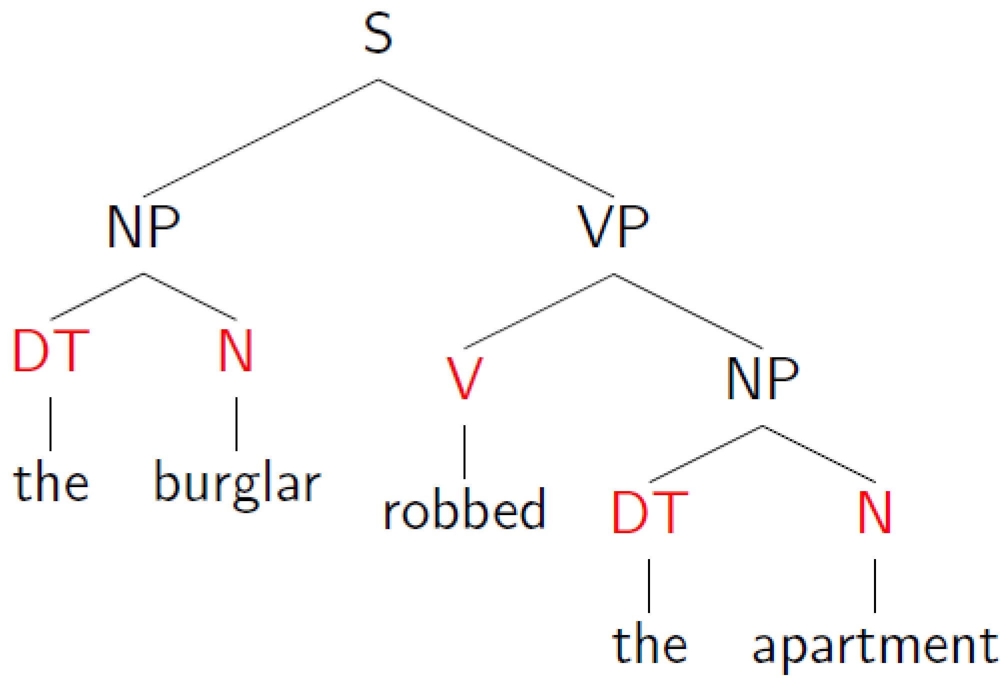
OUTPUT: 句法树



句法树中包含的信息

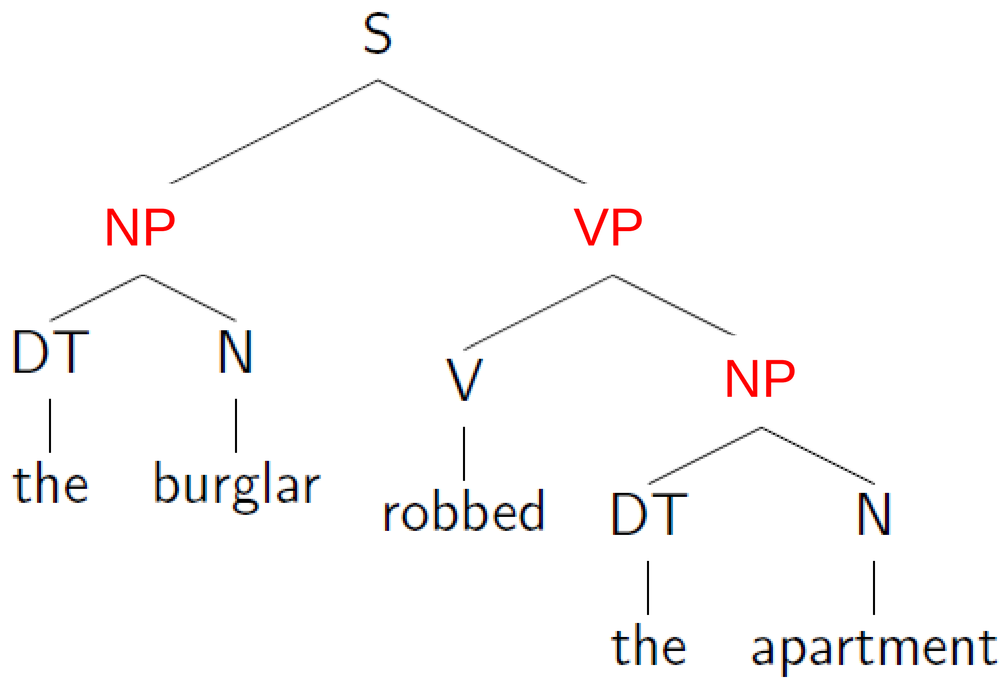
- (1) 词的词性类别

  (N = noun, V = verb, DT = determiner)



句法树中包含的信息

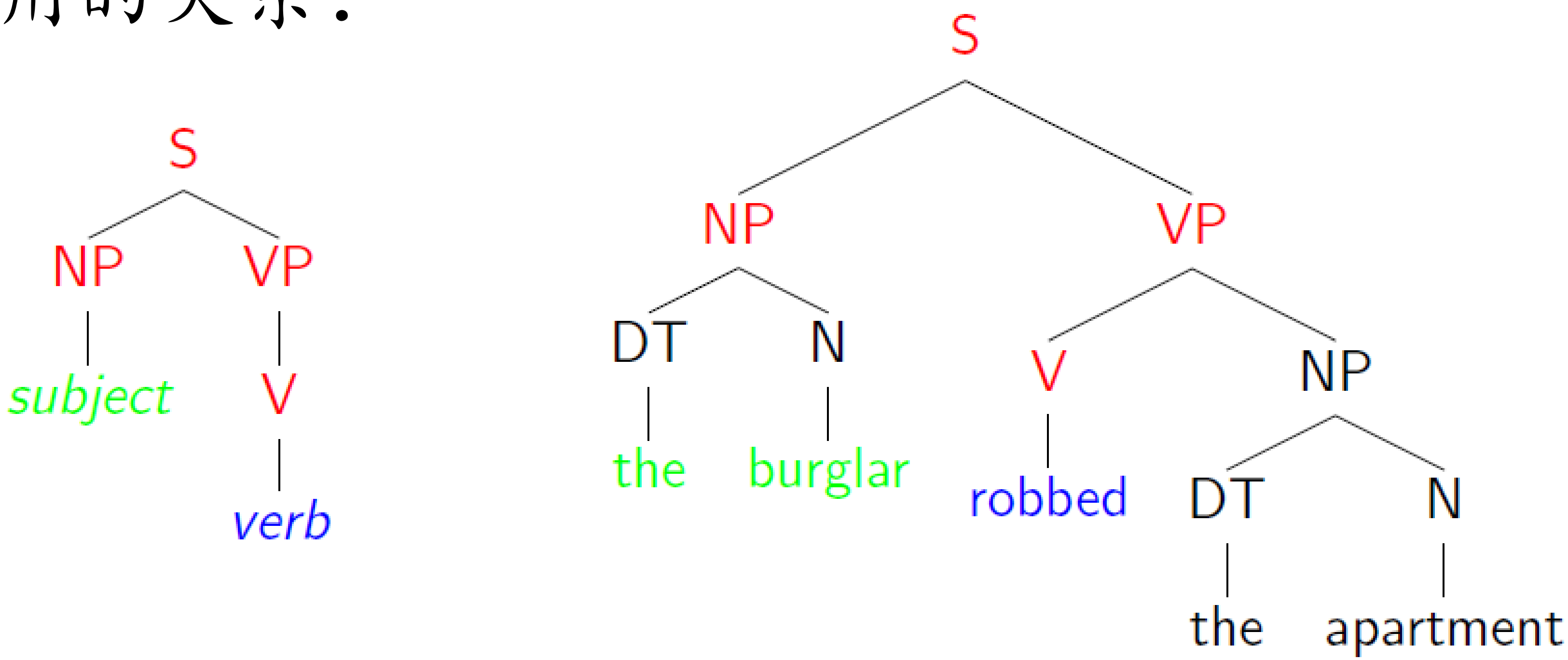
- (2) 短语



- 名词短语(NP): "the burglar", "the apartment"
- 动词短语 (VP): "robbed the apartment"
- 句子 (S): "the burglar robbed the apartment"

句法树中包含的信息

- 有用的关系：



- “the burglar”是“ robbed”的主语

句法分析

- 两个目的：
  - 判断输入句子是否合乎给定的语法
  - 识别句子各部分是如何依据语法规则组成合法句子，同时生成句法树
- 两个准备：
  - 语言的形式化描述（规定该语言中允许出现的结构）
  - 句法分析技术（根据语法来分析句子并确定其结构）

形式语法

- 形式语法是规定语言中允许出现的结构的形式化说明
- 形式语法可以追溯到1950s（Chomsky's PhD thesis）
- 几个主要的形式语法：
  - context-free grammar (CFG)
  - lexical functional grammar (LFG)
  - head-driven phrase-structure grammar (HPSG)
  - tree adjoining grammars (TAG)
  - combinatory categorical grammar (CCG)

上下文无关语法

- Context-free grammars (CFGs)
- Hopcroft and Ullman, 1979
- 假设一个语言L是由语法G生成，则G可以表示为一个四元组：G = (T, N, S, R)
  - T：终结符（terminal symbols）集合，通常包括句法树的叶子节点，如 *like, lecture*
  - N：非终结符（nonterminal symbols）集合，句法树的中间节点，如 *NP, S*
  - S：开始符号，特殊的非终结符($S \in N$)，表示句子
  - R：重写规则（或产生式），具有形式$X \rightarrow \gamma$，例如，*NP→DET JJ NN*
    - $X \in N$ 并且$\gamma \in (N \cup T)^*$

CFGs的特性

- 上下文无关特性：句法规则$X \rightarrow \gamma$的应用不依赖于X出现在什么上下文环境中
- 若$s \in T^*$是由CFGs定义的语言，则至少有一种重写规则可以生成s
- 由CFGs生成的语言可能有不止一个短语结构 (结构歧义)

一个简单的CFGs的例子

- T = {sleeps, saw, man, woman, telescope, the, with, in}
- N = {S, NP, VP, PP, DT, Vi, Vt, NN, IN}
- S = S
- R =

S $\rightarrow$ NP VP	Vi $\rightarrow$ sleeps
VP $\rightarrow$ Vi	Vt $\rightarrow$ saw
VP $\rightarrow$ Vt NP	NN $\rightarrow$ man
VP $\rightarrow$ VP PP	NN $\rightarrow$ woman
NP $\rightarrow$ DT NN	NN $\rightarrow$ telescope
NP $\rightarrow$ NP PP	DT $\rightarrow$ the
PP $\rightarrow$ IN NP	IN $\rightarrow$ with
	IN $\rightarrow$ in
……	……

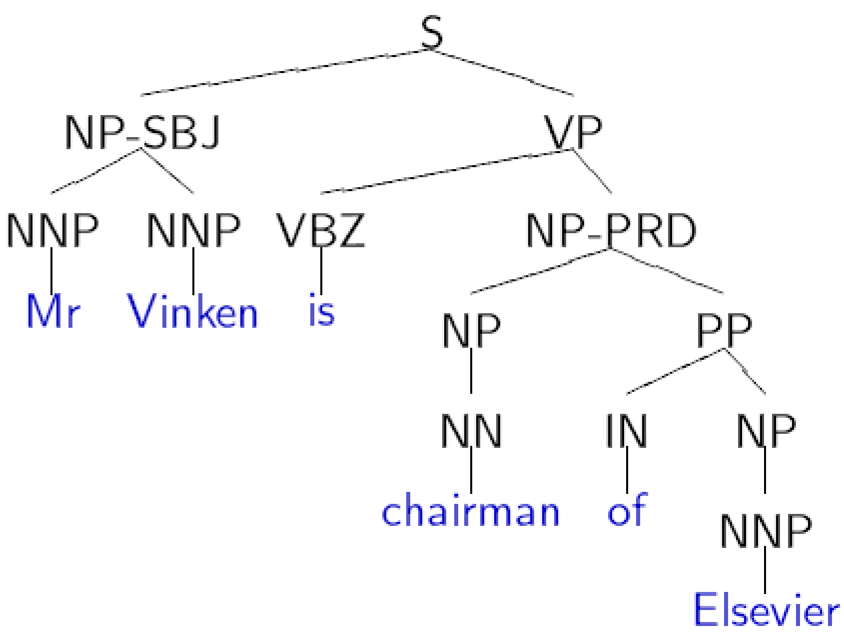
Chomsky 范式

- Chomsky Normal Form
- 一个受Chomsky范式约束的CFG句法 G = (T, N, S, R)，具有以下形式：
  - T：终结符集合
  - N：非终结符集合
  - S：开始符号，特殊的非终结符(S ∈ N)，表示句子
  - R：句法规则集合，具有以下两种形式：
    - $N^i \rightarrow N^j N^k$ for $N^i \in$ N, and N^j , $N^k \in$ N
    - $N^i \rightarrow w^j$ for $N^i \in$ N, and $w^j \in$ T

应用句法规则生成句子

Input	Rule	Output
S	S $\rightarrow$ NP VP	NP VP
NP VP	NP $\rightarrow$ PRO	PRO VP
PRO VP	PRO $\rightarrow$ *I*	*I* VP
I VP	VP $\rightarrow$ VP NP	*I* VP NP
I VP NP	VP $\rightarrow$ VB	*I* VB
I VB NP	VB $\rightarrow$ *like*	*I like* NP
I like NP	NP $\rightarrow$ DET JJ NN	*I like* DET JJ NN
I like DET JJ NN	DET $\rightarrow$ *the*	*I like the* JJ NN
I like the JJ NN	JJ $\rightarrow$ *interesting*	*I like the interesting* NN
I like the interesting NN	NN $\rightarrow$ *lecture*	*I like the interesting lecture*

应用句法规则构建句法树



S→NP-SBJ VP
NP-SBJ → NNP NNP
NNP → Mr
NNP → Vinken
VP → VBZ NP-PRD
VBZ → is
NP-PRD → NP PP
NP → NN
NN → chairman
PP → IN NP
IN → of
NP → NNP
NNP → Elsevier

应用句法规则构建句法树

- 可以表述为一个搜索过程
  - 搜索空间：语法规则
  - 搜索过程：检查各种语法规则所有可能的组合方式
  - 搜索目的：最终找到一种组合，其中的语法规则能够生成一个用来表示句子结构的句法树
  - 搜索方向：自顶向下 vs 自底向上

Cocke-Kasami-Younger (CKY) Parsing

- 已有一组上下文无关语法：
  - S→NP VP, NP → PRO, PRO → I, VP → VP NP, VP →VB, VB → like, NP →DET JJ NN, DET → the, JJ → interesting, NN → lecture
- 输入：句子
  - I like the interesting lecture
- CKY句法分析：
  - 自底向上的句法分析算法
  - 采用一个线图（chart）存储中间结果

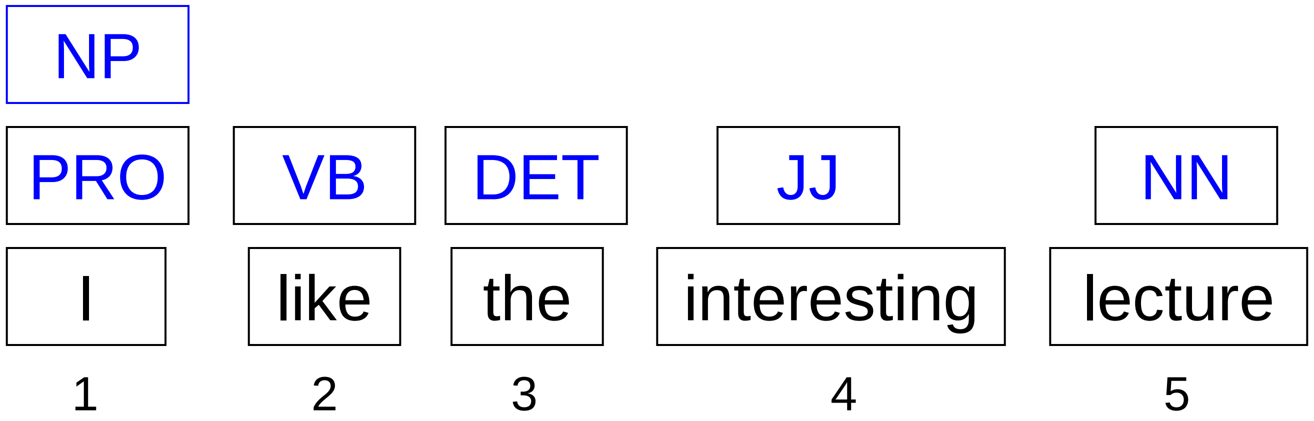
Example

- 初始化：采用词初始化线图
- 首先应用终结符的导出规则：PRO→I, VB → like ……

PRO	VB	DET	JJ	NN
I	like	the	interesting	lecture
1	2	3	4	5

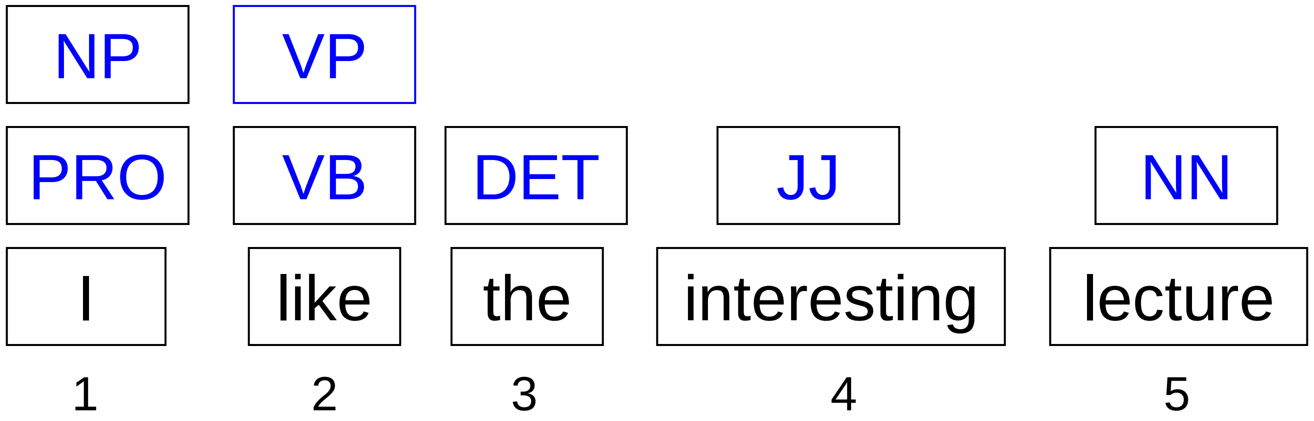
Example

- 先为第一个词搜索可能的非终结符重写规则：?→PRO
  - NP→PRO
- 递归：继续为第一个词搜索可能的非终结符重写规则：？→NP
  - 没有可匹配的规则



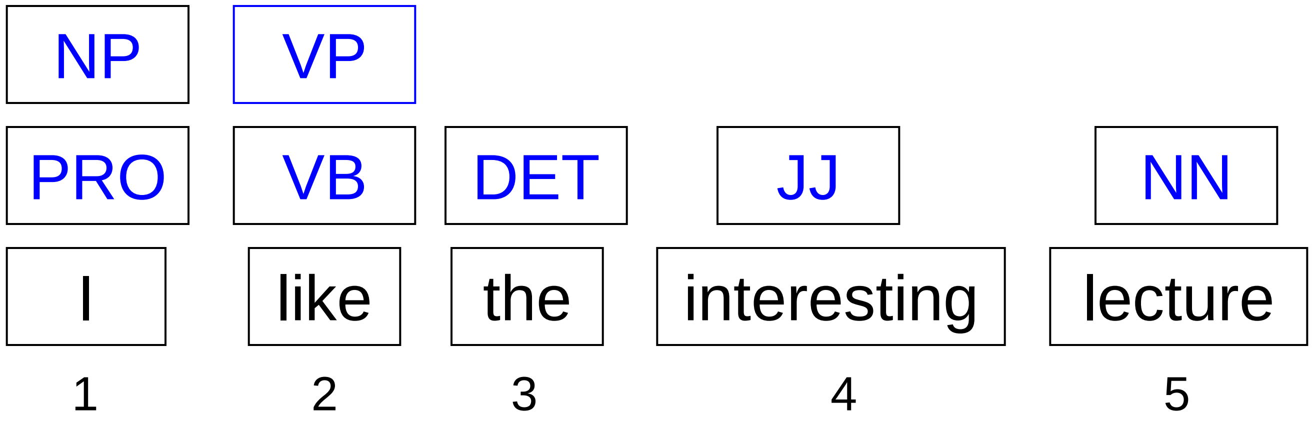
Example

- 为第二个词搜索可能的非终结符重写规则：?→VB
  – VP→VB
- 递归：继续为第二个词搜索可能的非终结符重写规则：？→VP
  – 没有可匹配的规则



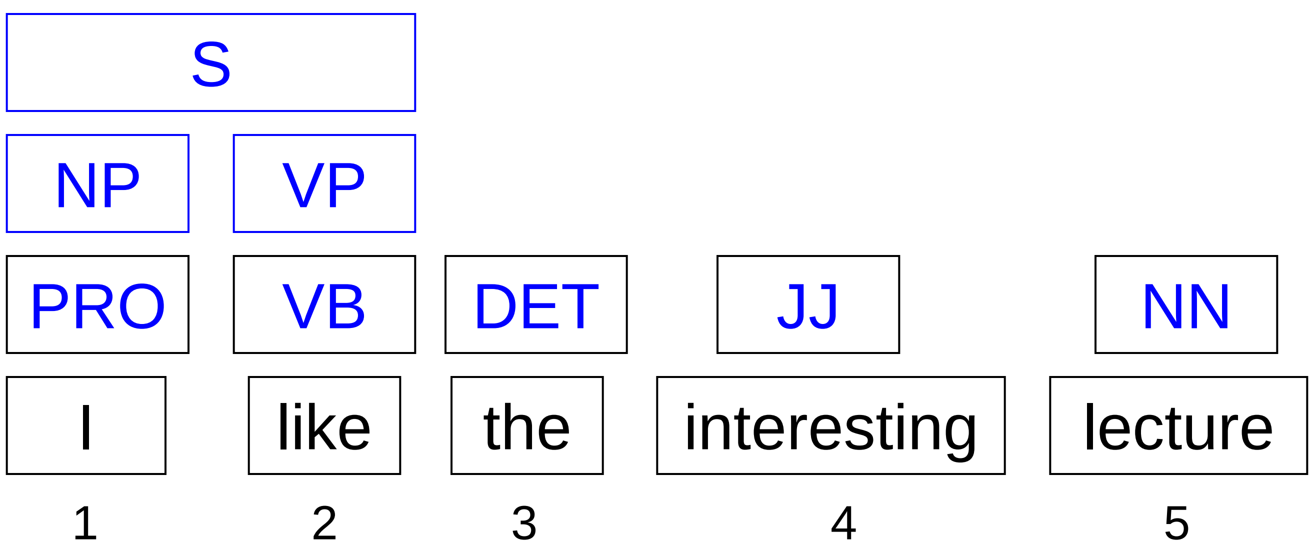
Example

- 为第三个词搜索可能的非终结符重写规则：?→DET
  - 没有可匹配的规则



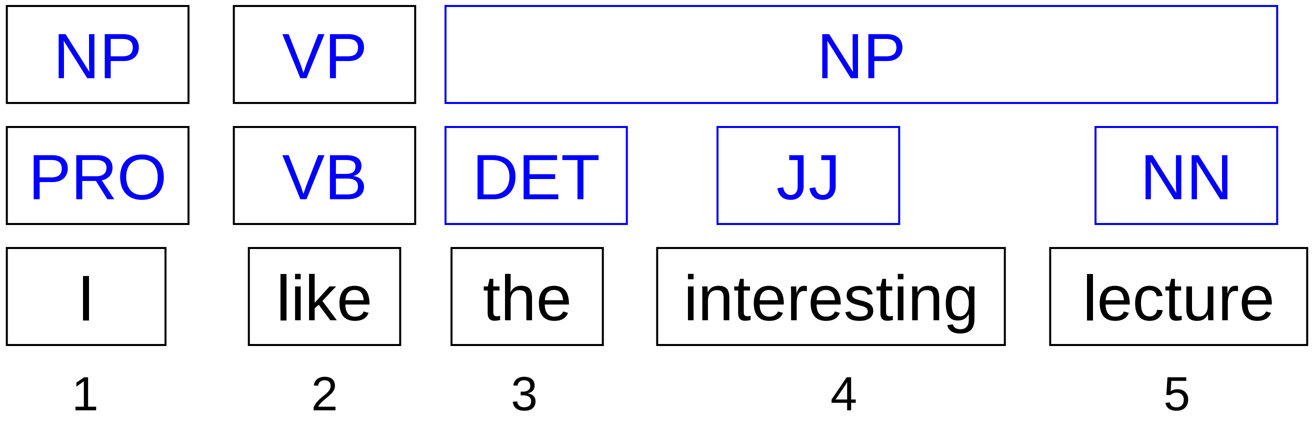
Example

- 为前两个词搜索非终结符重写规则：?→NP VP
  - S→NP VP
- 然而得不到一颗完整的树



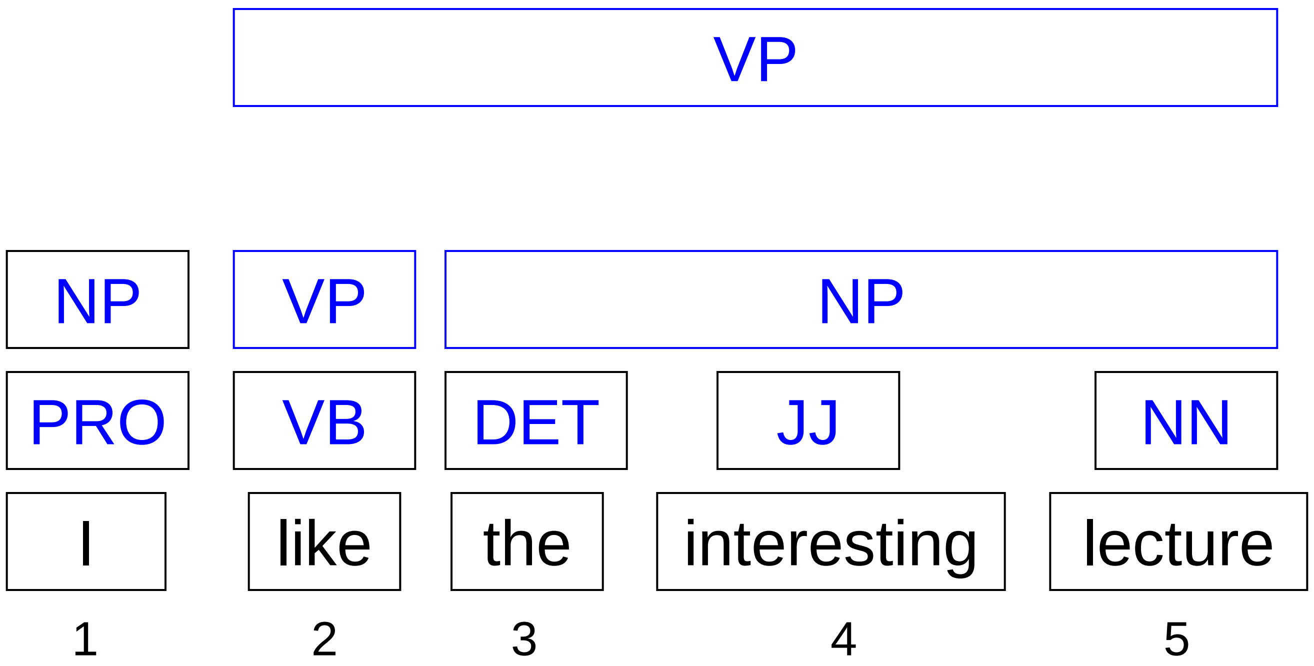
Example

- 为后三个词搜索非终结符重写规则：?→DET JJ NN
  - NP→DET JJ NN



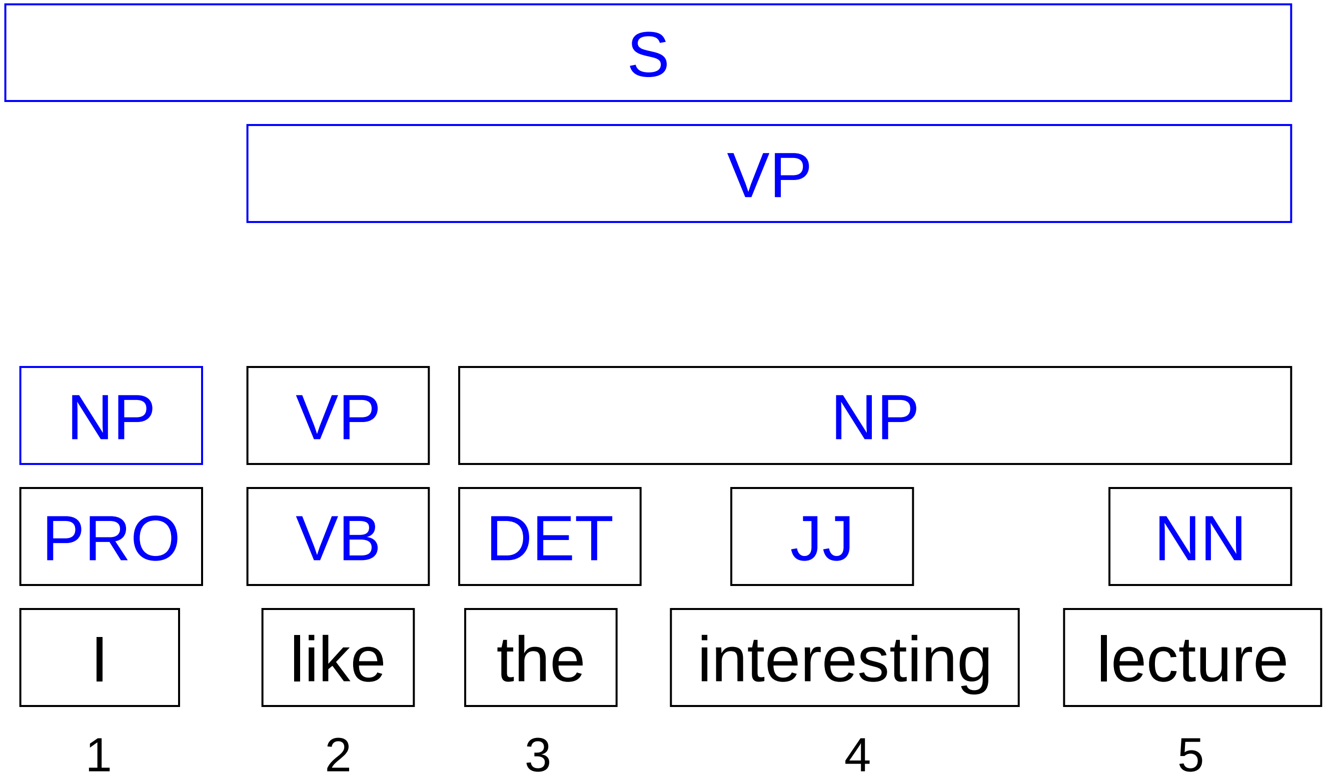
Example

- 为后四个词搜索非终结符重写规则：?→VP NP：
  - VP →VP NP



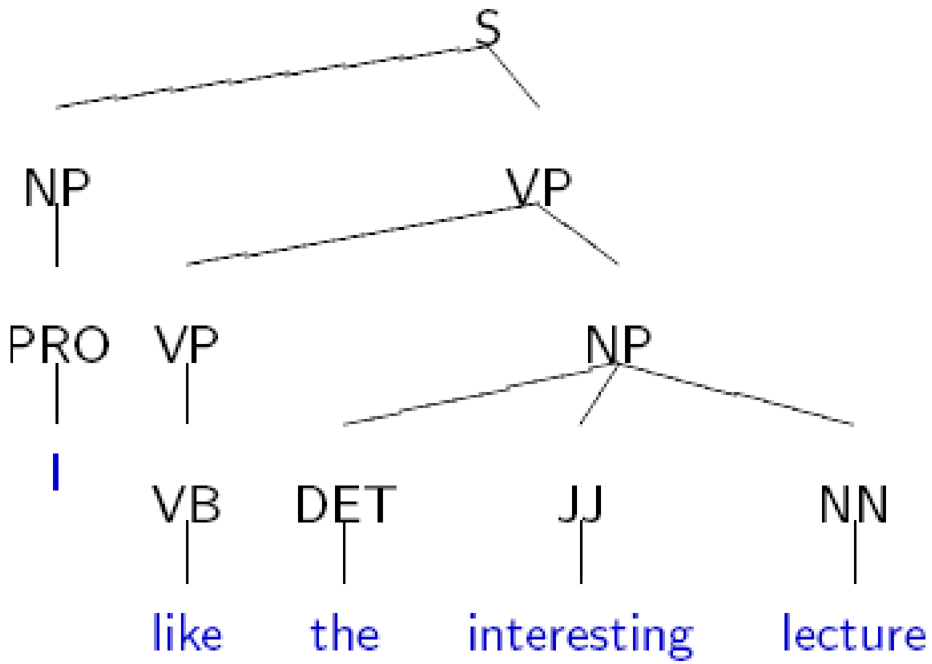
Example

- 为五个词搜索非终结符重写规则： ?→NP VP
  - S→NP VP



Example

- 最后得到完整句法树：



CKY算法

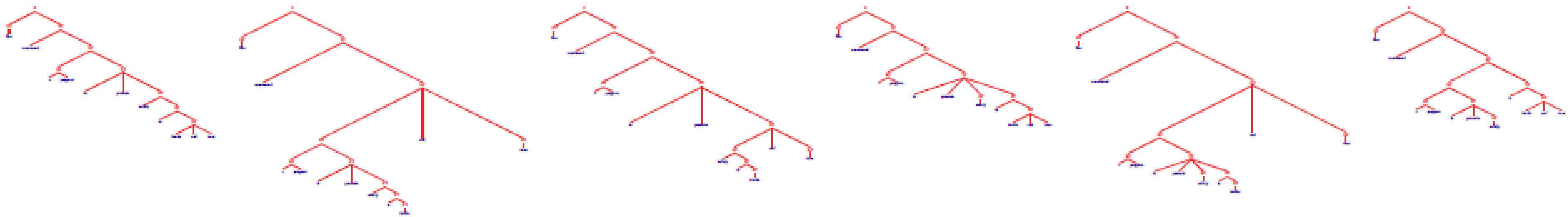
- for all words w_i: // terminal rules
  - for all rules $A \rightarrow w_i$: add new chart entry A at span [i, i]
- for length = 1 to sentence length n // non-terminal rules
  - for start = 1 to n − (length − 1)
    end = start + length − 1
    - for middle = start to end − 1: // binary rules
      for all non-terminals X in [start, middle]:
      for all non-terminals Y in [middle + 1, end]:
      for all rules $A \rightarrow X\ Y$:
      add new chart entry A at position [start, end]
  - for all non-terminals X in [start, end]: // unary rules
    for all rules $A \rightarrow X$:
    add new chart entry A at position [start, end]

Why is parsing hard?

- **输入：**

  She announced a program to promote safety in trucks and vans

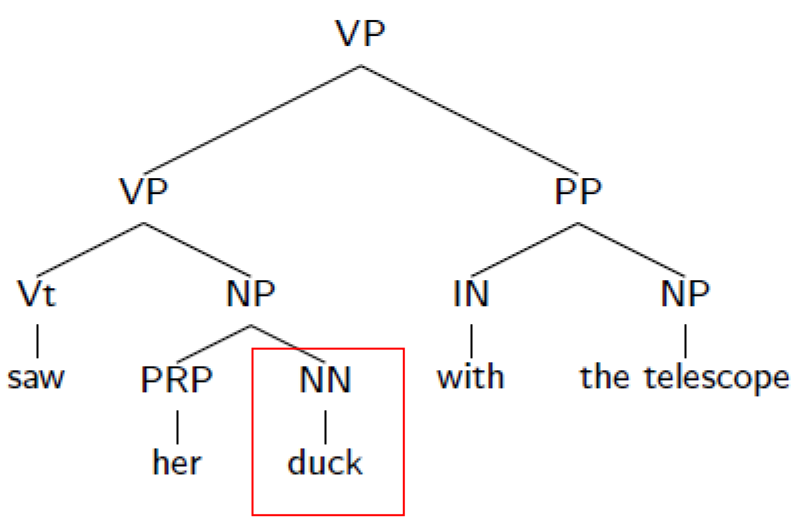
- **可能的输出：**

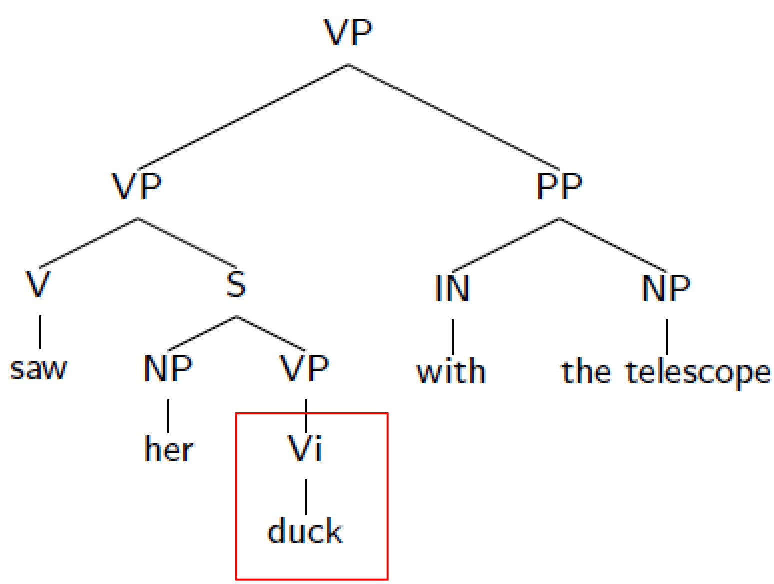


  – 还有很多…

几种常见的歧义

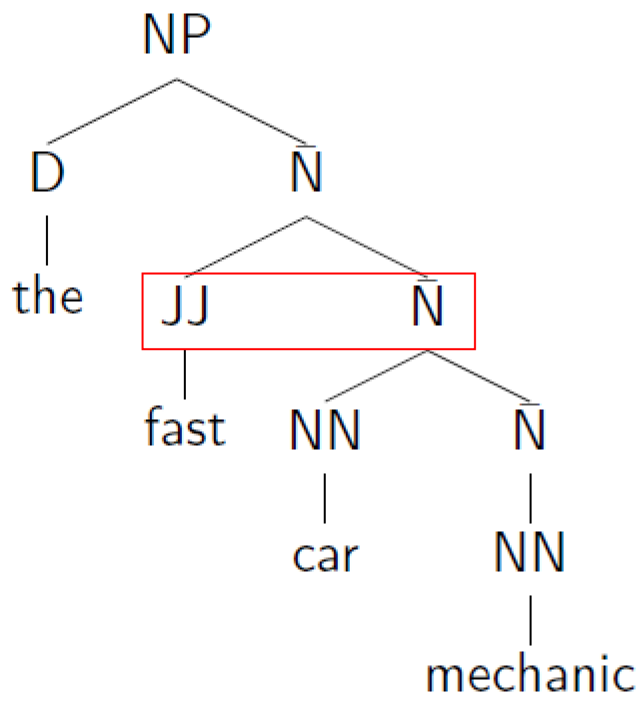
- 词性歧义：
  - NN → duck
  - Vi → duck

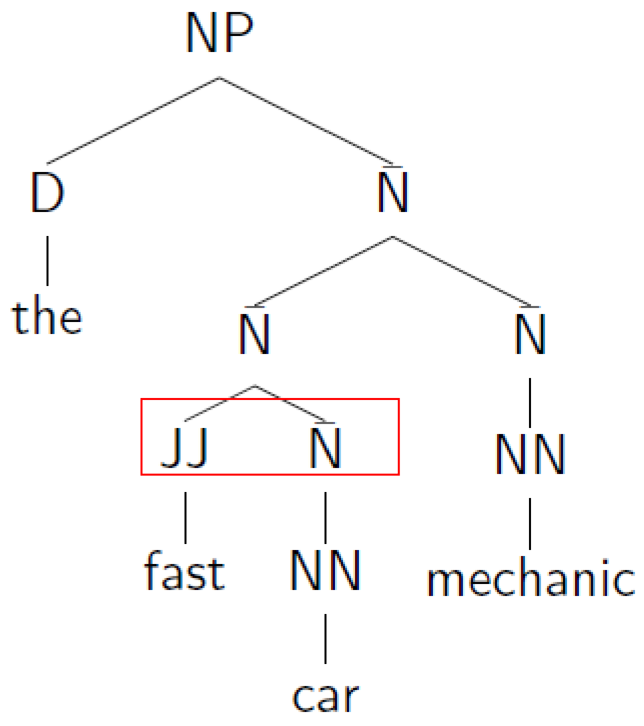




几种常见的歧义

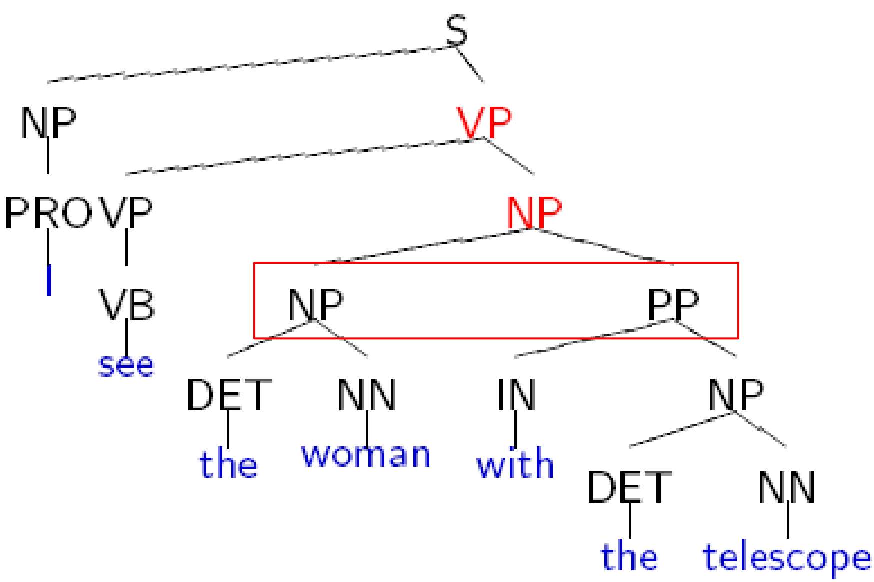
- 名词修饰语歧义：

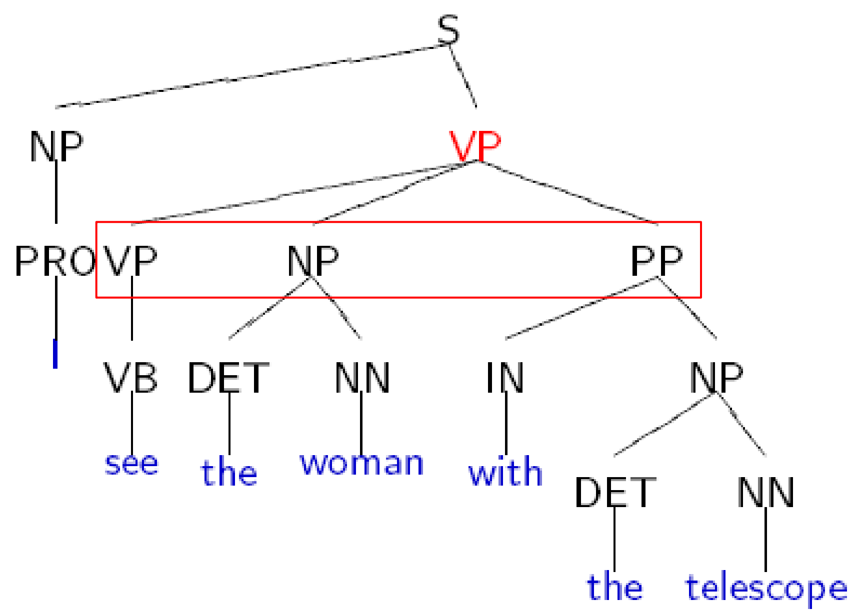




几种常见的歧义

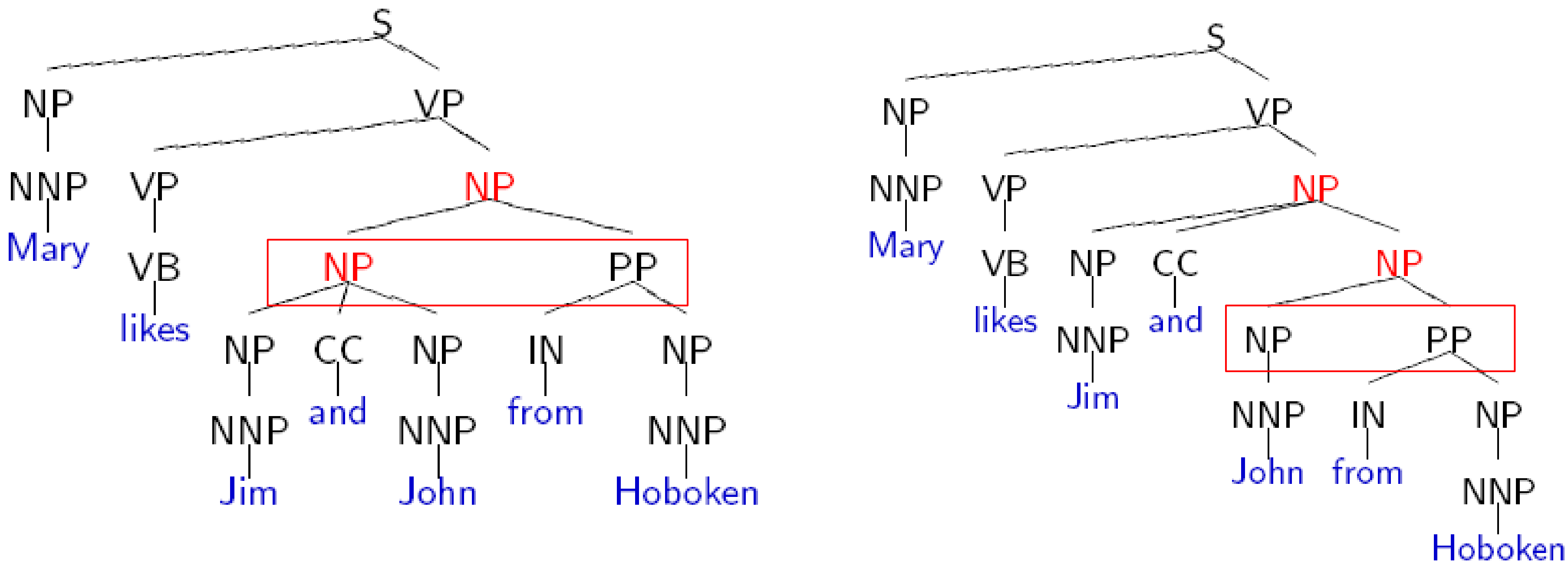
- 介词短语修饰语歧义：Who has the telescope?





几种常见的歧义

- 边界歧义： Is Jim also from Hoboken?



概率上下文无关文法

- Probabilistic context-free grammars (PCFGs)或– Stochastic context-free grammars (SCFGs)
- G = (T, N, S, R, P)
  - T：终结符（terminal symbols）集合
  - N：非终结符（nonterminal symbols）集合
  - S：开始符号，表示句子
  - R：重写规则（或产生式），具有形式X→γ，X ∈ N 并且γ ∈ (N ∪ T)*
  - P：概率函数，为每个重写规则赋予一个概率值
    - P: R → [0,1]

$$\forall X \in N, \sum_{X \to \gamma \in R} P(X \to \gamma) = 1$$

$$\sum_{\gamma \in T^*} P(\gamma) = 1$$

A simple PCFG

- S → *NP VP 1.0*
- PP → *P NP 1.0*
- VP → *V NP 0.7*
- VP → *VP PP 0.3*
- V → *saw 1.0*
- P → *with 1.0*
- *NP* → *NP PP 0.4*
- *NP* → *astronomers 0.1*
- *NP* → *saw 0.04*
- *NP* → *ears 0.18*
- *NP* → *stars 0.18*
- *NP* → *telescopes 0.1*

PCFG的特性

- 为CFG规则下的每一棵句法导出树赋予一个概率
- 设句法树t使用的规则有：$\alpha_1 \rightarrow \beta_1, \ldots, \alpha_n \rightarrow \beta_n$，规则$\alpha_i \rightarrow \beta_i$的概率为$q(\alpha_i \rightarrow \beta_i)$
- 则句法树t的概率为：

$$p(t) = \prod_{i=1}^{n} q(\alpha_i \rightarrow \beta_i)$$

- 对于句子s和其可能的句法导出树集合Γ(s)，PCFG为Γ(s)中的每棵树t赋予一个概率 p(t)，即得到候选树按照概率的排序
- 句子s最可能的句法树为：

$$\arg\max_{t \in \Gamma(s)} p(t)$$

两个问题

- 如何得到PCFG？
  – 句法规则学习
- 如何从多个候选树中找出一个概率最大的树？
  – 基于PCFG的句法分析

从treebank中学习语法

- Penn treebank：标注了句法树的英文句子
  - 由the University of Pennsylvania构建
  - 标注了the Wall Street Journal的真实文本
  - 40,000个英文句子，约100万个词

```
( (S (NP-SBJ The move)
     (VP followed
         (NP (NP a round)
             (PP of
                 (NP (NP similar increases)
                     (PP by
                         (NP other lenders))
                     (PP against
                         (NP Arizona real estate loans)))))
         ,
         (S-ADV (NP-SBJ *)
                (VP reflecting
                    (NP (NP a continuing decline)
                        (PP-LOC in
                                (NP that market))))))
     .))
```

从treebank中学习语法

- Penn treebank包括多种语言：
  - German
  - French
  - Spanish
  - Arabic
  - **Chinese:** Chinese Penn Treebank (CTB)

- 树库提供了非常多有用的信息：
  - 可重用性
    - 可以基于此得到不同的词性标注器、句法分析器等
    - 语言学的重要资源
  - 大量的统计信息：频次、分布等
  - 提供了一种用于系统评价的标准数据集

从treebank中学习语法

- 给定句法树样本 (树库，treebank)
- 从训练语料中统计观测到的重写规则，将其作为CFGs语法
- 并从中估计每个重写规则 $\alpha \rightarrow \beta$ 的概率：

$$q_{ML}(\alpha \rightarrow \beta) = \frac{Count(\alpha \rightarrow \beta)}{Count(\alpha)}$$

- 假设训练数据由其背后的PCFGs生成，则如果训练数据规模足够大，极大似然估计法得到的PCFG应该收敛于真实的PCFGs的概率分布

Parsing with a PCFG

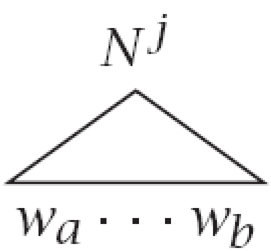
- 给定PCFG句法及句子s，定义Γ(s)为s的候选句法树构成的集合

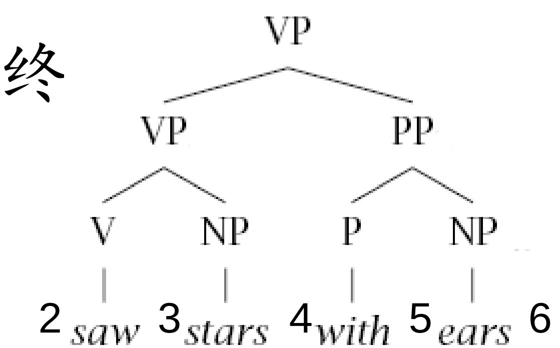
- 句法分析的目标：

$$\arg\max_{t \in \Gamma(s)} p(t)$$

PCFG notation

- G：PCFG语法
- L：G生成的或G能接受的语言
- t：句法树
- $\{N^1,\ldots, N^n\}$：非终结符集合（特殊的，N^1为开始符号）
- $\{w^1,\ldots, w^V\}$：*终结符集合*
- $\{w_1,\ldots,w_m\}$：要处理的句子
- N^j_{pq}：管辖位置p到q的词串的非终
- $\alpha(p, q, N^j)$：外向概率
- $\beta(p, q, N^j)$：内向概率





PCFG的假设

- 1. 位置不变性：

$$\forall k, P\left(N_{k(k+c)}^{j} \rightarrow \zeta\right) \text{ 不变}$$

- 2. 上下文无关：

$$P\left(N_{kl}^{j} \rightarrow \zeta \mid \text{words outside } w_k \ldots w_l\right) = P\left(N_{kl}^{j} \rightarrow \zeta\right)$$

- 3. 祖先节点无关：

$$P\left(N_{kl}^{j} \rightarrow \zeta \mid \text{ancestor nodes of } N_{kl}^{j}\right) = P\left(N_{kl}^{j} \rightarrow \zeta\right)$$

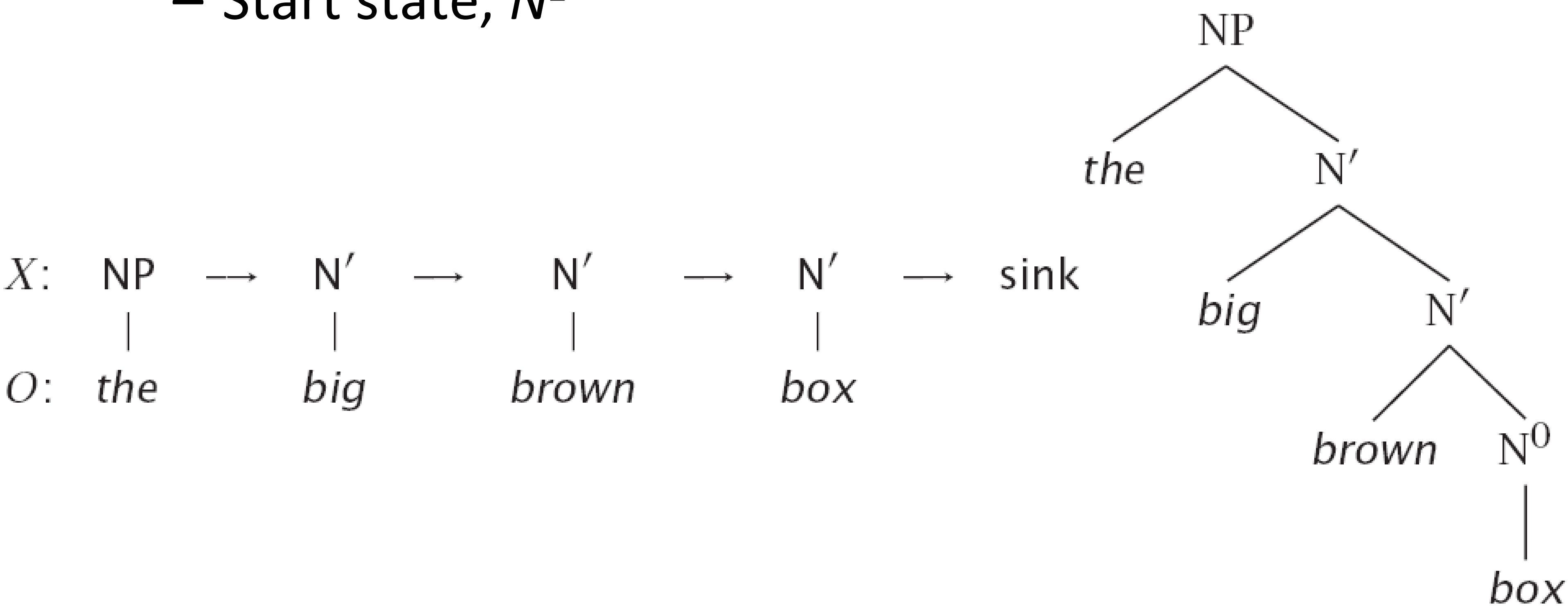
PCFG 参数

- CNF PCFG的句法规则：
  - $N^i \rightarrow N^j N^k$
  - $N^i \rightarrow w^j$
- CNF PCFG的参数：
  - *$P(N^i \rightarrow N^j N^k)$: A n^3 matrix of parameters*
  - *$P(N^i \rightarrow w^j)$: An nV matrix of parameters*
- 满足： *$j=1, \ldots, n,$*

$$\sum_{r,s} P\left(N^j \rightarrow N^r N^s\right) + \sum_k P\left(N^j \rightarrow w^k\right) = 1$$

HMMs与PCFGs的比较

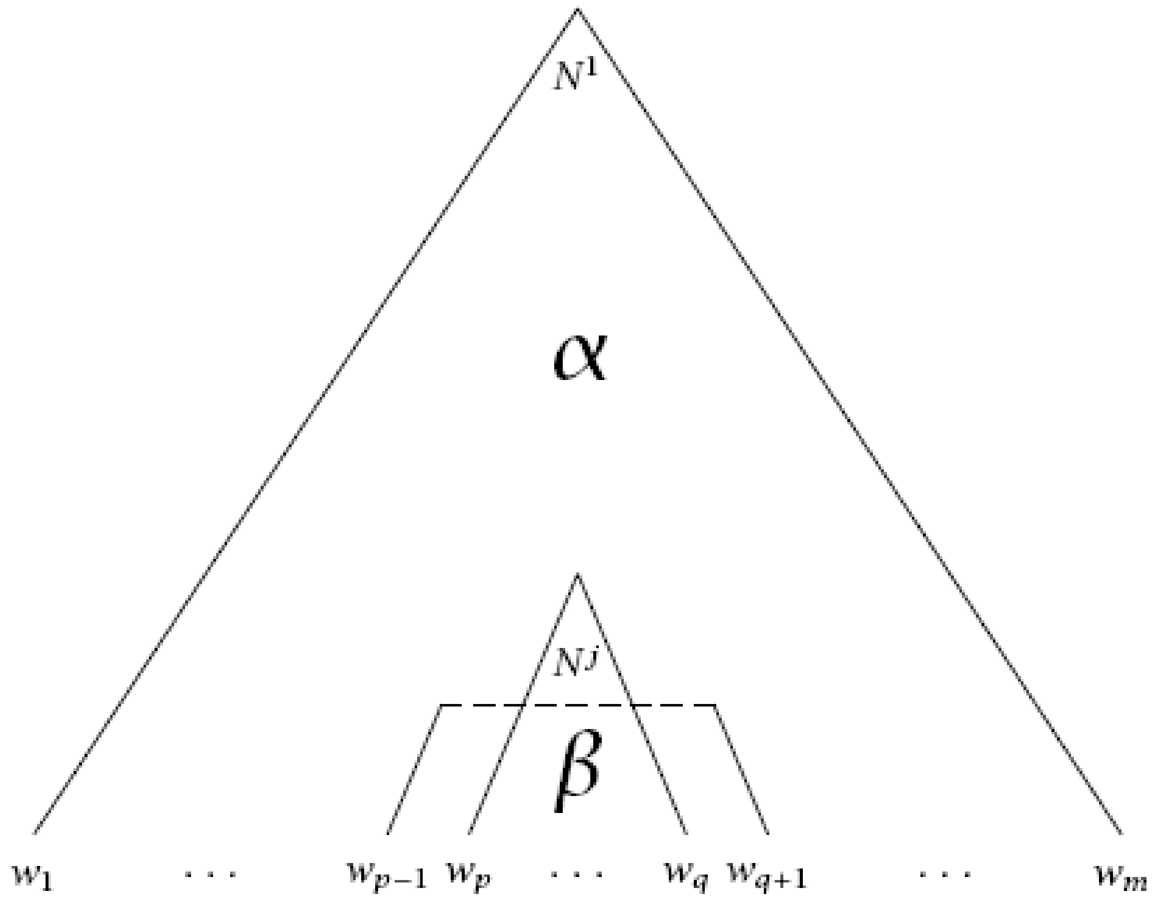
- *HMM:* **Probabilistic Regular Grammar**
  - $N^i \rightarrow w^j N^k$
  - $N^i \rightarrow w^j$
  - Start state, N^1



向内和向外概率

- HMM中定义了前向、后向概率：
  - Forwards = α_i (t)=$P(w_{1(t-1)}, X_t=i)$
  - Backwards = β_i(t)=P(w_{tT}|X_t=i)
- 同理，定义PCFG中的向外、向内概率：
  - Outside = $\alpha(p, q, N^j)=P(w_{1(p-1)}, N^j_{pq}, w_{(q+1)m}|G)$
  - Inside = $\beta(p, q, N^j)=P(w_{pq}|N^j_{pq}, G)$

向外和向内概率



$\alpha_j(p, q)=P(w_{1(p-1)}, N^j_{pq}, w_{(q+1)m}|G)$

$\beta_j(p, q)=P(w_{pq}|N^j_{pq}, G)$

PCFGs的三个问题

- 正如HMM的三个问题一样，PCFGs的三个基本问题如下：
  - 计算句子的概率：$P(w_{1m}|G)$
  - 为句子找到最优句法树：$\text{argmax}_t\, P(t|w_{1m};G)$
  - 参数学习：求解使得$P(w_{1m}|G)$ 最大的句法G

Problem2: Parsing

- 采用类似于Viterbi算法一样的思路，为句子找到最优句法树
- HMM：定义变量$\delta_j(t)$记录在t时刻到达状态j的最优路径对应的概率（所有可能路径的概率的最大值）
- PCFG：定义变量$\pi(i, j, X)$记录由非终结符X推导出子串$w_i, \ldots w_j$的最大可能树X_{ij}对应的概率（所有可能的导出结构的概率的最大值）

Problem2: Parsing

- 定义动态规划表：

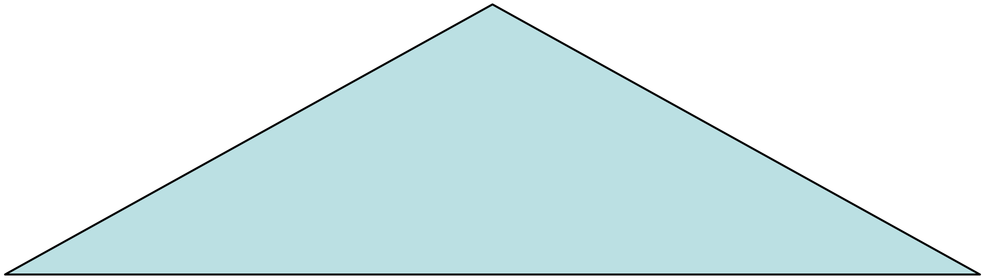
  $\pi(i, j, X)$ =由非终结符X推导出子串$w_i,\ldots w_j$的最大概率

  = 子树X_{pq}最大的向内概率

- 目标是计算：

$$\max_{t \in \Gamma(s)} p(t) = \pi(1, n, S)$$

$\pi(2,5,NP)$ $\pi(1,6,S)$



w_1 w_2 w_3 w_4 w_5 w_6

A Dynamic Programming Algorithm

- Base case definition: for all i=1, …, n, for X∊N

$$\pi(i,i,X) = q(X \to \omega_i)$$

  - Note define $P(X \rightarrow w_i) = 0$ if $P(X \rightarrow w_i)$ is not in the grammar

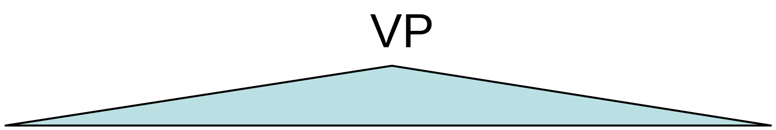
- Recursive definition: for all i=1…n-1, j=(i+1)…n, for X∊N

$$\pi(i,j,X) = \max_{\substack{X \to YZ \in R,\\ s \in \{i \ldots (j-1)\}}} (q(X \to YZ) \times \pi(i,s,Y) \times \pi(s+1,j,Z))$$

An Example

$$\pi(i,j,X) = \max_{\substack{X \to YZ \in R, \\ s \in \{i \ldots (j-1)\}}} \left(q(X \to YZ) \times \pi(i,s,Y) \times \pi(s+1,j,Z) \right)$$

To compute: π(3,8,VP)



The dog saw the man with the telescope

1 2 3 4 5 6 7 8

Suppose: q(VP→ V NP)=0.7, q(VP→ VP PP)=0.3

q(VP→ V NP) X π(3,3,V) X π(4,8,NP)
q(VP→ V NP) X π(3,4,V) X π(5,8,NP)
…
q(VP→ V NP) X π(3,7,V) X π(8,8,NP)

q(VP→ VP PP) X π(3,3,VP) X π(4,8,PP)
…
q(VP→ VP PP) X π(3,7,VP) X π(8,8,PP)

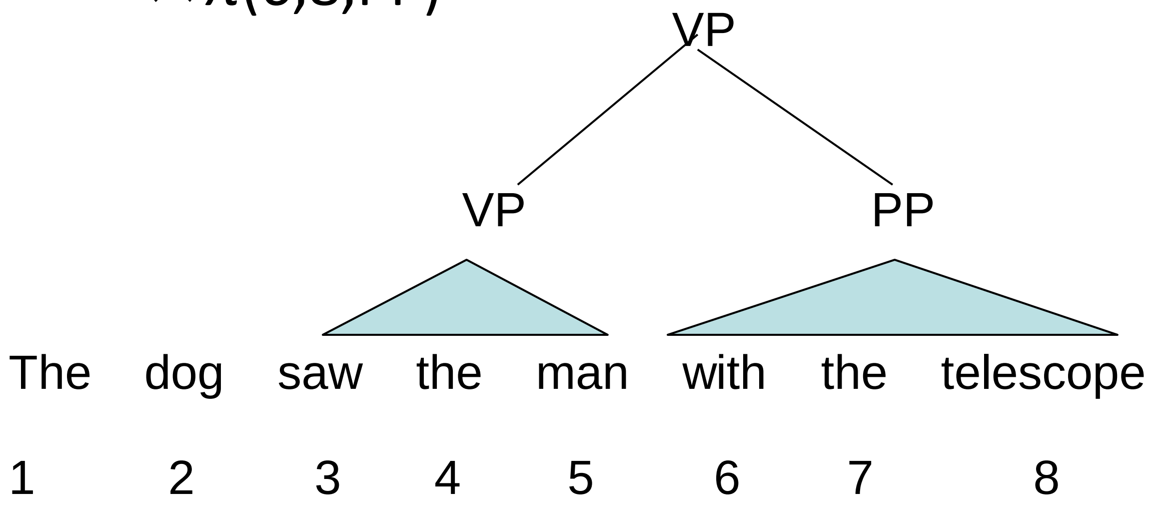
Exercise

- Consider the example sentence: the dog saw the man with the telescope
- Assume that we have π values such that
  - $\pi(3,3,\mathrm{V}) \times \pi(4,8,\mathrm{NP})=0.01$, $\pi(3,5,\mathrm{VP}) \times \pi(6,8,\mathrm{PP})=0.1$, $\pi(3,6,\mathrm{VP}) \times \pi(7,8,\mathrm{NP})=0.1$, $\pi(3,7,\mathrm{VP}) \times \pi(8,8,\mathrm{N})=0.01$
- For all other values of $s \in \{3\ldots7\}$ and $X \in N, Y \in N$, assume that $\pi(3,s,Y) \times \pi(s+1,8,X)=0$
- Also assume that the PCFG has the following parameters
  - $q(\mathrm{VP} \rightarrow \mathrm{V\ NP})=0.2$
  - $q(\mathrm{VP} \rightarrow \mathrm{VP\ PP})=0.5$
  - $q(\mathrm{VP} \rightarrow \mathrm{VP\ NP})=0.2$
  - $q(\mathrm{VP} \rightarrow \mathrm{VP\ N})=0.1$
- What is the value for $\pi(3,8,\mathrm{VP})$?

Exercise

- $\pi(3,8,\text{VP}) = 0.05$

$$= q(\text{VP} \rightarrow \text{VP PP})$$
$$\times \pi(3,5,\text{VP})$$
$$\times \pi(6,8,\text{PP})$$



The Full Dynamic Programming Algorithm

- Input: a sentence s=x1….xn, a PCFG G=(N, ∑, S, R, q)
- Initialization:

$$\pi(i,i,X) = \begin{cases} q(X \to x_i) & \text{if } X \to x_i \in R \\ 0 & \text{otherwise} \end{cases}$$

- Algorithm:

- For $l = 1 \ldots (n-1)$
  - For $i = 1 \ldots (n-l)$
    - Set $j = i + l$
    - For all $X \in N$, calculate

$$\pi(i,j,X) = \max_{\substack{X \to YZ \in R, \\ s \in \{i \ldots (j-1)\}}} \left(q(X \to YZ) \times \pi(i,s,Y) \times \pi(s+1,j,Z) \right)$$

and

$$bp(i,j,X) = \arg \max_{\substack{X \to YZ \in R, \\ s \in \{i \ldots (j-1)\}}} \left(q(X \to YZ) \times \pi(i,s,Y) \times \pi(s+1,j,Z) \right)$$

Problems with the Inside-Outside algorithm

- Slow
  - Each iteration is $O(m^3n^3)$, *where m=sum_{i=1, w}, and n is the* number of nonterminals in the grammar.
- Local maxima are much more of a problem
  - Charniak reports that on each trial a different local maximum was found
    - Use simulated annealing?
    - Restrict rules by initializing some parameters to zero?
    - Or HMM initialization?
    - Reallocate nonterminals away from “greedy” terminals?

Weakness of PCFG

- *Independence assumption* too strong
- Non-terminal rule applications do not use *lexical information*
- Not sufficiently sensitive to *structural differences* beyond parent/child node relationships

Some features of PCFGs

- Reasons to use a PCFG, and some idea of their limitations:
  - Partial solution for grammar ambiguity: a PCFG gives some idea of the plausibility of a sentence
  - But not a very good idea, as not lexicalized
  - Better for grammar induction (Gold 1967)
  - Robustness (Admit everything with low probability)

Some features of PCFGs

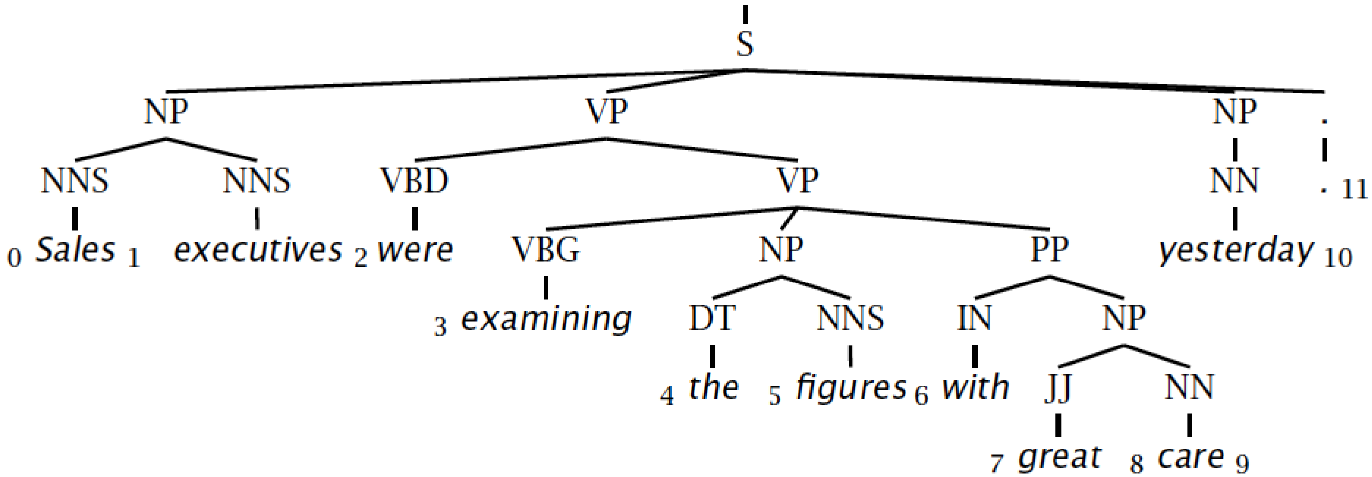
- Gives a probabilistic language model for English.
- In practice, a PCFG is a worse language model for English than a trigram model.
- Can hope to combine the strengths of a PCFG and a trigram model.
- PCFG encodes certain biases, e.g., that smaller trees are normally more probable.

Summary

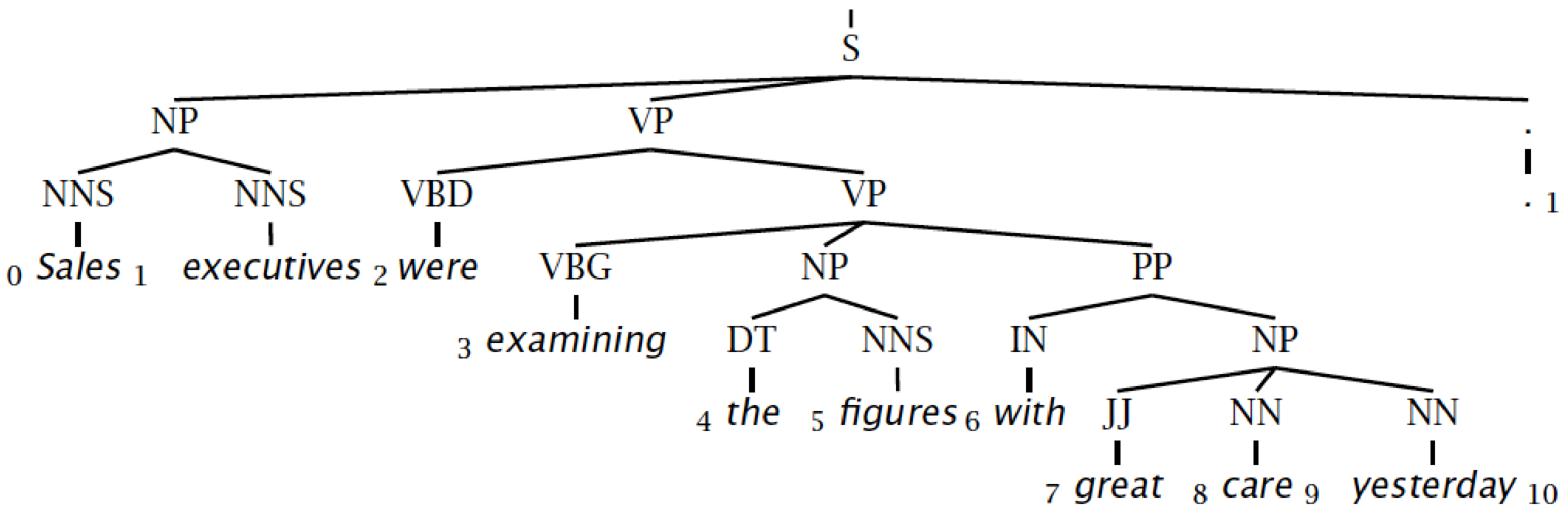
- PCFGs augments CFGs by including a probability for each rule in the grammar
- The probability for a parse tree is the product of probabilities for the rules in the tree
- To build a PCFG-parsed parser:
  - 1. Learn a PCFG from a treebank
  - 2. Given a test data sentence, use the CKY algorithm to compute the highest probability tree for the sentence under the PCFG

句法分析的评价

Gold standard brackets: **S-(0:11)**, **NP-(0:2)**, VP-(2:9), VP-(3:9), **NP-(4:6)**, PP-(6-9), NP-(7,9), NP-(9:10)



Candidate brackets: **S-(0:11)**, **NP-(0:2)**, VP-(2:10), VP-(3:10), **NP-(4:6)**, PP-(6-10), NP-(7,10)



句法分析的评价

标准结果：

S-(0:11), NP-(0:2), VP-(2:9), VP-(3:9), **NP-(4:6)**, PP-(6-9), NP-(7,9), NP-(9:10)

系统输出结果：

S-(0:11), **NP-(0:2)**, VP-(2:10), VP-(3:10), **NP-(4:6)**, PP-(6-10), NP-(7,10)

Labeled Precision	3/7 = 42.9%
Labeled Recall	3/8 = 37.5%
F1	40.0%
Tagging Accuracy	11/11 = 100.0%

More Topics for Parser

- PCFG vs language model: lexicalized PCFG, head-driven PCFG
- Agenda-based/history-based PCFG
- Dependency parser
- Unified approach for POS tagging and parsing

Several Free Parsers

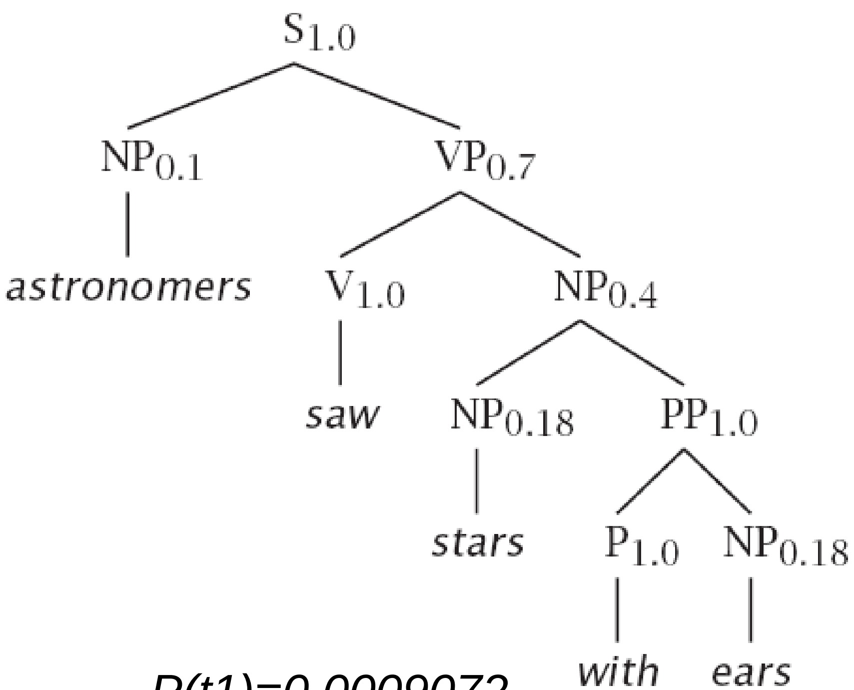
- Michael Collins's Parser
  - http://people.csail.mit.edu/mcollins/code.html
  - English
- Dan Bikel's Parser
  - http://www.cis.upenn.edu/~dbikel/software.html#stat-parser
  - English / Chinese / Arabic
- Stanford Parser
  - http://www-nlp.stanford.edu/software/lex-parser.shtml
  - English / Chinese / German
- David Chiang's Parser
  - http://www.isi.edu/~chiang/
  - English / Chinese

- Next 3 lectures: sentence/text representation

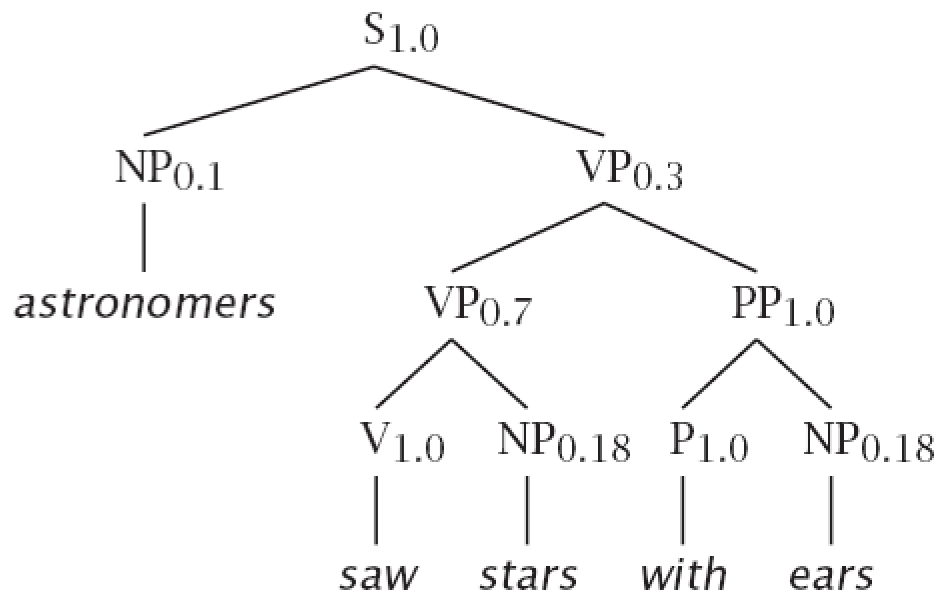
- 补充资料：
  - Problem1和3的求解，不讲解，不要求掌握
  - Training PCFGs：EM方法，不讲解，不要求掌握

Problem1: 计算词串的概率

$$\begin{aligned} P(w_{1n}) &= \sum_t P(w_{1n}, t) \quad t \text{ a parse of } w_{1n} \\ &= \sum_{\{t:\text{yield}(t)=w_{1n}\}} P(t) \end{aligned}$$



$P(t1)=0.0009072$

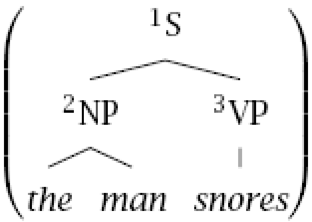


$P(t2)= 0.0006804$

$P(w15)=P(t1)+P(t2)=0.0015876$

Problem1:计算词串的概率

- Probability of a sub-tree: an example

$P\Big($  $\Big)$

$$= P\left({}^1S_{13} \rightarrow {}^2NP_{12}\,{}^3VP_{33},\ {}^2NP_{12} \rightarrow the_1 man_2,\ {}^3VP_{33} \rightarrow snores_3\right)$$

the chain rule

$$= P\left({}^1S_{13} \rightarrow {}^2NP_{12}\,{}^3VP_{33}\right) P\left({}^2NP_{12} \rightarrow the_1 man_2 \mid {}^1S_{13} \rightarrow {}^2NP_{12}\,{}^3VP_{33}\right)$$
$$P\left({}^3VP_{33} \rightarrow snores_3 \mid {}^1S_{13} \rightarrow {}^2NP_{12}\,{}^3VP_{33},\ {}^2NP_{12} \rightarrow the_1 man_2\right)$$

$$= P\left({}^1S_{13} \rightarrow {}^2NP_{12}\,{}^3VP_{33}\right) P\left({}^2NP_{12} \rightarrow the_1 man_2\right) P\left({}^3VP_{33} \rightarrow snores_3\right)$$

context-free assumption

$$= P(S \rightarrow NP\,VP)\, P(NP \rightarrow the\,man)\, P(VP \rightarrow snores)$$

position-nvariant assumption

Using inside probabilities

- Inside probability

$$P(w_{1m} \mid G) = P(N^1 \Rightarrow w_{1m} \mid G)$$

$$= P(w_{1m}, N^1_{1m}, G) = \beta_1(1, m)$$

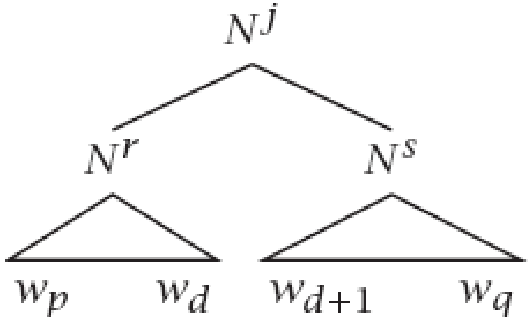
- **Base case**: We want to find $\beta_j(k,k)$ *(the probability of a rule* $N^j \rightarrow w_k$

$$\beta_j(k,k) = P(w_k \mid N^j_{kk}, G)$$

$$= P(N^j \rightarrow w_k \mid G)$$

Using inside probabilities

- **Inductive case**: We want to find $\beta_j(p, q)$*, for* $p < q$*. As this is the inductive* step using a Chomsky Normal Form grammar, the first rule must be of the form $N^j \rightarrow N^r N^s$*, so we can proceed by induction, dividing the* string in two in various places and summing the result:



- These inside probabilities can be calculated bottom up.

Using inside probabilities

- For all *j*,

$$\beta_j(p,q) = P\left(w_{pq} \mid N^j_{pq}, G\right)$$

$$= \sum_{r,s} \sum_{d=p}^{q-1} P\left(w_{pd}, N^r_{pd}, w_{(d+1)q}, N^s_{(d+1)q} \mid N^j_{pq}, G\right)$$

$$= \sum_{r,s} \sum_{d=p}^{q-1} P\left(N^r_{pd}, N^s_{(d+1)q} \mid N^j_{pq}, G\right) P\left(w_{pd} \mid N^j_{pq}, N^r_{pd}, N^s_{(d+1)q}, G\right)$$

$$\times P\left(w_{(d+1)q} \mid N^j_{pq}, N^r_{pd}, N^s_{(d+1)q}, w_{pq}, G\right)$$

$$= \sum_{r,s} \sum_{d=p}^{q-1} P\left(N^r_{pd}, N^s_{(d+1)q} \mid N^j_{pq}, G\right) P\left(w_{pd} \mid N^r_{pd}, G\right)$$

$$\times P\left(w_{(d+1)q} \mid N^s_{(d+1)q}, G\right)$$

Calculation of inside probabilities (CKY algorithm)

	1	2	3	4	5
1	$\beta_{NP} =$ 0.1		$\beta_S =$ 0.0126		$\beta_S =$ 0.0015876
2		$\beta_{NP} =$ 0.04 $\beta_V =$ 1.0	$\beta_{VP} =$ 0.126		$\beta_{VP} =$ 0.015876
3			$\beta_{NP} =$ 0.18		$\beta_{NP} =$ 0.01296
4				$\beta_P =$ 1.0	$\beta_{PP} =$ 0.18
5					$\beta_{NP} =$ 0.18
	astronomers	*saw*	*stars*	*with*	*ears*

Cell *(p, q) shows non-zero* probabilities $\beta_i(p, q)$*. calculated via the inside algorithm*

$$\beta_i(2,5) =$$

$$P(VP \to V\,NP)\beta_V(2,2)\beta_{NP}(3,5) + P(VP \to VP\,PP)\beta_{VP}(2,3)\beta_{PP}(4,5)$$

Using outside probabilities

- Probability of a string: For any *k, 1≤k≤m,*

$$P(w_{1m} \mid G) = \sum_j P\left(w_{1(k-1)}, w_k, w_{(k+1)m}, N^j_{kk} \mid G\right)$$

$$= \sum_j P\left(w_{1(k-1)}, N^j_{kk}, w_{(k+1)m} \mid G\right)$$

$$\times P\left(w_k \mid w_{1(k-1)}, N^j_{kk}, w_{(k+1)n}, G\right)$$

$$= \sum_j \alpha_j(k,k) P\left(N^j \rightarrow w_k\right)$$

- **Base Case:**

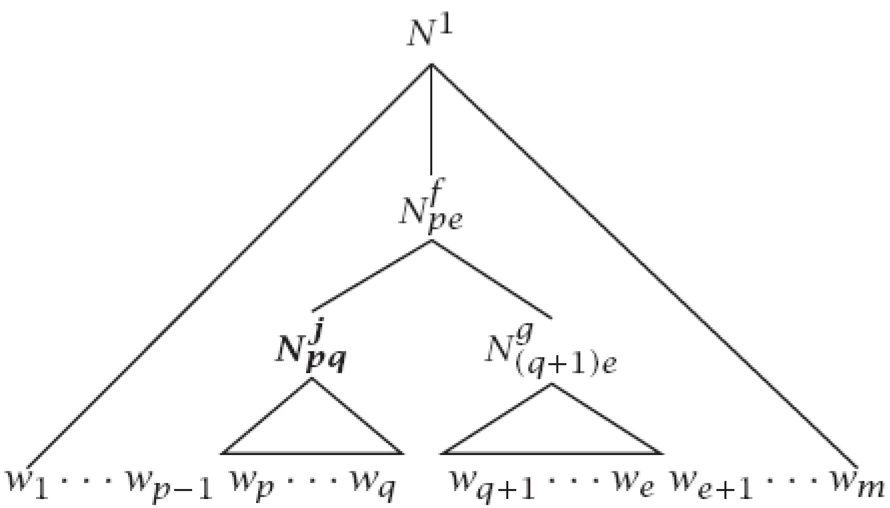
$$\alpha_1(1,m) = 1$$

$$\alpha_j(1,m) = 0 \quad for \quad j \neq 1$$

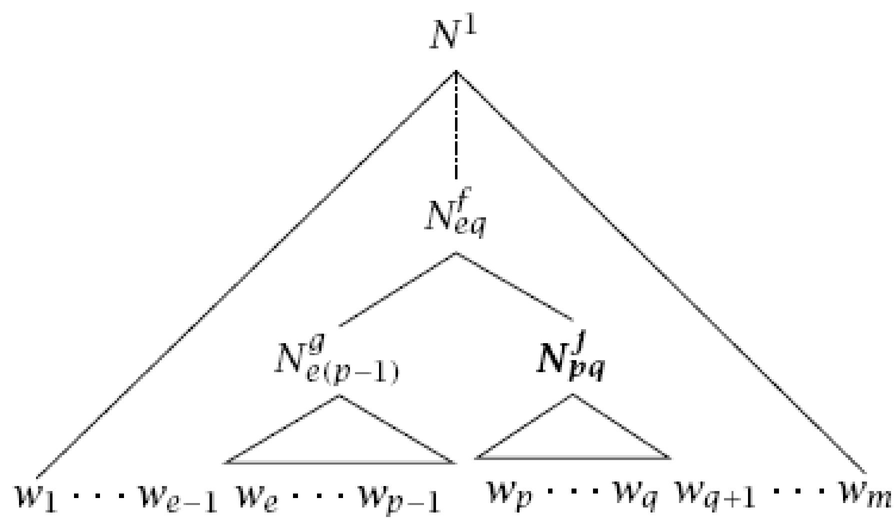
- Inductive calculation: One calculates the outside probabilities top down (after determining the inside probabilities).

Using outside probabilities

- **Inductive Case**: we want to find $\alpha_j(p, q)$*, for* $p < q$
  - N^i_{pq} is either a left or right branch
  - we will sum over both possibilities and calculate using outside and inside probabilities



N^j_{pq}: left branch of the parent node



N^j_{pq}: right branch of the parent node

Using outside probabilities

$$\alpha_j(p,q)=\left[\sum_{f,g\neq j}\sum_{e=q+1}^{m}P\left(w_{1(p-1)},w_{(q+1)m},N_{pe}^{f},N_{pq}^{j},N_{(q+1)e}^{g}\right)\right]$$

$$+\left[\sum_{f,g}\sum_{e=1}^{p-1}P\left(w_{1(p-1)},w_{(q+1)m},N_{eq}^{f},N_{e(p-1)}^{g},N_{pq}^{j}\right)\right]$$

$$=\left[\sum_{f,g\neq j}\sum_{e=q+1}^{m}P\left(w_{1(p-1)},w_{(e+1)m},N_{pe}^{f}\right)P\left(N_{pq}^{j},N_{(q+1)e}^{g}\mid N_{pe}^{f}\right)\right.$$

$$\left.\times P\left(w_{(q+1)e}\mid N_{(q+1)e}^{g}\right)\right]+\left[\sum_{f,g}\sum_{e=1}^{p-1}P\left(w_{1(e-1)},w_{(q+1)m},N_{eq}^{f}\right)\right.$$

$$\left.\times P\left(N_{e(p-1)}^{g},N_{pq}^{j}\mid N_{eq}^{f}\right)P\left(w_{e(p-1)}\mid N_{e(p-1)}^{g}\right)\right]$$

$$=\left[\sum_{f,g\neq j}\sum_{e=q+1}^{m}\alpha_f(p,e)P\left(N^f\rightarrow N^jN^g\right)\beta_g(q+1,e)\right]$$

$$+\left[\sum_{f,g}\sum_{e=1}^{p-1}\alpha_f(e,q)P\left(N^f\rightarrow N^gN^j\right)\beta_g(e,p-1)\right]$$

Overall probability of a node existing

- As with a HMM, we can form a product of the inside and outside probabilities. This time:

$$\alpha_j(p,q)\beta_j(p,q)$$

$$= P\left(w_{1(p-1)}, N^j_{pq}, w_{(q+!)m} \mid G\right) P\left(w_{pq} \mid N^j_{pq}, G\right)$$

$$= P\left(w_{1m}, N^j_{pq} \mid G\right)$$

- Therefore,

$$p\left(w_{1m}, N_{pq} \mid G\right) = \sum_j \alpha_j(p,q)\beta_j(p,q)$$

Problem 3: Training a PCFG

- We would like to calculate how often each rule is used:

$$\hat{P}\left(N^j \to \zeta\right) = \frac{C\left(N^j \to \zeta\right)}{\sum_\gamma C\left(N^j \to \gamma\right)}$$

- Have data=>*count; else work iteratively from expectations of current* model (construct an EM training algorithm, as for HMMs).

Training a PCFG

- Consider:

$$\alpha_j(p,q)\beta_j(p,q) = P\left(N^1 \overset{*}{\Rightarrow} w_{1m}, N^j \overset{*}{\Rightarrow} w_{pq} \mid G\right)$$

$$= P\left(N^1 \overset{*}{\Rightarrow} w_{1m} \mid G\right) P\left(N^j \overset{*}{\Rightarrow} w_{pq} \mid N^1 \overset{*}{\Rightarrow} w_{1m}, G\right)$$

- We have already solved how to calculate *P(N*[1] *=>w*$_{1m}$*); let us call this* probability π. Then:

$$P\left(N^j \overset{*}{\Rightarrow} w_{pq} \mid N^1 \overset{*}{\Rightarrow} w_{1m}, G\right) = \frac{\alpha_j(p,q)\beta_j(p,q)}{\pi}$$

- And

$$E\left(N^j \text{ is used in the derivation}\right) = \boxed{\sum_{p=1}^{m}\sum_{q=p}^{m}} \frac{\alpha_j(p,q)\beta_j(p,q)}{\pi}$$

Training a PCFG

- In the case where we are not dealing with a preterminal, we substitute the inductive definition of β, and for any *r and s, p > q:*

$$P\left(N^j \rightarrow N^r N^s \Rightarrow w_{pq} \mid N^1 \Rightarrow w_{1n}, G\right) =$$

$$\frac{\sum_{d=p}^{q-1} \alpha_j(p,q) P\left(N^j \rightarrow N^r N^s\right) \beta_r(p,d) \beta_s(d+1,q)}{\pi}$$

- Therefore the expectation is:

$$E\left(N^j \rightarrow N^r N^s, N^j used\right) =$$

$$\frac{\boxed{\sum_{p=1}^{m-1} \sum_{q=p+1}^{m}} \sum_{d=p}^{q-1} \alpha_j(p,q) P\left(N^j \rightarrow N^r N^s\right) \beta_r(p,d) \beta_s(d+1,q)}{\pi}$$

Training a PCFG

- Now for the maximization step, we want:

$$P\left(N^j \rightarrow N^r N^s\right) = \frac{E\left(N^j \rightarrow N^r N^s, N^j used\right)}{E\left(N^j used\right)}$$

- Therefore, the reestimation formula,

$$\hat{P}\left(N^j \rightarrow N^r N^s\right) =$$

$$\frac{\sum_{p=1}^{m-1}\sum_{q=p+1}^{m}\sum_{d=p}^{q-1}\alpha_j(p,q)P\left(N^j \rightarrow N^r N^s\right)\beta_r(p,d)\beta_s(d+1,q)}{\sum_{p=1}^{m}\sum_{q=1}^{m}\alpha_j(p,q)\beta_j(p,q)}$$

Training a PCFG

- Similarly, for rules like N^j->w_k

$$E\left(N^j \to w^k \mid N^1 \Rightarrow w_{1m}, G\right) = \frac{\sum_{h=1}^{m} \alpha_j(h,h) P\left(N^j \to w_h, w_h = w^k\right)}{\pi}$$

- Therefore,

$$\hat{P}\left(N^j \to w^k\right) = \frac{\sum_{h=1}^{m} \alpha_j(h,h) P\left(N^j \to w_h, w_h = w^k\right)}{\sum_{p=1}^{m} \sum_{q=1}^{m} \alpha_j(p,q) \beta_j(p,q)}$$

- Inside-Outside algorithm: repeat this process until the estimated probability change is small.

Training a PCFG

- Multiple training instances: if we have training sentences $W=(W_1, \ldots, W_\omega)$, with $W_i=(w_{1i}, \ldots, w_{mi})$ *and we let u and v be the common subterms* from before:

$$u_i(p,q,j,r,s) = \frac{\sum_{d=p}^{q-1} \alpha_j(p,q) P(N^j \to N^r N^s) \beta_r(p,d) \beta_s(d+1,q)}{P(N^1 \Rightarrow W_i \mid G)}$$

- and

$$v_i(p,q,j) = \frac{\alpha_j(p,q)\beta_j(p,q)}{P(N^1 \Rightarrow W_i \mid G)}$$

- Assuming the observations are independent, we can sum contributions:

$$\hat{P}(N^j \to N^r N^s) = \frac{\sum_{i=1}^{\omega} \sum_{p=1}^{m_i-1} \sum_{q=p+1}^{m_i} u_i(p,q,j,r,s)}{\sum_{i=1}^{\omega} \sum_{p=1}^{m_i-1} \sum_{q=p}^{m_i} v_i(p,q,j)}$$

- and

$$\hat{P}(N^j \to w^k) = \frac{\sum_{i=1}^{\omega} \sum_{\{h: w_h = w^k\}} v_i(h,h,j)}{\sum_{i=1}^{\omega} \sum_{p=1}^{m_i} \sum_{q=p}^{m_i} v_i(p,q,j)}$$