Problem sheet 2

Issued 31 October 2022, due 21 November 2022.

Coursework forms 25% of the assessment for this unit and will be comprised of 2 problem sheets of which this is the second. In this sheet each of five problems carries the same weight.

1.

(i) How many maps are there from $\{1,2\}$ to $\{1,2,3\}$?
(ii) How many injective maps are there from $\{1,2\}$ to $\{1,2,3\}$?
(iii) How many bijective maps there are from $\{1,2\}$ to $\{1,2,3\}$?
(iv) How many maps are there from $\{1,2,3\}$ to $\{2,3\}$?
(v) How many surjective maps are there from $\{1,2,3\}$ to $\{2,3\}$?

2. Let $\mathbf{N}$ denote the set of all *positive* (greater than zero) integers.

(i) Give an example of a bijective map from $\mathbf{N}$ to $\mathbf{N}$ which is not the *identity* map.
(ii) Give an example of a bijective map from the set of all even integers to the set of all odd integers.
(iii) Give an example of a surjective map from $\mathbf{N}$ to $\mathbf{Z}$.
(iv) Give an example of an injective map from $\mathbf{Z}$ to $\mathbf{N}$.

3. Among the following relations R on $\mathbf{Z}$ identify those that are maps from $\mathbf{Z}$ to $\mathbf{Z}$. Justify your answer in each case.

(i) $R = \{(x,y) |\, x = y + 2\}$;
(ii) $R = \{(x,y) |\, x = y^2\}$;
(iii) $R = \{(x,y) |\, x^2 = y\}$;
(vi) $R = \left\{(x,y) |\, x = \frac{y^{\frac{1}{3}}}{2}\right\}$.

4. Each of the following defines a relation on $\mathbf{Z}$. In each case determine if the relation is reflexive, symmetric, or transitive. Justify your answers.

(i) $x + y$ is an odd integer;
(ii) $x + y$ is an even integer;
(iii) xy is an odd integer;
(iv) $x + xy$ is an even integer.

5. Consider the subset relation $\subset$ on the set $A = \{\{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$.
(a) Is $\subset$ a partial order on A, a strict partial order on A, or neither? Justify your answer.
(b) Is $\subset$ a total order on A? Justify your answer.
(c) Decide whether there are maximal elements, and whether there are minimal elements, in A with respect to $\subset$. If such elements exist, list them all.