## MATHEMATICS FOR COMPUTATION 2024 PROBLEM SHEET 1

ISSUED 12 FEBRUARY 2024, DUE 11 MARCH 2024

Coursework forms 25% of the assessment for this unit and will be comprised of 2 problem sheets equally weighted. On this sheet, each question is equally weighted.

1. Solve the following system of linear equations using the Cramer's rule.

$$3X - Y = 8$$
  
 $-2X + Y + Z = 9$   
 $2X - Y + 4Z = -5$ .

The solution should be represented by simple fractions. Show your calculations of necessary determinants.

2. Solve the same system of equations by first inverting its matrix

$$A = \left(\begin{array}{rrr} 3 & -1 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & 4 \end{array}\right).$$

Compute the inverse of A using the calculation of the adjugate matrix  $A^*$ . The elements of  $A^{-1}$  should be represented by simple fractions. Show your working, including all intermediate steps.

3. Given a matrix

$$A = \left(\begin{array}{rrr} 1 & 4 & 1 \\ -1 & 2 & 2 \\ 3 & 1 & -1 \end{array}\right),$$

use Gaussian elimination to compute the determinant det(A) of A and to solve the system of linear equations AX = b, where  $X = (X_1, X_2, X_3)^T$  is the vector of unknowns and  $b = (1, 0, 1)^T$ . The solution should be represented by simple fractions. Show your working.

**4.** Using Gaussian elimination, compute the rank of the following  $(4 \times 4)$ -matrix

$$\left(\begin{array}{ccccc}
-1 & 2 & 1 & 2 \\
2 & 1 & 0 & 1 \\
1 & 3 & 1 & 3 \\
-2 & 4 & 2 & 4
\end{array}\right)$$

Show your working.

5. Consider the following system of linear equations.

$$3X_1 - 2X_2 + 3X_3 - X_4 = 1$$
  
 $X_2 + X_4 = 3$   
 $X_1 + X_2 - 2X_3 + 4X_4 = 1$ 

Using Gaussian elimination, show that it has at least one solution. Represent the general solution as an affine map from one vector space to another, in matrix/vector form. Find one specific solution. Show your working.