# Fitting genotype by environment models in sommer

## Giovanny Covarrubias-Pazaran

#### 2023-04-06

The sommer package was developed to provide R users a powerful and reliable multivariate mixed model solver. The package is focused on problems of the type p > n (more effects to estimate than observations) and its core algorithm is coded in C++ using the Armadillo library. This package allows the user to fit mixed models with the advantage of specifying the variance-covariance structure for the random effects, specifying heterogeneous variances, and obtaining other parameters such as BLUPs, BLUEs, residuals, fitted values, variances for fixed and random effects, etc.

The purpose of this vignette is to show how to fit different genotype by environment (GxE) models using the sommer package:

- 1) Single environment model
- 2) Multienvironment model: Main effect model
- 3) Multienvironment model: Diagonal model (DG)
- 4) Multienvironment model: Compund symmetry model (CS)
- 5) Multienvironment model: Unstructured model (US)
- 6) Multienvironment model: Random regression model (RR)
- 7) Multienvironment model: Other covariance structures for GxE
- 8) Multienvironment model: Finlay-Wilkinson regression
- 9) Multienvironment model: Factor analytic (reduced rank) model (FA)
- 10) Two stage analysis

When the breeder decides to run a trial and apply selection in a single environment (whether because the amount of seed is a limitation or there's no availability for a location) the breeder takes the risk of selecting material for a target population of environments (TPEs) using an environment that is not representative of the larger TPE. Therefore, many breeding programs try to base their selection decision on multi-environment trial (MET) data. Models could be adjusted by adding additional information like spatial information, experimental design information, etc. In this tutorial we will focus mainly on the covariance structures for GxE and the incorporation of relationship matrices for the genotype effect.

#### 1) Single environment model

A single-environment model is the one that is fitted when the breeding program can only afford one location, leaving out the possible information available from other environments. This will be used to further expand to GxE models.

```
data=DT, verbose = FALSE)
summary(ansSingle)
       Multivariate Linear Mixed Model fit by REML
## ************** sommer 4.2 ************
 _____
##
                    BIC Method Converge
       logLik
              AIC
## Value -78.80875 159.6175 162.8378 NR
## Variance-Covariance components:
             VarComp VarCompSE Zratio Constraint
                    2.202 2.965
## u:Name.Yield-Yield 6.529
## units.Yield-Yield 13.868
                     1.633 8.494
## Fixed effects:
         Effect Estimate Std.Error t.value
## 1 Yield (Intercept) 11.74 0.4876
## Groups and observations:
##
      Yield
## u:Name
## Use the '$' sign to access results and parameters
Ai <- as(solve(A), Class="dgCMatrix")
ansSingle <- mmec(Yield~1,
        random= ~ vsc(isc(Name), Gu=Ai),
        rcov= ~ units.
        data=DT, verbose = FALSE)
summary(ansSingle)
Multivariate Linear Mixed Model fit by REML
## **************** sommer 4.2 ************
logLik AIC
                    BIC Method Converge
## Value -359.0031 720.0062 723.2266 AI TRUE
## Variance-Covariance components:
           VarComp VarCompSE Zratio Constraint
## Name:Ai:isc:isc 6.495
                  1.479 4.392
                            Positive
            13.869
## units:isc:isc
                   1.799 7.711
                            Positive
## Fixed effects:
         Estimate Std.Error t.value
## (Intercept) 11.74
                0.4861
                      24.15
## Use the '$' sign to access results and parameters
```

In this model, the only term to be estimated is the one for the germplasm (here called Name). For the sake of example we have added a relationship matrix among the levels of the random effect Name. This is just a diagonal matrix with as many rows and columns as levels present in the random effect Name, but any other non-diagonal relationship matrix could be used.

#### 2) MET: main effect model

A multi-environment model is the one that is fitted when the breeding program can afford more than one location. The main effect model assumes that GxE doesn't exist and that the main genotype effect plus the fixed effect for environment is enough to predict the genotype effect in all locations of interest.

```
ansMain <- mmer(Yield~Env,</pre>
         random= ~ vsr(Name, Gu=A),
         rcov= ~ units,
         data=DT, verbose = FALSE)
summary(ansMain)
Multivariate Linear Mixed Model fit by REML
## ***************** sommer 4.2 ************
logLik
              AIC
                     BIC Method Converge
## Value -32.59421 71.18842 80.84949
                         NR
## Variance-Covariance components:
              VarComp VarCompSE Zratio Constraint
## u:Name.Yield-Yield
               4.856
                     1.5233 3.188
                                Positive
## units.Yield-Yield
               8.109
                     0.9615 8.434
                               Positive
## Fixed effects:
   Trait
          Effect Estimate Std.Error t.value
## 1 Yield (Intercept) 16.385 0.5849 28.012
## 2 Yield EnvCA.2012
               -5.688
                      0.5741 - 9.908
## 3 Yield EnvCA.2013 -6.218
                     0.6107 -10.182
## Groups and observations:
      Yield
## u:Name
        41
## Use the '$' sign to access results and parameters
# or
Ai <- as(solve(A), Class="dgCMatrix")
ansMain <- mmec(Yield~Env,</pre>
         random= ~ vsc(isc(Name), Gu=Ai),
         rcov= ~ units,
         data=DT, verbose = FALSE)
summary(ansMain)
Multivariate Linear Mixed Model fit by REML
## ************** sommer 4.2 ************
logLik
               AIC
                     BIC Method Converge
## Value -313.3005 632.6011 642.2621
                         ΑI
## Variance-Covariance components:
            VarComp VarCompSE Zratio Constraint
## Name:Ai:isc:isc 4.854
                    1.449 3.350
                             Positive
```

```
8.109
                     1.807 4.486
## units:isc:isc
                               Positive
 ______
## Fixed effects:
##
        Estimate Std.Error t.value
## Intercept
          16.385
                 0.5847
                      28.021
## CA.2012
          -5.688
                 0.5741 - 9.909
## CA.2013
          -6.219
                 0.6107 -10.183
## Use the '$' sign to access results and parameters
```

### 3) MET: diagonal model (DG)

A multi-environment model is the one that is fitted when the breeding program can afford more than one location. The diagonal model assumes that GxE exists and that the genotype variation is expressed differently at each location, therefore fitting a variance component for the genotype effect at each location. The main drawback is that this model assumes no covariance among locations, as if genotypes were independent (despite the fact that is the same genotypes). The fixed effect for environment plus the location-specific BLUP is used to predict the genotype effect in each locations of interest.

```
##
         Multivariate Linear Mixed Model fit by REML
  ****************** sommer 4.2 **************
  ______
##
         logLik
                  AIC
                         BIC Method Converge
 Value -21.04157 48.08315 57.74421
                               NR.
                                    TRUE
  ______
##
  Variance-Covariance components:
                     VarComp VarCompSE Zratio Constraint
## CA.2011:Name.Yield-Yield 17.493
                             6.1099 2.863
                                          Positive
## CA.2012:Name.Yield-Yield
                       5.337
                             1.7662
                                   3.022
                                          Positive
## CA.2013:Name.Yield-Yield
                       7.884
                             2.5526
                                   3.089
                                          Positive
## units.Yield-Yield
                       4.381
                             0.6493 6.747
                                          Positive
  _____
## Fixed effects:
   Trait
            Effect Estimate Std.Error t.value
## 1 Yield (Intercept)
                   16.621
                           0.948 17.532
                   -5.958
## 2 Yield EnvCA.2012
                           1.045
                                -5.699
## 3 Yield EnvCA.2013
                   -6.662
                           1.098 -6.067
## -----
## Groups and observations:
##
            Yield
## CA.2011:Name
              41
## CA.2012:Name
              41
## CA.2013:Name
## Use the '$' sign to access results and parameters
```

```
Ai <- as(solve(A), Class="dgCMatrix")
ansDG <- mmec(Yield~Env,</pre>
            random= ~ vsc(dsc(Env),isc(Name), Gu=Ai),
            rcov= ~ units,
            data=DT, verbose = FALSE)
summary(ansDG)
##
           Multivariate Linear Mixed Model fit by REML
  ****************** sommer 4.2 **************
  ______
          logLik
                     AIC
                             BIC Method Converge
## Value -301.7528 609.5057 619.1668
                                    ΑI
                                          TRUE
## Variance-Covariance components:
                           VarComp VarCompSE Zratio Constraint
## Env:Name:Ai:CA.2011:CA.2011
                            17.124
                                      3.479
                                           4.922
                                                   Positive
## Env:Name:Ai:CA.2012:CA.2012
                             5.351
                                      2.814 1.901
                                                   Positive
## Env:Name:Ai:CA.2013:CA.2013
                             7.904
                                      2.924 2.703
                                                   Positive
                             4.332
## units:isc:isc
                                      2.258 1.919
                                                   Positive
## Fixed effects:
           Estimate Std.Error t.value
                      0.9386 17.709
## Intercept
             16.621
## CA.2012
             -5.958
                      1.0367
                             -5.747
## CA.2013
             -6.662
                      1.0898 -6.113
## Use the '$' sign to access results and parameters
```

#### 4) MET: compund symmetry model (CS)

A multi-environment model is the one that is fitted when the breeding program can afford more than one location. The compound symmetry model assumes that GxE exists and that a main genotype variance-covariance component is expressed across all location. In addition, it assumes that a main genotype-by-environment variance is expressed across all locations. The main drawback is that the model assumes the same variance and covariance among locations. The fixed effect for environment plus the main effect for BLUP plus genotype-by-environment effect is used to predict the genotype effect in each location of interest.

```
## Variance-Covariance components:
                  VarComp VarCompSE Zratio Constraint
## u:Name.Yield-Yield
                    3.682
                          1.691 2.177
                                      Positive
## u:Env:Name.Yield-Yield 5.173
                           1.495 3.460
## units.Yield-Yield
                   4.366
                           0.647 6.748 Positive
## Fixed effects:
           Effect Estimate Std.Error t.value
## 1 Yield (Intercept) 16.496 0.6855 24.065
## 2 Yield EnvCA.2012 -5.777
                         0.7558 - 7.643
## 3 Yield EnvCA.2013 -6.380 0.7960 -8.015
## Groups and observations:
          Yield
## u:Name
            41
## u:Env:Name
          123
## -----
## Use the '$' sign to access results and parameters
E <- diag(length(unique(DT$Env)));rownames(E) <- colnames(E) <- unique(DT$Env)
Ei <- solve(E)
Ai <- solve(A)
EAi <- kronecker(Ei, Ai, make.dimnames = TRUE)
Ei <- as(Ei, Class="dgCMatrix")</pre>
Ai <- as(Ai, Class="dgCMatrix")
EAi <- as(EAi, Class="dgCMatrix")
ansCS <- mmec(Yield~Env,
          random= ~ vsc(isc(Name), Gu=Ai) + vsc(isc(Env:Name), Gu=EAi),
          rcov= ~ units,
          data=DT, verbose = FALSE)
summary(ansCS)
        Multivariate Linear Mixed Model fit by REML
## **************** sommer 4.2 ************
logLik
                 AIC
                       BIC Method Converge
## Value -300.8517 607.7034 617.3645 AI TRUE
## Variance-Covariance components:
                 VarComp VarCompSE Zratio Constraint
##
## Name:Ai:isc:isc
                  3.683 1.881 1.958 Positive
## Env:Name:EAi:isc:isc 5.174
                          2.420 2.138
                                    Positive
## units:isc:isc
                  4.360
                          2.270 1.920
## Fixed effects:
         Estimate Std.Error t.value
## Intercept 16.496 0.6856 24.062
## CA.2012
          -5.777
                  0.7558 - 7.643
## CA.2013
          -6.380
                  0.7960 -8.015
## Use the '$' sign to access results and parameters
```

## 5) MET: unstructured model (US)

A multi-environment model is the one that is fitted when the breeding program can afford more than one location. The unstructured model is the most flexible model assuming that GxE exists and that an environment-specific variance exists in addition to as many covariances for each environment-to-environment combinations. The main drawback is that is difficult to make this models converge because of the large number of variance components, the fact that some of these variance or covariance components are zero, and the difficulty in choosing good starting values. The fixed effect for environment plus the environment specific BLUP (adjusted by covariances) is used to predict the genotype effect in each location of interest.

```
ansUS <- mmer(Yield~Env,
           random= ~ vsr(usr(Env), Name, Gu=A),
           rcov= ~ units,
           data=DT, verbose = FALSE)
summary(ansUS)
  ______
           Multivariate Linear Mixed Model fit by REML
  ______
##
                  AIC
                          BIC Method Converge
         logLik
## Value -14.20951 34.41901 44.08008
                                NR
  ______
## Variance-Covariance components:
##
                            VarComp VarCompSE Zratio Constraint
## CA.2011:Name.Yield-Yield
                             15.994
                                      5.381
                                           2.972
## CA.2012:CA.2011:Name.Yield-Yield
                             6.172
                                      2.503
                                           2.465
                                                 Unconstr
## CA.2012:Name.Yield-Yield
                              5.273
                                      1.750
                                           3.013
                                                 Positive
## CA.2013:CA.2011:Name.Yield-Yield
                             6.366
                                      3.069
                                           2.074
                                                 Unconstr
## CA.2013:CA.2012:Name.Yield-Yield
                             0.376
                                      1.535
                                           0.245
                                                 Unconstr
## CA.2013:Name.Yield-Yield
                             7.689
                                      2.490
                                           3.088
                                                 Positive
## units.Yield-Yield
                              4.386
                                      0.650
                                           6.748
                                                 Positive
  ______
## Fixed effects:
   Trait
            Effect Estimate Std. Error t.value
## 1 Yield (Intercept)
                   16.341
                           0.8141
## 2 Yield EnvCA.2012
                   -5.696
                           0.7406
                                 -7.692
## 3 Yield EnvCA.2013
                   -6.286
                           0.8202 -7.664
## Groups and observations:
                  Yield
## CA.2011:Name
                     41
## CA.2012:CA.2011:Name
                     82
## CA.2012:Name
                     41
## CA.2013:CA.2011:Name
                     82
## CA.2013:CA.2012:Name
                     82
## CA.2013:Name
                     41
## Use the '$' sign to access results and parameters
# adjust variance BLUPs by adding covariances
# ansUS$U[1:6] <- unsBLUP(ansUS$U[1:6])
# or
Ai <- solve(A)
```

```
Ai <- as(Ai, Class="dgCMatrix")
ansUS <- mmec(Yield~Env,
          random= ~ vsc(usc(Env),isc(Name), Gu=Ai),
          rcov= ~ units,
          data=DT, verbose = FALSE)
summary(ansUS)
##
         Multivariate Linear Mixed Model fit by REML
## ***************** sommer 4.2 **************
  ______
##
                  AIC
                        BIC Method Converge
         logLik
## Value -295.2125 596.4249 606.086
                              AΙ
  ______
## Variance-Covariance components:
##
                       VarComp VarCompSE Zratio Constraint
## Env:Name:Ai:CA.2011:CA.2011 14.8143
                                3.513 4.2167
                                            Positive
## Env:Name:Ai:CA.2011:CA.2012 5.7221
                                1.957 2.9241
                                            Unconstr
## Env:Name:Ai:CA.2012:CA.2012 4.9599
                                2.263 2.1913
                                            Positive
## Env:Name:Ai:CA.2011:CA.2013 6.1289
                                2.726 2.2481
                                            Unconstr
## Env:Name:Ai:CA.2012:CA.2013 0.6439
                                1.899 0.3391
                                            Unconstr
## Env:Name:Ai:CA.2013:CA.2013 7.5772
                                2.905 2.6084
                                            Positive
## units:isc:isc
                        4.0134
                                2.171 1.8483
                                            Positive
## Fixed effects:
         Estimate Std.Error t.value
## Intercept 16.344
                  0.7847 20.828
## CA.2012
           -5.693
                   0.7150 - 7.963
## CA.2013
           -6.277
                   0.7918 -7.927
## Use the '$' sign to access results and parameters
```

#### 6) MET: random regression model

A multi-environment model is the one that is fitted when the breeding program can afford more than one location. The random regression model assumes that the environment can be seen as a continuous variable and therefore a variance component for the intercept and a variance component for the slope can be fitted. The number of variance components will depend on the order of the Legendre polynomial fitted.

```
library(orthopolynom)
DT$EnvN <- as.numeric(as.factor(DT$Env))</pre>
ansRR <- mmer(Yield~Env,</pre>
          random= ~ vsr(leg(EnvN,1),Name),
          rcov= ~ units,
          data=DT, verbose = FALSE)
summary(ansRR)
        Multivariate Linear Mixed Model fit by REML
## ***************** sommer 4.2 *************
  ______
##
         logLik
                  AIC
                        BIC Method Converge
## Value -27.70318 61.40636 71.06743
```

```
## Variance-Covariance components:
##
                  VarComp VarCompSE Zratio Constraint
## leg0:Name.Yield-Yield 10.392 3.1473 3.302 Positive
                          0.9792 2.123 Positive
## leg1:Name.Yield-Yield 2.079
## units.Yield-Yield
                    6.297
                          0.8442 7.459
                                      Positive
## Fixed effects:
   Trait
           Effect Estimate Std.Error t.value
## 1 Yield (Intercept) 16.541 0.6770 24.432
## 2 Yield EnvCA.2012 -5.832 0.6425 -9.078
## 3 Yield EnvCA.2013 -6.472 0.8239 -7.854
## Groups and observations:
         Yield
##
## leg0:Name
           41
## leg1:Name
## Use the '$' sign to access results and parameters
# or
ansRR <- mmec(Yield~Env,
          random= ~ vsc(dsc(leg(EnvN,1)),isc(Name)),
          rcov= ~ units,
          data=DT, verbose = FALSE)
summary(ansRR)
        Multivariate Linear Mixed Model fit by REML
## ************** sommer 4.2 ************
AIC
                        BIC Method Converge
        logLik
## Value -308.4098 622.8195 632.4806 AI TRUE
## Variance-Covariance components:
                 VarComp VarCompSE Zratio Constraint
## EnvN:Name:leg0:leg0 10.355
                        2.319 4.465 Positive
                         1.759 1.182 Positive
## EnvN:Name:leg1:leg1 2.079
## units:isc:isc
                  6.304
                          2.005 3.145
## Fixed effects:
        Estimate Std.Error t.value
## Intercept 16.541 0.6761 24.467
## CA.2012 -5.833
                  0.6421 -9.084
## CA.2013
        -6.472 0.8233 -7.861
## -----
## Use the '$' sign to access results and parameters
In addition, an unstructured, diagonal or other variance-covariance structure can be put on top of the
polynomial model:
library(orthopolynom)
DT$EnvN <- as.numeric(as.factor(DT$Env))</pre>
ansRR <- mmer(Yield~Env,</pre>
          random= ~ vsr(usr(leg(EnvN,1)),Name),
       rcov= ~ units,
```

```
data=DT, verbose = FALSE)
summary(ansRR)
## =========
        Multivariate Linear Mixed Model fit by REML
## ************** sommer 4.2 ************
AIC
                       BIC Method Converge
        logLik
## Value -25.56967 57.13935 66.80042 NR
                                 TRUF.
## Variance-Covariance components:
                    VarComp VarCompSE Zratio Constraint
                     10.791
                           3.2745 3.295
## leg0:Name.Yield-Yield
## leg1:leg0:Name.Yield-Yield -2.428
                            1.3699 -1.772
                                       Unconstr
## leg1:Name.Yield-Yield 2.286 1.0404 2.197 Positive
## units.Yield-Yield
                     6.260 0.8421 7.434 Positive
## Fixed effects:
           Effect Estimate Std.Error t.value
   Trait
## 1 Yield (Intercept) 16.501 0.7778 21.216
## 2 Yield EnvCA.2012 -5.791
                        0.6704 -8.638
## 3 Yield EnvCA.2013 -6.476
                       0.8554 -7.570
## Groups and observations:
##
            Yield
## leg0:Name
              41
## leg1:leg0:Name
              82
## leg1:Name
              41
## Use the '$' sign to access results and parameters
# or
ansRR <- mmec(Yield~Env,</pre>
         random= ~ vsc(usc(leg(EnvN,1)),isc(Name)),
         rcov= ~ units,
         data=DT, verbose = FALSE)
summary(ansRR)
##
        Multivariate Linear Mixed Model fit by REML
## ************** sommer 4.2 ************
AIC
        logLik
                      BIC Method Converge
## Value -308.7042 623.4085 633.0695 AI
                                 TRUF.
## Variance-Covariance components:
                VarComp VarCompSE Zratio Constraint
## EnvN:Name:leg0:leg0 10.6512 2.242 4.75048 Positive
## EnvN:Name:leg0:leg1 0.1204 1.445 0.08328
                                   Unconstr
## EnvN:Name:leg1:leg1 2.0947 1.920 1.09114
                                   Positive
## units:isc:isc
                6.6403
                        2.042 3.25168 Positive
## Fixed effects:
```

#### 7) Other GxE covariance structures

Although not very commonly used in GxE models, the autoregressive of order 1 (AR1) and other covariance structures could be used in the GxE modeling. Here we show how to do it (not recommending it).

```
E \leftarrow AR1(DT\$Env) \# can be AR1() or CS(), etc.
rownames(E) <- colnames(E) <- unique(DT$Env)</pre>
EA <- kronecker(E,A, make.dimnames = TRUE)
ansCS <- mmer(Yield~Env,
          random= ~ vsr(Name, Gu=A) + vsr(Env:Name, Gu=EA),
          rcov= ~ units,
          data=DT, verbose = FALSE)
summary(ansCS)
Multivariate Linear Mixed Model fit by REML
## ***************** sommer 4.2 *************
  ______
##
        logLik
                 AIC
                       BIC Method Converge
## Value -19.39067 44.78134 54.4424
                             NR
## Variance-Covariance components:
##
                   VarComp VarCompSE Zratio Constraint
## u:Name.Yield-Yield
                    2.225
                           1.7536 1.269
                                       Positive
## u:Env:Name.Yield-Yield
                    6.424
                           1.8293 3.512
                                       Positive
## units.Yield-Yield
                    4.334
                           0.6418 6.752
                                       Positive
## Fixed effects:
   Trait
           Effect Estimate Std.Error t.value
## 1 Yield (Intercept)
                 16.484
                         0.6735 24.474
## 2 Yield EnvCA.2012
                  -5.780
                         0.7365
                               -7.848
## 3 Yield EnvCA.2013
                 -6.372
                         0.7799 -8.170
## Groups and observations:
##
          Yield
## u:Name
            41
## u:Env:Name
           123
## Use the '$' sign to access results and parameters
```

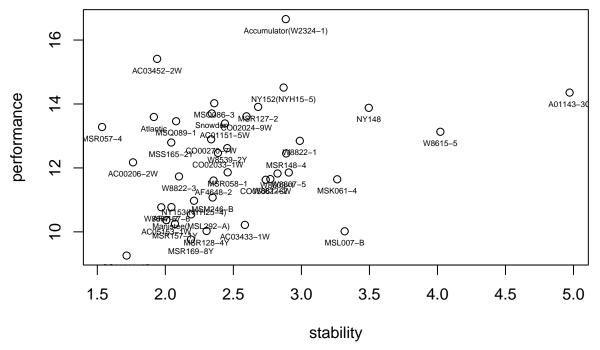
## 8) Finlay-Wilkinson regression

```
data(DT_h2)
DT <- DT_h2</pre>
```

```
## build the environmental index
ei <- aggregate(y~Env, data=DT,FUN=mean)</pre>
colnames(ei)[2] <- "envIndex"</pre>
ei$envIndex <- ei$envIndex - mean(ei$envIndex, na.rm=TRUE) # center the envIndex to have clean VCs
ei <- ei[with(ei, order(envIndex)), ]</pre>
## add the environmental index to the original dataset
DT2 <- merge(DT,ei, by="Env")
# numeric by factor variables like envIndex: Name can't be used in the random part like this
# they need to come with the vsc() structure
DT2 <- DT2[with(DT2, order(Name)), ]
mix2 <- mmec(y~ envIndex,</pre>
              random=~ Name + vsc(dsc(envIndex),isc(Name)), data=DT2,
              rcov=~vsc(dsc(Name),isc(units)),
              tolParConvNorm = .0001,
              nIters = 50, verbose = FALSE
)
## as(<dgeMatrix>, "dgCMatrix") is deprecated since Matrix 1.5-0; do as(., "CsparseMatrix") instead
# summary(mix2)$varcomp
b=mix2$uList$`vsc(dsc(envIndex), isc(Name))` # adaptability (b) or genotype slopes
mu=mix2$uList$`vsc( isc( Name ) )` # general adaptation (mu) or main effect
e=sqrt(summary(mix2)$varcomp[-c(1:2),1]) # error variance for each individual
## general adaptation (main effect) vs adaptability (response to better environments)
plot(mu[,1]~b[,1], ylab="general adaptation", xlab="adaptability")
text(y=mu[,1],x=b[,1], labels = rownames(mu), cex=0.5, pos = 1)
                                                                                         0
                                                                                   Accumulator(W2
      က
                                                           0
                                                        AC03452-2W
general adaptation
                                                                                 0
                                                                               MSQ086-3
                                                                           0
                                              W8615-8 ○ AC01151
                                                                        MSQ089-1 O
                                                                             MSR057-4
                                                                   0
                               0
       0
                            AC00206-210
                                                      CO02033-1W
                                        867-5 U MSR058-1
463098061-6WMSK061-4 W8822-3
                                 W8603
                              0
                          NY153(NYH29-4) O MSM246-B
                              AF4157\\\0008867-7\\0008
              0
            03433-1W
                           MSR157-1Y O AC05153-1V Manistee(MSL292-A)
                                 MSL007-B
                                         MSR128-4Y
                        MSR169-8Y
                            -0.2
                                                                  0.2
                                               0.0
                                                                                     0.4
                                              adaptability
## prediction across environments
```

Dt <- mix2\$Dtable

```
Dt[1,"average"]=TRUE
Dt[2,"include"]=TRUE
Dt[3,"include"]=TRUE
pp <- predict(mix2,Dtable = Dt, D="Name")
preds <- pp$pvals
# preds[with(preds, order(-predicted.value)), ]
## performance vs stability (deviation from regression line)
plot(preds[,2]~e, ylab="performance", xlab="stability")
text(y=preds[,2],x=e, labels = rownames(mu), cex=0.5, pos = 1)</pre>
```



#### 9) Factor analytic (reduced rank) model

When the number of environments where genotypes are evaluated is big and we want to consider the genetic covariance between environments and location-specific variance components we cannot fit an unstructured covariance in the model since the number of parameters is too big and the matrix can become non-full rank leading to singularities. In those cases is suggested a dimensionality reduction technique. Among those the factor analytic structures proposed by many research groups (Piepho, Smith, Cullis, Thompson, Meyer, etc.) are the way to go. Sommer has a reduced-rank factor analytic implementation available through the rrc() function. Here we show an example of how to fit the model:

```
verbose=FALSE,
    data=DT)

summary(ans1b)$varcomp
```

```
## VarComp VarCompSE Zratio Constraint

## Env:Name:y:PC1:PC1 14.7797212 2.1735853 6.7996970 Positive

## Env:Name:y:PC1:PC2 0.5721396 1.3828997 0.4137246 Unconstr

## Env:Name:y:PC2:PC2 3.0682493 2.9985559 1.0232423 Positive

## units:isc:isc 7.6250662 0.8331028 9.1526112 Positive

Lam=with(DT, rrc(Env, Name, y, returnLam = TRUE, nPC = 2))$Lam # extract loadings

score.mat <- ans1b$uList[[1]]; # extract factor scores

BLUP = score.mat %*% t(Lam) # BLUPs for all environments
```

As can be seen BLUPs for all environments can be recovered by multiplying the loadings (Lam) by the factor scores (score.mat). This is a parsomonious way to model an unstructured covariance.

# 10) Two stage analysis

It is common then to fit a first model that accounts for the variation of random design elements, e.g., locations, years, blocks, and fixed genotype effects to obtain the estimated marginal means (EMMs) or best linear unbiased estimators (BLUEs) as adjusted entry means. These adjusted entry means are then used as the phenotype or response variable in GWAS and genomic prediction studies.

```
#########
## stage 1
## use mmer for dense field trials
#########
data(DT_h2)
DT <- DT_h2
head(DT)
##
                             Env Loc Year
                                               Block y
                    Name
## 1
                W8822-3 FL.2012 FL 2012 FL.2012.1 2
## 2
                W8867-7 FL.2012 FL 2012 FL.2012.2 2
## 3
               MSL007-B M0.2011 M0 2011 M0.2011.1 3
## 4
             C000270-7W FL.2012 FL 2012 FL.2012.2 3
## 5 Manistee(MSL292-A) FL.2013 FL 2013 FL.2013.2 3
               MSM246-B FL.2012 FL 2012 FL.2012.2 3
envs <- unique(DT$Env)</pre>
BLUEL <- list()</pre>
XtXL <- list()</pre>
for(i in 1:length(envs)){
  ans1 <- mmer(y~Name-1,
                random=~Block,
                verbose=FALSE,
                data=droplevels(DT[which(DT$Env == envs[i]),]
  )
  ans1$Beta$Env <- envs[i]
  BLUEL[[i]] <- ans1$Beta
  # to be comparable to 1/(se^2) = 1/PEV = 1/Ci = 1/[(X'X)inv]
```

```
XtXL[[i]] <- solve(ans1$VarBeta)</pre>
}
DT2 <- do.call(rbind, BLUEL)
OM <- do.call(adiag1,XtXL)</pre>
##########
## stage 2
## use mmec for sparse equation
#########
m <- matrix(1/var(DT2$Estimate, na.rm = TRUE))</pre>
ans2 <- mmec(Estimate~Env,
             random=~Effect + Env:Effect,
             rcov=~vsc(isc(units,thetaC = matrix(3), theta = m)),
             W=OM,
             verbose=FALSE,
             data=DT2
             )
```

## Using the weights matrix

summary(ans2)\$varcomp

```
## VarComp VarCompSE Zratio Constraint
## Effect:isc:isc 2.076920 0.5758219 3.606880 Positive
## Env:Effect:isc:isc 3.333792 0.1850479 18.015831 Positive
## units:m: 1.000000 0.1850479 5.404005 Fixed
```

#### Literature

Covarrubias-Pazaran G. 2016. Genome assisted prediction of quantitative traits using the R package sommer. PLoS ONE 11(6):1-15.

Covarrubias-Pazaran G. 2018. Software update: Moving the R package sommer to multivariate mixed models for genome-assisted prediction. doi: https://doi.org/10.1101/354639

Bernardo Rex. 2010. Breeding for quantitative traits in plants. Second edition. Stemma Press. 390 pp.

Gilmour et al. 1995. Average Information REML: An efficient algorithm for variance parameter estimation in linear mixed models. Biometrics 51(4):1440-1450.

Henderson C.R. 1975. Best Linear Unbiased Estimation and Prediction under a Selection Model. Biometrics vol. 31(2):423-447.

Kang et al. 2008. Efficient control of population structure in model organism association mapping. Genetics 178:1709-1723.

Lee, D.-J., Durban, M., and Eilers, P.H.C. (2013). Efficient two-dimensional smoothing with P-spline ANOVA mixed models and nested bases. Computational Statistics and Data Analysis, 61, 22 - 37.

Lee et al. 2015. MTG2: An efficient algorithm for multivariate linear mixed model analysis based on genomic information. Cold Spring Harbor. doi: http://dx.doi.org/10.1101/027201.

Maier et al. 2015. Joint analysis of psychiatric disorders increases accuracy of risk prediction for schizophrenia, bipolar disorder, and major depressive disorder. Am J Hum Genet; 96(2):283-294.

Rodriguez-Alvarez, Maria Xose, et al. Correcting for spatial heterogeneity in plant breeding experiments with P-splines. Spatial Statistics 23 (2018): 52-71.

Searle. 1993. Applying the EM algorithm to calculating ML and REML estimates of variance components. Paper invited for the 1993 American Statistical Association Meeting, San Francisco.

Yu et al. 2006. A unified mixed-model method for association mapping that accounts for multiple levels of relatedness. Genetics 38:203-208.

Tunnicliffe W. 1989. On the use of marginal likelihood in time series model estimation. JRSS 51(1):15-27.