Week 5: Networks of Neurons-Introduction

Biological Modeling of Neural Networks

Week 5
NETWORKS of NEURONS and ASSOCIATIVE MEMORY

Wulfram Gerstner

EPFL, Lausanne, Switzerland

5.1 Introduction
- networks of neuron
- systems for computing
- associative memory

5.2 Classification by similarity

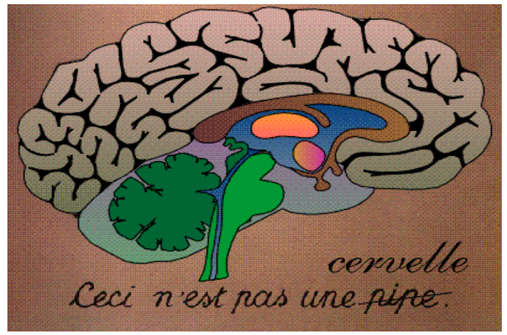
5.3 Detour: Magnetic Materials

5.4 Hopfield Model

5.5 Learning of Associations

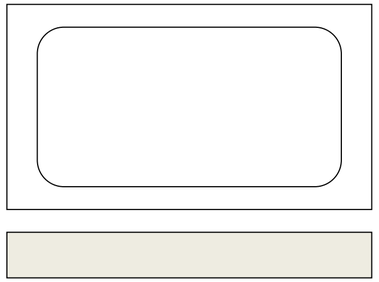
5.6 Storage Capacity

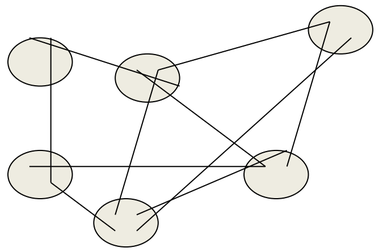
Systems for computing and information processing

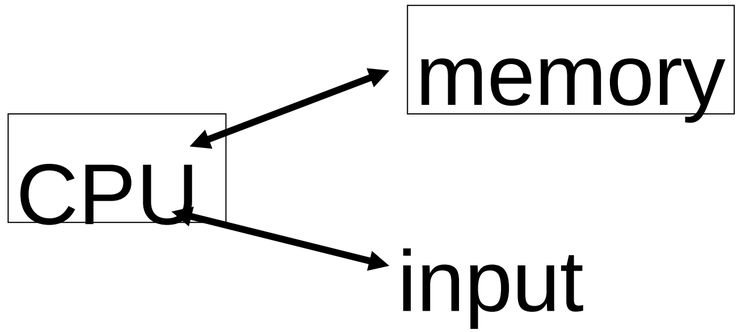


Brain

Computer







Distributed architecture

(10 ^{10} proc. Elements/neurons)

No separation of

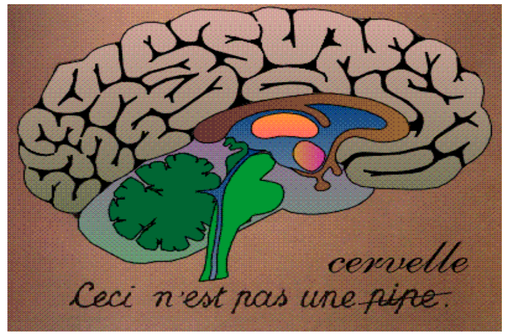
processing and memory

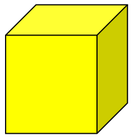
Von Neumann architecture

1 CPU

(10 ^{10} transistors)

Systems for computing and information processing

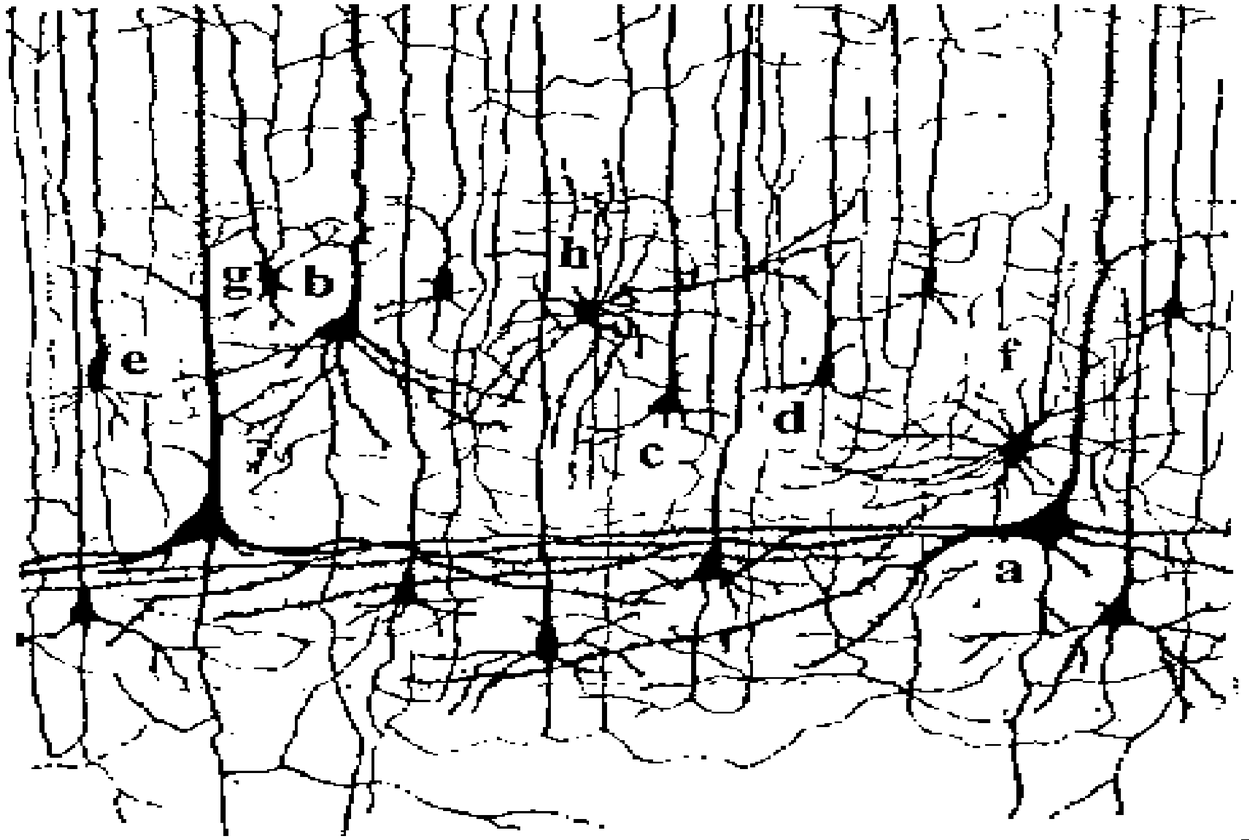




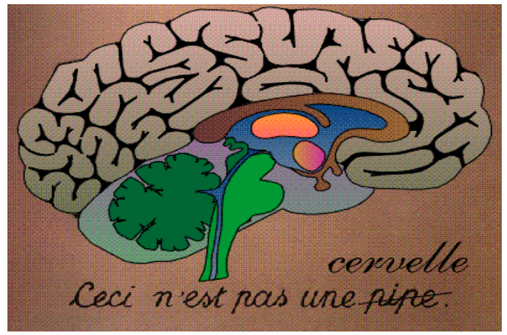
1mm

10 000 neurons

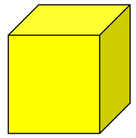
3 km wire



Systems for computing and information processing



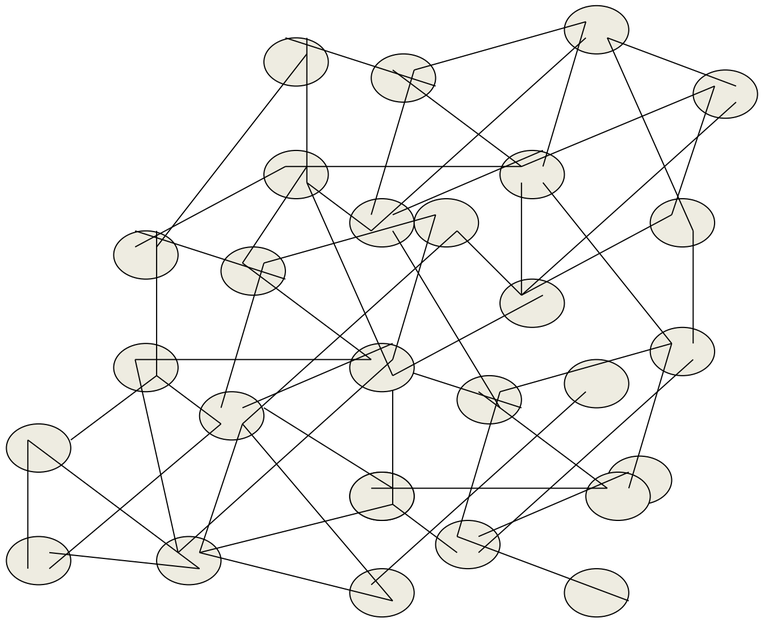
Brain



1mm

10 000 neurons

3 km wire



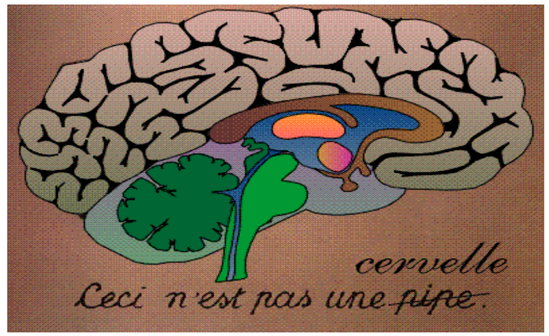
Distributed architecture

10^{10} neurons

10^{4} connections/neurons

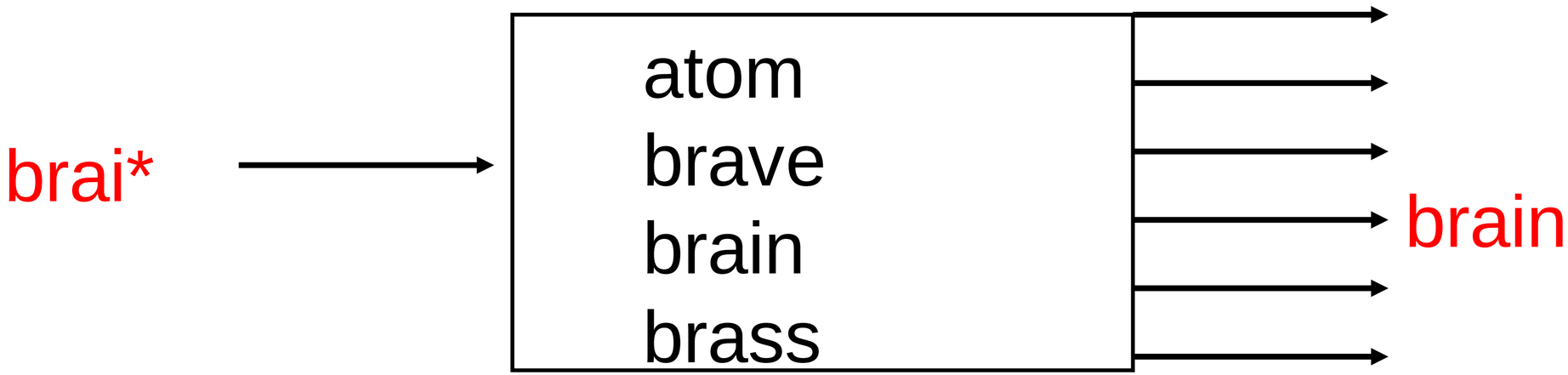
No separation of processing and memory

Associations, Associative Memory



*Read this text **NOW**!*

pattern completion/word recognition



Noisy word

List of words

Output the closest one

Your brain fills in missing information: 'associative memory'

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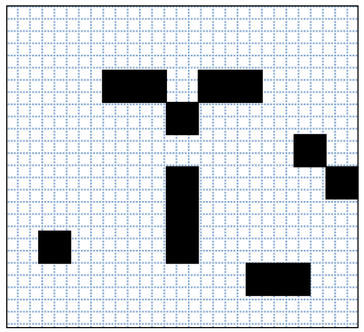
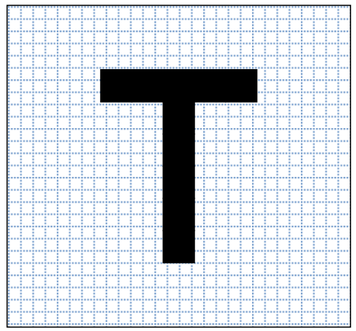
5.2 Classification by similarity: **pattern recognition**

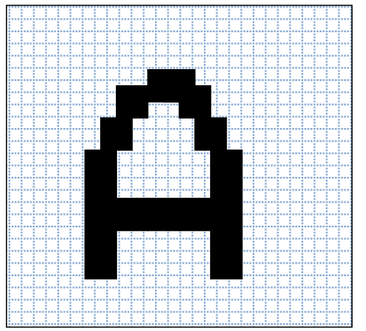
image

Classification:
comparison
with prototypes

A
B
T
Z







Noisy image

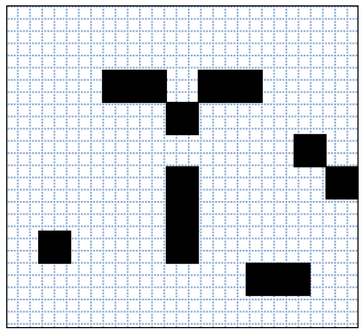
Prototypes

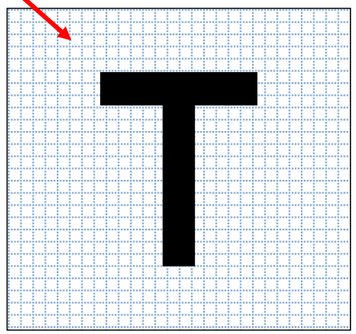
5.2 Classification by similarity: **pattern recognition**

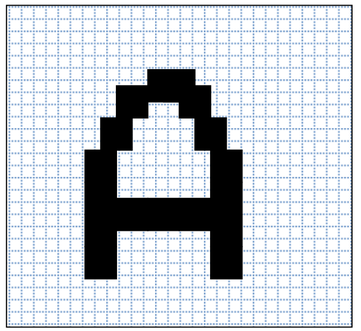
Classification by closest prototype

Blackboard:

$$\left|x - p^{T}\right| \leq \left|x - p^{A}\right|$$







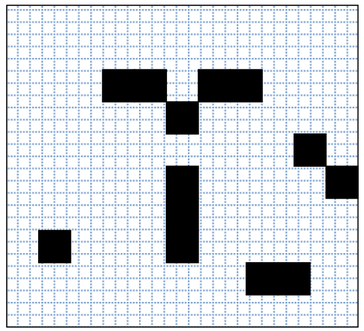
Noisy image

Prototypes

5.2 pattern recognition and Pattern completion

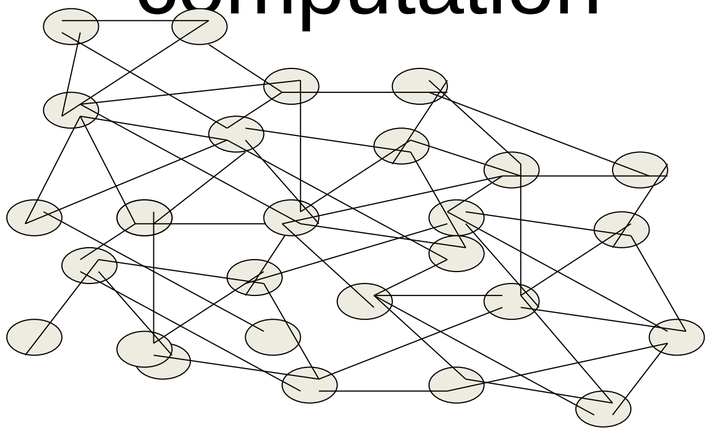
Aim: Understand Associative Memory

Noisy image



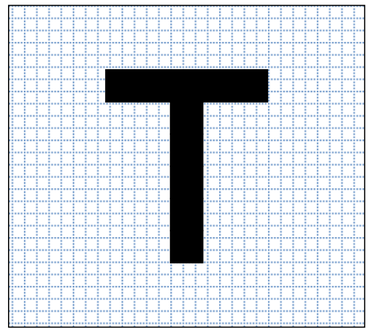
Partial word

Associative memory/ collective computation



Brain-style computation

Full image



Full word

Quiz 5.1: Connectivity

A typical neuron in the brain makes connections
-To 6-20 neighbors
-To 100-200 neurons nearby
-To more than 1000 neurons nearby
-To more than 1000 neurons nearby or far away.

In a typical cristal in nature, each atom interacts
-with 6-20 neighbors
-with 100-200 neurons nearby
-with more than 1000 neurons nearby
-with more than 1000 neurons nearby or far away.

Week 5: Networks of Neurons-Introduction

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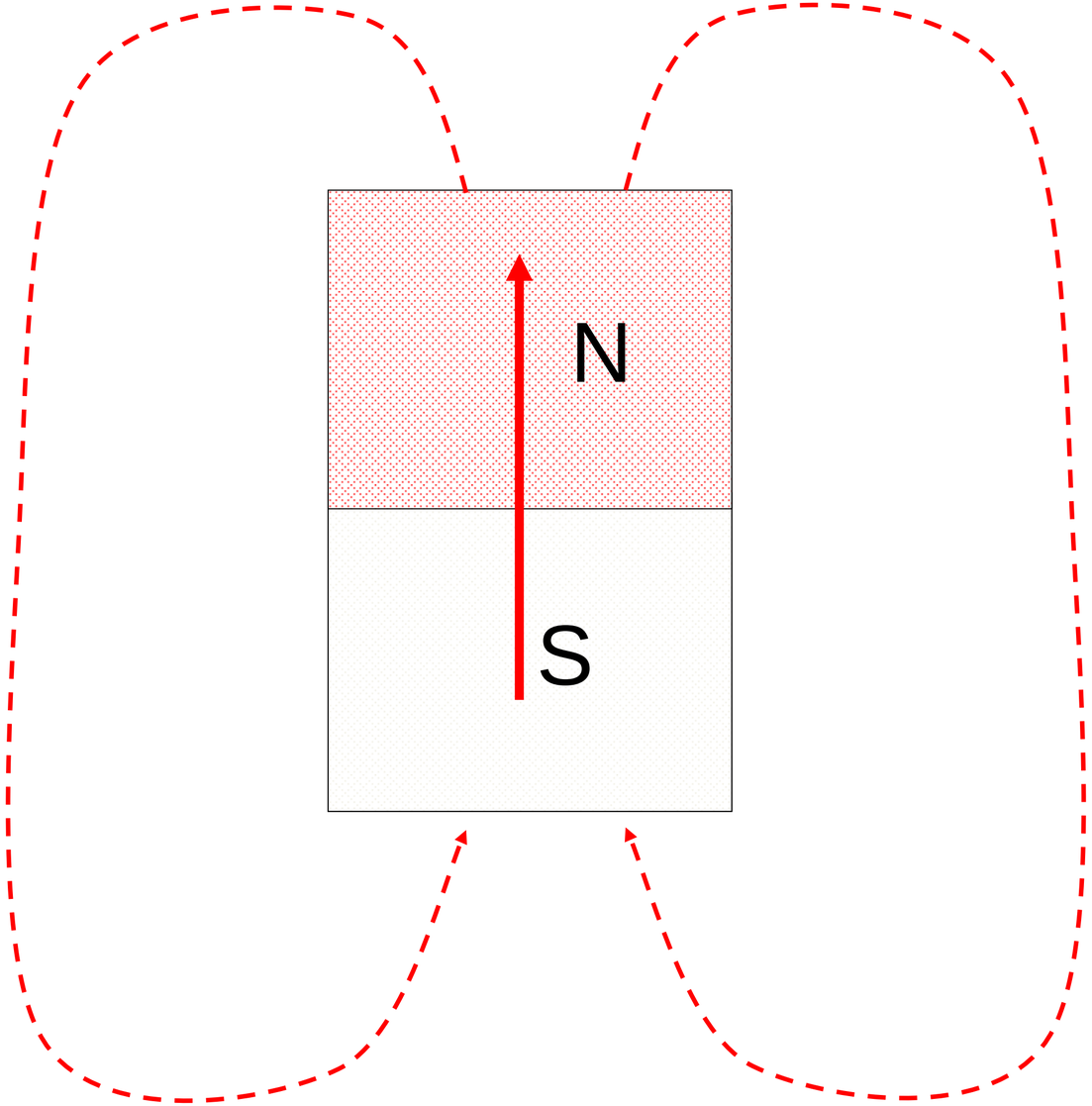
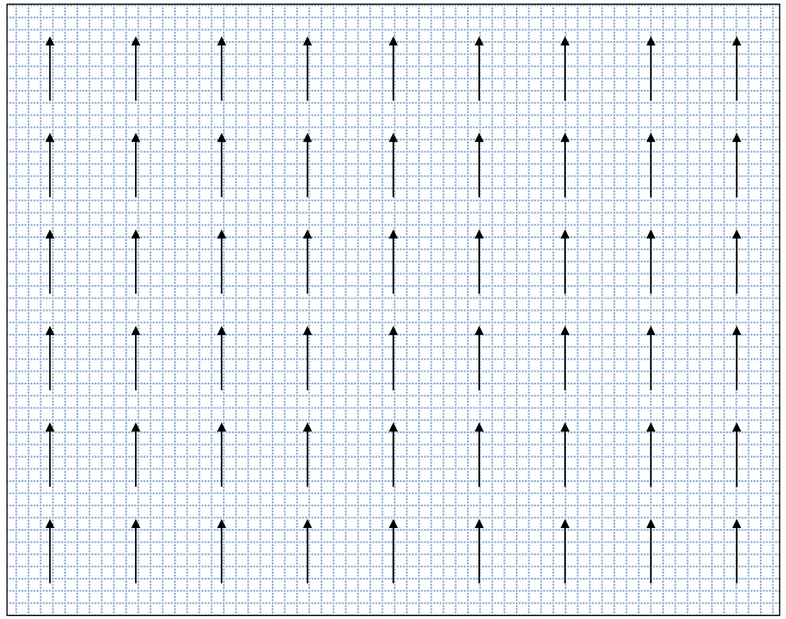
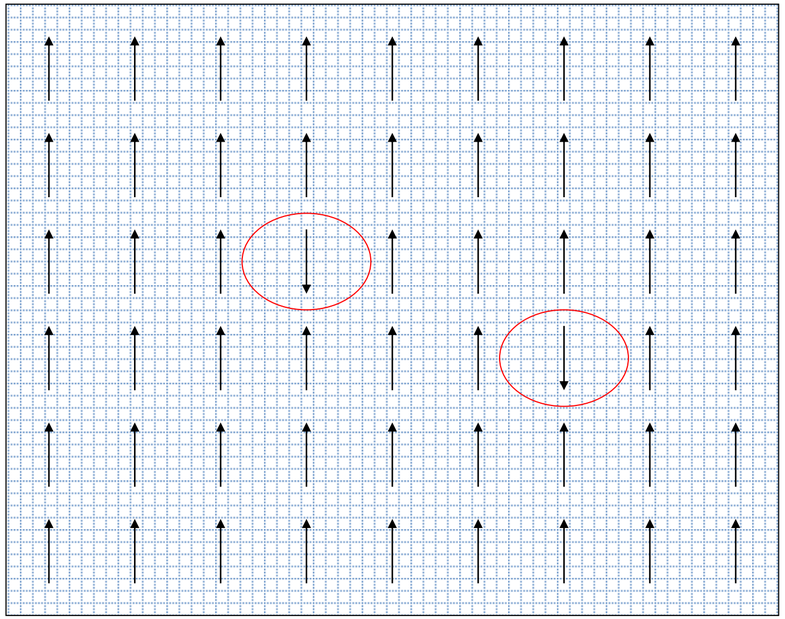
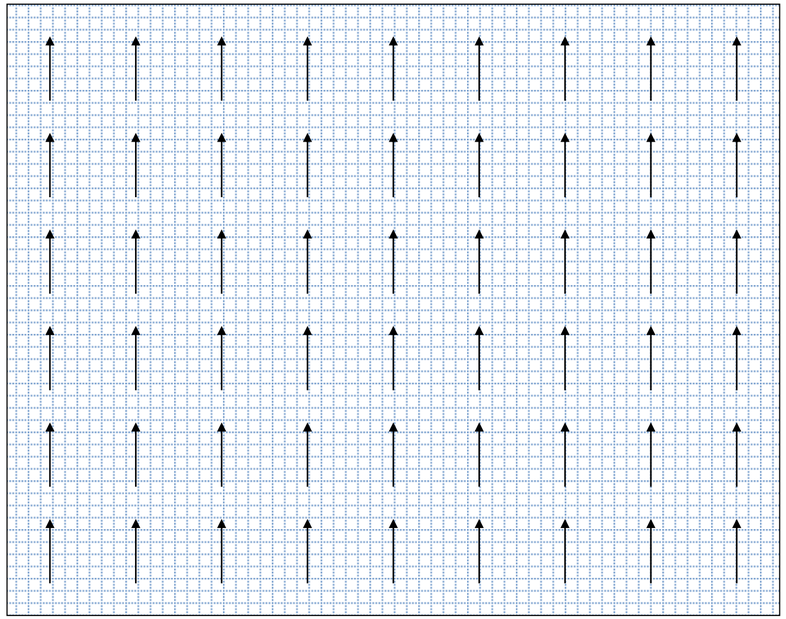
5.3 Detour: magnetism





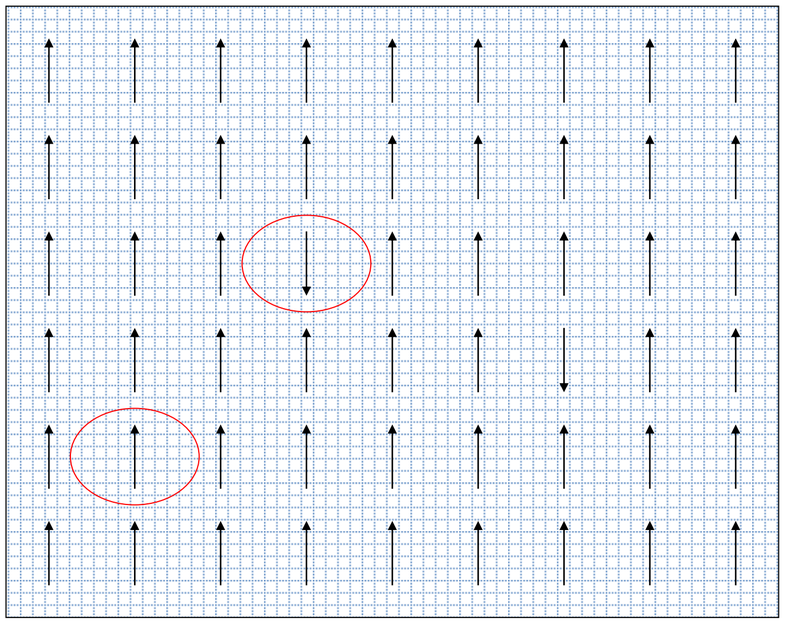
5.3 Detour: magnetism





Noisy magnet ⟶ pure magnet

5.3 Detour: magnetism



Elementary magnet

↑ $S_i = +1$

↓ $S_i = -1$

Blackboard: example

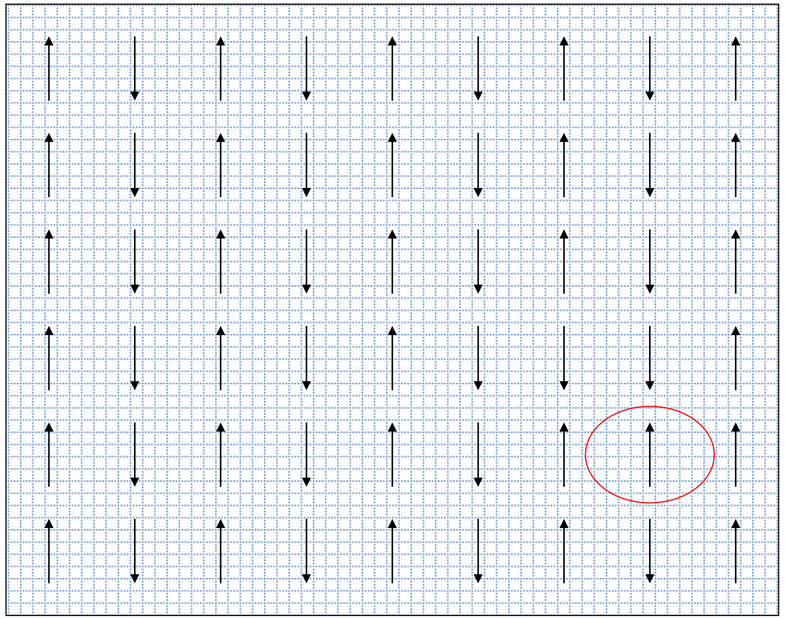
dynamics

$$S_i\,(t+1) = \mathrm{sgn}\left(\sum S_j\right)$$

Sum over all interactions with i

5.3 Detour: magnetism

Anti-ferromagnet



Elementary magnet

↑ $S_i = +1$

↓ $S_i = -1$

↑↑ $w_{ij} = +1$

↑↓ $w_{ij} = -1$

dynamics

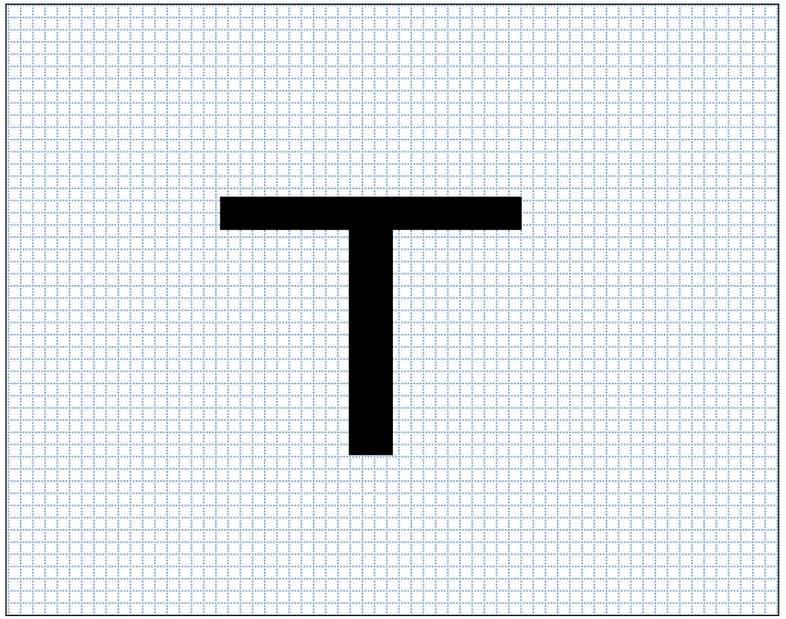
$$S_i(t+1) = \mathrm{sgn}\left(\sum_j w_{ij} S_j\right)$$

Sum over all interactions with i

blackboard

5.3 Magnetism and memory patterns

blackboard



Elementary pixel

- $S_i = +1$
- $S_i = -1$

$w_{ij} = +1$

$w_{ij} = +1$

$w_{ij} = -1$

dynamics

$$S_i\,(t+1) = \mathrm{sgn}\left(\sum_j w_{ij} S_j\right)$$

Sum over all interactions with i

Hopfield model:
Several patterns→ next section

Exercise 1: Associative memory (1 pattern)

Next lecture at
10h15

Elementary pixel

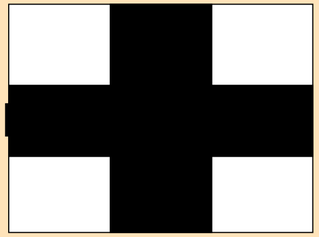
■ $S_i = +1$

□ $S_i = -1$

■↔■ $w_{ij} = +1$

□↔□ $w_{ij} = +1$

□ ■ $w_{ij} = -1$



dynamics

$$S_i\,(t+1) = \mathrm{sgn}\left(\sum_j w_{ij} S_j\right)$$

9 neurons
- define appropriate weights
- what happens if one neuron wrong?
- what happens if *n* neurons wrong?

Sum over all interactions with i

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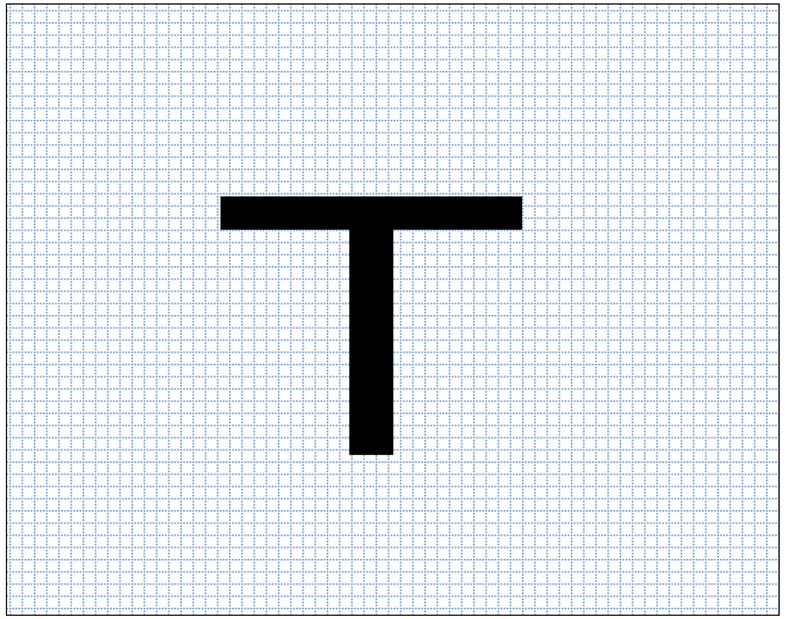
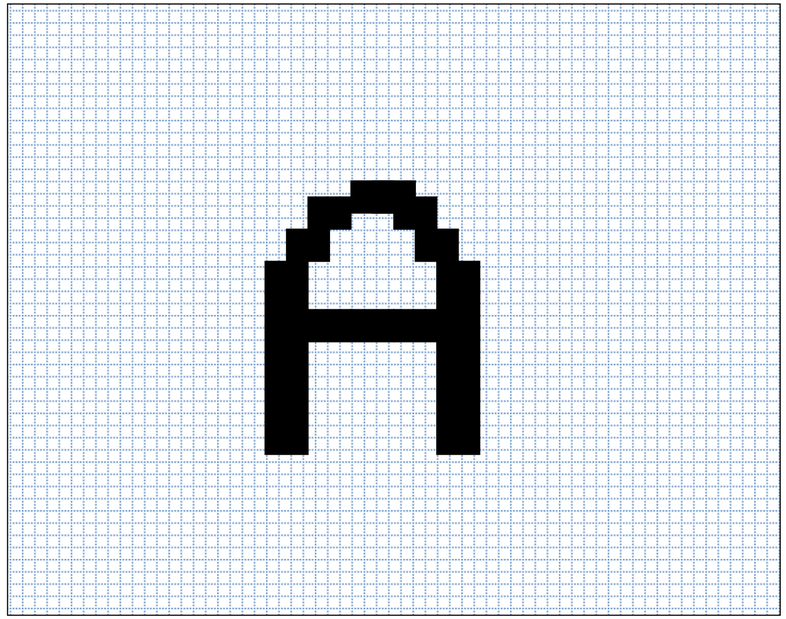
5.4 Hopfield Model of Associative Memory





Prototype $\vec{p}^1$

Prototype $\vec{p}^2$

Hopfield model

interactions

$$w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

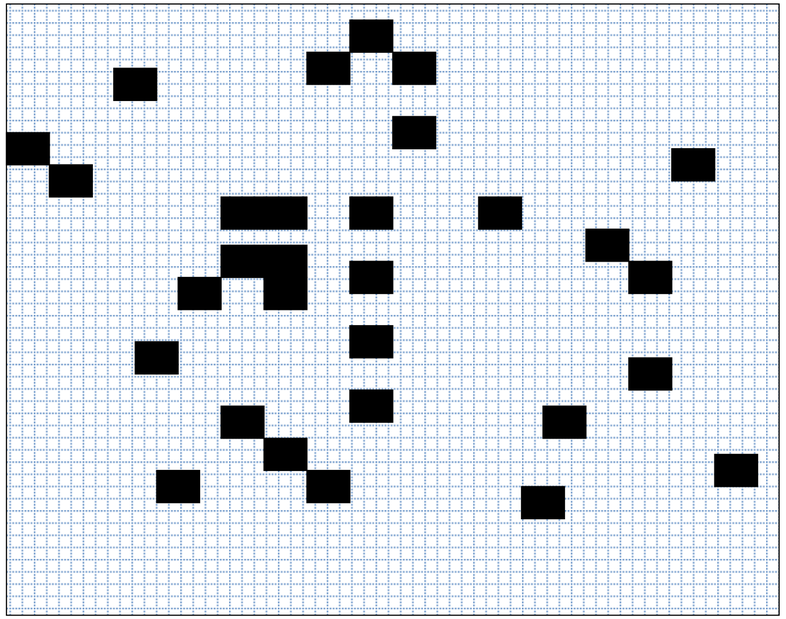
Sum over all prototypes

dynamics

$$S_i(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j\right]$$

Sum over all interactions with i

5.4 Hopfield Model of Associative Memory



Prototype

$\vec{p}^{1}$

interactions

$$w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

Sum over all prototypes

This rule is very good for **random** patterns

It does not work well for correlated patters

DEMO

dynamics

$$S_i(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j \quad t\right]$$

all interactions with i

Random patterns, fully connected:
Hopfield model

5.4 Hopfield Model of Associative Memory

$$S_i(t+1) = \operatorname{sgn}\left(\sum_j w_{ij} S_j\right)$$

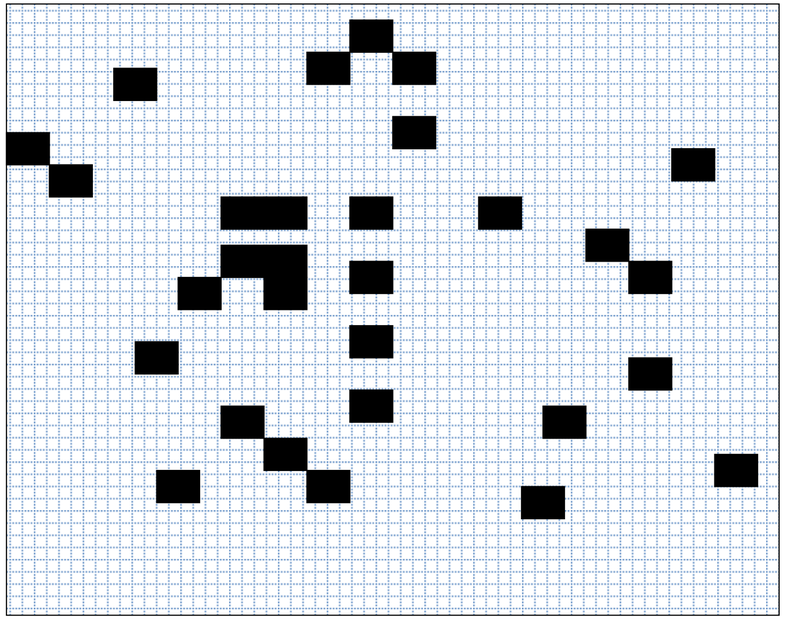
$$w_{ij} = \sum_\mu p_i^\mu p_j^\mu$$

Blackboard

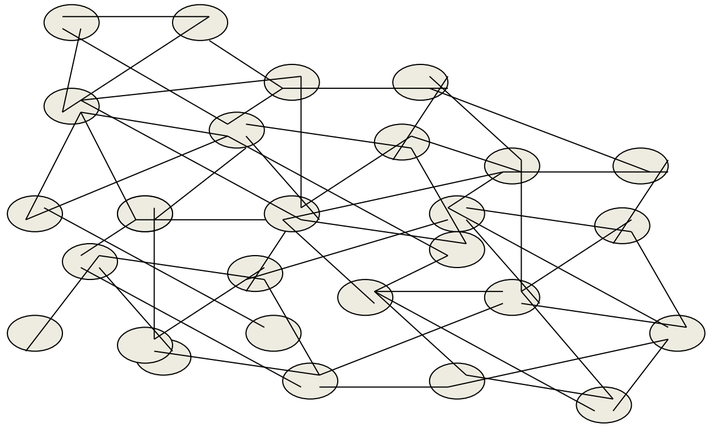
overlap $m^\mu(t) = \frac{1}{N}\sum_j \xi_j^\mu S_j \quad t$

$$m^\mu(t+1) = \frac{1}{N}\sum_j \xi_j^\mu S_j \quad t+1$$

5.4 Hopfield Model of Associative Memory



Interacting neurons



Prototype

$\vec{p}^{1}$

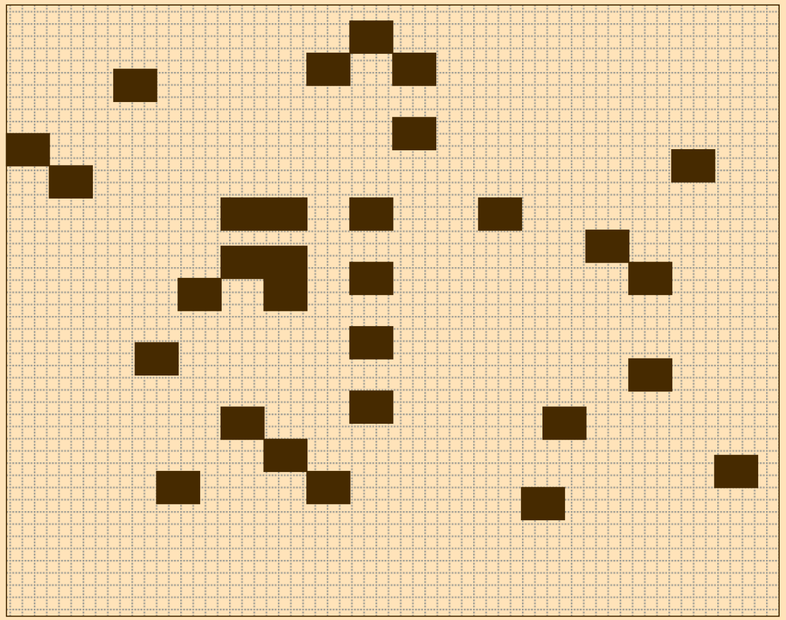
Finds the closest prototype
i.e. maximal overlap
(similarity) m^{μ}

Hopfield model

Computation

- without CPU,
- without explicit memory unit

Exercise 3 (now)



Prototype

$\vec{p}^{1}$

Next lecture at
11h15

$$w_{ij} = \frac{1}{N} \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

$$S_i\,(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j\right]$$

Sum over all
interactions with i

Assume 4 patterns. At time t=0, overlap with Pattern 3, no overlap with other patterns.
discuss temporal evolution (of overlaps)
(assume that patterns are orthogonal)

Week 5-5: Learning of Associations

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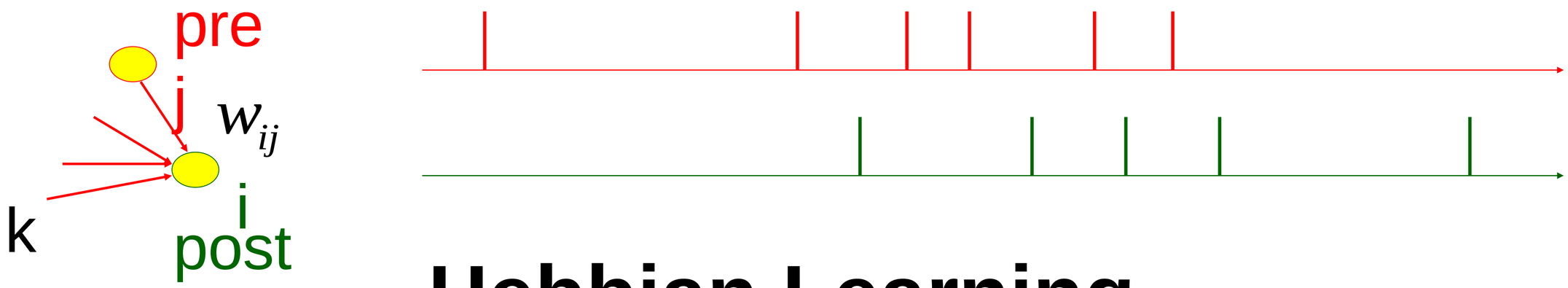
5.5 Learning of Associations

Where do the connections come from?



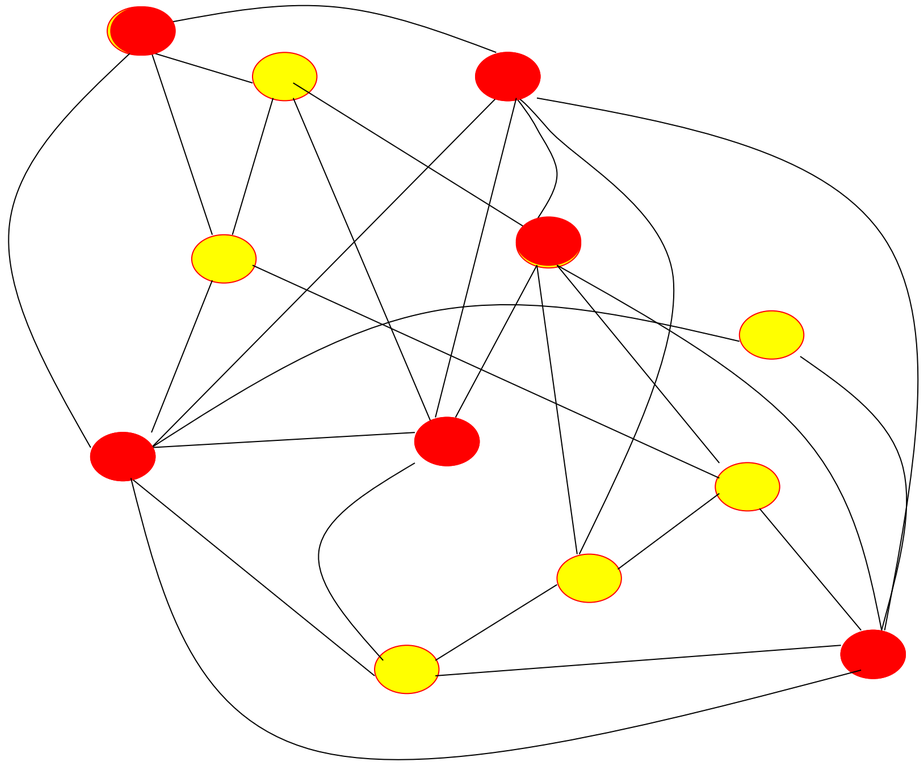
Hebbian Learning

When an axon of cell **j** repeatedly or persistently takes part in firing cell **i**, then j's efficiency as one of the cells firing i is increased

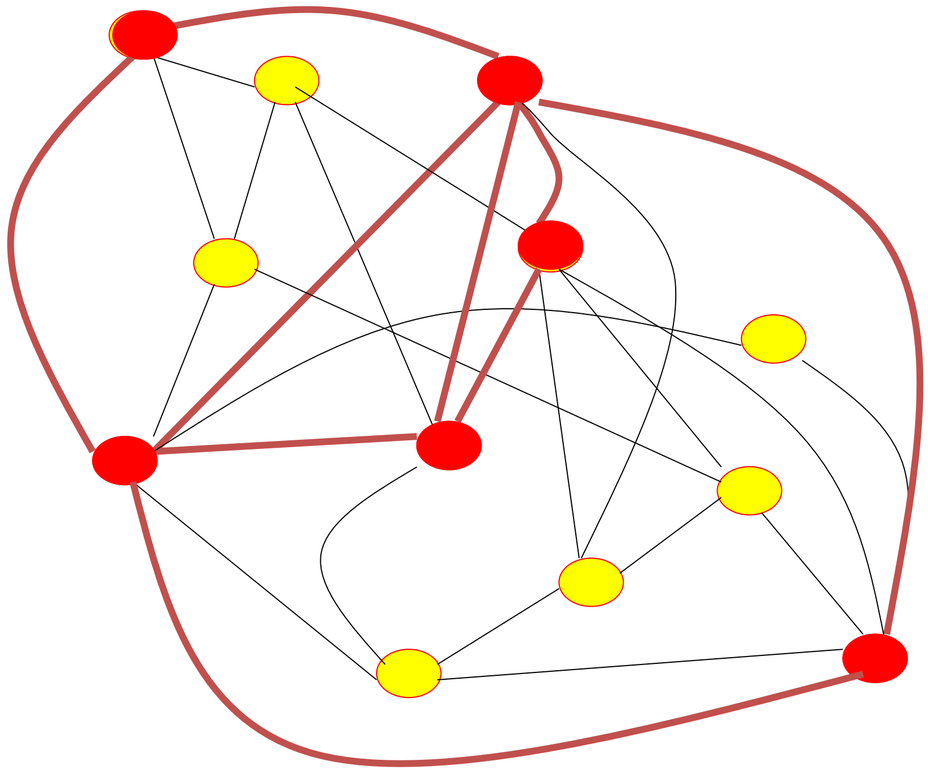
Hebb, 1949

- local rule
- simultaneously active (correlations)

5.5 Hebbian Learning of Associations



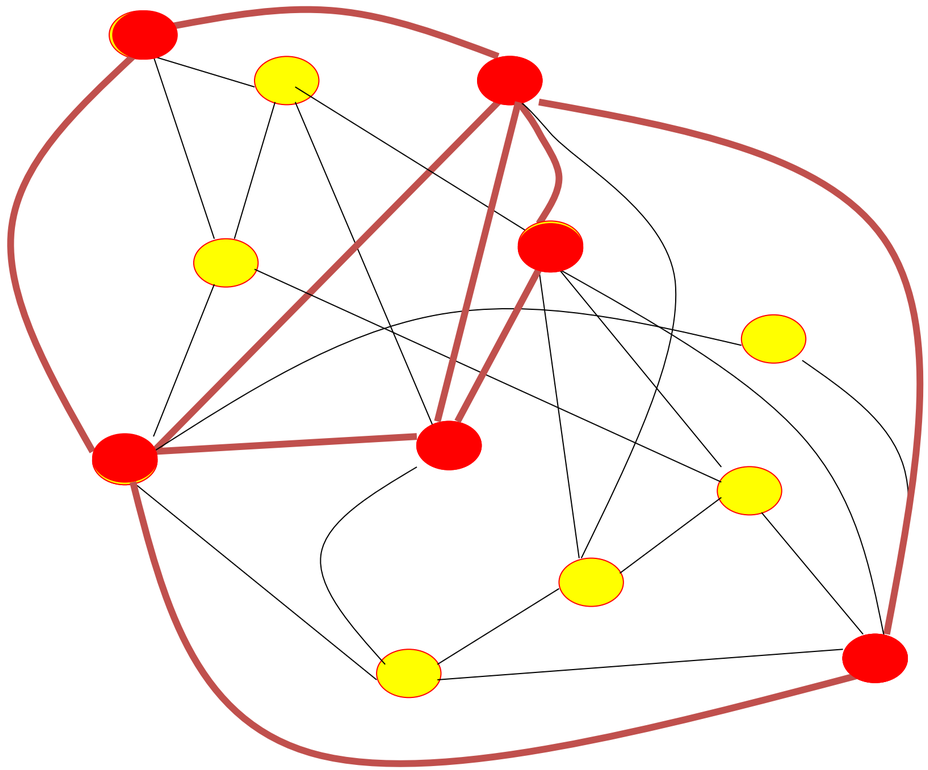
5.5 Hebbian Learning of Associations



item memorized

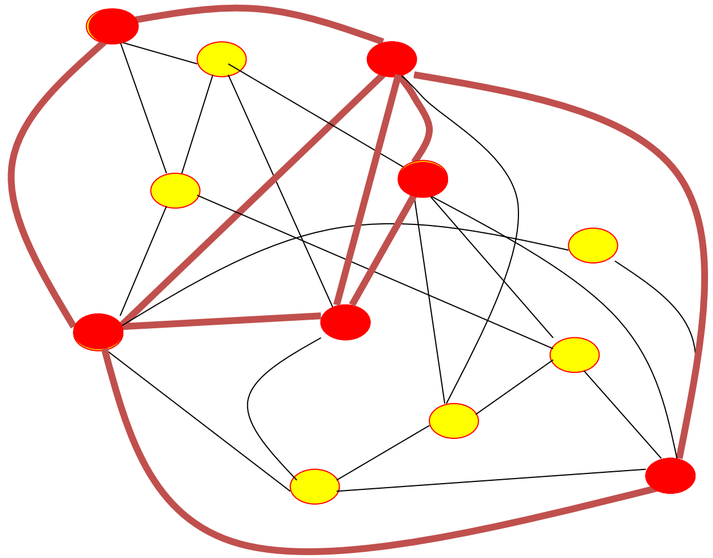
5.5 Hebbian Learning: Associative Recall

Recall:
Partial info

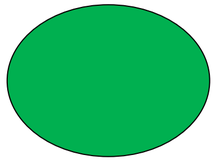


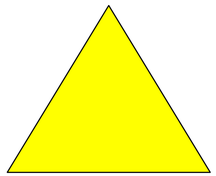
item recalled

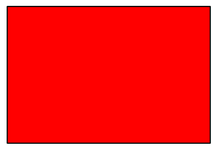
5.5 Associative Recall



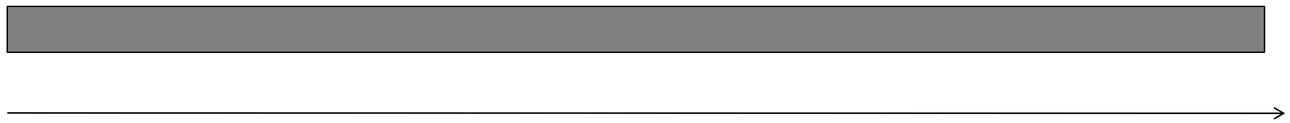
Tell me the ~~color~~ **shape**
for the following list of 5 items:





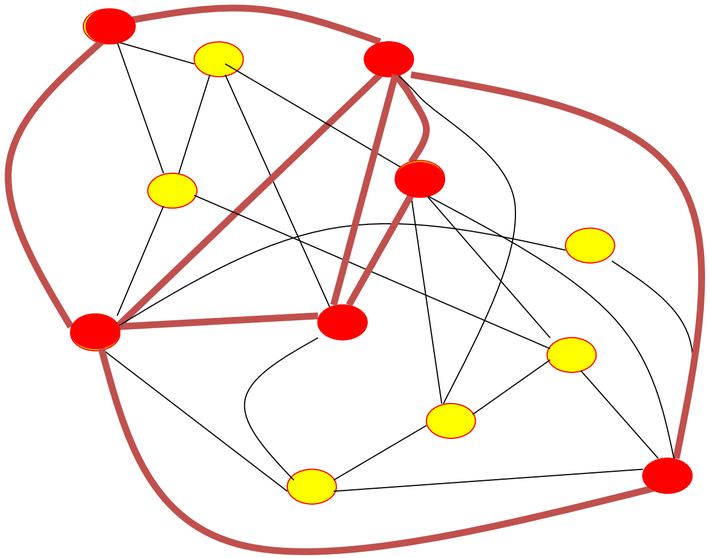


be as fast as possible:



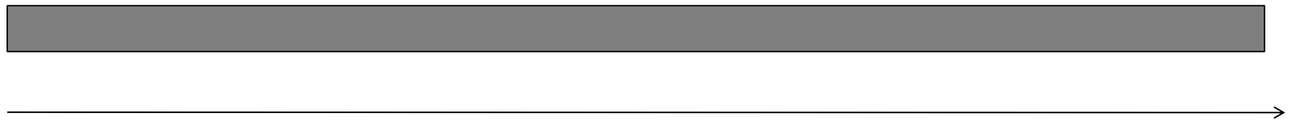
time

5.5 Associative Recall



Tell me the **color**
for the following list of 5 items:

be as fast as possible:



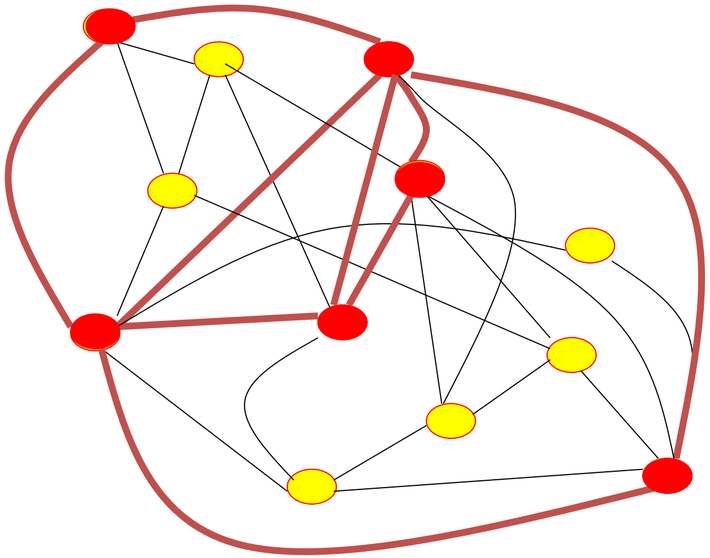
time

***Stroop effect**:*

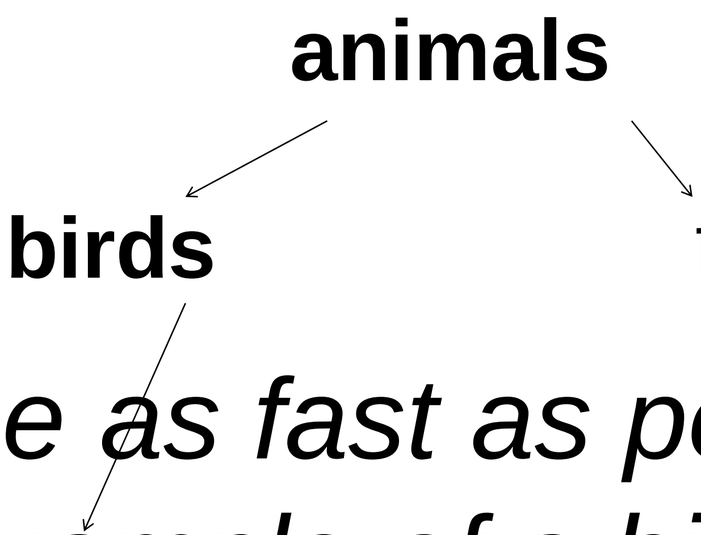
Slow response: hard to work

Against natural associations

5.5 Associative Recall



Hierarchical organization of
Associative memory



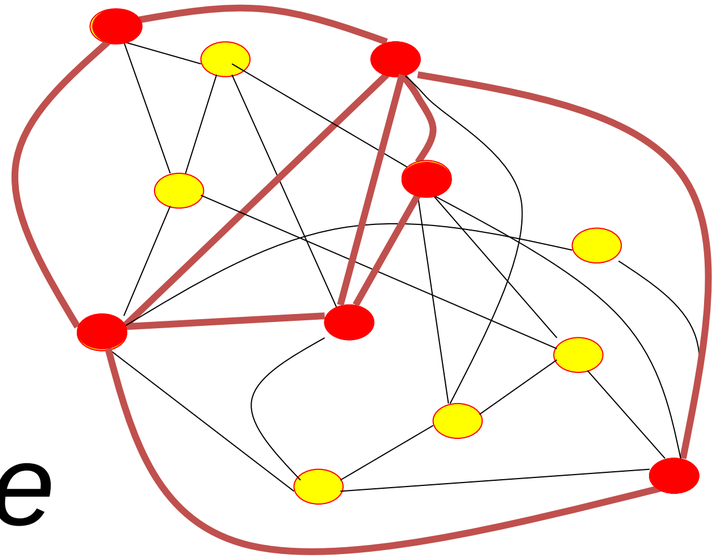
Name as fast as possible

an example of a bird

swan (or goose or raven or …)

Write down first letter: *s* for *swan* or *r* for *raven* …

5.5 Associative Recall



Nommez au plus vite possible un exemple d'un /d'une

name as fast as possible an example of a

outil **tool**

couleur **color**

fruit **fruit**

instrument de musique **music instrument**

Week 5-5: Learning of Associations

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Biological Modeling of Neural Networks

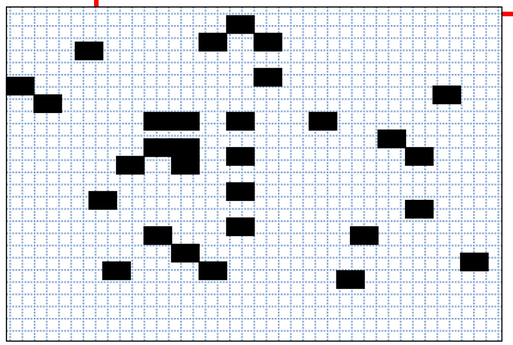
Week 5
NETWORKS of NEURONS and ASSOCIATIVE MEMORY

Wulfram Gerstner

EPFL, Lausanne, Switzerland

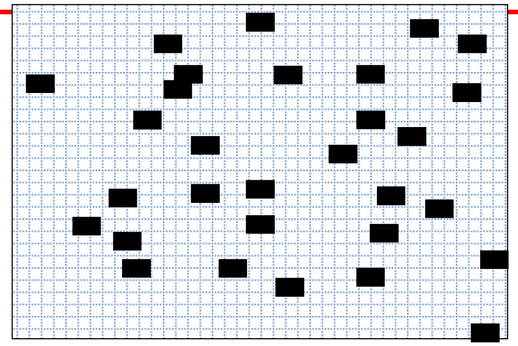
- √ 5.1 Introduction
  - networks of neuron
  - systems for computing
  - associative memory
- √ 5.2 Classification by similarity
- √ 5.3 Detour: Magnetic Materials
- √ 5.4 Hopfield Model
- √ 5.5 Learning of Associations
- 5.6 Storage Capacity

learning of prototypes



Prototype

$\vec{p}^{1}$



Prototype

$\vec{p}^{2}$

interactions

(1) $$w_{ij} = \frac{1}{N} \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

Sum over all prototypes

Q; How many prototypes can be stored?

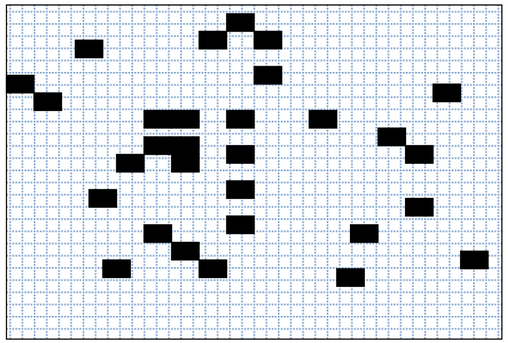
dynamics

$$S_i(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j \right]$$

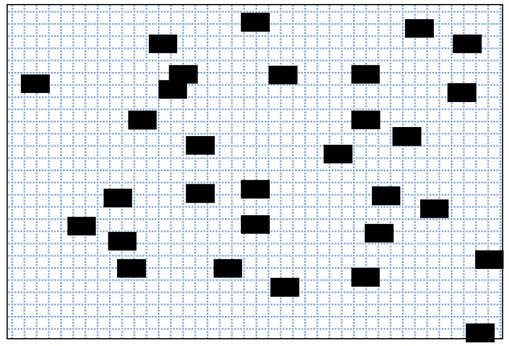
all interactions with i

Q; How many prototypes can be stored?

blackboard



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

Random patterns

Interactions (1) $$w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

Dynamics (2) $$S_i(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j\right]$$

Minimal condition: pattern is fixed point of dynamics

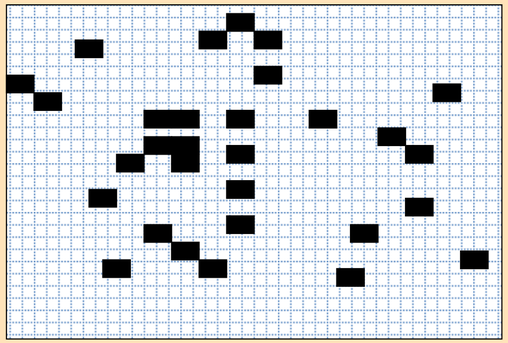
-Assume we start directly in one pattern

-Pattern stays

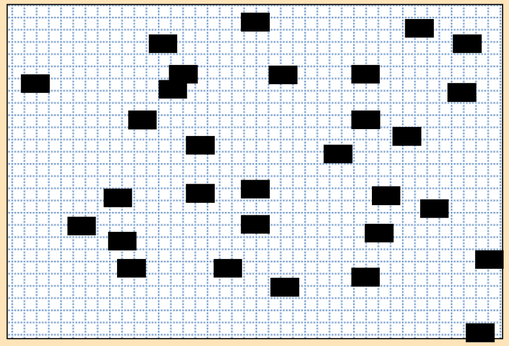
Attention: Retrieval requires more (pattern completion)

Exercise 4 now: Associative memory

Q; How many prototypes can be stored?



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

End of lecture, exercise+
Computer exercise : 12:00

Random patterns

Interactions (1) $w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$

Dynamics (2) $S_i(t+1) = \mathrm{sgn}\left(\sum_j w_{ij} S_j\right)$

Random patterns → random walk

a) show relation to erf function: importance of p/N

b) network of 1000 neurons – allow at most 1 wrong pixel?

c) network of N neurons – at most 1 promille wrong pixels?

The end