# Reminder on Signals and Systems

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# I. REMINDER ON SIGNALS AND SYSTEMS

## A. Signal basics

Any physically realisable real electrical signal $x(t)$ can be seen either as a voltage $v(t)$ or as a current $i(t)$, with **instantaneous power** : $p(t) = \frac{v^2(t)}{R} = i^2(t)R$. When studying communication systems, it is convenient to normalize the power with respect to the unit impedance $R = 1\ \Omega$ :

$$p(t) = x^2(t)\ . \tag{1}$$

The energy dissipated by this signal on the unit impedance during the time interval $[0, T)$, and its average power over this interval are :

$$E_x = \int_0^T x^2(t)\ dt,\ \text{[Joules]};\quad P_x = \frac{E_x}{T} = \frac{1}{T}\int_0^T x^2(t)\ dt,\ \text{[Joule/s]=[Watt]}\ . \tag{2}$$

Note, that :

— The **signal energy** at the receiver determines the quality of the reception : the signals with higher energy are easier detected with less errors.

— The **signal power** is the "rate" of the energy transfer through the channel, $P = E/T$ [Watt = Joule/s]. It defines the signal voltage applied at the transmitter.

## B. Energy signals and power signals

*1) Energy signals:* They have a finite non-zero energy computed over the infinite time period :

$$E_x = \lim_{T\to\infty}\int_0^T x^2(t)dt = \int_{-\infty}^{\infty} x^2(t)dt,\quad 0 < E_x < \infty. \tag{3}$$

The average power of the energy signal is zero : $P_x^E = \lim_{T\to\infty}\frac{E_x}{T} = 0$. Examples : i) non-periodic signals, single pulse ; ii) deterministic signals. More generally, **any real physical signal used to communicate (waveform) is the energy signal**.

*2) Energy spectral density:* Consider a non-periodic signal $x(t)$. The Parseval's relationship describes its energy in time and frequency domains :

$$E_x = \int_{-\infty}^{\infty} x^2(t)\ dt = \int_{-\infty}^{\infty} \underbrace{|X(f)|^2}_{ESD} df,\quad X(f) = \mathcal{F}\{x(t)\}. \tag{4}$$

The Energy Spectral Density (ESD) describes the distribution of the signal energy in frequency domain. For a typical communication waveform signal, the ESD is the squared magnitude $|X(f)|^2$ of its Fourier Transform (FT) $X(f)$, and the energy is the surface under the ESD :

$$\Phi^E(f) = |X(f)|^2\ ,\ \text{[Joules/Hz]}. \tag{5}$$

FIGURE 1: © *Signal's energy and ESD.*

*3) Power signals:* This term is used to describe the signals with infinite energy, $E_x = \infty$, but finite power :

$$P_x = \lim_{T\to\infty}\frac{1}{T}\int_0^T x^2(t)dt,\quad 0 < P_x < \infty. \tag{6}$$

Examples : i) periodic signals, of infinite duration ; pulse train ii) random signals with infinite energy, for instance, realizations of stochastic processes.

*4) Power spectral density (PSD):* Periodic signals have infinite energy, therefore, their Fourier transform can not be defined. They are analysed by using a Fourier series decomposition. Its coefficients describe the distribution of the signal power in frequency domain.

The average power of a real periodic signal with period $T$ can be computed through Parseval's relationship :

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_0^T x^2(t)dt = \sum_{n=-\infty}^{\infty} |c_n|^2 = 2\int_0^{\infty} G_x(f)\ df\ , \tag{7}$$

where

$$G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2\ \delta(f - nf_0),\ \text{[Watts/Hz]} \tag{8}$$

is this signal's PSD, that describes the power distribution with frequency. The PSD of a periodic signal is a positive real even function of frequency $G_x(f) = G_x(-f)$, which explains the factor of 2.

For non-periodic signals, such as stochastic processes, the Fourier transform may not exist, again, due to their infinite energy. To analyse such signals, the approach consists to consider their truncated time versions, $x_T(t)$, of finite energy. The FT can be computed, and their PSD defined throught the limit :

$$G_x(f) = \lim_{T\to\infty} \frac{1}{T}|X_T(f)|^2,\ \text{[Watts/Hz]}\ . \tag{9}$$

### *C. Autocorrelation*

Autocorrelation is :

— a measure of the ressemblance of the signal to its time delayed version ;
— not a function of the absolute time, but of the time shift $\tau$ between the two versions of the signal ;
— $\tau$ is often called lag, or scanning parameter.

*a) Autocorrelation of the energy signals.:* For a real signal $x(t)$ it is defined as :

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)dt\ . \tag{10}$$

The following relationship holds : $R_x(\tau) \overset{FT}{\leftrightharpoons} \Phi^E(f)$, or

$$R_x(\tau) = \int_{-\infty}^{\infty} \Phi^E(f)e^{2\pi f\tau}df\ . \tag{11}$$

By combining this result with the expression for the ESD and using the Parseval's relationship, the following important result can be introduced :

$$R_x(0) = \int_{-\infty}^{\infty} x^2(t)dt = \int_{-\infty}^{\infty} |X(f)|^2\, df = \int_{-\infty}^{\infty} \Phi^E(f)\ df = E_x. \tag{12}$$

*b) Autocorrelation of the periodic signal (power signal).:* The time average can be computed over a single period $T$ (instead of using the limit as in the equation (7), if the signal is not periodic) :

$$R_x(\tau) = \frac{1}{T}\int_0^T x(t)x(t+\tau)\ dt\ . \tag{13}$$

Another example of the power signal (with infinite energy) is a stationary stochastic process, that may not have a closed form analytic expression $x(t)$.

For these two types of signals, their autocorrelation is defined as the inverse Fourier transform of their PSD : (9)

$$R_x(\tau) = \int_{-\infty}^{\infty} G(f)e^{j2\pi f\tau}\ df. \tag{14}$$

Thus, at zero time shift $\tau = 0$, the autocorrelation of the periodic signal is equal to its average power :

$$R_x(0) = \int_{-\infty}^{\infty} G(f)e^{j2\pi f\tau}df\Big|_{\tau=0} = \int_{-\infty}^{\infty} G(f)df = P_x \geq 0\ . \tag{15}$$

### D. Random signals

From the receiver point of view, every message is random, as it does not know which one is transmitted ; moreover, channel effects and receiver noise are random too. All those factors determine the random nature of the received signal.

Follows a reminder on probability. Some errors (eq 1.25d, see later).

### E. Random processes

The correlation between two phenomenæ or signals, describes how closely resemblant they are, in behaviour or appearance, how well they match. The autocorrelation is computed by comparing the signal to its time shifted version by $\tau$, it is therefore a function of this shift.

For a non-periodic power signal (infinite energy), such as random process, the autocorrelation is computed for a truncated version of a realization (or sample) of this random process, as :

$$R_X(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T X(t)X(t+\tau)dt. \tag{16}$$

Often, this is done by averaging the result over a sliding window.

The **autocorrelation conveys frequency information** about the random process or signal, and thus its relation through the FT to the PSD of the signal has a direct physical meaning.

— Indeed, a very slowly changing process is described by the low frequency content, and therefore, its autocorrelation function remains just slightly smaller then its maximum value $R_X(\tau = 0)$ for a longer time, that is there is a strong match between the time shifted versions with big time shifts $\tau$.
  — In the limit, a constant signal has its autocorrelation almost constant, and thus its PSD tends to Dirac
— On the opposite, rapidly varying signal with higher frequency content, is described by autocorrelation falling to zero for very small values of time shift $\tau$, since for such signal only slightly time shifted versions can be completely different.
  — In the limit, a very short time spike signal that tends to Dirac has its autocorrelation that tends to Dirac too, and its PSD is therefore almost constant. Ex : AWGN - White Gaussian Noise.

*a) Example. Random sequence of bipolar pulses (ex : NRZ).:* It has an autocorrelation in the form :

$$R_X(\tau) = \begin{cases} 1 - \frac{|\tau|}{T} \ , & \text{for } |\tau| \le T, \\ 0 \ , & \text{for } |\tau| > T. \end{cases} \tag{17}$$

The previous observation on autocorrelation describing the frequency content can be applied to such signal. Specifically, for shorter pulse durations (higher pulse rate, or frequency), the time correlation is shorter, and the bandwidth occupancy is higher then for shorter time pulses.

### F. Noise

Noise is due to many sources, both natural (galactic noise) and man-made (receiver electronics, motors). Some of these sources can be eliminated by communication system design—using specific filters, modulation. Yet, the thermal noise that is due to the random electron motion in the dissipative passive components of the receiver, and can not be suppressed. It can be described as additive noise, statistically characterized as Gaussian, by the central limit theorem.

*1) White noise:* The term white noise is appropriated, since the PSD of the thermal noise is constant over all positive and negative frequencies (same power per unit bandwidth), from DC to $\approx 10^{12}$ Hz. The following theoretical model is used, for its PSD :

$$G_n(f) = \frac{N_0}{2}, \ \text{Watt/Hz} \tag{18}$$

The autocorrelation function of white noise is therefor the Dirac delta function, weighted by $N_0/2$ (zero for $\tau \neq 0$), which means that any two noise samples are uncorrelated :

$$R_n(\tau) = \mathcal{F}^{-1}\{G_n(f)\} = \frac{N_0}{2}\delta(\tau). \tag{19}$$

In this theoretical model, the average power of white noise is infinite, since its BW is infinite. In practical systems, the effect of this noise can only be observed at the output of the receiver finite bandwidth filter, and since $BW_{noise} \gg BW_{signal}$, the noise still can be considered as having infinite bandwidth compared to that of the signal.

The thermal noise is a Gaussian process, therefore, since the samples are uncorrelated, they are also independent. Such *Additive White Gaussian Noise (AWGN)* channel is memoryless, and affects all the data symbols independently.

## *G. Linear Systems*

Systems (channels) can be characterized either in time or in frequency domain. We are interested in the effect the system produces both on signals and noise, and more specifically, in characterizing the time or frequency response of the system to some arbitrary signal.

We consider Linear Time-Invariant (LTI) systems.

*1) Impulse Response:* When a unit impulse function $\delta(t)$ (theoretical, physically non-realizable signal) is applied at the input of the LTI system, the output signal $y(t)$ is equal to the system impulse response $h(t)$ to this input. Thus $h(t)$ is a time-domain characteristic of the system.

For some arbitrary signal $x(t)$, the output of the LTI system is related to its input by convolution operation :

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \tag{20}$$

*2) Frequency transfer function:* It can be obtained by taking the Fourier transform of the equation (20) :

$$Y(f) = H(f)X(f), \tag{21}$$

where

$$H(f) = \mathcal{F}\{h(t)\} = |H(f)|e^{j\theta(f)} \tag{22}$$

is the frequency transfer function (or frequency response) of the system, which in general is complex. It can be measured by sounding the system with a sinusoidal signal by varying its frequency.

*a) Random processes and linear systems:* If the input of the LTI system is a random process, the output is also a random process. The input and the output PSD are linked by important relationship :

$$G_Y(f) = G_X(f) \cdot |H(f)|^2 \tag{23}$$

In particular, an important property of Gaussian process is that if it is presented at the input of the LTI system, the output is also Gaussian.

*3) Distortionless transmission:* In ideal transmission line, the only 2 possible effects on the input signal are amplitude scaling and delay :

$$y(t) = Kx(t-t_0); \quad K, t_0 = const\ . \tag{24}$$

Taking the FT, and dividing by $X(f)$ (it is supposed that $X(f) \neq 0, \forall \boldsymbol{f}$, over infinite bandwidth), we can obtain the frequency response of such ideal system, of constant magnitude $K$, and phase shift linear in frequency :

$$H(f) = \frac{Y(f)}{X(f)} = Ke^{-j2\pi f t_0} \tag{25}$$

The interpretation of linear phase shift requirement is as follows : all frequency components must have identical delay at the system output, independent on frequency $f$. Consider the following common measure of delay distortion called *envelope or group delay* :

$$\tau(f) = -\frac{1}{2\pi}\frac{d\theta(f)}{df} \tag{26}$$

From this expression, it can be easily observed that if the phase $\theta(f)$ is linear in $f$, as for example, in equation (25), the envelope delay is constant independent on frequency, here $\tau(f) = t_0$.

*a) Ideal filter:* The ideal system described by equation (25) can not be physically realized, since it requires infinite bandwidth. Consider a system with a truncated double sided frequency response with constant magnitude, contained between the lower and upper cutoff frequencies $f_\ell$ and $f_u$, having the filter bandwidth $W_f = (f_u - f_\ell)$ Hz. Outside this system's passband $f_\ell < |f| < f_u$, the ideal filter response magnitude is zero. Depending on the values of cutoff frequencies, the filter can be bandpass (BPF), lowpass (LPF) or highpass (HPF).

Example : ideal LPF[1], $f_\ell = 0$. It can be shown, that the impulse response of such a filter (for simplicity $|H(f)| = K = 1$, and thus $H(f) = e^{-j2\pi f t_0}$), is equal to :

$$\begin{aligned} h(t) = \mathcal{F}^{-1}\{H(f)\} &= \int_{-\infty}^{\infty} H(f) e^{j2\pi f t} df \\ &= \int_{-f_u}^{f_u} e^{j2\pi f(t-t_0)} df = 2f_u \text{ sinc } 2f_u(t-t_0) \end{aligned} \tag{27}$$

This impulse response centered at $t_0 \geq 0$ is non-causal, that is, delivers a non-zero output signal even before the input signal is applied at $t = 0$, and thus can not be realized in practice.

*b) Effect of ideal filter on white noise:* The PSD of the noise at the output is :

$$G_{nf}(f) = G_n(f) \cdot |H(f)|^2 = \begin{cases} \frac{N_0}{2}, & \text{for } |f| < f_u, \\ 0\ , & \text{elsewhere .} \end{cases} \tag{28}$$

By using inverse Fourier transform, the autocorrelation of noise passed through the ideal LPF is :

$$R_{nf}(\tau) = \mathcal{F}^{-1}\{G_{nf}(f)\} = N_0 f_u \text{ sinc } 2f_u\tau. \tag{29}$$

**Thus, the filtered noise is not white any more**, since its autocorrelation is no more a weighted delta function $\delta(\tau)$, but rather an infinite support $sinc$ function. Still, with the appropriate sampling this can be leveraged. Specifically, the noise samples taken at time shifts $\tau = \frac{n}{2f_u}, n \in \mathbb{Z}$, are uncorrelated. The underlying condition is perfect syncrhonisation, ensuring that the sampling is done at these exact time instants.

*c) Realizable filters:* A classical example of the realizable low-pass filter is RC-filter, its transfer function is :

$$H(f) = \frac{Y(f)}{X(f)} = \frac{1}{1 + j2\pi fRC} = \frac{1}{\sqrt{1 + (2\pi fRC)^2}} e^{-j\arctan(2\pi fRC)} \tag{30}$$

The bandwidth of the LPF is defined by its half-power cutoff frequency[2], where the output power is half of its maximal power ($-3$ dB), or equivalently, the output voltage magnitude is $1/\sqrt{2} = 0.707$ of the maximal. For RC LPF, the half-power point and thus the bandwidth is at $f = 1/(2\pi RC) = W_{LPF}$, Hz.

The filter **shape factor** defines how closely it approximates the ideal filter ; it is defined as the ratio of the filter bandwidths at $-60$ dB and $-6$ dB amplitude response points, with respect to its maximum (0 dB wrt itself). To take an example, it is equal to 600 for a simple RC filter, and can be as small as 2 for very sharp cutoff filters. Butterworth filters of order $n$ are frequent choice.

*d) Effect of an RC filter on white noise.:* The PSD of the noise at the output is :

$$G_{nf}(f) = G_n(f) \cdot |H(f)|^2 = \frac{N_0}{2} \frac{1}{1 + (2\pi fRC)^2} \tag{31}$$

By using inverse Fourier transform, the autocorrelation of noise passed through the RC LPF is :

$$R_{nf}(\tau) = FT^{-1}\{G_{nf}(f)\} = \frac{N_0}{4RC} e^{-\frac{|\tau|}{RC}} \tag{32}$$

As previously observed for ideal filter, **the filtered noise is not white any more**, since its autocorrelation is infinite support exponential function. Note, that for a narrowband filter (large $RC$), the autocorrelation is stronger for a given time shift, than for a wideband filter.

1. HW : make the same calculations for HPF and BPF
2. This is one of many possible definitions of system BW, more on this later.

### H. *Signals, circuits and spectra*

An important question to be addressed now is : how the system or the channel frequency transfer function affects the spectrum content of the signal which passes through it ?

Since filtering in frequency domain is a multiplication of the signal spectrum by the system (filter) frequency response, it becomes clear that when the input signal bandwidth is bigger than the system bandwidth, the output signal is seen as a distorted (filtered) variant of the input signal.

Equivalently, the effect of a filter on the transmitted waveform can be studied in time domain.

Example [p.44 Sklar] discusses the transmission of an ideal pulse with bandwidth $W_p = 1/T$ through RC filter with half-power bandwidth $W_f = 1/(2\pi RC)$. One can distinguish 3 cases :

— $W_p \ll W_f$, the filter has almost no effect on signal frequency content, and the output signal $y(t)$ fairly ressembles to the input signal $x(t)$, with good fidelity. In time domain, it could be seen as transmission of a pulse of much longer duration than the impulse response of the channel, which results in only a slight alteration of the pulse amplitude and a small stretch of pulse duration. This is due to the nature of time domain convolution—the resulting signal is the superposition of time shifted replicas of the same input pulse, each multiplied by the impulse response value at the corresponding time shift.
Application example : radar ranging, where the pulse time shift relates to the distance to the target, and thus, a very steep front received pulse is desired.

— $W_p \approx W_f$. The output signal is altered but still recognizable. This is suitable for binary digital communications, since the objective there is to determine in which of two states is the received pulse, not its exact shape.

— $W_p \gg W_f$. Useless for communication systems.

### I. *Digital bandwidth measures*

*1) Baseband vs carrier modulation:* Consider a baseband signal with bandwidth $f_m$. The simplest modulation is done by a **heterodyne** by multiplying the baseband signal by the carrier (or mixing) signal $2\pi f_c t$, so that $f_c \gg f_m$. The result is the dual sideband (DSB) signal, with its spectrum obtained from the spectrum of the original signal by transposing and mirroring it around $\pm f_c$, and scaling by a factor $1/2$, since the total signal power is constant (surface under the PSD).

The important consequence of carrier modulation is that the bandwidth of the DSB signal required is twice the bandwidth of the baseband signal : $W_{DSB} \approx 2f_m$ (see Fig. 2).

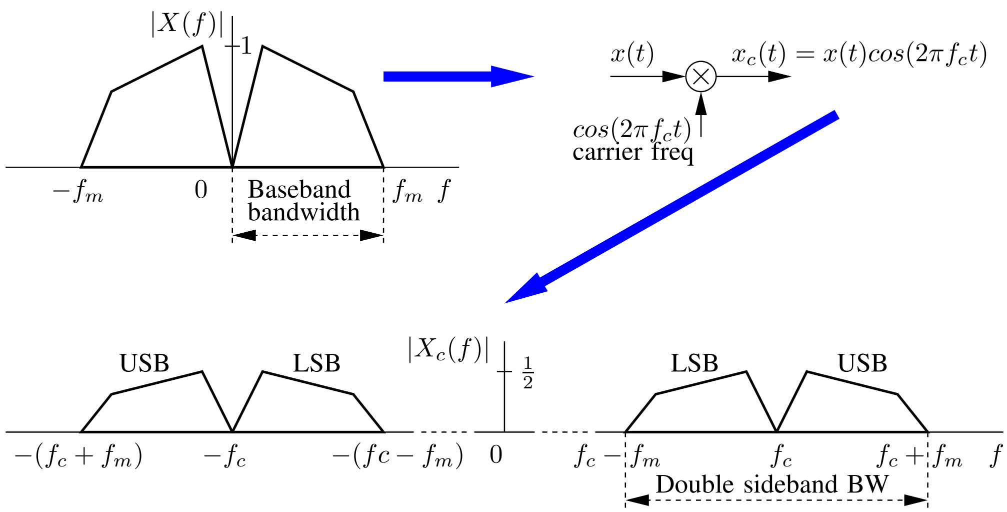


FIGURE 2: © *[Sklar]*

*2) The bandwidth dilemma:* It can be resumed as follows :

— The infinite duration signals that have finite bandwidth or bandlimited, are not physically realizable. Ex : ideal LPF, that is bandlimited, but has infinite and non-causal impulse response.

— The finite duration signals can be realized, but have infinite bandwidth.

Thus, the **mathematical abstract model of a signal can not be strictly duration and band limited**. Hence, considering the second case of realizable signals, any bandwidth criterion aims at defining a measure of the width $W$ of the infinite spectrum density of such signal. Moreover, many theorems in communication theory apply to bandlimited signals.

The most common definitions of bandwidth are now provided [Sklar]. The Fig. 3 presents the general form and expression for the single-sided spectral density $G_x(f)$ for a single modulated (heterodyned) pulse $x_c(t)$, and is also true for a random pulse sequence[3].

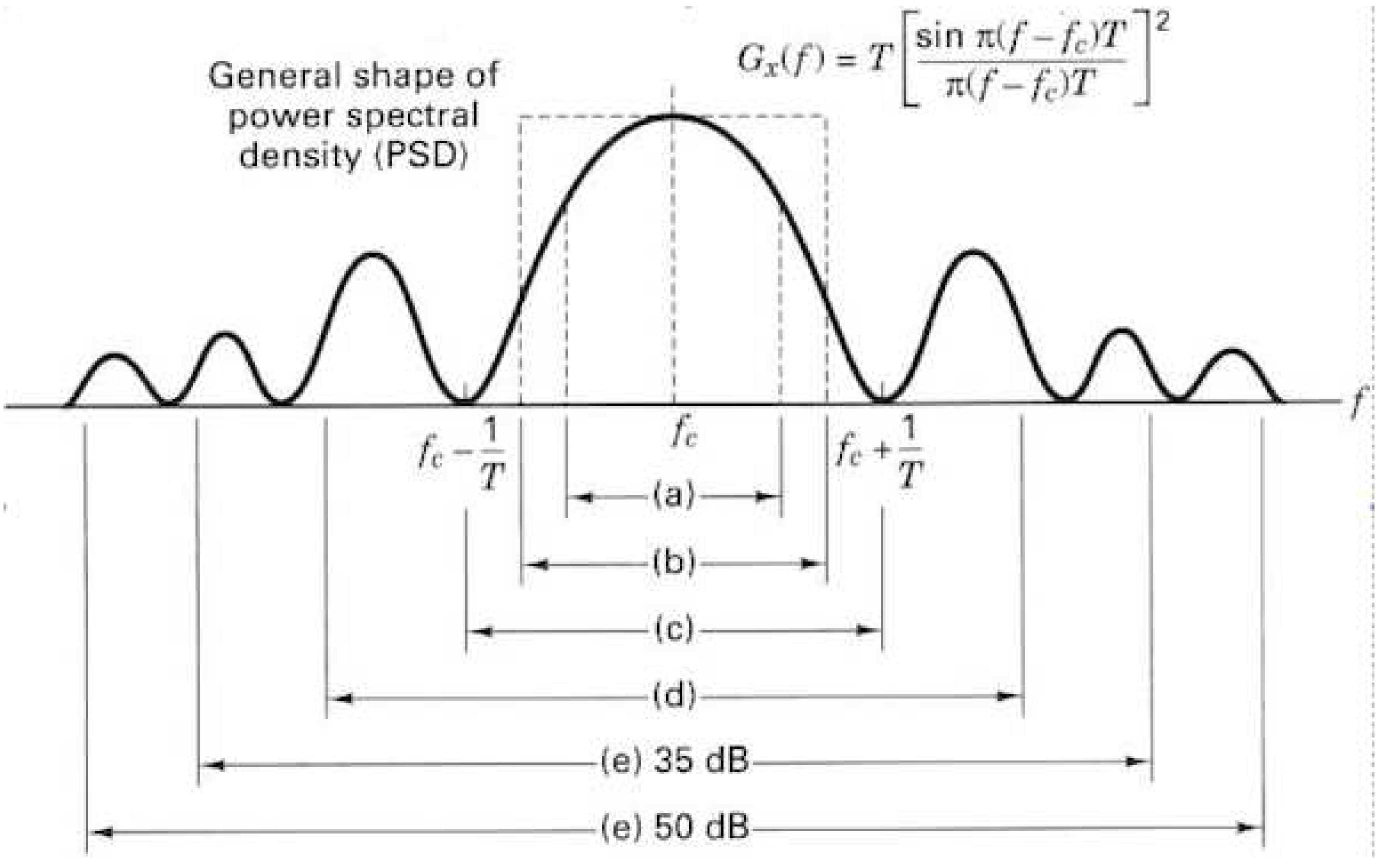


**Figure 1.20** Bandwidth of digital data. (a) Half-power. (b) Noise equivalent. (c) Null to null. (d) 99% of power. (e) Bounded PSD (defines attenuation outside bandwidth) at 35 and 50 dB.

FIGURE 3: © *[Sklar]*

Comments :

a) Half-power with respect to the maximum of PSD at $f_c$.

b) Rectangular bandwidth or noise equivalent BW of the spectral window. Originally, it was defined to compute the power of the noise $P_n = G_n(f)W_n$, that would have been produced at the output of the amplifier, with the wideband input noise with known constant PSD $G_n(f)$ applied at its input, if the amplifier's ideal filter (rectangular) BW was set to $W_n$.

   In bandwidth definition context, the signal power $P_x$ is known. Thus, $W_n$ is interpreted as the bandwidth of the receiver filter, that would have produced at its output the same power $P_x$ as contained in the modulated signal pulse, if the wideband noise with the constant PSD equal to the maximum of the signal's PSD $G_x(f_c)$ at $f_c$, was applied at its input[4].

3. This general shape is valid for many modulation formats, however, some of them do not have well-defined spectral lobes.
4. In other terms, $W_n$ is the width of the rectangle with the area equal to $P_x$, and the height equal to $G_x(f_c)$.

The noise equivalent BW is defined as :

$$W_n = \frac{P_x}{G_x(f_c)} = \frac{\int_{-\infty}^{\infty} G_x(f)df}{G_x(f_c)}, \tag{33}$$

c) **Most used, width of the main spectral lobe $2/T$, containing $\approx 90$ % of signal power**, also named nul-to-nul bandwidth.
d) Fractional power containment BW : 99% of power.
e) Bounded PSD : bw beyond which the PSD attenuation compared to the maximum $G_x(f_c)$ is more then some level : 35 dB, 50 dB.

Any of the definitions of the signal bandwidth intrinsically determines the **degree of accuracy** that is, the percentage of the signal's average power preserved after the bandwidth limitation, and the **distortion** introduced into the signal due to the cutoff of the higher spectral lobes.

**NB : The need to define the signal bandwidth and thus to limit its spectral content is determined by two factors : physical realizability of the signal and band-limitedness of the communication channels which is due to the system design (realizable filters) or regulatory constraints.**

Suppose that the source produces the binary output at the rate $R_b = 1/T_b$ [bits/s]. After the modulation (either line coding or passband), the BW of the transmitted signal depends on the modulation parameter—the pulse duration $\tau$, with respect to the bit interval $T_b$. Shorter pulses require higher bandwidth, and the resulting BW expansion factor is $\eta_B = T_b/\tau = B/R_b \geq 1$. Therefore, the signal BW relative to the source rate is $B = (T_b/\tau)R_b = \eta_B R_b$.

Now, let us see the inverse problem. Suppose a bandlimited channel[5] with a $BW = 1/\tau$, that is the shortest pulse duration is $\tau$. Therefore, if the line coding is used with $\tau < T_b$, the bit duration has to be increased. For instance, if $T_b/\tau = \eta_B = 2$, as for the RZ code, the bandwidth expansion factor is $\eta_B = 2$. As a consequence of such system design, the useful bit rate is decreased by this same factor : $R_b = B/\eta_b = B/2$. The source has to adapt its rate to be able to transmit over the bandlimited channel without distortion.

*a) Ex : Accuracy—bandwidth tradeoff.:* Consider a rectangular pulse with finite energy, and fixed duration $\tau$. Suppose that this signal is transmitted over a bandlimited channel with $B = 1/\tau = 10000$ kHz, defined by its first spectral lobe, which is equivalent to 90% accuracy for the signal. Now, suppose that the accurancy requirement has been set so as to preserve 98% of the signal's average power within the bandwidth of the channel. The required BW expansion factor is $\eta_B = 5$. Therefore, the maximum possible binary rate is $R_{b,max} = B/\eta_b = 2000$ kHz. That is, the pulse duration $\tau$ is unchanged, but the bit interval had to be increased by a factor $\eta_B$, so that $T_b = 5\tau$, and the spectral content of the resulting data signal is shrinked, with the first lobe at $1/T_b = R_{b,max}$.

In conclusion, it can be said that the definition of the signal bandwidth via approximation is dictated by the application, since realistic finite length signals have infinite spectral content.

### *J. Reminder on data units*

- Analog source, waveform : any physical signal. Ex : sensor output.
- Discret source : finite alphabet. Ex : $A, ..., Z$.
- Message : character, text : "A", "HELLO, WORLD !". Can be encoded into a binary word. Ex : ASCII, $[01100001] = [A]_{ASCII}$.
- Bit-BInary digiT, $\{0, 1\}$.
- Bit stream. [...0010110010000101...]. Information rate of a bit stream is $R_b = 1/T_b$ [bit/s]. This is the result of either analog or digital source digitization.
  - $T_b$ is the **bit interval** between the successive pulses, and is not always equal to the
  - **pulse duration** $\tau$. Typically : $\tau/T_b = 1, 1/2, 1/4, \ldots$. $\tau_{max} = T_b$ to avoid the successive pulse overlapping. The shorter the pulse (smaller ratios $\tau/T_b$), the more bandwidth is required to transmit the data, while preserving the rate $R_b$.
- Baseband digital message (symbol) of $k$ bits : $m_i = [0110]_{k=4}$. Grouping is arbitrary, not related to the source.
- Baseband symbol is mapped to waveform $m_i \Leftrightarrow g_i(t)$, line coding, typically baseband signal.
- Symbol in RF or carrier digital modulation $s_i(t)$. Several bits are mapped onto it.

5. Most communication systems have bandlimited channels !

  - — Channel (or symbol, or pulse) rate $R_s = 1/T_s = 1/(kT_b) = R_b/k$ [Baud] = [symbols/s].
- — Digital waveform $g_i(t)$ or $s_i(t)$ parameters
  - — Baseband : shape, amplitude, width, duration of pulses.
  - — Carrier : amplitude $A$, frequency $f$, phase $\varphi$ for the carrier $A\sin(2\pi f_c t + \varphi)$.

Bit stream pulses :

- — If the pulse duration is $< T_b$ (ex : Manchester line coding), this would $\nearrow$ bandwidth, required to transmit the data.
- — Or, if channel BW is limited to $1/T$, shorter data pulses would require wider bandwidth. Thus since the data pulse duration should be $T_b \geq T$.

### K. *Notations*

- — $H(X)$ entropy of the source
- — $R$ [bits/s] rate. Different notations for :
  - — $R$ source coding rate, or uncoded data rate.
  - — $R_c$ coded data rate. If directly transmitted becomes raw physical channel bitrate.
  - — $R = \frac{k}{n} R_c$ for block codes, where $\frac{k}{n}$ is the code rate. NB : in some sources the code the latter is denoted $R_c$, confusion !