Numerical Analysis - HW3

1. (a) Obtain a Gaussian quadrature rule for approximating $\int_\Omega f(x,y)dxdy$ where Ω is the square $[-1,1]\times[-1,1]$ using the 3-point Gaussian quadrature formula in each dimension. Verify what the highest degree of polynomial is that your rule integrates exactly. Use your rule to integrate

$$\int_0^{1/2}\int_{1/4}^{1/2} e^{xy}dxdy$$

   (b) Approximate the above integral using Monte Carlo method.

2. Write a code for the Romberg integration to approximate the following integral without using built in integrations.

$$\int_0^5 \frac{dx}{1+(x-\pi)^2}\ dx = tan^{-1}(5-\pi)+tan^{-1}(\pi)$$

   Evaluate the approximation $R(n,m)$ to fill in the table. Check the convergence rate and compare with the theory.

(n,m)		2	$\cdots$			7	
	approx.	error	rate	$\cdots$	approx.	error	rate
2				$\cdots$			
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
7				$\cdots$			

3. Use Midpoint rule and Trapezoid rule as a pair to implement adaptive quadrature rule to approximate

$$\int_0^\pi e^x cos(4x)\ dx$$

   with the tolerance of $\epsilon = 10^{-6}$.

   - Find the approximation of integral and the error.
   - Display the partial sum of results over all subintervals completed as follows: