Hazma Documentation

Release 1.2

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CHAPTER ONE

DESCRIPTION

1.1 Introduction

This package is used to compute the gamma ray spectra $\frac{dN}{dE}$ for light particles, such as, pions, kaon, electrons and muons, in an energy regime where the mass effects are important, i.e. is the MeV energy range. The code has been written in python/cython.

1.2 Decay spectra

In this section, we describe how the radiative decay spectra are computed for the muon, charged pion and neutral pion.

1.2.1 Muon

The dominant contribution to the radiative decay of the muon comes from $\mu^{\pm} \to e^{\pm}\nu\bar{\nu}\gamma$. The unpolarized differential branching fraction of this decay mode in the *muon rest frame* can be written as [1]

$$\frac{dB}{dy\, d\cos\theta_{\gamma}^{R}} = \frac{1}{y}\frac{\alpha}{72\pi}(1-y)\left[12\left(3-2y(1-y)^{2}\right)\log\left(\frac{1-y}{r}\right)+y(1-y)(46-55y)-102\right]$$

where $r=(m_e/m_\mu)^2$, $0 \leq y = 2E_{\gamma}^{R\mu}/m_\mu \leq 1-r$, ($E_{\gamma}^{R\mu}$ is the energy of the photon in the muon rest frame) and θ_{γ}^{R} is the angle the photon makes with respect to some axis in the muon rest frame. In order to obtain the decay spectrum in the laboratory frame, we need to boost the above spectrum. In other words, we need to change variables from the gamma ray energy and angle in the muon rest frame to those in the lab frame. We then integrate over the angle to compute dN/dE_γ. The Jacobian for this change of variables is

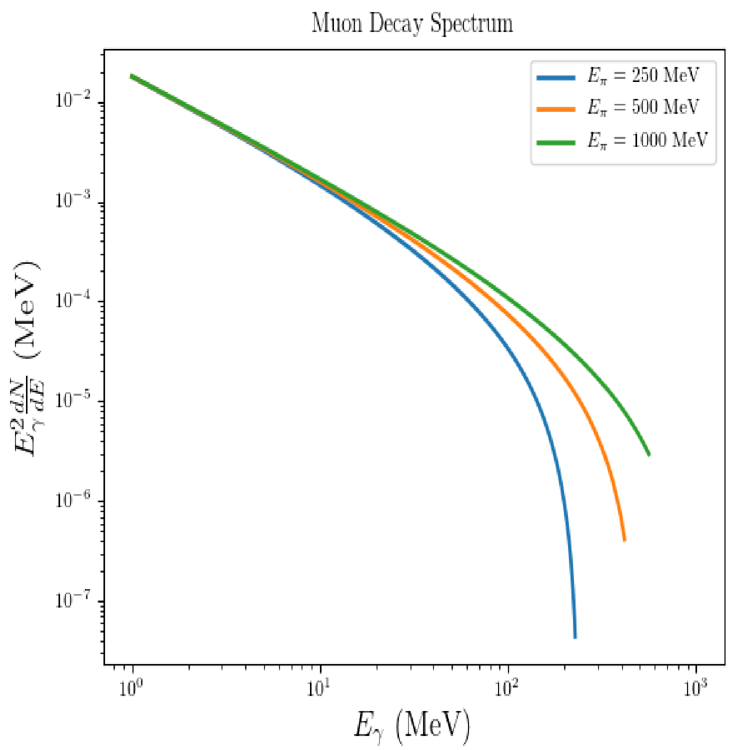
$$J = \frac{1}{2\gamma(1-\beta\cos\theta_{\gamma}^{L})}$$

where the boost parameters are

$$\gamma = E_\mu/m_\mu, \qquad \beta = \sqrt{1-\left(\frac{m_\mu}{E_\mu}\right)^{2}}$$

Integrating over angles yields the gamma ray spectrum in the lab frame:

$$\frac{dN}{dE_{\gamma}^{L}} = \int_{-1}^{1} d\cos\theta_{\gamma}^{L}\,\frac{1}{2\gamma(1-\beta\cos\theta_{\gamma}^{L})}\,\frac{dB}{dE_{\gamma}^{R\mu}}$$



1.2.2 Charged Pion

To compute the gamma ray spectrum from a charged pion, one considers to possible decay modes. These decay modes are $\pi^\pm \to \mu^\pm \nu_\mu \gamma$ and $\pi^\pm \to \mu^\pm \nu_\mu \to e^\pm \nu_\mu \nu_\mu \nu_e \gamma$. To compute the gamma ray spectrum from the first decay mode, one uses results from [2]. It turns out that the spectrum from this decay mode is roughly a factor of 100 times smaller than the spectrum from the second decay mode. We thus ignore the contributions from $\pi^\pm \to \mu^\pm \nu_\mu \gamma$.

To compute the γ-ray spectrum from $\pi^\pm \to \mu^\pm \nu_\mu \to e^\pm \nu_\mu \nu_\mu \nu_e \gamma$, we first take the muon decay spectra (see section on muon decay spectra) and boost the muon into the pion rest frame use the following:

$$\gamma_1 = E_{R\mu}/m_\mu \qquad \beta_1 = \sqrt{1 - \left(\frac{m_\mu}{E_{R\mu}}\right)^2} \qquad E_{R\mu} = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}$$

where $E_{R\mu}$ is the energy of the muon in the pion rest frame. The photon spectrum in the charged pion rest frame, $dN/dE_\gamma^{R\pi}$, is obtain by integrating the differential branching ratio times a Jacobian factor $1/2\gamma_1(1-\beta_1\cos\theta_\gamma^{R\pi})$ over the angle the photon makes with the muon. Once this integration is completed, one then boosts into the laboratory frame of reference. The steps are nearly identical to boosting from the muon rest frame to the pion rest frame. The only thing that changes in the boost factor and the Jacobian. In going from the charged pion rest frame to the laboratory frame, the Jacobian and boost factor are

$$J = \frac{1}{2\gamma_2(1-\beta_2\cos\theta_\gamma^L)} \qquad \gamma_2 = E_\pi/m_\pi \qquad \beta_2 = \sqrt{1 - \left(\frac{m_\mu}{E_\pi}\right)^2}$$

The gamma-ray spectrum in the laboratory frame will thus be

$$\frac{dN}{dE_\gamma^L} = \int_{-1}^{1} d\cos\theta_\gamma^L \frac{1}{2\gamma_2(1-\beta_2\cos\theta_\gamma^L)} \times \left(\int_{-1}^{1} d\cos\theta_\gamma^{R\pi} \frac{1}{2\gamma_1(1-\beta_1\cos\theta_\gamma^L)} \frac{dB}{dE_\gamma^{R\mu}}\right)$$

where

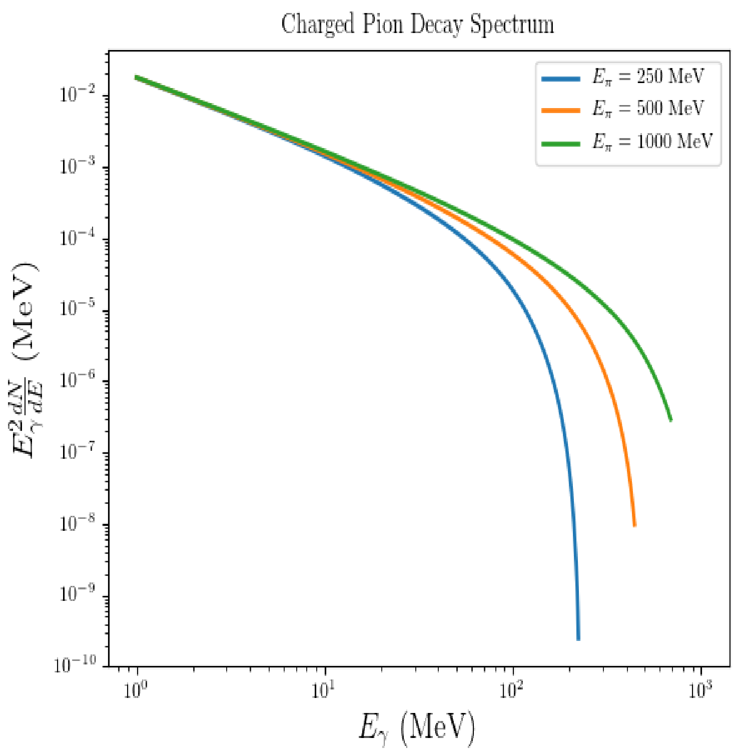
$$E_\gamma^{R\mu} = \gamma_1 E_\gamma^{R\pi}\left(1 - \beta_1 \cos\theta_\gamma^{R\pi}\right)$$

and

$$E_\gamma^{R\pi} = \gamma_2 E_\gamma^{L}\left(1 - \beta_2 \cos\theta_\gamma^{L}\right)$$

The limits on the photon energy are given by

$$0 \leq E_\gamma^L \leq \frac{m_\mu^2 - m_e^2}{2m_\mu}\gamma_1\gamma_2(1+\beta_1)(1+\beta_2)$$



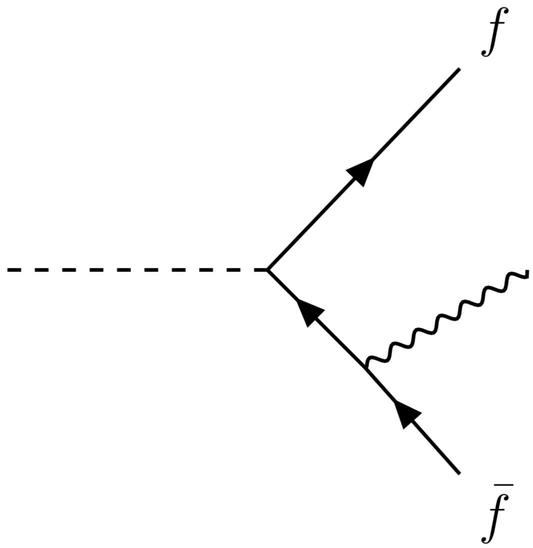
1.2.3 Neutral Pion

The dominant decay mode of the neutral pion is $\pi^0 \to \gamma\gamma$. In the laboratory frame, the spectrum is

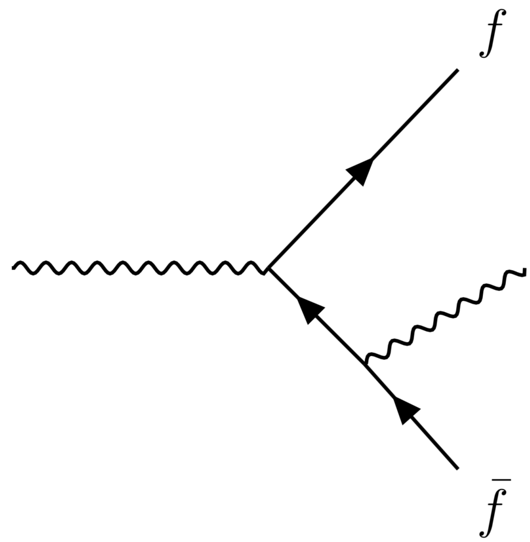
$$\frac{dN}{dE_\gamma} = \frac{2}{m_\pi \gamma\beta}$$

1.3 Final State Radiation

Along with computing decay spectra, hazma is able to compute final state radiation spectra from decays of off-shell mediators (scalar, psuedo-scalar, vector or axial-vector.) The relavent diagrams for such processes are

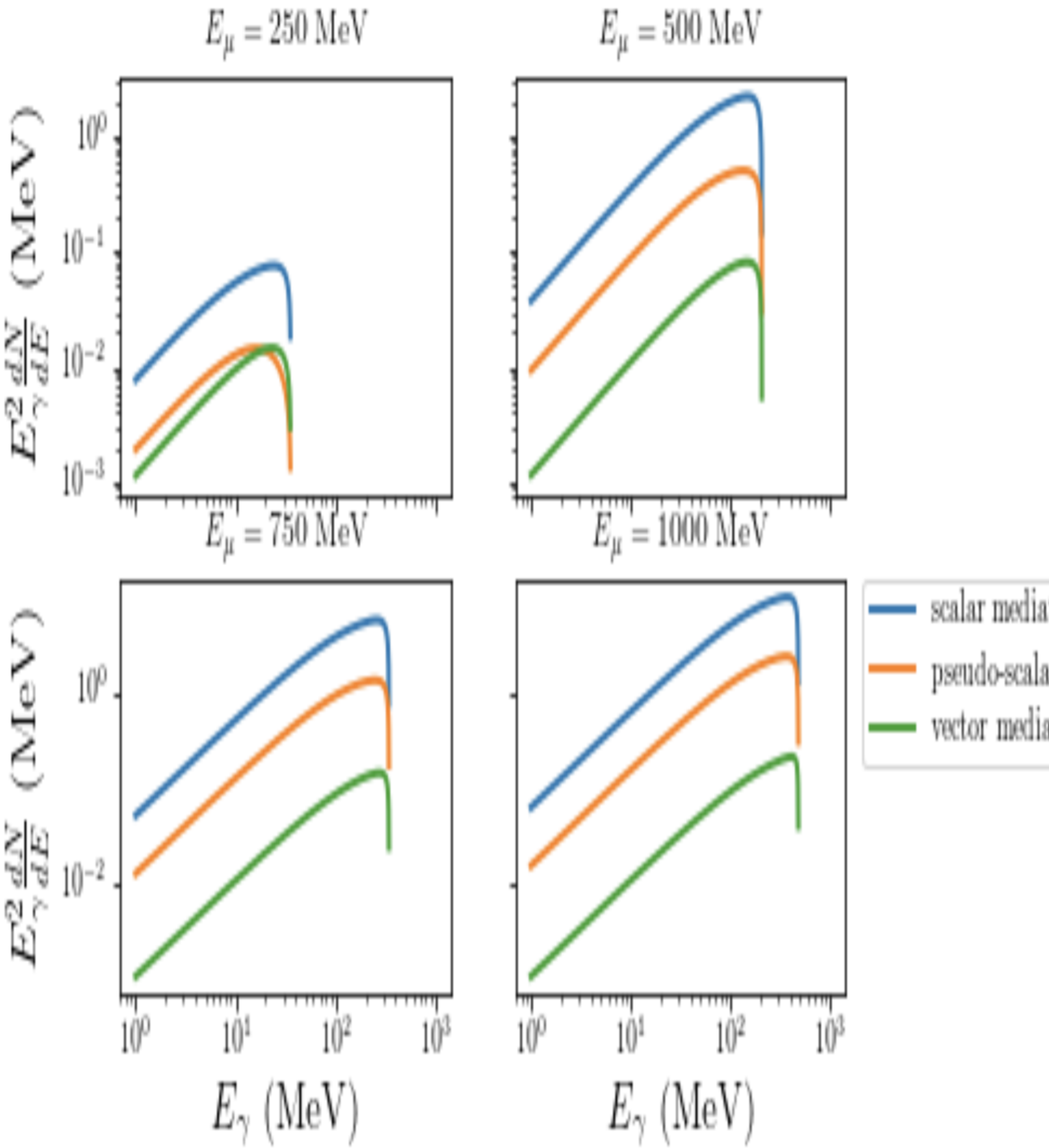


1. Scalar mediator



2. Vector mediator

Computing the matrix elements squared of these diagrams (including diagrams with the photon attached to the other fermion leg) and integrating over all variables except the photon energy yields $d\sigma(M^* \to \mu^+\mu^-\gamma)/dE$. To compute dN/dE, we divide $d\sigma(M^* \to \mu^+\mu^-\gamma)/dE$ by $\sigma(M^* \to \mu^+\mu^-)$.



1.3.1 References

CHAPTER

TWO

GAMMA RAY SPECTRA GENERATOR (HAZMA.GAMMA_RAY)

2.1 Description

Sub-package for generating gamma ray spectra given a multi-particle final state.

Hazma includes two different methods for computing gamma ray spectra. The first is done by specifying the final state of a process. Doing so, the particle decay spectra are then computed. The second method `gamma_ray_rambo` takes in the tree-level and radiative squared matrix elements and runs a Monte-Carlo to generate the gamma ray spectra.

2.2 Functions

Generate spectrum from builtin functions	func_gamma_ray
Generate spectrum using Monte Carlo	func_gamma_ray_rambo

CHAPTER

THREE

RAMBO (HAZMA.RAMBO)

3.1 Description

Sub-package for generating phases space points and computing phase space integrals using a Monte Carlo algorithm called RAMBO.

3.2 Functions

Computing annihilation cross sections	func_compute_annihilation_cross_section
Computing decay widths	func_compute_decay_width
Computing energy histograms for final state particles	func_generate_energy_histogram
Compute a single relativistic phase space point	func_generate_phase_space_point
Compute many relativistic phase space points	func_generate_phase_space

CHAPTER

FOUR

DECAY (HAZMA.DECAY)

4.1 Description

4.2 Functions

Muon	func_muon_decay
Neutral Pion	func_neutral_pion_decay
Charged Pion	func_charged_pion_decay
Short Kaon	func_short_kaon_decay
Long Kaon	func_long_kaon_decay
Charged Kaon	func_charged_kaon_decay

CHAPTER FIVE

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