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Seminar - 4. letnik

Aharonov-Bohm effect

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Abstract

In this seminar I present Aharonov-Bohm effect, a quantum phenomenon in which a particle is effected by electomagnetic fields even when traveling through a region of space in which both electric and magnetic field are zero. I will describe theoretical background of the effect, present some experimental verifications and show how this phenomenon can be practicaly used in modern devices for precise measurement of magnetic field.

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1 Introduction

In classical mechanics, the motion of particles is described by action of forces. Newton's second law $\vec{F} = m\vec{a}$ tells us how the particle will move through space under the influence of force $\vec{F}$, which is, in general, always Lorentz force which describes interaction between charged particle and electric and magnetic fields [1].

Electric and magnetic fields are uniquely described by Maxwell's equations, which we will describe in detail in next section. As we will see, description of electromagnetic phenomena can be simplified by introduction of electromagnetic potentials: scalar potential ϕ and vector potential $\vec{A}$. To write electric and magnetic field in form of potentials is a usefull because we need only four components (one scalar field and three components of vector field) to describe electromagnetic field, which in genaral consists of six components (three components for each vector field $\vec{E}$ and $\vec{B}$). Until the beginnig of the 20. century it was widely belived that potentials are only a mathematical construct to simplify calculations and that they contain no physical significance.

With the development of quantum mechanics in the early 20. century, this view was put under question, because Schrödinger equation, basic equation of quantum mechanics, doesn't contain fields but potentials. So the question arise, which description of electromagnetic phenomena is more fundamental, through electric and magnetic fields or through scalar and vector potentials. In 1959, Yakir Aharonov and his doctoral advisor David Bohm, proposed an experiment to resolve this issue. The hearth of the experiment is the effect in which wavefunction aquire some additional phase when traveling through space with no electromagnetic fields, only potentials. This is called Aharonov-Bohm effect and will be the main topic of this seminar.

First we will take a closer look at the electromagnetic potentials and their important property of gauge symmetry. We will examine some important properities of wavefunction of a charged particle in electromagnetic field, show how they lead to Aharonov-Bohm effect and describe it's properties in detail. In second part of seminar we will take a look at some experimental evidence of the effect and discuss it's importance and weather it can be use in some practical way.

2 Maxwell's equations and gauge symmetry

2.1 Maxwell's equations

In 1861, Scottish physicist and mathematician James Clerk Maxwell wrote four partial differential equations which represents a foundation of classical electrodinamics [1].

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \tag{1}$$

$$\nabla \cdot \vec{B} = 0 \tag{2}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3}$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{4}$$

These are so-called Maxwell's equations. They describe dynamics of electromagnetic field and together with Lorentz force on charged particle

$$\vec{F} = e(\vec{E} + \vec{v} \times \vec{B}), \tag{5}$$

they describe most of the classic physics.

Equations tells us relations between electric and magnetic fields $\vec{E}$ and $\vec{B}$ and their sources, electric current density $\vec{j}$ and charge density ρ. If we take a look at the second Maxwell equation, Equation (2), we see that magnetic field is always divergentless. This means that, using Helmholz theorem [1], we can write magnetic field in a form

$$\vec{B} = \nabla \times \vec{A}. \tag{6}$$

Because divergence of a curl of some vector field is always zero, this clearly satisfy Equation (2). $\vec{A}$ is called vector potential.

We can do similar thing with the third Maxwell equation (Equation (3)). If we write electric field in a form

$$\vec{E} = -\nabla \phi - \frac{\partial \vec{A}}{\partial t} \tag{7}$$

and use Equation (6) for magnetic field, we always satisfy Equation (3).

We see that if we write electric and magnetic field as described above, two Maxwell equations are automaticaly satisfied, so number of equations is reduced by half. We use other two Maxwell equations to determine relations between potentials and sources of electromagnetic field and we have all electromagnetic phenomena described in terms of potentials, which are much easier to work with than fields. If we want to calculate the force on charged particle, we only use Equations (6) and (7) to determine fields and Lorentz force law (Equation(5)) to calculate force.

2.2 Gauge transformations

Electromagnetic potentials have another important property. If we look at Equations (6) and (7) again, we see that potentials don't stand for themselves but allways in a form of derivative. This mean that if we transform potentials ϕ and $\vec{A}$ in the following way:

$$\vec{A}' = \vec{A} + \nabla\chi \tag{8}$$

$$\phi' = \phi - \frac{\partial\chi}{\partial t}, \tag{9}$$

potentials ϕ' and $\vec{A}'$ correspond to the same electric and magnetic field that ϕ and $\vec{A}$. This is easy to show, using the identity $\nabla \times \nabla f \equiv 0$ and the fact that spatial and time derivatives commute. Transformation, written above, is ussualy called "gauge transformation" and function χ is called "gauge function". Because we can satisfy Maxwell's equations with different potentials we say that the equations are "gauge invariant".

Gauge invariance of Maxwell's equations is the main reason why potentials were ussualy considered as purely mathematical construct without any physical significance. That view changed with development of quantum mechanics in 20. century and especially with introduction of Aharonov-Bohm effect.

3 Charged particle in proximity of solenoid magnet

3.1 Schrödinger equation in electromagnetic field

In classical mechanics, we can describe the motion of a particle either using a Lorentz force law, which contains fields, or by using canonical formalism and hamilton function, which is expressed in terms of potentials. But if we want to describe dynamics of particle in quantum mechanics, we are forced to use canonical formalism because Schrödinger equation explicitly contains Hamilton function. For a charged particle in electromagnetic field, it is of a form [2]

$$H = \frac{1}{2m}(\vec{p} - e\vec{A}(\vec{r}))^2 + e\phi(\vec{r}) + V(\vec{r}), \tag{10}$$

where V stands for possible non-electric potential. If we write momentum in form $\vec{p} = -i\hbar\nabla$ and put Hamiltonian in Schrödinger equation, we get

$$[\frac{1}{2m}(i\hbar\nabla + e\vec{A}(\vec{r}))^2 + e\phi(\vec{r}) + V(\vec{r})]\Psi = i\hbar\frac{\partial\Psi}{\partial t}, \tag{11}$$

which describes dinamics of charged particle in electromagnetic field.

3.2 Vector potential of solenoid magnet

Now let us consider a charged particle in vicinity of a long solenoid, carrying magnetic field $\vec{B}$. If solenoid is extremely long, the field inside is uniform and the field outside is zero. We will use polar coordinate sistem with z axis in the middle of solenoid and pointing in direction of magnetic field. To solve Schrödinger equation, we must first determine the potentials $\vec{A}$ and ϕ. Because solenoid is uncharged, the electric field $\vec{E} = -\nabla\phi = 0$, so we choose $\phi = 0$, which clearly satisfy previous equation. Vector potential $\vec{A}$ outside solenoid must satisfy two conditions. First, $\vec{B} = \nabla \times \vec{A} = 0$, is simply the definition of vector potential. The second is a consequence of Stokes theorem and states

$$\oint_C \vec{A} \cdot d\vec{r} = \int_S (\nabla \times \vec{A}) \cdot d\vec{S} = \int_S \vec{B} \cdot d\vec{S} = \Phi_m, \tag{12}$$

if path of integration C is contracted curve around solenoid and Φ_m is total magnetic flux through solenoid. Usually we choose vector potential to be

$$\vec{A} = \frac{\Phi_m}{2\pi r}\hat{\Phi}, \tag{13}$$

where r is distance from the z axis and $\hat{\Phi}$ is unit vector in direction Φ of our polar coordinate sistem. It is easy to show that vector potential, given by Equation (), satisfies both conditions. We see that even though magnetic field $\vec{B}$ is confined to the interior of solenoid, the vector potential $\vec{A}$ outside solenoid is not zero. If we use proper gauge function χ (Equation (8)), we can put it to zero almost everywhere outside solenoid, but we still have to satisfy the condition from Equation (12).

3.3 Wavefunction in vector potential

To describe wavefunction of a charged particle, we have to solve Equation (11). In our case, it can be simplified by writing the wavefunction in a form

$$\Psi(\vec{r}, t) = e^{ig(\vec{r})}\Psi'(\vec{r}, t), \tag{14}$$

where

$$g(\vec{r}) \equiv \frac{e}{\hbar}\int_0^{\vec{r}} \vec{A}(\vec{r}') \cdot d\vec{r}' \tag{15}$$

Initial point of integration O is chosen arbitrary, which is consequence of gauge freedom for electomagnetic potentials. At this point it is crucial that potential $\vec{A}$ is irottational (field $\vec{B}$ is zero), otherwise $g(\vec{r})$ is dependent of path of integration in Equation (15), so it is not a function of $\vec{r}$.

In terms of Ψ', the gradient of Ψ is

$$\nabla\Psi = e^{ig(\vec{r})}(i\nabla g(\vec{r}))\Psi' + e^{ig(\vec{r})}(\nabla\Psi'). \tag{16}$$

Because $\nabla g(\vec{r}) = (e/\hbar)\vec{A}$,

$$(-i\hbar\nabla - e\vec{A})\Psi = -i\hbar e^{ig(\vec{r})}\nabla\Psi' \tag{17}$$

and further

$$(-i\hbar\nabla - e\vec{A})^2\Psi = -\hbar^2 e^{ig(\vec{r})}\nabla^2\Psi' \tag{18}$$

Putting this into Equation (11) and canceling the factor $e^{ig(\vec{r})}$, we are left with

$$-\frac{\hbar^2}{2m}\nabla^2\Psi' - V\Psi' = i\hbar\frac{\partial\Psi'}{\partial t}. \tag{19}$$

We see that wavefunction Ψ' is a solution of Schrödinger equation in absence of vector potential $\vec{A}$. So if we can solve Equation (19), the solution in presence of vector field is the same wavefunction, multiplied by phase factor $e^{ig(\vec{r})}$.

3.4 Magnetic Aharonov-Bohm effect

Now we make a thought experiment. We take a beam of electrons, split it in to two and send each beam past solenoid on different side of it (Figure 1).

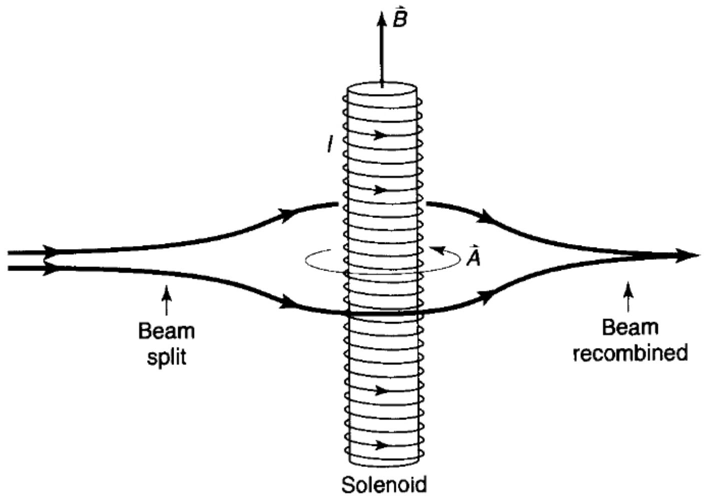

Figure 1: Schematic picture of magnetic Aharonov-Bohm effect [2]

What we do is in fact very similar to double slit experiment, so we expect that electron beams will make intreference pattern when they meet on the other side of solenoid. To describe interference, we write beams in form of plane waves.

$$\Psi_1 = Ae^{ikx_1}, \Psi_2 = Ae^{ikx_2}, \tag{20}$$

where k is wave vector of electron beams and x_1 and x_2 are lengths each beam travels. Of course this is not exact solution, but it helps us understand whath happens with beams. If solenoid contains no magnetic field, vector potential outside solenoid is set to zero, so the phase shift beetween Ψ_1 and Ψ_2 and consequetly interference pattern will depend only on difference between traveled paths: $\Delta\Phi_0 = k(x_2 - x_1)$.

But when we turn magnetic field on, vector potential $\vec{A}$ is of a form Equation (3.2), so the wavefunctions Ψ_1 and Ψ_2 will aquire additional phase factors as shown in Equation

(14). Consequently, interference pattern will shift for additional phase $\Delta\Phi = g_1 - g_2$. To calculate the difference, we use Equation (15) and write

$$\Delta\Phi = g_1 - g_2 = \frac{e}{\hbar}[\int_{C_1} \vec{A}(\vec{r}') \cdot d\vec{r}' - \int_{C_2} \vec{A}(\vec{r}') \cdot d\vec{r}'] = \frac{e}{\hbar} \oint_C \vec{A}(\vec{r}') \cdot d\vec{r}' = \frac{e\Phi_m}{\hbar} \tag{21}$$

C_1 and C_2 stands for paths each beam travels when passing past solenoid. Since they form a closed path around solenoid, total phase difference between beams will be propotional to magnetic flux inside solenoid. So if we change magnetic field in solenoid, we change phase difference between beams and interference pattern will shift. This is called Aharonov-Bohm effect.

Let us again point out that when electron beams are traveling past solenoid, they never pass through regions of space with non-zero magnetic field, so in classical electrodinamics, we would expect no interaction between electron and magnetic field. Nevertheless we saw that if we describe particles in terms of quantum mechanics, magnetic field in region isolated from particles produces measurable effect on their motion. So we have to conclude that either interaction between field and particle is not local or that potentials are more fundamental description of physical reality than fields.

The main objection against physical relevance of potentials, their gauge freedom, represents no problem in treatment of our experiment. If we change vector potential to $\vec{A'} = \vec{A} + \nabla\chi$ and carry out integration in Equation (21), we see that phase difference $\Delta\Phi$ doesn't change because we integrate gradient of a function over a closed loop. The result is identically zero, which means that phase difference is gauge invariant.

3.5 Electric Aharonov-Bohm effect

When we mention Aharonov-Bohm effect, we ussualy refer to phenomenon, described in previous section, but in fact, there exists two types of the effect. In magnetic effect, described earlier, wavefunctions gain phase difference due to the vector potential, which ussualy describes magnetic field. But in their arcticle, Aharonov and Bohm also described another version of same phenomenon, when particles travel through region where electric field $\vec{E}$ is zero, but scalar potential ϕ is not.

In fact, this version of effect is easier to explain. As we know, wavefunction with energy E evolves with time as

$$\Psi(\vec{r}, t) = \Psi(\vec{r}, t = 0)e^{i\frac{Et}{\hbar}}. \tag{22}$$

The evolution is similar to Equation (14) if we put $E = e\phi$ and write

$$g = \frac{e}{\hbar}\int_{t'=0}^{t'=t} \phi dt \tag{23}$$

We see that Equations (15) and (23) has the same form, only that in first we integrate vector potential over space and in second scalar potential over time, which is direct consequence of special relativity, where vector and scalar potential form the same physical entity $A_\mu = (\frac{\phi}{c}, \vec{A})$. In electric Aharonov-Bohm effect, electron beams should travel through regions of space with different scalar potential to aquire phase difference.

Electric Aharonov-Bomh effect could be realized in a form, presented on Figure 2.

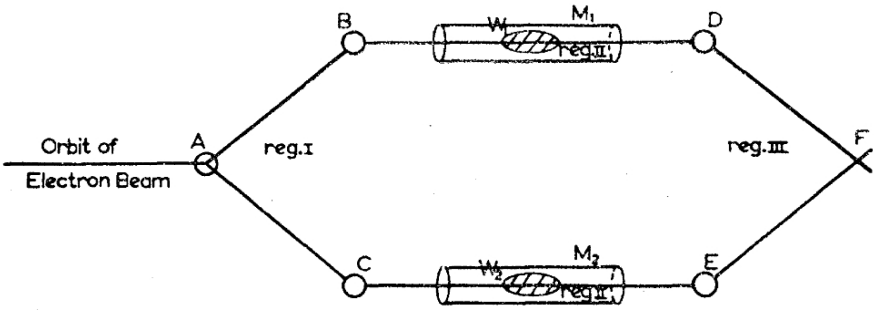

Figure 2: Schematic picture of electric Aharonov-Bohm effect [3]

Each beam travels through different conductive cylinder at potential ϕ_1 or ϕ_2. It is important that cylinder is sufficently long that field inside is zero and potential is constant. If potentials of cylinders are different, beams aquire phase difference of

$$\Delta\Phi = \frac{et}{\hbar}\Delta\phi, \tag{24}$$

where $\Delta\phi$ is potential difference between cylinders and t is time electron needs to pass through cylinder. The result of experiment would be similar as in magnetic Aharonov-Bohm effect, a fringe shift would appear in interference pattern. Main problem in this type of experiment is that it is difficult to carry out and results are harder to interpretate because we can't achieve situation where electrons wouldn't have to pass through electric field, which is inevetably present at beginning and end of cylinders. In case of magnetic effect, field can really be localised, so we usally use that type of experiment to measure the effect.

3.6 Aharonov-Bohm effect and superconductors

In addition to zero resistivity, superconductors posses some other important properties. One of them is Meissner effect [4], which means that magnetic field can't penetrate into superconductor. If magnetic field is to strong, some types of superconductors form special structures, called flux lines, which enables field to penetrate through superconductor, but only in form of thin lines. Interesting property of such lines is that the magnetic flux in each of them is quantized in units of

$$\Phi_m = \frac{h}{2e_0}, \tag{25}$$

where e_0 is charge of electron. This phenomenon can be explained using formalism, developed previously in this section.

Wavefunction of electron Ψ in superconductor is defined in a plane which is penetrated by a flux line. Magnetic field is localised to flux line and is zero in rest of superconductor, which is the situation we described in Section 3.1. If we move along the path around flux line with same starting and ending, we see that value of wavefunction is changed from $\Psi(\vec{r}_0)$ to $e^{ig}\Psi(\vec{r}_0)$. We want wavefunction Ψ to be single-valued, which means that

$e^{ig} = 1$. If we use Equation (15) to calculate phase difference g, we get condition

$$\frac{e\Phi_m}{\hbar} = 2\pi m, \tag{26}$$

$$\Phi_m = \frac{h}{e}m, \tag{27}$$

with m being integer. We see that we get the correct result of we put $e = 2e_0$, which means that particles in superconductor has charge twice larger than electron. It can be explained by Cooper pairs [4], which are composed of 2 electrons. Quantisation of flux in superconductor is important for proper interpretation of experimental results when measuring Aharonov-Bohm effect at low temperatures.

4 Experimental evidence of Aharonov-Bohm effect

Aharonov-Bohm effect was first described in 1959 in an article [3], written by Yakir Aharonov and his doctoral advisor David Bohm and recieved various responses. Many physicist claimed that the effect in fact cannot be measured and that it is, like potentials, only a mathematical construct, so experimental conformation was needed. In fact, when developing their idea, Aharonov and Bohm consulted experimental physicist Robert G. Chambers and in their article, they discribed the experiment which had to be carried out to prove their theory. Only a year later, in 1960, Chambers performed the proposed experiment and proved that effect does exist. In following years, effect was confirmed by more and more precise experiments so today only a few people still doubt it's existance.

In this section we will discribe Chambers's experiment and discuss his results. We will also take a look at a more advanced experiment from year 1986, which was performed in deferent geometry, using superconducting toroidal magnet.

4.1 Solenoid magnet experiment

Geometry, used in experiment, performed by Robert G. Chambers in 1960 [5], is practically identical to geometry described in Section 3. The first problem in designing the experiment was how to sepatate electron beams to sufficient distance. Spatial coherence of electrons is determined by the size of electron source. If source is infinitely small, separation of beams can be arbitrary, but in real experiment, separation is limited by finite size of source. For succesful experiment, we need sufficient spatial coherence to separate beams enough to put magnet between them. Experimental geometry is shown on Figure 3.

Beam of electrons, used in experiment, was produced by electron microscope. Electron wavelenght was smaller than $1nm$, which is much smaller than the size of solenoid, so diffraction can be neglected. Beam is split into two by a electrostatic biprism (e and f on figure) and interfering on observation plane o. Biprism consists of aluminized quartz fibre f and two earthed metal plates e. Effective angle of biprism can be altered by applying positive potential on fibre f. In our calculations in Section 3, we assumed that solenoid is infinitely long so that magnetic field outside is identically zero. In experimental situation, this cannot be achieved, so Chambers performed two experiments to distinguish the effects

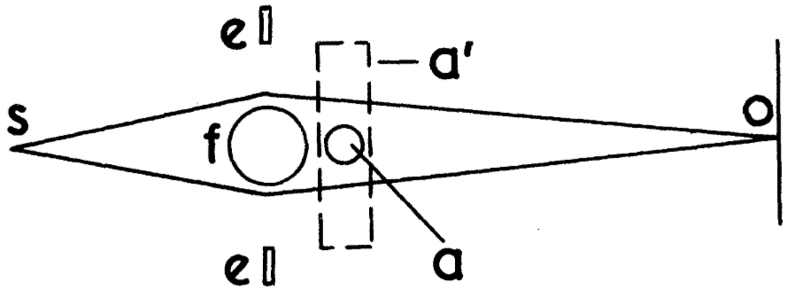

Figure 3: Schematic diagram of inferometer, used by Chambers [5]

of magnetic field, leaking from solenoid, from Aharonov-Bohm effect. In first experiment, electrons travel through magnetic field, extended over region a' (Figure 3). As shown in Figure 4, field, extended over region a', leaves fringes on interference pattern unchanged, it only displaces entire patteren over the screen, which can be explained classicaly by Lorentz force.

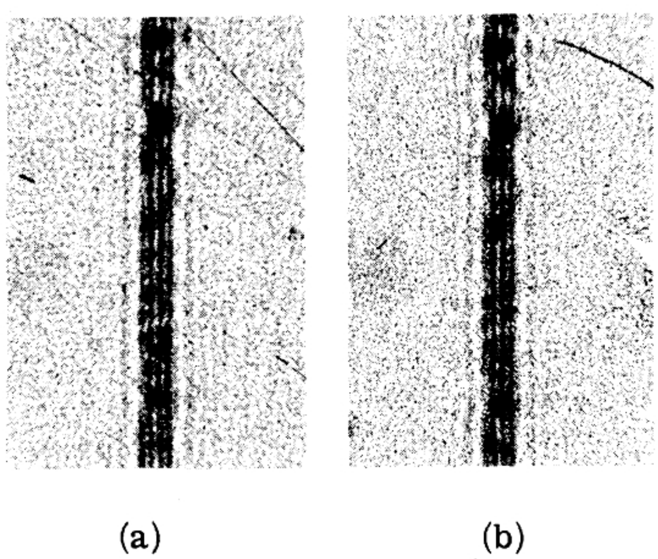

Figure 4: Interferogram due to a) biprism alone and b) magnetic field in region a' [5]

In the second experiment, magnetic field is localised to region a on Figure 3, which is a situation in which we expect Aharonov-Bohm effect. Instead of solenoid, which is hard to made so tiny, iron whisker 1 μm wide and 0.5 mm long was used. The flux in such wisker decreases with along it's lenght with the slope approximately $(hc/e)/1\mu m$, which means that in case of Aharonov-Bohm effect, interference pattern will shift for about 1 fringe per μm. Whisker is positioned in shadow of cylinder f, so no electron pass through it. Results of this type of experiment is shown on Figure 5.

We see that fringes in fact shift in vertical direction, which is clear evidence of Aharonov-Bohm effect.

Even though Chambers's experimental results confirmed predictions of Aharonov and

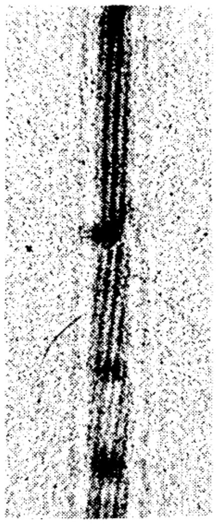

Figure 5: Interferogram due to magnetic field in region a, produced by iron whisker [5]

Bohm, some still argued that the effect was not caused by electromagnetic potentials. Because the flux in iron whisker is decreasing, it is obvious that magnetic field leaks out in the region where electrons travel, so many belived that the observed effect could be explained by interaction of electrons with magnetic field. Further experiments were needed to undisputably prove Aharonov-Bohm effect.

4.2 Toroidal magnet experiment

In this section, we will describe an experiment, performed by a group of japanese physicist from Hitachi laboratory in year 1986 [6]. They used a bit different geometry of experiment and supermagnetic properties of materials to get more precise results of Aharonov-Bohm effect.

Instead of solenoid magnet, proposed in original article in 1959 and used in first experiments, toroidal magnet was used. In case of solenoid, magnetic field around it is never precisly zero because magnetic lines of force have to be closed, so they have to link both poles of the magnet. Magnetic field outside of solenoid is decrising with it's length, but we would need infinitely long magnet to get field exactly zero. Toroidal magnet has north and south pole attached, lines of force are circular so in theory we can achieve magnetic field outside of it to be identically zero.

To furher reduce possibility of field leaking into surrounding area, toroidal magnet was covered with Nb, which is superconducting at temperatures under $T_c = 9K$. Because of Meissner effect, magnetic field is unable to penetrate through superconductor and it's value outside is zero. The entire structure is additionaly covered by a layer of Cu, shielding superconductor from electrons.

The scheme of experiment is presented on Figure 6. Part of electron beam is traveling through a hole in toroidal magnet, so together with reference beam it forms a closed circuit around magnetic flux, which causes phase shift. The rest of electrons interfere with reference beam without any phase shift, so we can use that pattern as a reference. Toroidal

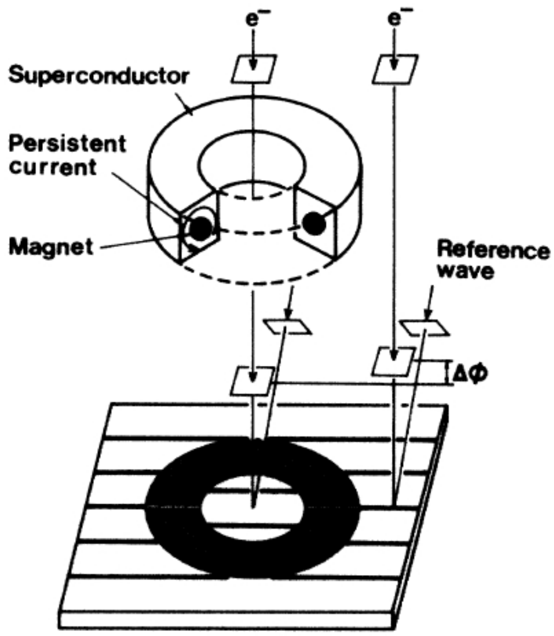

Figure 6: Conceptual diagram of toroidal magnet experiment [6]

magnet was made of ferromagnetic Permalloy. Due to the phase transition, magnetisation of Permalloy is increasing when we lower the temperature and magnetic flux through magnet is rising. In experiment, phase shift was observed while cooling magnet from room temperature to 5 K. Figure 7 shows us interference patterns for different magnets at temperatures 15 K and 5 K.

In first column, we see interference pattern at $T = 15K$. At this temperature, Nb is not in superconducting state, so flux in magnet can take any value and phase shift can be arbitrary. Below Nb phase transition, magnetic flux in toroid is quantized: $\Phi_m = \frac{h}{2e_0}n$, which means that phase shift of electrons is also quantized

$$\Delta\Phi = \frac{e_0\Phi_m}{\hbar} = \frac{e_0}{\hbar}\frac{h}{2e_0}n = \pi n \tag{28}$$

Such quantization means that interference pattern can be shifted by either half fringe if n is odd or not shifted at all if n is even. Half fringe shift can be observed only at phase amplification $\times 1$, which is reason why we see no difference in pattern between top and bottom picture in Figure 7. In third column, phase amplification is $\times 1$ and we can clearly see that phase shift is 0 in top picture and π in the bottom picture.

Experiment clearly shows that phase shift between electron beams is present even when magnetic field outside magnet is identicaly zero, which is evidence that electrons are influenced by potentials, not fields.

5 Practical use of Aharonov-Bohm effect

In addition to it's phylosophical value in understanding electromagnetic and quantum phenomena, Aharonov-Bohm effect also has some important practical implications. We

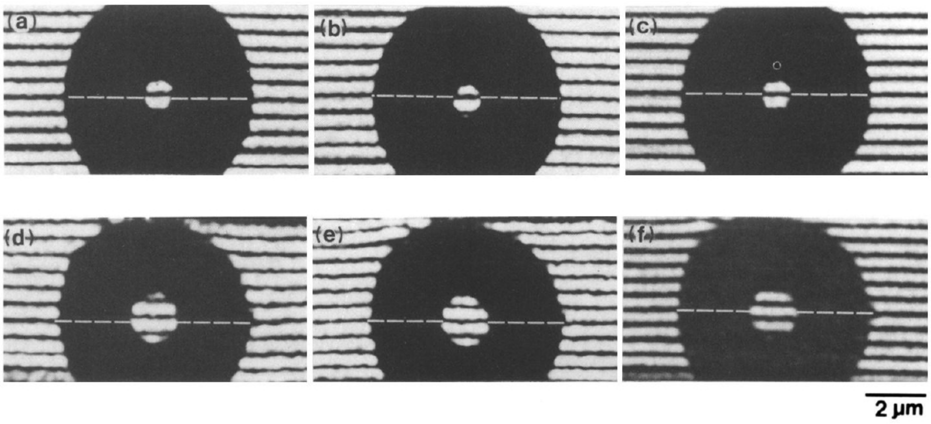

Figure 7: Interference pattern for 6 different situatuations. In top line, ((a),(b) and (c)), flux is quantized with even n, and in bottom line, ((d),(e) and (f), flux is quantized with odd n at $T < T_c$. First column shows results at $T = 15K$, second at $T = 5K$ and phase amplification $\times 2$ and third at $T = 5K$ and phase amplification $\times 1$. [6]

saw that phase shift between electron beams strongly depends on enclosed magnetic flux. Interference patteren shifts one fringe for every $\Delta\Phi_m = h/e_0 = 4.1 \times 10^{-15} Tm^2$, which is very small value. In principle, the effect enables us measurement of extremely small differences in magnetic flux. The most simple case of such magnetometer would be two wires with electron current, forming a closed loop, and we would count oscilations of current through structure when magnetic flux through loop would change.

In fact, this basic idea can be reallized in even better way, using Josephson effect. In that case, loop is formed of two superconductors separated by small insulating barrier. The actual reallizaton of idea is to complicated to describe in this seminar, but it is much easier to understand with basic knowledge of Aharonov-Bohm effect and behaviour of wavefunction around magnetic fluxes.

6 Conclusion

The main objective of this seminar was to show that in quantum mechanics, electromagnetic potentials appear to be more fundamental physical entities than fields. Aharonov-Bohm effect is phenomenon which can't be describe in terms of classical mechanics and is of purely quantum origin. The effect was confirmed by many different experiments and today it's existence is widely accepted. Anyhow, some physicist belive that mechanism of the effect is not properly described [7]. They argue that the effect is not purely quantum, described by phase shift, but that it is consequence of forces, acting on electron wave packets when traveling past magnet. That problem will probably be resolved by further experiments.

Anyway, we can conclude that electomagnetic interaction can take place in regions of space where field is zero and that Aharonov-Bohm effect does exist. Yet again we see that laws of quantum mechanics often contradict our intuition, which is a reason why *New Scientist* magazine proclaimed Aharonov-Bohm effect to be one of the "seven wonders of the quantum world" [8].

7 Literature

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