Path Integral Study Note

Refer to U.Mosel, Path Integrals in Field Theory

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The Path Integral in Quantum Theory

1.1 Propagator of the Schrödinger Equation

The non-relativistic Schrödinger equation with one dimensional potential $V(x)$:

$$H\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V(x)\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t} \tag{1}$$

What can obtain from schrödinger equation?

- the wave function $\psi(x,t)$ if we know $\psi(x,t_0)$ at the earlier time $t_0 < t$.

Rewrite Schrödinger eq :

$$\left(i\hbar\frac{\partial}{\partial t} - H\right)\psi(x,t) = 0 \tag{2}$$

Next, we consider the function $K(x,t;x_i,t_i)$ which is defined as a solution of the equation :

$$\left(i\hbar\frac{\partial}{\partial t} - H\right)K(x,t;x_i,t_i) = i\hbar\delta(x-x_i)\delta(t-t_i) \tag{3}$$

What is K?

- the **Green's function** of the Schrödinger equation
- or often called the **propagator**

Initial condition of K :

$$K(x,t_i+0;x_i,t_i) = \delta(x-x_i) \tag{4}$$

The solution for $t > t_i$ (Huygen's principle) of the Schrödinger equation :

$$\psi(x,t) = \int K(x,t:x_i,t_i)\psi(x_i,t_i)dx_i \tag{5}$$