Top quark mass from Primordial Black Holes in Critical Higgs Inflation

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We study the case when the primordial black holes produced from the Higgs inflation would take a dominant portion of the dark matter in the universe so that the seemingly unrelated phenomena, cosmological inflation, dark matter and the precision particle physics are closely related. A prediction of top quark mass is made.

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INTRODUCTION

The standard model (SM) of particle physics has been established by the observation of the Higgs boson at the LHC and no new physics signal has shown up so far [1]. Conventional wisdom, however, often states that the SM cannot address the most notable observational problems in cosmology, namely the dark matter(DM) problem and the cosmic inflation problem thus the extension of the SM is unavoidable [2]. Even though we agree that the SM should be regarded as an effective description of a more fundamental theory, we do not think that all the possibilities are exhausted. We'd like to check the possibility *within* the SM that the SM Higgs field may explain the cosmic inflation and the primordial black holes (PBH) produced during the inflation may explain the majority of DM in our universe. The production of PBH from the inflection point of the inflaton was discussed earlier in Refs [3, 4][1].

PBH have been intensively searched for by ...There exists no rigorous treatment of how to apply the PBH bounds for an extended mass functions and different study conclude differently. Here are some recent results:

$$\frac{M_{\rm PBH,DM}}{M_\odot} \simeq \begin{cases} 10^{-10} - 10^{-8} \\ 10^{-13} - 10^{-9} \\ 5\times 10^{-14} \\ 5\times 10^{-16} \sim 2\times 10^{-14} \end{cases} \tag{1}$$

NOTE::: for texing reason, let me put the references here: [6, 7] [8, 9] [10] [11]:::

where $M_\odot \simeq 2.0\times 10^{33}$g is the solar mass even though other values are also allowed in [11]. See also a bound from X-ray observation [12].

It is suggested that the super-slow role in the critical Higgs inflation (CHI) may provide enough PBH for all dark matter. [9].

In this letter we study the Higgs inflation model requesting the PBH as all dark matter and predict the allowed range of top quark mass. See earlier works [13, 14].[2]

HIGGS INFLATION

The Higgs inflation [17, 18] (see also [19, 20]) is described by the action

$$S = \int d^4x\,\sqrt{-g}\,\left[\frac{M_{\rm P}^2}{2}R + \frac{1}{2}\xi\phi^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{4}\lambda\phi^4\right], \tag{2}$$

where ξ is the nonminimal coupling, λ the quartic coupling and ϕ is the Higgs field in unitary gauge. The reduced Planck mass is denoted as $M_P^2 = 1/(8\pi G) \approx (2.4\times 10^{18}{\rm GeV})^2$. It is convenient to analyse the model in the Einstein frame which can be obtained via Weyl rescaling: $g_{\mu\nu} \to g^{\rm E}_{\mu\nu} = \Omega^2 g_{\mu\nu}$, where $\Omega^2 = 1 + \xi\phi^2/M_{\rm P}^2$. The resultant Einstein-frame action is then given by

$$S^{\rm E} = \int d^4x\,\sqrt{-g^{\rm E}}\,\left[\frac{M_{\rm P}^2}{2}R^{\rm E} - \frac{1}{2}g^{{\rm E}\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V\right], \tag{3}$$

where φ is the canonically normalised field, which is related to ϕ by

$$\left(\frac{d\varphi}{d\phi}\right)^2 = \frac{1+(1+6\xi)\xi\phi^2/M_{\rm P}^2}{(1+\xi\phi^2/M_{\rm P}^2)^2} \tag{4}$$

and V is the potential in the Einstein frame, which is related to the Jordan-frame potential by

$$V = \frac{\lambda\phi^4}{4\left(1+\xi\phi^2/M_{\rm P}^2\right)^2}\,. \tag{5}$$

The potential in the Einstein frame becomes flat $V \to \lambda M_p^4/(4\xi^2)$ at high field value $\phi \gg M_p/\sqrt{\xi}$.

[1] The effects of Hawking radiation for a light PBH in $M < 10^{17}$ g ($M < 10^{-16}M_\odot$) was shown in [5]

[2] The naturalness problem in the classical non-minimal inflation has been addressed in [15]. Also see [16].

Within the framework of slow-roll inflation, cosmological observables such as the scalar power spectrum $\mathcal{P}_s$, scalar spectral index n_s and tensor-to-scalar ratio r are conveniently expressed in terms of the slow-roll parameters, which are defined by

$$\epsilon \equiv \frac{M_{\rm P}^2}{2}\left(\frac{V_{,\varphi}}{V}\right)^2 , \quad \eta \equiv M_{\rm P}^2 \frac{V_{,\varphi\varphi}}{V} , \tag{6}$$

where the single and double derivatives of the potential are $V_{,\varphi} \equiv \partial V/\partial\varphi$ and $V_{,\varphi\varphi} \equiv \partial^2 V/\partial\varphi^2$, respectively.

Explicitly,

$$\epsilon(\phi) = 8\frac{M_P^2}{\phi^2}\frac{\left[1+\frac{\lambda'}{4\lambda}\left(1+\xi\frac{\phi^2}{M_P^2}\right)\frac{\phi}{M_P}-\frac{1}{2}\xi'\frac{\phi^3}{M_P^3}\right]^2}{1+(1+6\xi)\xi(\frac{\phi}{M_P})^2}, \tag{7}$$

$$\eta(\phi) = ... \tag{8}$$

Here we include the derivatives of the effective quartic coupling, $\lambda' = M_P d\lambda/d\phi$ and the non-minimal coupling $\xi' = M_P d\xi/d\phi$.

From Eq. 7, we can find a condition for 'super slow roll' (SSR) $\epsilon = 0$:

$$1+\frac{\lambda'}{4\lambda}\left(1+\xi\frac{\phi^2}{M_P^2}\right)\frac{\phi}{M_P}-\frac{1}{2}\xi'\frac{\phi^3}{M_P^3}=0, \text{ (SSR)} \tag{9}$$

where the scalar amplitude is greatly enhanced by $1/\epsilon$,

$$\left.\frac{\phi}{M_P}\right|_{SSR} \approx x_* \tag{10}$$

where x_* is a real, positive solution to the Eq. 9.

::FOR NUMERICS:: I think we can solve SSR equation by the following steps:

1. The SSR eq. is of the form of cubic equation:

$$1+\alpha x_* + \beta x_*^3 = 0$$

where $\alpha = \lambda'/4\lambda$ and $\beta = \lambda'\xi/4\lambda - \xi'/2$. Here α and β are implcit functions of x. So, I want to see $\alpha(x)$ and $\beta(x)$ first to see the rough range of $x_* < 1$.

2. The number of efoldings from $\varphi_{start}(=\varphi_0)$ to φ is obtained by integration

$$N(\varphi) = \int_\varphi^{\varphi_0}\frac{d\varphi}{\sqrt{2\epsilon}} \quad \text{(slow-roll)}$$
$$= \int_\phi^{\phi_0}\frac{d\phi}{4}\frac{\phi\left(1+(1+6\xi)\xi\phi^2\right)}{[1+\frac{\lambda'}{4\lambda}(1+\xi\phi^2)\phi-\frac{1}{2}\xi'\phi^3][1+\xi\phi^2]}$$

where $\varphi < \varphi_{start}$ for a large field inflation evolving down to a small field value φ. We conveniently use dimensionless parameter $\phi \to \phi/M_P$ and change of variable by $d\varphi = (d\varphi/d\phi)d\phi$. **We determine** the end of inflation, φ_{end} by the condition $\text{Max}(|\eta|,|\epsilon|) = 1$ and request a largish efolding number $N(\varphi_{end}) \approx 60(50)$ for successful inflation.

3.

The observables are determined at the CMB scale by slow-roll parameters:

$$\mathcal{P}_s(k=k_{CMB}) = A_s(k)\left(\frac{k}{k_\star}\right)^{n_s(k)-1}, \tag{11}$$
$$n_s = 1-6\epsilon+2\eta\,, \tag{12}$$
$$r = 16\epsilon\,. \tag{13}$$

where $A_s(k) = \left.\frac{V}{24\pi^2 M_{\rm P}^4\epsilon}\right|_k$ is the scalar amplitude at the scale k. The pivot scale is $k_\star = aH$ given at the horizon crossing.

Explicitly,

$$A_s(\phi) = \frac{V}{24\pi^2 M_P^4\epsilon(\phi)} \tag{14}$$

The Planck result determines the value at ϕ_{60} or the beginning of the inflation and the value at the stage of PBH production ϕ_{PBH}:

$$A_s(\phi_{60}) = \frac{V(\phi_{60})}{24\pi^2 M_P^4\epsilon(\phi_{60})} = 2.2\times10^{-9}. \tag{15}$$

$$A_s(\phi_{PBH}) = \frac{V(\phi_{PBH})}{24\pi^2 M_P^4\epsilon(\phi_{PBH})} = ?? \tag{16}$$

where ?? is from the condition $\mathcal{P}_s(k_{PBH}) \sim 10^{-4}$ or 10^{-2} for enough production of PBH. If we request that ϕ_{PBH} should be near the criticality, we request $\phi_{PBH} = \phi_c$ and $\lambda(\phi_c) = 0 = \lambda'(\phi_c)$.

NOTE::: We should relate k_{PBH} with $\phi_{PBH} = \phi_c$ but How?? $k_{PBH} = \phi_{PBH}$ is good?

From the latest Planck result for the cosmic microwave background radiation (CMB) [21],

$$\mathcal{P}_s(k_\star = 0.05\text{Mpc}^{-1}) \approx 2.2\times10^{-9}\text{(Planck)},$$

one can relate λ and ξ as $\lambda/\xi^2 \approx 4.3\times10^{-10}$. With the given ratio λ/ξ^2, the other observables are completely determined as

$$n_s \approx 0.966\,, \quad r \approx 0.0032 \quad \text{(without RGE)}. \tag{17}$$

To compare with the measured values at the Planck, we present the values when the running of the running is allowed to float. The Planck TT+lowP (Planck TT, TE, EE+lowP) data give:

$$n_s = 0.9569 \pm 0.0077\ (0.9586 \pm 0.0056), \tag{18}$$
$$dn_s/d\ln k = 0.011^{+0.014}_{-0.013}\ (0.009 \pm 0.010), \tag{19}$$
$$d^2n_s/d\ln k^2 = 0.029 + 0.015\ (0.025 \pm 0.013), \tag{20}$$

at the pivot scale $k_\star = 0.05\text{Mpc}^{-1}$ with 1σ uncertainties.

So far we have sketched the Higgs inflation at classical level and did not take the renomalization group running of the parameters. However, as inflation scale is much higher than the LHC scale i.e. electroweak scale, it is crucial to take into account quantum corrections [22–24]. In order words, one has to consider renormalization group (RG) running of parameters e.g. λ and ξ. To do so, we shall quantise the theory in the original Jordan frame. The RG-improved effective action is then given by [25]

$$\Gamma = \int d^4x\,\sqrt{-g}\,\left[\frac{M_{\rm P}^2}{2}\Omega^2 R - \frac{1}{2}g^{\mu\nu}G^2\partial_\mu\phi\partial_\nu\phi - U_{\rm eff}\right], \tag{21}$$

where

$$\begin{aligned} U_{\rm eff} &= \frac{1}{4}\lambda(t)G^4(t)\phi^4(t)\,, \\ \Omega^2(t) &= 1+\xi(t)G^2(t)\frac{\phi^2(t)}{M_{\rm P}^2}\,, \\ G(t) &= \exp\left(-\int dt\,\frac{\gamma}{1+\gamma}\right)\,, \end{aligned}$$

with $\gamma(t)$ being the Higgs field anomalous dimension, $t = \ln(\mu/M_t)$ and μ the renormalization scale. In this paper we choose the renormalization scale to be $\mu = (y_t/\sqrt{2})\phi$ (y_t the top Yukawa coupling) so that the radiative correction is minimised.

The RG-improved Einstein-frame effective potential reads

$$V_{\rm eff} = \frac{\lambda(t)G^4(t)\phi^4(t)}{4(1+\xi(t)G^2(t)\phi^2(t)/M_{\rm P}^2)^2}\,. \tag{22}$$

We summarise the full RG equations up to two-loop order in appendix. The initial conditions for RG equations are chosen as [26]

$$\begin{aligned} \lambda(\mu=M_t) &= 0.12604 + 0.00206\left(\frac{M_H}{\rm GeV} - 125.15\right) \\ &\quad - 0.00004\left(\frac{M_t}{\rm GeV} - 173.34\right)\,, \qquad (23) \\ y_t(\mu=M_t) &= 0.93690 + 0.00556\left(\frac{M_t}{\rm GeV} - 173.34\right) \\ &\quad - 0.00003\left(\frac{M_H}{\rm GeV} - 125.15\right) \\ &\quad - 0.00042\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right)\,, \qquad (24) \\ g_3(\mu=M_t) &= 1.1666 + 0.00314\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \\ &\quad - 0.00046\left(\frac{M_t}{\rm GeV} - 173.34\right)\,, \qquad (25) \\ g_2(\mu=M_t) &= 0.64779 + 0.00004\left(\frac{M_t}{\rm GeV} - 173.34\right) \\ &\quad + 0.00011\left(\frac{M_W/{\rm GeV} - 80.384}{0.014}\right)\,, \qquad (26) \\ g_1(\mu=M_t) &= 0.35830 + 0.00011\left(\frac{M_t}{\rm GeV} - 173.34\right) \\ &\quad - 0.00020\left(\frac{M_W/{\rm GeV} - 80.384}{0.014}\right)\,, \qquad (27) \end{aligned}$$

with the latest PDG values [27]:

$$M_W = 80.385\,{\rm GeV}\,, \qquad M_Z = 91.1876\,{\rm GeV}\,, \tag{28}$$
$$M_H = 125.09\,{\rm GeV}\,, \qquad \alpha_s(M_Z) = 0.1182\,. \tag{29}$$

We treat M_t as a parameter. So far, the most precise measurements of the M_t have involved fitting the reconstructed top decay products to a theoretical curve, which is usually produced using Monte-Carlo (MC) event generators. This 'Monte-Carlo mass', $M_t^{\rm MC}$, is often assumed to be identical to the pole mass, which cannot be the case as $M_t^{\rm MC}$ depends on which MC is used and tuned. See e.g. recent proposal to improve the uncertainty in $e^+e^- \to t\bar{t}$ with Jet Grooming [28]. At a hadron machine such as the LHC, the situation is much worse.

Note that the nonminimal coupling ξ is not a free parameter as explained earlier. Also the running effect of $\xi(\mu)$ is minor [14]. We thus take into account RG runnings of ξ up to one-loop level, while those of other Standard Model parameters are considered up to two-loop level. We then compute the slow-roll parameters and cosmological observables numerically.

FORMATION OF PRIMORDIAL BLACK HOLES

The fraction of the Universe which gravitationally collapses into primordial black holes of mass M at the formation time is given by [29–31]

$$\beta(M) = \int_{\zeta_c}^{\infty} \frac{2}{\sqrt{2\pi \mathcal{P}_\zeta}} \exp\left(-\frac{\zeta^2}{2\mathcal{P}_\zeta}\right) d\zeta \,, \tag{30}$$

where we have assumed a Gaussian probability distribution function for the curvature fluctuations with ζ_c being the threshold value. There is a large uncertainty in ζ_c (typically from 0.01 to 1, see [32]) which drew vast studies of analytical as well as numerical methods. Thus we shall take ζ_c to be a free parameter. The primordial black hole mass M is given by the mass inside the horizon at the formation time [29]; that is,

$$M = \gamma \frac{4\pi M_{\rm P}^2}{H_I} e^{2N} \,, \tag{31}$$

where N is the number of e-folds when a mode crossed the horizon, H_I is the Hubble rate at the horizon crossing and γ is a numerical factor which represents an efficiency of the gravitational collapse. In this work we choose $\gamma \simeq 0.4$ instead of the conventional value $\gamma = 0.2$ [6] (see also [33]).

In order for the primordial black holes to play the role of dark matter today, the evaporation should last long enough. We require the primordial black holes to survive until the time of matter-radiation equality $t_{\rm eq}$. In other words,

$$t_{\rm life} \approx 2.1 \times 10^{67}\,{\rm yr} \left(\frac{M}{M_\odot}\right)^3 \gtrsim t_{\rm eq} \,, \tag{32}$$

which sets the lower limit of the primordial black holes: $M_{\rm lower} \approx 1.0 \times 10^{12}\,{\rm g}$. Those primordial black holes that are formed at the horizon re-entry behave like matter. Thus the fraction β is proportional to the scale factor. The abundance of the primordial black holes at the radiation-matter equality is given by

$$\Omega_{\rm PBH}^{\rm eq} = \int_{M_{\rm lower}}^{M_{\rm eq}} d\ln M \, \beta^{\rm eq} \,, \tag{33}$$

where $\beta^{\rm eq} = \beta\sqrt{M_{\rm eq}/M}$ and $M_{\rm eq} \approx 3.5 \times 10^{17} M_\odot$ is the horizon mass at the equality [33]. In our study we use the number of e-folds N as a time variable which is related to the mass by $dN = dM/(2M)$ [30, 34]. If $\Omega_{\rm PBH}^{\rm eq} = \Omega_{\rm DM}^{\rm eq}$, where $\Omega_{\rm DM}^{\rm eq} = 0.42$ is the dark matter abundance at the equality, then the primordial black holes constitute all of the dark matter.

CONCLUSION

The top quark mass is predicted in the Higgs inflation scenario by requesting the successful inflation as well as the production of enough primordial black holes (PBH) so that the dark matter problem is simultaneously resolved. The predicted mass is

$$M_t^{th} = blah... \;\; \text{(CHI + PBH as DM)}, \tag{34}$$

which should be compared with the current measurement coming mostly from the kinematics of $t\bar{t}$ events:

$$M_t^{exp} = 173 \pm 0.51 \pm 0.71 \;\; \text{(PDG 2016)}, \tag{35}$$

which is sensitive to the top quark mass used in the MC generator that is often mis-interpreted as the pole mass, but the theoretical uncertainty in this interpretation is still to be largely improved and hard to quantify at the moment.

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APPENDIX: RENORMALIZATION GROUP EQUATIONS

We present the two-loop RG equations of the Standard Model used in our analysis, following the arguments given in Ref. [22]. The beta functions are given by

$$
\begin{aligned}
\beta_\lambda =& \frac{1}{16\pi^2}\left[6(1+3s_\phi^2)\lambda^2 + 12\lambda y_t^2 - 6y_t^4 - 3\lambda(3g_2^2+g_1^2) + \frac{3}{8}\left(2g_2^4 + (g_2^2+g_1^2)^2\right)\right] \\
&+ \frac{1}{(16\pi^2)^2}\Bigg\{\frac{1}{48}\left[(912+3s_\phi)g_2^6 - (290-s_\phi)g_1^2g_2^4 - (560-s_\phi)g_1^4g_2^2 - (380-s_\phi)g_1^6\right] \\
&+ (38-8s_\phi)y_t^6 - y_t^4\left[\frac{8}{3}g_1^2 + 32g_3^2 + (12-117s_\phi+108s_\phi^2)\lambda\right] \\
&+ \lambda\bigg[-\frac{1}{8}(181+54s_\phi-162s_\phi^2)g_2^4 + \frac{1}{4}(3-18s_\phi+54s_\phi^2)g_1^2g_2^2 + \frac{1}{24}(90+377s_\phi+162s_\phi^2)g_1^4 \\
&+ (27+54s_\phi+27s_\phi^2)g_2^2\lambda + (9+18s_\phi+9s_\phi^2)g_1^2\lambda - (48+288s_\phi-324s_\phi^2+624s_\phi^3-324s_\phi^4)\lambda^2\bigg] \\
&+ y_t^2\left[-\frac{9}{4}g_2^4 + \frac{21}{2}g_1^2g_2^2 - \frac{19}{4}g_1^4 + \lambda\left(\frac{45}{2}g_2^2 + \frac{85}{6}g_1^2 + 80g_3^2 - (36+108s_\phi^2)\lambda\right)\right]\Bigg\}, \\
\beta_{g_1} =& \frac{1}{16\pi^2}\left[\frac{81+s_\phi}{12}g_1^3\right] + \frac{1}{(16\pi^2)^2}\left[\frac{199}{18}g_1^5 + \frac{9}{2}g_1^3g_2^2 + \frac{44}{3}g_1^3g_3^2 - \frac{17}{6}s_\phi g_1^3y_t^2\right], \\
\beta_{g_2} =& \frac{1}{16\pi^2}\left[\frac{s_\phi-39}{12}g_2^3\right] + \frac{1}{(16\pi^2)^2}\left[\frac{3}{2}g_1^2g_2^3 + \frac{35}{6}g_2^5 + 12g_2^3g_3^2 - \frac{3}{2}s_\phi g_2^3y_t^2\right], \\
\beta_{g_3} =& -\frac{7}{16\pi^2}g_3^3 + \frac{1}{(16\pi^2)^2}\left[\frac{11}{6}g_1^2g_3^3 + \frac{9}{2}g_2^2g_3^3 - 26g_3^5 - 2s_\phi g_3^3y_t^2\right], \\
\beta_{y_t} =& \frac{y_t}{16\pi^2}\left[\left(\frac{23}{6}+\frac{2}{3}s_\phi\right)y_t^2 - \left(8g_3^2 + \frac{9}{4}g_2^2 + \frac{17}{12}g_1^2\right)\right] \\
&+ \frac{y_t}{(16\pi^2)^2}\bigg[-\frac{23}{4}g_2^4 - \frac{3}{4}g_1^2g_2^2 + \frac{1187}{216}g_1^4 + 9g_2^2g_3^2 + \frac{19}{9}g_1^2g_3^2 - 108g_3^4 \\
&+ \left(\frac{225}{16}g_2^2 + \frac{131}{16}g_1^2 + 36g_3^2\right)s_\phi y_t^2 + 6\left(-2s_\phi^2y_t^4 - 2s_\phi^3y_t^2\lambda + s_\phi^2\lambda^2\right)\bigg],
\end{aligned}
\tag{A-1}
$$

where y_t is the top Yukawa coupling, $g_{1,2,3}$ are the SM gauge couplings, and

$$
s_\phi = \frac{1+\xi\phi^2/M_{\rm P}^2}{1+(1+6\xi)\xi\phi^2/M_{\rm P}^2}
\tag{A-2}
$$

is the suppression factor. For the nonminimal couplings we use one-loop beta functions,

$$
16\pi^2\beta_\xi = \left[6(1+s_\phi)\lambda + 6y_t^2 - \frac{3}{2}(3g_2^2+g_1^2)\right]\left(\xi+\frac{1}{6}\right).
\tag{A-3}
$$

Note that the beta functions are defined by, for any coupling constant g except the nonminimal couplings,

$$
\beta_g = (1+\gamma)\frac{dg}{dt},
\tag{A-4}
$$

where

$$
\begin{aligned}
\gamma =& -\frac{1}{16\pi^2}\left(\frac{9}{4}g_2^2 + \frac{3}{4}g_1^2 - 3y_t^2\right) + \frac{1}{(16\pi^2)^2}\bigg[\frac{271}{32}g_2^4 - \frac{9}{16}g_1^2g_2^2 - \frac{431}{96}s_\phi g_1^4 \\
&- \frac{5}{2}\left(\frac{9}{4}g_2^2 + \frac{17}{12}g_1^2 + 8g_3^2\right)y_t^2 + \frac{27}{4}s_\phi y_t^4 - 6s_\phi^3\lambda^2\bigg].
\end{aligned}
\tag{A-5}
$$

is the Higgs field anomalous dimension and $t = \ln(\mu/M_t)$.

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