Resuelve cada ED por medio de variación de parámetros

1- $y'' + y = \sec x$

$m^2 + 1 = 0$ $\qquad \frac{-b \pm \sqrt{b^2 - 4ac}}{2} = \frac{\sqrt{-4}}{2} = \pm i$

$y = C_1 \cos(x) + C_2 \sin(x)$

Para y_p: $y_1 = \cos(x)$ y $y_2 = \sin(x)$ $\qquad u_1 ? u_2 \Rightarrow u_1 y_1 + u_2 y_2$

$W \begin{vmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{vmatrix} = \cos^2(x) + \sin^2(x) = 1$

$W_1 \begin{vmatrix} 0 & \sin(x) \\ \sec(x) & \cos(x) \end{vmatrix} = -\frac{\sin(x)}{\cos(x)} = -\tan(x)$

$W_2 \begin{vmatrix} \cos(x) & 0 \\ -\sin(x) & \sec(x) \end{vmatrix} = \frac{\cos(x)}{\cos(x)} = 1$

$u_1' = \frac{W_1}{W} = \frac{\tan(x)}{1} = \tan(x) \qquad u_2' = \frac{W_2}{W} = 1$

$\int 1 = x + c$

$-\int \frac{\sin(x)}{\cos(x)} dx \qquad u = \cos(x)$
$\qquad du = -\sin(x) dx$

$+\int \frac{1}{u} du = \ln(u) = +\ln|\cos(x)| + c$

$y_p = +\ln|\cos(x)| \cos(x) + x \sin(x)$

$y = C_1 \cos(x) + C_2 \cos(x) + x\sin(x) + \cos(x) \ln|\cos(x)|$

$2.- \; y'' + y = \operatorname{sen}(x)$

$m^2+1 = \frac{-0 \pm \sqrt{(0)^2-4(1)(1)}}{2} = \frac{\pm\sqrt{-4}}{2} = \pm i$

$y = C_1\cos(x) + C_2\sin(x)$

Para y_p $\quad y_1 = \cos(x) \quad y_2 = \sin(x) \quad u_1 = ? \quad u_2 = ? \quad y_p = u_1y_1 + u_2y_2$

$W \begin{vmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{vmatrix} = \cos^2(x) - (-\sin^2(x)) = 1$

$W_1 \begin{vmatrix} 0 & \sin(x) \\ \sin(x) & \cos(x) \end{vmatrix} = -\sin^2(x)$

$W_2 \begin{vmatrix} \cos(x) & 0 \\ -\sin(x) & \sin(x) \end{vmatrix} = \cos(x)\sin(x)$

$u_1' = \frac{W_1}{W} = \frac{-\sin^2(x)}{1} = -\sin^2(x) \qquad u_2' = \frac{W_2}{W} = \frac{\cos(x)\sin(x)}{1}$

$u_1 = \int -\sin^2(x) \qquad \int \sin^n(x)dx \qquad u_2 = \int \cos(x)\sin(x) \qquad u = \sin(x)$

$u_1 = \frac{\cos(x)\sin(x)}{2} - \frac{1}{2}\int 1 \qquad u_2 = \int u\,du = \frac{u^2}{2} \qquad du = \cos(x)dx$

$u_1 = \frac{\cos(x)\sin(x) - x}{2} + C \qquad u_2 = \frac{\sin^2(x)}{2}$

$y_p = \frac{\cos^2(x)\sin(x) - x}{2} + \frac{\sin^3(x)}{2}$

$y = C_1\cos(x) + C_2\sin(x) + \frac{\cos^2(x)\sin(x) - x}{2} + \frac{\sin^3(x)}{2}$

3- $y'' + 3y' + 2y = \operatorname{Sen} e^x$

$m^2 + 3m + 2 = 0$ $m = -1$

$(m+1)(m+2)$ $m = -2$

$y_c = C_1 e^{-x} + C_2 e^{-2x}$

para y_p $y_1 = e^{-x}$ $y_2 = e^{-2x}$ $u_1 = ?$ $u_2 = ?$ $y_p = u_1 y_1 + u_2 y_2$

$$W \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -2e^{-3x} + e^{-3x} = -e^{-3x}$$

$$W_1 \begin{vmatrix} 0 & e^{-2x} \\ \operatorname{Sen}(e^x) & -2e^{-2x} \end{vmatrix} = -\operatorname{Sen}(e^x) e^{-2x}$$

$$W_2 \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & \operatorname{Sen}(e^x) \end{vmatrix} = e^{-x} \operatorname{Sin}(e^x)$$

$$u_1' = \frac{W_1}{W} = \frac{-\operatorname{Sen}(e^x) e^{-2x}}{-e^{-3x}} = \frac{\operatorname{Sen}(e^x)}{-e^{-x}} = \operatorname{Sen}(e^x)(-e^x)$$

$$u_2' = \frac{W_2}{W} = \frac{e^{-x} \sin(e^x)}{-e^{-3x}} \quad \frac{\sin(e^x)}{-e^{-2x}} = -e^{2x} \sin(e^x)$$

$u_1 = \int \operatorname{Sen}(e^x)(-e^x)$ $u = e^x$ $du = e^x dx$ $u_2 = \int -e^{2x} \sin(e^x)$ $u = e^x$ $du = e^x dx$

$u_1 = -\int \operatorname{Sen}(u)\, du = \cos u$ $u_2 = -\int u \sin(u)\, du$ $w = u$ $dw = 1$ $dv = \sin(u)$ $v = \cos(u)$

$u_1 = \cos(e^x)$ $-\left[u \cos(u) - \int \cos(u)\, du\right.$

$-u \cos(u) + \sin(u)\, du$

$y_p = e^{-x} \cos(e^x) +$

$e^{-2x}(-e^x \cos(e^x) + \sin(e^x))$ $-e^x \cos(e^x) + \sin(e^x)\, dx$

$y_p = e^{-x} \cos(e^x) - e^{-x} \cos(e^x) + e^{-2x} \sin(e^x) = e^{-2x} \sin(e^x)$

$y = C_1 e^{-x} + C_2 e^{-2x} + e^{-2x} \sin(e^x)$

4- $y'' + 2y' + y = e^{-t} \ln t$

$m^2 + 2m + 1 \qquad m = -1$

$(m+1)(m+1) \qquad m = -1$

$y_c = c_1 e^{-t} + c_2 t e^{-t}$

para y_p $\quad y_1 = e^{-t} \quad y_2 = t e^{-t} \quad u_1 = ? \; u_2 = ? \quad y_p = u_1 y_1 + u_2 y_2$

$$W \begin{vmatrix} e^{-t} & t e^{-t} \\ -e^{-t} & (1-t)e^{-t} \end{vmatrix} = e^{-t}\left[(1-t)e^{-t}\right] - (-e^{-t})(t e^{-t})$$

$$= e^{-2t}(1-t) + t e^{-2t} = e^{-2t}$$

$$W_1 \begin{vmatrix} 0 & t e^{-t} \\ e^{-t} \ln t & (1-t)e^{-t} \end{vmatrix} = -t \ln t \, e^{-2t}$$

$$W_2 \begin{vmatrix} e^{-t} & 0 \\ -e^{-t} & e^{-t} \ln t \end{vmatrix} \; e^{-2t} \ln t$$

$$u_1' = \frac{W_1}{W} = \frac{-t \ln t \, e^{-2t}}{e^{-2t}} = -t \ln t$$

$$u_2' = \frac{W_2}{W} = \frac{e^{-2t} \ln t}{e^{-2t}} = \ln t$$

$u = \int -t \ln(t)\, dt \qquad u = \ln t \;\; dv = t \;\; du = \frac{1}{t} \;\; v = \frac{t^2}{2}$ $\qquad u_2 = \int \ln(t) \qquad u = \ln(t) \;\; v = t \;\; du = \frac{1}{t} \;\; dv = 1$

$u = \frac{t^2 \ln(t)}{2} - \int \frac{t^2}{2} \frac{1}{t}\, dt \qquad u_2 = t \ln(t) - \int 1\, dt$

$u = \frac{t^2 \ln(t)}{2} - \frac{t^2}{4} \qquad u_2 = t \ln(t) - t$

$$y_p = u_1 y_1 + u_2 y_2 = \left(\frac{t^2 \ln(t)}{2} - \frac{t^2}{4}\right) e^{-t} + (t \ln(t) - t)\, t e^{-t}$$

$$y = c_1 e^{-t} + c_2 t e^{-t} + \frac{1}{2} t^2 e^{-t} \ln(t) - \frac{3t^2}{4} e^{-t}$$

5 - $3y'' - 6y' + 6y = e^x \sec x$

$3m^2 - 6m + 6$ $\quad \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = \frac{2 \pm i\sqrt{4}}{2} = 1 \pm i$

$m^2 - 2m + 2$

$y_c = e^x C_1 \cos(x) + e^x C_2 \sin(x)$

para $y_p = u_1 y_1 + u_2 y_2$, $y_1 = e^x \cos(x)$ $\quad y_2 = e^x \sin(x)$

$$W \begin{vmatrix} e^x \cos(x) & e^x \sin(x) \\ e^x \cos(x) - e^x \sin(x) & e^x \sin(x) + e^x \cos(x) \end{vmatrix}$$

$e^x \cos(x)(e^x \sin(x) + e^x \cos(x)) - e^x \sin(x)(e^x \cos(x) - e^x \sin(x))$

$= e^{2x}(\cos^2(x) + \sin^2(x)) = e^{2x}$

$$W_1 \begin{vmatrix} 0 & e^x \sin(x) \\ e^x \sec x & e^x \sin(x) + e^x \cos(x) \end{vmatrix} = -e^{2x} \frac{\sin(x)}{\cos(x)}$$

$$W_2 \begin{vmatrix} e^x \cos(x) & 0 \\ e^x \cos(x) - e^x \sin(x) & e^x \sec(x) \end{vmatrix} = e^{2x} \cos(x) \sec(x)$$

$u_1' = \frac{W_1}{W} = \frac{\cancel{e^{2x}} \sin(x) \sec(x)}{\cancel{e^{2x}}} = \frac{\sin(x)}{\cos(x)}$

$u_2' = \frac{W_2}{W} = \frac{\cancel{e^{2x}} \cos(x) \sec(x)}{\cancel{e^{2x}}} \quad \cos(x) \sec(x) = 1$

$u_1 = \int \frac{\sin(x)}{\cos(x)} dx \quad u = \cos(x)$ $\quad\quad u_2 = \int 1 = x$

$du = -\sin(x) dx$

$u_1 = -\int \frac{1}{u} du = -\ln|u| = -\ln|\cos(x)|$

$y_p = -\ln|\cos(x)|(e^x \cos(x)) + x e^x \sin(x)$

$y = e^x C_1 \cos(x) + e^x C_2 \sin(x) + x e^x \sin(x) + \ln|\cos(x)|(e^x \cos(x))$

6- $y'' + 3y' + 2y = \dfrac{1}{1+e^x}$ $\quad m^2+3m+2 \quad (m+1)(m+2) \quad m = -1 \quad m_2 = -2$

$y_c = C_1 e^{-x} + C_2 e^{-2x}$

para $y_p = u_1 y_1 + u_2 y_2$; $y_1 = e^{-x}$ $\quad y_2 = e^{-2x}$

$W \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -2e^{-3x} + e^{-3x} = -e^{-3x}$

$W_1 \begin{vmatrix} 0 & e^{-2x} \\ \frac{1}{1+e^x} & -2e^{-2x} \end{vmatrix} = \dfrac{-e^{-2x}}{1+e^x}$

$W_2 \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & \frac{1}{1+e^x} \end{vmatrix} = \dfrac{e^{-x}}{1+e^x}$

$u_1' = \dfrac{W_1}{W} = \dfrac{-e^{-2x}}{-e^{-3x}(1+e^x)} = \dfrac{e^x}{(1+e^x)}$

$u_2' = \dfrac{W_2}{W} = \dfrac{e^{-x}}{-e^{-3x}(1+e^x)} = \dfrac{-e^{2x}}{(1+e^x)}$

$u_1 = \int \dfrac{e^x}{(1+e^x)}$ $\quad u = (1+e^x)$ $\quad du = e^x\,dx$ $\qquad u_2 = \int \dfrac{-e^{2x}}{(1+e^x)}$ $\quad u = (1+e^x)$ $\quad du = e^x\,dx$

$u_1 = \int \frac{1}{u}\,du = \ln|u|$ $\qquad u_2 = \int \dfrac{u-1}{u} = \int 1\,du - \int \frac{1}{u}\,d$

$u_1 = \ln(1+e^x)$ $\qquad u_2 = u - \ln|u| = (1+e^x - \ln|1+e^x|)$

$y_p = \ln(1+e^x)e^{-x}(-1-e^x + \ln|1+e^x|)e^{-2x}$

$y = C_1 e^{-x} + C_2 e^{-2x} - e^{-2x} - e^{-x} \ln|1+e^x| e^{-2x} + \ln(1+e^x)e^{-x}$

7.- $y'' - y = \cosh x$

$(m^2-1)=0 \quad m_1=1 \quad m_2=-1$

$y_c = c_1 e^{x} + c_2 e^{-x}$

$y_p = u_1 y_1 + u_2 y_2 \quad y_1 = e^{x} \quad y_2 = e^{-x}$

$W \begin{vmatrix} e^{x} & e^{-x} \\ e^{x} & -e^{-x} \end{vmatrix} = -1-1 = -2$

$W_1 \begin{vmatrix} 0 & e^{-x} \\ \cosh x & -e^{-x} \end{vmatrix} = -e^{-x}\cosh x$

$W_2 \begin{vmatrix} e^{x} & 0 \\ e^{x} & \cosh x \end{vmatrix} = e^{x}\cosh x$

$u_1 = \dfrac{-e^{-x}\cosh x}{-2} = \dfrac{e^{-x}\cosh(x)}{2}$ $\qquad \cosh(x) = \dfrac{e^{x}+e^{-x}}{2}$

$u_2 = \dfrac{e^{x}\cosh(x)}{-2} = -\dfrac{e^{x}\cosh(x)}{2}$

$u_1 = \int \dfrac{e^{-x}\cosh(x)}{2} = \int \dfrac{\frac{e^{x}+e^{-x}}{2} e^{-x}}{2} = \int \dfrac{e^{0}+e^{-2x}}{4}\,dx = \int \frac{1}{4}\,dx + \int \frac{e^{-2x}}{4}\,dx$

$u_1 = \dfrac{x}{4} - \dfrac{e^{-2x}}{8}$

$u_2 = \int -\dfrac{e^{x}\cosh(x)}{2}\,dx = \int -\dfrac{e^{2x}+e^{0}}{4}\,dx = -\int \dfrac{e^{2x}}{4} - \int \frac{1}{4}\,dx = -\dfrac{e^{2x}}{8} - \dfrac{x}{4}$

$y_p = e^{x}\left(\dfrac{x}{4} - \dfrac{e^{-2x}}{8}\right) + e^{-x}\left(-\dfrac{e^{2x}}{8} - \dfrac{x}{4}\right)$

$= \frac{1}{4}xe^{x} - \dfrac{e^{-x}}{8} - \dfrac{e^{x}}{8} - \dfrac{xe^{-x}}{4} = \dfrac{x}{4}(e^{x} - e^{-x}) - \frac{1}{8}(e^{x} + e^{-x})$

$= \dfrac{x}{2}\sinh(x) - \dfrac{1}{4}\cosh(x)$

$c_1 e^{x} + c_2 e^{-x} + \dfrac{1}{2}x\sinh(x) - \dfrac{1}{4}\cosh(x)$

$4y''-y=xe^{x/2}$ $4m-1=\frac{-0\pm\sqrt{0+16}}{8}=\pm\frac{1}{2}$

$y_c=C_1e^{x/2}+C_2e^{-x/2}$

Para $y_p=y_1=e^{x/2}, y_2e^{-x/2}$

$w\begin{vmatrix} e^{x/2} & e^{-x/2} \\ \frac{e^{x/2}}{2} & -\frac{e^{-x/2}}{2} \end{vmatrix}=-\frac{1}{2}^{x}-\frac{1}{2}^{x}=-1^{x}$

$w_1\begin{vmatrix} 0 & e^{-x/2} \\ xe^{x/2} & -\frac{e^{-x/2}}{2} \end{vmatrix}=-x$

$w_2\begin{vmatrix} e^{x/2} & 0 \\ \frac{e^{x/2}}{2} & xe^{x/2} \end{vmatrix}=xe^{x}$

$u_1'=+x$ $u_2'=-xe^{x}$

$u_1=\int x=\frac{x^2}{2}$ $u_2=\int -xe^{x}$ $u=x$ $dv=e^{x}$
$du=1$ $v=e^{x}$

$-[xe^{x}-\int e^{x}\,dx]=-xe^{x}+e^{x}$

$y_p=e^{x/2}\frac{x^2}{2}+e^{-x/2}[-xe^{x}+e^{x}]=e^{x/2}\frac{x^2}{2}+e^{x/2}-xe^{x/2}$

$y=C_1e^{x/2}+C_2e^{x/2}+e^{x/2}\frac{x^2}{2}+e^{x/2}-xe^{x/2}=0$

$C_1+C_2+1=0$ $C_1=-1-C_2$

$y'=\frac{C_1e^{x/2}}{2}+\frac{C_2e^{x/2}}{2}+\frac{e^{x/2}}{2}-xe^{x/2}+\frac{x(x+4)e^{x/2}}{4}-$

$y'=\frac{C_1}{2}+\frac{C_2}{2}+\frac{1}{2}-0-0=0$ $C_2=$

$\frac{-C_2-1}{2}+\frac{C_2}{2}+\frac{1}{2}=0$ $\frac{-C_2-1+C_2}{2}=\frac{1}{2}$ $C_2-1+C_2=1$
$C_2+C_2=0$

$2.-\ y'' + 2y' - 8y = 2e^{-2x} - e^{-x} \quad y(0)=1 \quad y'(0)=0$

$m^2 + 2m - 8 = 0, \quad \frac{-2 \pm \sqrt{2^2 - 4(1)(8)}}{2(1)} \quad \frac{-2 \pm \sqrt{36}}{2} = \frac{-2 \pm 6}{2} = -1 \pm 3$

$y_c = c_1 e^{2x} + c_2 e^{-4x} \qquad m_1 = 2, \ m_2 = -4$

para $y_p = u_1 y_1 + u_2 y_2 \quad y_1 = e^{2x} \quad y_2 = e^{-4x}$

$$W = \begin{vmatrix} e^{2x} & e^{-4x} \\ 2e^{2x} & -4e^{-4x} \end{vmatrix} = -4e^{-2x} - 2e^{-2x} = -6e^{-2x}$$

$$W_1 \begin{vmatrix} 0 & e^{-4x} \\ 2e^{-2x} - e^{-x} & -4e^{-4x} \end{vmatrix} = -2e^{-6x} + e^{-5x}$$

$$W_2 \begin{vmatrix} e^{2x} & 0 \\ 2e^{2x} & 2e^{-2x} - e^{-x} \end{vmatrix} = 2 - e^{x}$$

$u_1' = \frac{W_1}{W} = \frac{-2e^{-6x} + e^{-5x}}{-6e^{-2x}} = \frac{2e^{-4x} + e^{-3x}}{6}$

$u_2' = \frac{W_2}{W} = \frac{2 - e^{x}}{-6e^{-2x}}$

$u^1 = \frac{2}{6}\left[\int e^{-4x} + e^{-3x}\right] = \frac{2}{6}\left[\frac{e^{-4x}}{4} + \frac{e^{-3x}}{3}\right] = \frac{2}{24}e^{-4x} + \frac{e^{-3x}}{18}$

$u^2 = \int \frac{2 - e^{x}}{-6e^{-2x}} = \frac{1}{6}\int (2 - e^{x}) e^{2x} \quad u = e^{x} \quad du = e^{x} = \frac{1}{6}\int (2u - 1)\,du = u^2 - u$

$u^2 = \frac{e^{x}}{6} - \frac{e^{2x}}{6}$

$y_p = \left[\frac{2}{24}e^{-4x} + \frac{e^{-3x}}{18}\right]e^{2x} + \left[\frac{e^{x}}{6} - \frac{e^{2x}}{6}\right]e^{-4x} = \frac{2e^{-2x}}{24} + \frac{e^{-x}}{18} + \frac{e^{-3x}}{6} - \frac{e^{-2x}}{6}$

$y = C_1 e^{-4x} + C_2 e^{2x} - \frac{1}{4}e^{-2x} + \frac{1}{4}e^{-x}$

$c_1 + c_2 - \frac{1}{4} + \frac{1}{9} = 1$ $\qquad c_1 + c_2 = \frac{1}{4} - \frac{1}{9} + 1$

$y' = -4c_1 e^{-4x} + 2c_2 e^{2x} + \frac{1}{2} e^{-2x} - \frac{1}{9} e^{-x}$

$-4c_1 + 2c_2 + \frac{1}{2} - \frac{1}{9} = 0$ $\qquad -4c_1 + 2c_2 = \frac{-7}{18}$

$c_1 + c_2 = \frac{5}{36}$ $\qquad c_1 = \frac{5}{36} - c_2$

$-4c_1 + 2c_2 = \frac{-7}{18}$ $\qquad -4\left(\frac{5}{36} - c_2\right) + 2c_2 = \frac{-7}{18}$ $\qquad c_2 = \frac{1}{36}$

$-\frac{20}{36} - 4c_2 + 2c_2 = \frac{-7}{18}$

$\frac{20}{36} - 2c_2 = \frac{-7}{18}$

$-2c_2 = \frac{-7}{18} + \frac{20}{36}$

$c_1 = \frac{5}{36} - \frac{2}{6} = \frac{7}{36}$ $\qquad -2c_2 = \frac{6}{36} = \frac{1}{6}$

$-c_2 = \frac{2}{6}$

$y = \frac{7}{36} e^{-4x} + \frac{25}{36} e^{2x} - \frac{1}{4} e^{-2x} + \frac{1}{9} e^{-x}$