solución general a las siguientes EDs mediante coeficientes indeterminados

$y'' + y = 2x\,\text{sen}\,x$

Para y_c

$m^2 + 1$

$m = \frac{-0 \pm \sqrt{0-4}}{2}$ $m = \pm i$

$y_c = e^0[C_1\cos(x) + C_2\sin(x)] = C_1\cos(x) + C_2\sin(x)$

A B

Para $y_p =$

$y_p\ (Ax+B)\,\text{sen}(x) + (Cx+D)\cos(x)$

$y_p' = A\,\text{sen}(x) + Ax\cos(x) + B\cos(x) + C\cos(x) - Cx\sin(x) - D\sin(x)$

$(Ax+B)\cos(x)$ $(Cx+D)\cos(x)$

$y_p'' = A\cos(x) + A\cos(x) - Ax\sin(x) - B\sin(x) - C\sin(x) - C\sin x -$

$Cx\cos(x) - D\cos(x)$

$2A\cos(x) - Ax\sin(x) - B\sin(x) - 2C\sin(x) - Cx\cos(x) - D\cos(x)$

$+ A\,\text{sen}(x) + Ax\cos(x) + B\cos(x) + C\cos(x) - Cx\sin(x) - D\sin(x)$

$= 2x\,\text{sen}\,x$

$(2A + B + C - D + Ax - Cx)\cos x + (A - B - 2C - D - Ax - Cx)\sin(x) =$

$2x\,\text{sen}\,x$

$A = C$ $-A - A = 2$

$A + C - A - C = 2$ $-2A = 2$

$-2C = 2$ $A = -1$

$-A - C = 2$ $C = 1$ $C = -1$

$2(1) + B + (1) - D = 0$ $3 + B - D = 0$ $B - D = 3$

$(1) - B - 2(1) - D = 1$ $-2 - B - D = 1$ $B = 3 + D$

$-2 - 3 - D - D = 1$

$-5 - 2D = 1$

$-2D = 6$

$D = -3$

$2\ y'' - 2y' + 5y = e^x \cos 2x$

$m^2 - 2m + 5$ $\quad m = \frac{+2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm \sqrt{-16}}{2}$

$m = 1 \pm 2i$

$y_c = e^x [C_1 \sin(2x) + C_2 \cos(2x)]$

$y_p = Ae^x \cos(2x) + Be^x \sin(2x)$

$y_p' = Ae^x \cos(2x) - 2Ae^x \sin(2x) + Be^x \sin(2x) + 2Be^x \cos(2x)$

$y_p' = e^x [(A+2B)\cos(2x) + (B-2A)\sin(2x)]$

$y_p'' = Ae^x \cos(2x) - 2Ae^x \sin(2x) - 2Ae^x \sin(2x) - 4Ae^x \cos(2x) +$

$Be^x \sin(2x) + 2Be^x \cos(2x) + 2Be^x \cos(2x) - 4Be^x \sin(2x)$

$y_p'' = -3Ae^x \cos(2x) - 4Ae^x \sin(2x) - 3Be^x \sin(2x) + 4Be^x \cos(2x)$

$y_p' =$

$-3Ae^x \cos(2x) - 4Ae^x \sin(2x) - 3Be^x \sin(2x) + 4Be^x \cos(2x) -$
$2(Ae^x \cos(2x) - 2Ae^x \sin(2x) + Be^x \sin(2x) + 2Be^x \cos(2x)) +$
$5(Ae^x \cos(2x) + Be^x \sin(2x)) = e^x \cos 2x$

3 3

$0 = e^x \cos 2x$

3:- $y'' + 2y' + y = \operatorname{sen} x + 3\cos 2x$

$m^2 + 2m + 1 \qquad m = \dfrac{-2 \pm \sqrt{4-4}}{2} = -1$

$Y_c = C_1 e^{-x} + C_2 x e^{-x}$

$Y_p = A\cos(x) + B\sin(x) + C\cos(2x) + D\sin(2x)$

$Y_p' = -A\sin(x) + B\cos(x) - 2C\sin(2x) + 2D\cos(2x)$

$Y_p'' = -A\cos(x) - B\sin(x) - 4C\cos(2x) - 4D\sin(2x)$

$-3 \qquad -3$

$-A\cos(x) - B\sin(x) - 4C\cos(2x) - 4D\sin(2x) + 2[-A\sin(x) + B\cos(x) + 2C\sin(2x) + 2D\cos(2x)] + A\cos(x) + B\sin(x) + C\cos(2x) + D\sin(2x)$

$-2A\sin(x) + 2B\cos(x) - 4C\sin(2x) + 4D\cos(2x) - 3C\cos(2x) - 3D\sin(2x) = \sin(x) + 3\cos(2x)$

$\cos(x): 2B = 0 \quad B = 0$

$\sin(x): -2A = 1 \quad A = -\dfrac{1}{2}$

$\cos(2x): -3C + 4D = 3$

$\sin(2x): -4C - 3D = 0 \quad D = -\dfrac{4}{3}C$

$-3C + 4\left(-\dfrac{4}{3}C\right) = 3$

$-3C - \dfrac{16}{3}C = 3$

$-\dfrac{25}{3}C = 3$

$C = -\dfrac{9}{25}$

$D = -\dfrac{4}{3}\left(-\dfrac{9}{25}\right)$

$= \dfrac{36}{75}$

$= \dfrac{12}{25}$

$Y_p = -\dfrac{1}{2}\cos(x) - \dfrac{9}{25}\cos(2x) + \dfrac{12}{25}\sin(2x)$

$y = C_1 e^{-x} + C_2 x e^{-x} - \dfrac{1}{2}\cos(x) - \dfrac{9}{25}\cos(2x) + \dfrac{12}{25}\sin(2x)$

4.- $y''' - 6y'' = 3 - \cos x$

$m^3 - 6m^2$

$m^2(m-6)$ $m_1 = 0$ $m_2 = 0$ $m_3 = 6$

$y_c = C_1 + C_2 x + C_3 e^{6x}$

$y_p = A + B\cos(x) + C\sin(x)$

$y_p' = -B\sin(x) + C\cos(x)$

$y_p'' = -B\cos(x) - C\sin(x)$

$y_p''' = B\sin(x) - C\cos(x)$

$B\sin(x) - C\cos(x) + 6B\cos(x) + 6C\sin(x) = 3 - \cos x$

$\sin(x): B + 6C = 0 \quad B = -6C$

$\cos(x): -C + 6B = -1 \quad -C + 6(-6C) = -1 \quad B = -6\left(\frac{1}{37}\right) = -\frac{6}{37}$

$-37C = -1$

$C = \frac{1}{37}$

$y_p = 3 - \frac{6}{37}\cos(x) + \frac{1}{37}\sin(x)$

$y = C_1 + C_2 x + C_3 e^{6x} + 3 - \frac{6}{37}\cos(x) + \frac{1}{37}\sin(x)$

5.- $y''' - 3y'' + 3y' - y = x - 4e^x$

$m^3 - 3m^2 + 3m - 1$

$(m-1)^3 = 0 \qquad m = 1$

$y_c = c_1 e^x + c_2 x e^x + c_3 x^2 e^x$

$\cancel{y_p = Ax + Be^x}$

$y_p = Ax + B + Cx^3 e^x$

$y_p' = A + 3Cx^2 e^x + Cx^3 e^x$

$y_p'' = 6Cxe^x + 3Cx^2e^x + 3Cx^2e^x + Cx^3e^x = 6Cxe^x + 6Cx^2e^x + Cx^3e^x$

$y_p''' = 6Ce^x + 6Cxe^x + 6Cxe^x + 3Cx^2e^x + 6Cxe^x + 3Cx^2e^x + 3Cx^2e^x + Cx^3e^x$

$y_p''' = 6Ce^x + 18Cxe^x + 9Cx^2e^x + Cx^3e^x$

$6Ce^x + \cancel{18Cxe^x} + 9Cx^2e^x + \cancel{Cx^3e^x} - \cancel{18Cxe^x} - 18Cx^2e^x - 3Cx^3e^x + 3A + 9Cx^2e^x + 3Cx^3e^x - Ax - B - \cancel{Cx^3e^x}$

$6Ce^x + 3A - Ax - B = x - 4e^x$

$e^x: 6C = -4 \quad C = -\frac{2}{3}$

$3A - B = 0 \quad B = 3A \quad B = 3(-1) \quad B = -3$

$x: -A = 1 \quad A = -1$

$y_p = -x - 3 - \frac{2}{3}x^3 e^x$

$y = c_1 e^x + c_2 x e^x + c_3 x^2 e^x - x - 3 - \frac{2}{3} x^3 e^x$

Valores en la frontera mediante coeficientes indeterminados

r- $y'' + y = x^2 + 1 \quad y(0) = 5,\ y(1) = 0$

$m^2 + 1$

$m = \frac{0 \pm \sqrt{0-4}}{2} \quad m = \pm i$

$y_c = C_1 \cos(x) + C_2 \sin(x)$

$y_p = Ax^2 + Bx + C$

$y_p' = 2Ax + B$

$y_p'' = 2A$

$2A + Ax^2 + Bx + C = x^2 + 1$

$x^2: A = 1$

$x: B = 0$

$2A + C = 1 \quad C = 1 - 2A = 1 - 2(1) = -1$

$1x^2 + 0 - 1 = x^2 - 1$

$y = C_1 \cos(x) + C_2 \sin(x) + x^2 - 1$

$C_1 \cos(0) + C_2 \sin(0) + (0)^2 - 1 = 5$

$C_1 - 1 = 5 \quad C_1 = 6$

$6\cos(1) + C_2 \sin(1) + (1)^2 - 1 = 0$

$6\cos(1) + C_2 \sin(1) + 0 = 0$

$C_2 \sin(1) = -6\cos(1)$

$C_2 = \frac{-6\cos(1)}{\sin(1)} = -6\cot(1) = -3.8$

$y = 6\cos x - 6\cot 1)\sin(x) + x^2 - 1$

2- $y'' + 3y = 6x, \quad y(0) = 0, \quad y(1) + y'(1) = 0$

$m^2 + 3 \quad m = \frac{0 \pm \sqrt{0 - 4(3)}}{2} = \pm i\sqrt{3}$

$y_c = C_1 \cos(\sqrt{3}x) + C_2 \sin(\sqrt{3}x)$

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

$3Ax + 3B = 6x$

$x: 3A = 6, \; A = 2$

constante: $3B = 0, \; B = 0$

$y_p = 2x$

$y = C_1 \cos(\sqrt{3}x) + C_2 \sin(\sqrt{3}x) + 2x$

$C_1 \cos(0) + C_2 \sin(0) + 2(0) = 0$

$C_1 \cos(0) = 0 \quad C_1 = 0$

$y' = \frac{\sqrt{3}\, C_2 \cos(\sqrt{3}x)}{2\sqrt{x}} + 2$

$\frac{\sqrt{3}\, C_2 \cos(\sqrt{3(1)})}{2\sqrt{1}} + 2 + C_2 \sin(\sqrt{3(1)}) + 2(1) =$

$2C_2 \sin(\sqrt{3}) + 4 = -\frac{\sqrt{3}\, C_2 \cos(\sqrt{3}(1))}{2} - 4$

$2 \sin(\sqrt{3}) + 4$

$2\sin(\sqrt{3}) + \frac{4}{C_2} = -\frac{\sqrt{3}\cos(\sqrt{3})}{2}$

$\frac{4}{} = \left(-\frac{\sqrt{3}\cos(\sqrt{3})}{2} - 2\sin(\sqrt{3})\right) c^2 =$

$c^2 = \frac{8}{-\sqrt{3}\cos(\sqrt{3}) - 2\sin(\sqrt{3})}$

$y = \frac{-4 \sin\sqrt{x}}{-\sqrt{3}\cos(\sqrt{3}) - \sin(\sqrt{3})}$

Encuentre la solución a las siguientes PVIs mediante el método de coeficientes indeterminados

1. $5y'' + y' = -6x$, $y(0) = 0$, $y'(0) = -10$

$5m^2 + m = 0$

$m(5m+1) = 0$ $\quad m_1 = 0$ $\quad m_2 = -\frac{1}{5}$

$y_c = C_1 + C_2 e^{-\frac{x}{5}}$

$y_p = Ax + B$

$y_p' = A$

$y_p'' = 0$

$5(0) + A = -6x$ $\quad A = -6x$ $\quad y_p = -6x$

$y = C_1 + C_2 e^{-\frac{x}{5}} - 6x = 0$ $\quad y' = -\frac{C_2 e^{-\frac{x}{5}}}{5} - 6$

$y\ C_1 + C_2 e^{-\frac{0}{5}} - 6(0) = 0$ $\quad -\frac{C_2 e^{-\frac{0}{5}}}{5} - 6 = -10$

$C_1 + C_2 = 0$

$C_1 = -C_2$ $\quad -\frac{C_2}{5} - 6 = -10$

$y = 20 - 20e^{-\frac{x}{5}} - 6x = 0$ $\quad -\frac{C_2}{5} = -4$

$-C_2 = -20$

$C_2 = 20$

2: $y'' + 4y' + 5y = 35e^{-4x}$ $\quad y(0) = -3, \; y'(0) = 1$

$m^2 + 4m + 5 = 0 \qquad m = \frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm \frac{\sqrt{-4}}{2} = -2 \pm i$

$r = -2+i, \; r = -2-i$

$y_c = C_1 e^{-2x}\cos(x) + C_2 e^{-2x}\sin(x)$

$y_p = Ae^{-4x}$

$y_p' = -4Ae^{-4x}$

$y_p'' = 16Ae^{-4x}$

$16Ae^{-4x} - 16Ae^{-4x} + 5Ae^{-4x} = 35e^{-4x}$

$5Ae^{-4x} = 35e^{-4x}$

$e^{-4x}: \; 5A = 35, \; A = \frac{35}{5}, \; A = 7$

$y_p = 7e^{-4x}$

$y = C_1 e^{-2x}\cos(x) + C_2 e^{-2x}\sin(x) + 7e^{-4x}$

$C_1 e^{0}\cos(0) + C_2 e^{0}\sin(0) + 7e^{0} = -3$

$C_1 + 7 = -3 \qquad C_1 = -10$

$y' = -2C_1 e^{-2x}\cos(x) - C_1 e^{-2x}\sin(x) - 2C_2 e^{-2x}\sin(x) + C_2 e^{-2x}\cos(x) - 28e^{-4x}$

$y' = -2(-10)e^{-2(0)}\cos(0) - (-10)e^{-2(0)}\sin(0) - 2C_2 e^{-2(0)}\sin(0) + C_2 e^{-2(0)}\cos(0) - 28e^{-4(0)} = 1$

$y' = 20 + C_2 - 28 = 1 \qquad C_2 = 9$

$y = -10e^{-2x}\cos x + 9e^{-2x}\sin(x) + 7e^{-4x}$

3.- $\frac{d^2x}{dt^2} + \omega^2 x = F_0 \operatorname{sen}\omega t,\ x(0)=0,\ x'(0)$

$x'' + \omega^2 x = F_0 \operatorname{sen}\omega t$

$m^2 + \omega^2 = 0 \qquad m = \pm\sqrt{-\omega^2} = \pm i\omega$

$x_c = C_1 [\cos(\omega t) + C_2 \operatorname{sen}(\omega t)]$

$x_p = (A \operatorname{sen}(\omega t) + B\cos(\omega t))t$

$x_p = At\operatorname{sen}(\omega t) + Bt\cos(\omega t)$

$x_p' = A\operatorname{sen}(\omega t) + At\omega\cos(\omega t) + B\cos(\omega t) - Bt\omega\sin(\omega t)$

$x_p'' = A\omega\cos(\omega t) + A\omega\cos(\omega t) - At\omega^2\operatorname{sen}(\omega t) - B\omega\operatorname{sen}(\omega t)$
$- B\omega\operatorname{sen}(\omega t) - Bt\omega^2\cos(\omega t) = 2A\omega\cos(\omega t) - 2B\operatorname{sen}(\omega t)$
$- At\omega^2\operatorname{sen}(\omega t) - Bt\omega^2\cos(\omega t)$

$x_p'' + \omega^2 x_p = F_0 \operatorname{sen}(\omega t) \qquad A = 0 \qquad B = \frac{F_0}{2\omega}$

$2A\omega\cos(\omega t) - 2B\operatorname{sen}(\omega t) - At\omega^2\operatorname{sen}(\omega t) - Bt\omega^2\cos(\omega t) +$
$\omega^2(At\operatorname{sen}(\omega t) + Bt\cos(\omega t))$

$2A\omega\cos(\omega t) - 2B\omega\operatorname{sen}(\omega t) = F_0\operatorname{sen}\omega t$

$\operatorname{sen}\omega t = -2B\omega = F_0 \qquad B = \frac{F_0}{2\omega}$

$\cos(\omega t)\ 2A\omega = 0$

$x = C_1\cos(\omega t) + C_2\operatorname{sen}(\omega t) - F_0\operatorname{sen}(\omega t) = F\ \operatorname{sen}(\omega t)$

$C_1 = 0$

$x' = -C_1\omega\sin(\omega t) + C_2\omega\cos(\omega t) - F_0\omega\cos(\omega t)$

0

$C_2\omega - F_0\omega = 0$

$C_2\omega = F_0\omega \qquad C_2 = F_0$

$x = F_0\operatorname{sen}(\omega t) - F_0\operatorname{sen}(\omega t)$