TEMA FECHA

Encuentre la solución general a las ecuaciones de Cauchy-Euler

1- $xy^{(4)}+6y'''=0$ $\quad Y=x^m \quad y'=mx^{m-1} \quad y''=m(m-1)x^{m-2} \quad y'''=m(m-1)(m-2)x^{m-3}$

$y^{(4)}=m(m-1)(m-2)(m-3)x^{m-4}$

$x\,m(m-1)(m-2)(m-3)x^{m-4}+6m(m-1)(m-2)x^{m-3} \qquad x\neq 0$

$m(m-1)(m-2)(m-3)x^{m-3}+6m(m-1)(m-2)x^{m-3}$

$=m(m-1)(m-2)[(m-3)+6]=0$

$=m(m-1)(m-2)(m+3)=0$

$m=0,\ m=1,\ m=2,\ m=-3$

$y_c=c_1+c_2x^1+c_3x^2+c_4x^{-3}$

2- $x^3y'''-6y=0$

$x^3m(m-1)(m-2)x^{m-3}-6x^m=0$

$m(m-1)(m-2)x^m-6x^m=0$

$m(m-1)(m-2)-6=0$

$m(m^2-3m+2)=m^3-3m^2+2m$

$m^3-3m^2+2m-6=0$

$m=3$

$$\begin{array}{cccc|c} 1 & -3 & +2 & -6 & 3 \\ & 3 & 0 & 6 & \\ \hline 1 & 0 & 2 & 0 & \end{array}$$

$m^2+2 \qquad \dfrac{-0\pm\sqrt{0^2-4(1)(2)}}{2(1)}=\dfrac{\sqrt{-8}}{2}$

$=\dfrac{\sqrt{-2}\ \sqrt{4}}{2}=\dfrac{\sqrt{-2}(2)}{2}=i\sqrt{2}$

$m_1=3,\ m_2=+i\sqrt{2}\ \ m_3=-i\sqrt{2}$

$y_c=c_1x^3+c_2\cos(\sqrt{2}\ln(x))+c_3\sin(\sqrt{2}\ln(x))$

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3.- $3x^2y''+6xy'+y=0$

$x^m[3m^2+m(6-3)+1]=0$

$x^m(3m^2+3m+1)=0$

$\frac{-3\pm\sqrt{(3)^2-4(3)(1)}}{2(3)}=\frac{-3\pm\sqrt{9-12}}{6}$

$\frac{-3\pm\sqrt{-3}}{6}=\frac{-1}{2}\pm\frac{i\sqrt{3}}{6}$

$y_c=x^{-1/2}\left[c_1\cos\left(\frac{1}{6}\sqrt{3}\ln x\right)+c_2\operatorname{sen}\left(\frac{1}{6}\sqrt{3}\ln x\right)\right]$

4.- $x^2y''+5xy'+4y=0$

$m^2+m(5-1)+4=0$

$m^2+4m+4=0 \qquad m_1=-2$

$(m+2)\,(m+2)=0 \qquad m_2=-2$

$y_c=c_1x^{-2}+c_2x^{-2}\ln x$

5.- $25x^2y''+25xy'+y=0$

$25m^2+m(25-25)+1$

$25m^2+1$

$\frac{-0\pm\sqrt{0^2-4(25)(1)}}{2(25)}=\frac{\pm\sqrt{-100}}{50}$

$=\pm i\,\frac{10}{50}=\pm i\,\frac{1}{5}$

$y_c=c_1\cos\left(\frac{1}{5}\ln(x)\right)+c_2\cos\left(\frac{1}{5}\ln(x)\right)$

TEMA FECHA

Encentre la solución a los siguientes ecuaciones diferenciales que están sujetas a condiciones iniciales o en la frontera

1- $x^2y'' + 3xy' = 0, \quad y(1) = 0, \ y'(1) = 4 \quad y' = mx^{m-1} \quad y'' = m(m-1)x^{m-2}$

$x^2 m(m-1)x^{m-2} + 3x m x^{m-1}$

$m(m-1)x^m + 3mx^m$

$(m^2 - 1m)x^m + 3mx^m$

$x^m(m^2 + 2m)$

$$\frac{-2 \pm \sqrt{2^2 - 4(1)(0)}}{2} \quad \frac{-2 \pm 2}{2} = 0, -2$$

$m^2 + 2m = 0$

$m = -1$

$y_c = c_1 x^0 + c_2 x^{-2} = c_1 + c_2 x^{-2}$

$c_1 + c_2(1)^{-2} = 0 \quad c_1 + \frac{c_2}{1} = 0 \quad c_1 = -c_2$

$y_c' = -2c_2 x^{-3} = 4$

$\frac{-2c_2}{(1)^3} = 4 \quad -2c_2 = 4 \quad c_2 = -2$

$y_c = 2 - 2x^{-2}$

$y_c = -4x \ln x$

TEMA

FECHA

2.- $4x^2y''+y=0,\ y(-1)=2,\ y'(-1)=4$

$4m^2+m(0-4)+1$

$4m^2-4m+1$

$\frac{4\pm\sqrt{(-4)^2-4(4)(1)}}{2(4)}=\frac{4\pm\sqrt{16-16}}{8}$

$m=\frac{1}{2}$

$y_c=c_1x^{1/2}+c_2x^{1/2}\ln(x)$

$c_1(-x)^{1/2}+c_2(-x)^{1/2}\ln(-x)$

$c_1(1)^{1/2}+c_2(1)^{1/2}\ln(1)=2$

$c_1=2$

$y'=\frac{-c_1}{2(-x)^{1/2}}+\left(\frac{-c_2}{2(-x)^{1/2}}\ln(x)+c_2(-x)^{1/2}\frac{1}{x}\right)$

$=\frac{-2}{2(1)^{1/2}}+\left(\cancel{\frac{-c_2}{2(1)^{1/2}}\ln(1)}_{0}+c_2(1)^{1/2}\frac{1}{1}\right)$

$=-1+c_2=4\quad c_2=5$

$y_c=2x^{1/2}+5(-x)^{1/2}\ln(-x)$

TEMA

FECHA

Encuentre la solución general a las siguientes ecuaciones diferenciales, utilice variación de parámetros para la solución particular

1. $x^2y'' - xy' + y = 2x$

$m^2 + m(-1-1) + 1$ $\quad m_1 = 1$, $m_2 = 1$

$m^2 - 2m + 1 \quad (m-1)(m-1)$

$y_c = C_1 x + C_2 x \ln x$

Para $y_p = u_1 y_1 + u_2 y_2 \quad y_1 = x \quad y_2 = x\ln(x)$

$w \begin{vmatrix} x & x\ln(x) \\ 1 & \ln(x)+1 \end{vmatrix} x(\ln(x)+1) - x\ln(x) = x\cancel{\ln(x)} + x - x\cancel{\ln(x)} = x$

$w_1 \begin{vmatrix} 0 & x\ln(x) \\ \frac{2x}{x^2} & \ln(x)+1 \end{vmatrix} -2x^2\ln(x)$

$w_2 \begin{vmatrix} x & 0 \\ 1 & \frac{2x}{x} \end{vmatrix} = 2x^2$

$u_1' = \frac{w_1}{w} = \frac{-2x^2\ln(x)}{x} = \frac{-2\ln(x)}{x} \qquad u_2' = \frac{2x^2}{x} = 2x$

$u_1 = \int \frac{-2\ln(x)}{x}dx \quad u = \ln(x) \quad du = \frac{1}{x}dx \qquad u_2 = \int \frac{2x}{x} = 2\ln(x)x^2$

$du = \frac{1}{x} \qquad v = x^2$

$-2\int u\, du = \frac{-2u^2}{2} = -\ln^2(x)$

$\int \left(\frac{x^2\ln(x)}{2} - \int \frac{x^2}{2}dx\right) = \frac{x^2\ln(x)}{2} - \frac{x^2}{4}$

$y_p = x\ln(x) - \frac{x^2}{2}$

$y_p = -x\ln^2(x) + 2x\ln^2(x) + x\ln^2(x)\ln(x) - \frac{x^3}{2} + x^2\ln(x)$

$y = C_1 x + C_2 x\ln x + x\ln^2(x)$

TEMA

FECHA $\frac{\ln(x)x}{2x^{-1}}$ $\frac{\ln(x)x^2}{2}$

$\frac{\ln(x)x^{-1}}{2x^{-1}} - \ln(x)$

$2.- x^2y'' + xy' - y = \ln x$

$m^2 + m(1-1) + (-1)$

$m^2 - 1 \quad (m-1)(m+1) \quad m_1 = 1 \quad m_2 = -1$

$y_c = C_1 x^{-1} + C_2 x$

Para $y_p = U_1 y_1 + U_2 y_2 = \quad y_1 = x^{-1} \quad y_2 = x$

$w \begin{vmatrix} x^{-1} & x \\ -x^{-2} & 1 \end{vmatrix} \quad x^{-1} + x^{-1} = 2x^{-1}$

$w_1 \begin{vmatrix} 0 & x \\ \frac{\ln x}{x^2} & 1 \end{vmatrix} = -\frac{x\ln(x)}{x}$

$w_2 \begin{vmatrix} x^{-1} & 0 \\ -x^{-2} & \frac{\ln x}{x^2} \end{vmatrix} = \frac{\ln(x)}{x^3}$

$u_1' = \frac{w'}{W} = \frac{\ln(x)x^{-1}}{2x^{-1}} = \frac{\ln(x)x^{2}}{2}$ $\quad u_2' = \frac{w^2}{W} = \frac{\ln(x)x^{-3}}{2x^{-1}} = \frac{\ln(x)x^{-2}}{2}$

$= \frac{\ln(x)}{2x^{-2}}$

$U_1 = \int \frac{\ln(x)}{2} = \frac{1}{2}\int \ln(x) \quad w = \ln(x) \quad v = x$

$dw = \frac{1}{x} \quad dv = 1$

$u_1 = x\ln(x) - \int 1dx = \frac{x\ln(x) - x}{2}$

$u_2 = \frac{1}{2}\int \frac{\ln(x)}{x^2} \quad w = \ln(x) \quad v = \frac{-1}{x}$

$dw = \frac{1}{x} \quad dv = \frac{1}{x^2}$

$-\frac{\ln(x)}{x} - \int -\frac{1}{x^2}dx$

$\frac{\ln(x)}{x} + \frac{1}{x} = \frac{\ln(x)+1}{x}$

$y_p = \frac{x\ln(x) - x}{2}(x^{-1}) + \frac{\ln(x)+1}{x}(x)$

$y_p = \frac{-\ln(x)+1}{2} + \ln(x) + 1$

$y = C_1 x^{-1} + C_2 x + \ln(x)$

TEMA FECHA

3.- $xy'' - 4y' = x^4$

$m^2 + m(-4-1) + 0$ $\qquad \frac{5 \pm \sqrt{(-5)^2 - 4(1)(0)}}{2(1)} = \frac{5 \pm \sqrt{25}}{2} \quad \frac{5 \pm 5}{2}$

$m^2 + 5m + 0$ $\qquad 0, 5$

$y_c = C_1 + C_2 x^5$

para $y_p = u_1 y_1 + u_2 y_2 \quad y_1 = 1 + y_2 = x^5$

$w \begin{vmatrix} 1 & x^5 \\ 0 & 5x^4 \end{vmatrix} = 5x^4$

$w_1 \begin{vmatrix} 0 & x^5 \\ x^4 & 5x^4 \end{vmatrix} = x^9 \quad 0\ x^7$

$w_2 \begin{vmatrix} 1 & 0 \\ 0 & x^4 \end{vmatrix} = -x^4 \quad 0 - x^2$

$u_1' \frac{w_1}{w} = \frac{x^9}{5x^4} = \frac{x^5}{5}$ $\qquad u_2' = \frac{w_2}{w} = \frac{-x^4}{5x^4} = \frac{1}{5x^2}$

$u = \frac{1}{5} \int x^5 dx = \frac{1x^6}{30}$ $\qquad u_2 = \int \frac{1}{5} \int -\frac{1}{x^8} x - -\frac{1}{5x}$

$y_p = \frac{x^6}{36}(1) \frac{x}{5}(x^5) = \frac{x^6}{30} + \frac{1x^9}{5} = \frac{x^6}{6}$

$y = C_1 + C_2 x^5 + \frac{x^6}{6}$