Intelligent Systems

Lecture 1:

Approximate Reasoning

Uncertainty Handling

Fuzzy Logic theory

Bassam Kurdy Ph.D

Outline

- Motivation
- Basic concepts of fuzzy set theory and fuzzy logic
- Linguistic fuzzy rules
- Procedure of fuzzy decision making
- Fuzzy Expert System (fuzzification – defuzzification)

Motivation

- Complex, ill-defined processes difficult for description and analysis by exact mathematical techniques.
- Approximate and inexact nature of the real word; vague concepts easily dealt with by humans in daily life.
- Thus, we need other technique, as supplementary to conventional quantitative methods, for manipulation of vague and uncertain information, and to create systems that are much closer in spirit to human thinking.

Fuzzy Logic Is a Strong Candidate For This Purpose.

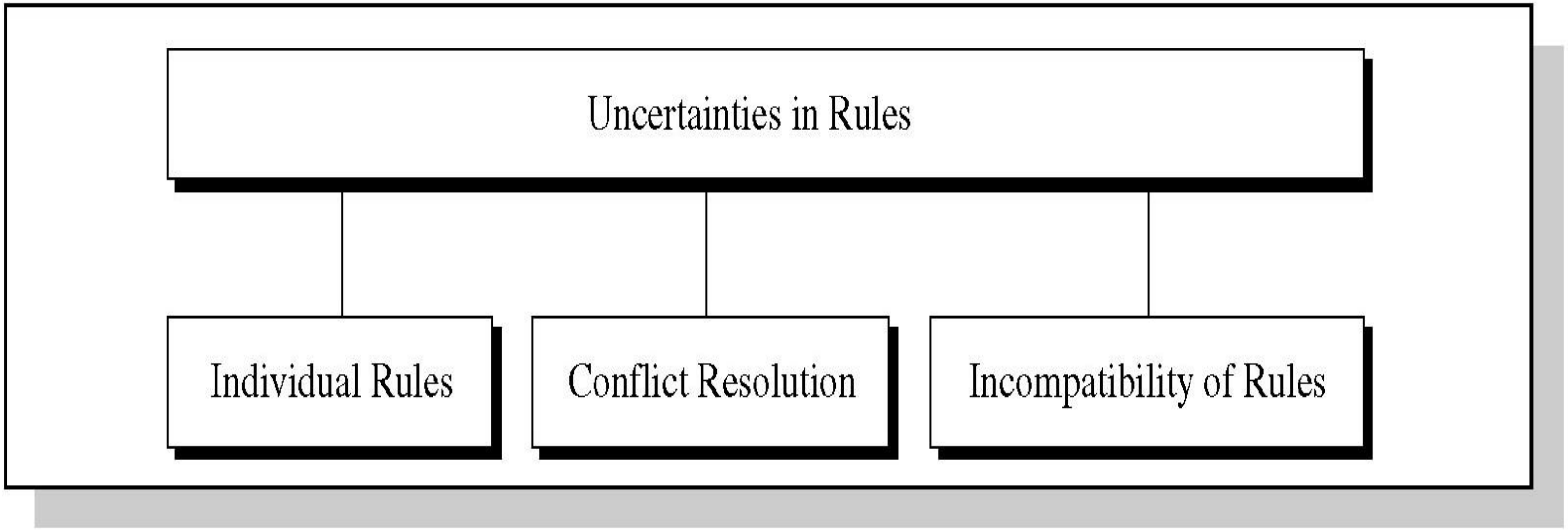
Goal of Knowledge Engineer

- The knowledge engineer endeavors to minimize, or eliminate, uncertainty if possible.
- Minimizing uncertainty is part of the verification of rules.
- Verification is concerned with the correctness of the system's building rules.

Verification vs. Validation

- Even if all the rules are correct, it does not necessarily mean that the system will give the correct answer.
- Verification refers to minimizing the local uncertainties.
- Validation refers to minimizing the global uncertainties of the entire expert system.
- Uncertainties are associated with creation of rules and also with assignment of values.

Major Uncertainties in Rule-Based Expert Systems



Uncertainty Handling Techniques

- ***certainty factor***
- ***Bayesian probability theory I & II***
- **fuzzy set theory**

Some Fuzzy Terms of Natural Language

tall
hot
low
medium
high
very
not
little
several
few
many
more
most
about
approximately
left-winger

Some Applications of Fuzzy Theory

- Control Algorithms
- Medical Diagnosis
- Decision Making
- Economics
- Engineering
- Environmental
- Literature
- Operations Research
- Pattern Recognition
- Psychology
- Reliability
- Security
- Science

what is fuzzy thinking?

- Experts rely on common sense when they solve problems.
- How can we represent expert knowledge that uses vague and ambiguous terms in a computer?
- Fuzzy logic is not logic that is fuzzy, but logic that is used to describe fuzziness.
- Fuzzy logic is the theory of fuzzy sets, sets that calibrate vagueness and uncertainty.
- Fuzzy logic is based on the idea that all things admit of degrees. Temperature, height, speed, distance, beauty – all come on a sliding scale. The motor is running really hot. Tom is a very tall guy.

Boolean logic Vs Fuzzy logic

- Boolean logic uses sharp distinctions. It forces us to draw lines between members of a class and non-members. For instance,
- Tom is tall because his height is 181 cm. If we drew a line at 180 cm,
  - David, who is 179 cm, is small.
    - Is David really a small man or we have just drawn an arbitrary line in the sand?

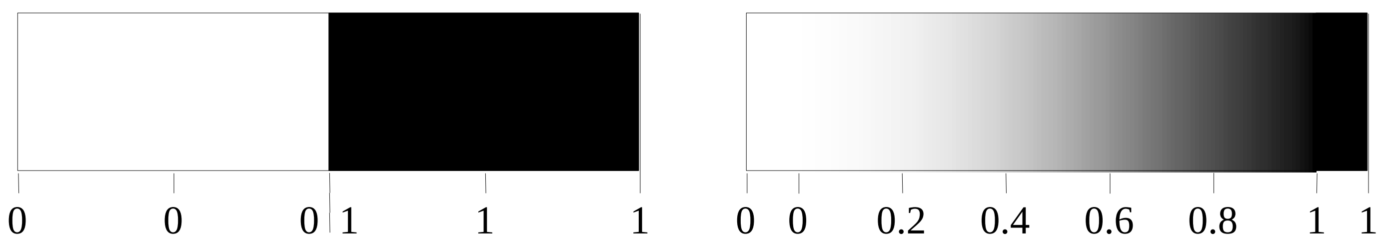
- Fuzzy logic reflects how people think. It attempts to model our sense of words, our decision making and our common sense.

- it is leading to new, more human, intelligent systems.

Boolean logic Vs Fuzzy logic

- Classical logic operates with only two values 1 (true) and 0 (false)
- Fuzzy logic, extended the range of truth values to all real numbers in the interval between 0 and 1.
- For example, the possibility that a man 181 cm tall is really tall might be set to a value of 0.86. It is likely that the man is tall. This work led to an inexact reasoning technique often called possibility theory.

Boolean logic Vs Fuzzy logic



(*a*) Boolean Logic. (*b*) Multi-valued Logic.

Today, fuzzy logic and Bayesian theory are most often used for uncertainty.

History Of FL

- In 1965 Lotfi Zadeh, published his famous paper “Fuzzy sets”.
  - He extended the work on possibility theory into a formal system of mathematical logic.
  - He introduced a new concept for applying natural language terms

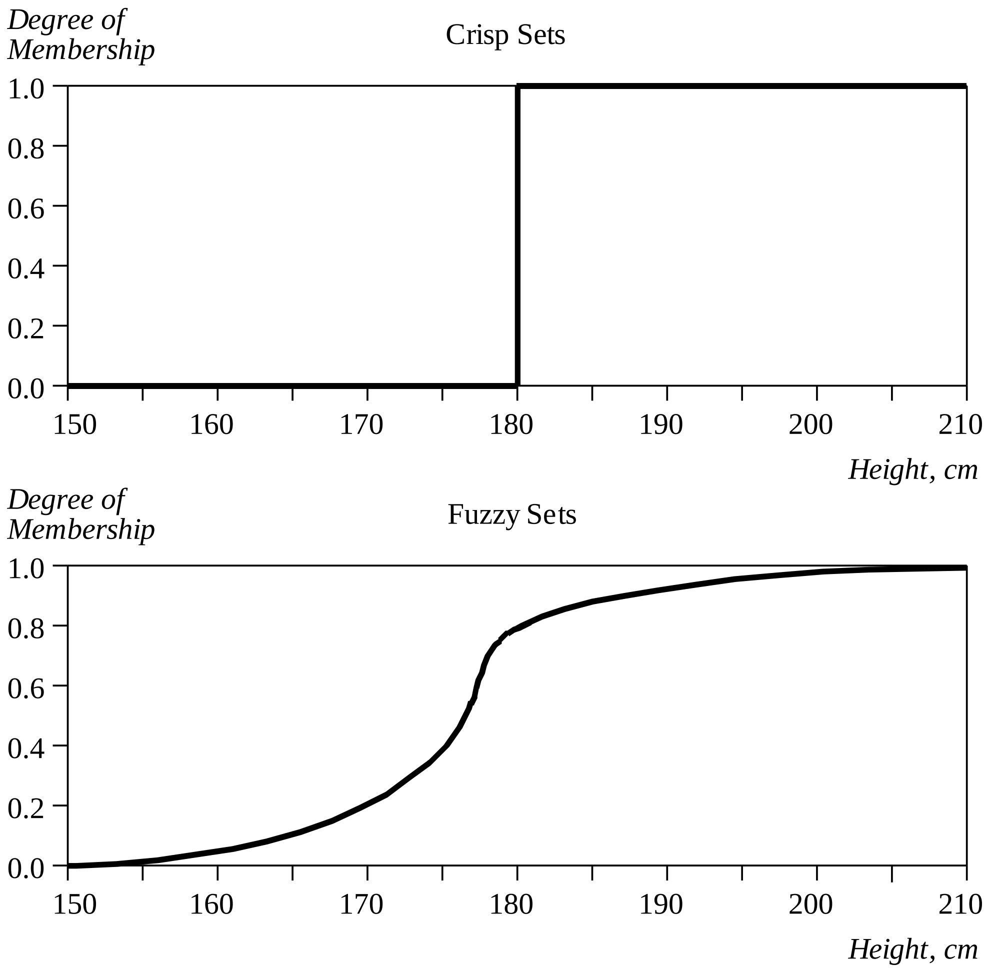
    - Zadeh became the Master of fuzzy logic.

Fuzzy sets

- The classical example in fuzzy sets is tall men. The elements of the fuzzy set "tall men" are all men, but their degrees of membership depend on their height.

Name	Height, cm	Degree of Membership	
		Crisp	*Fuzzy*
Chris	208	1	1.00
Mark	205	1	1.00
John	198	1	0.98
Tom	181	1	0.82
David	179	0	0.78
Mike	172	0	0.24
Bob	167	0	0.15
Steven	158	0	0.06
Bill	155	0	0.01
Peter	152	0	0.00

Crisp and fuzzy sets of "tall men"



- The x-axis represents the universe of discourse – the range of all possible values applicable to a chosen variable.

- The y-axis represents the membership value of the fuzzy set.

A fuzzy set is a set with fuzzy boundaries.

- Let X be the universe of discourse and its elements be denoted as x. In the classical set theory, crisp set A of X is defined as function $f_A(x)$ called the characteristic function of A

$$f_A(x) : X \rightarrow \{0, 1\}, \text{ where}$$

$$f_A(x) = \begin{cases} 1, \text{if } x \in A \\ 0, \text{if } x \notin A \end{cases}$$

- In the fuzzy theory, fuzzy set A of universe X is defined by function $\mu_A(x)$ called the membership function of set A

$$\mu_A(x): X \to [0, 1], \text{ where } \begin{cases} \mu_A(x) = 1 \text{ if } x \text{ is totally in } A; \\ \mu_A(x) = 0 \text{ if } x \text{ is not in } A; \\ 0 < \mu_A(x) < 1 \text{ if } x \text{ is partly in } A. \end{cases}$$

- This set allows a continuum of possible choices.
- For any element x of universe X, membership function $\mu_A(x)$ equals the degree to which x is an element of set A.
- This degree, a value between 0 and 1, represents the degree of membership, also called membership value, of element x in set A.

Fuzzy vs. Crisp Sets

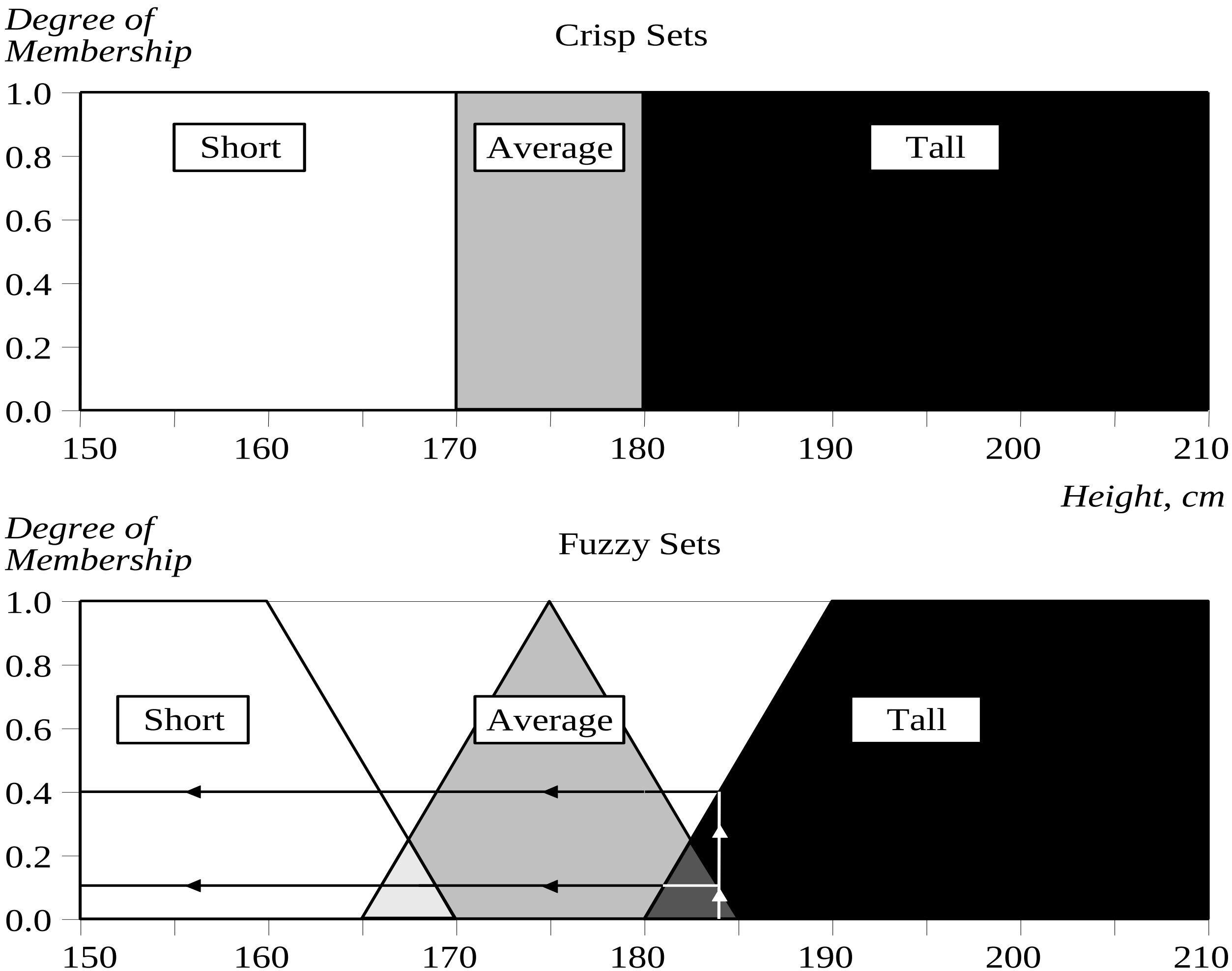
- Fuzzy set indicates how much (to which extent) an element belongs to it.

- Crisp set indicates whether an element belongs to it

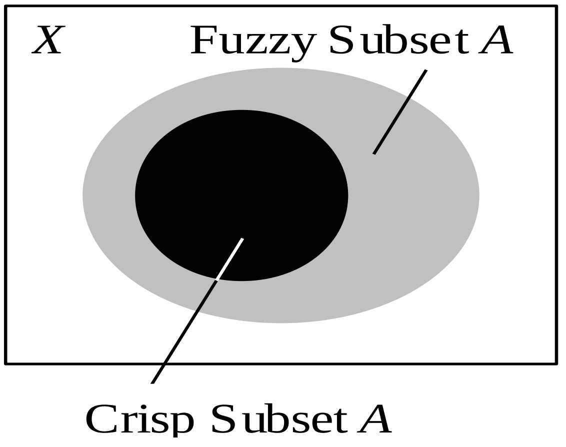
How to represent a fuzzy set in a computer?

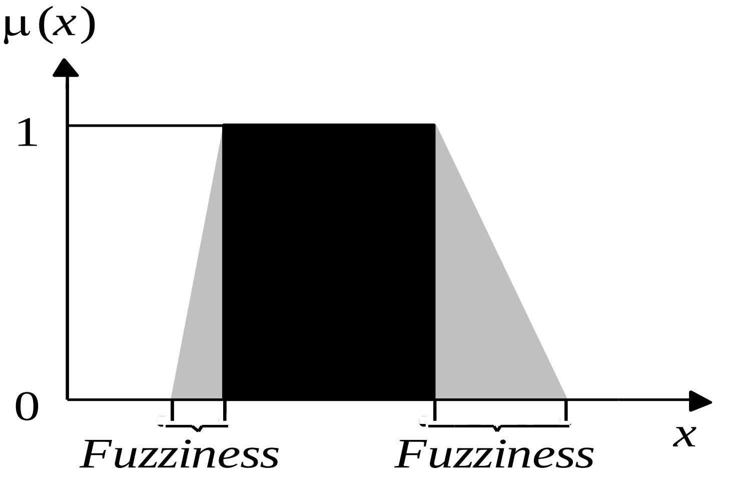
- First, we determine the membership functions.
  - In our “*tall men*” example, we can obtain fuzzy sets of *tall*, *short* and *average* men.
- The universe of discourse – the men’s heights – consists of three sets: *short*, *average* and *tall men*.
  - A man who is 184 cm tall is a member of the *average men* set with a degree of membership of 0.1, and at the same time, he is also a member of the *tall men* set with a degree of 0.4.

Crisp and fuzzy sets of short, average and tall men



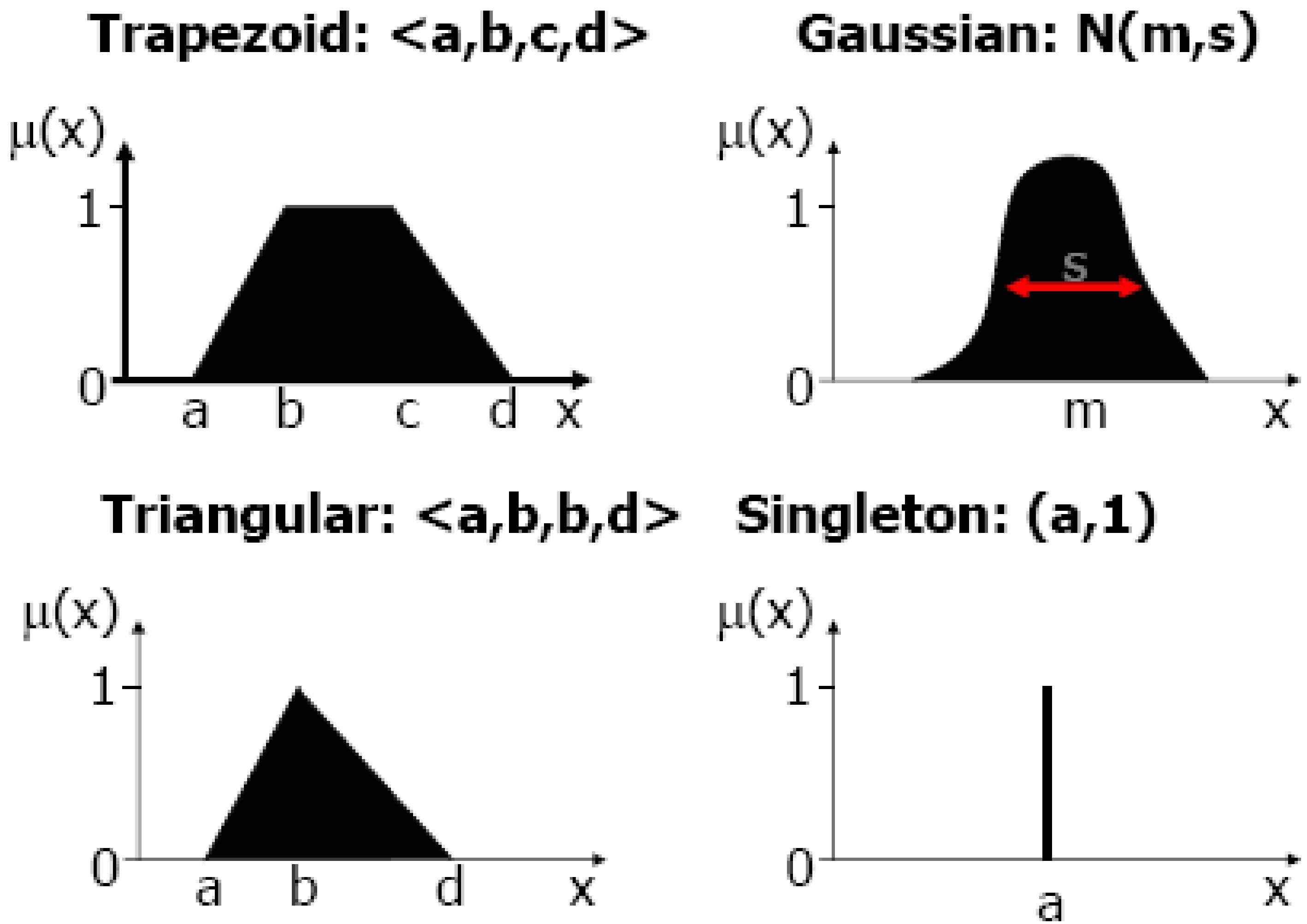
Representation of crisp and fuzzy subsets





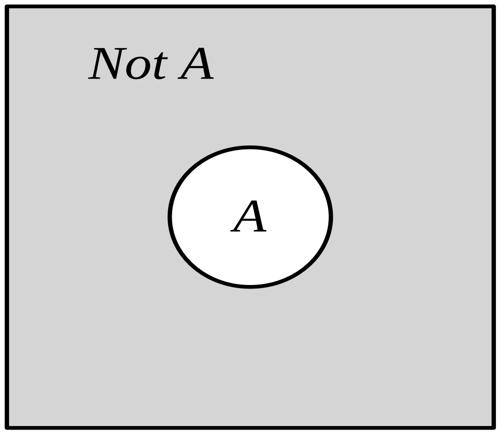
- Typical functions that can be used to represent a fuzzy set are sigmoid, gaussian and pi. However, these functions increase the time of computation. Therefore, in practice, most applications use **linear fit functions**.

Types of Membership Functions

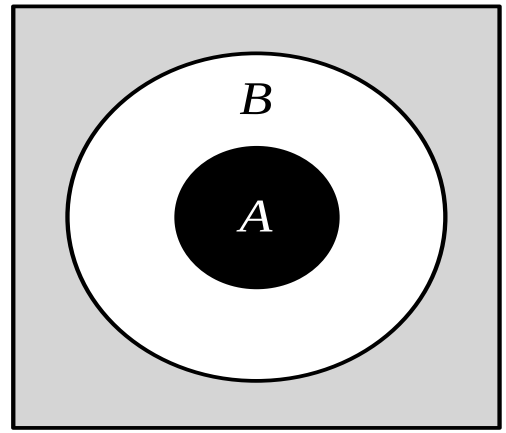


Operations of fuzzy sets

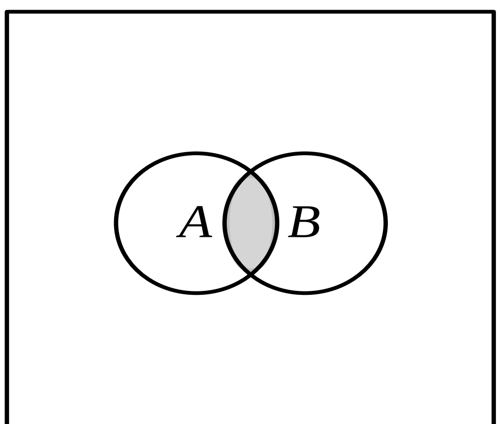
The classical set theory developed by Cantor describes how crisp sets can interact. These interactions are called **operations**.



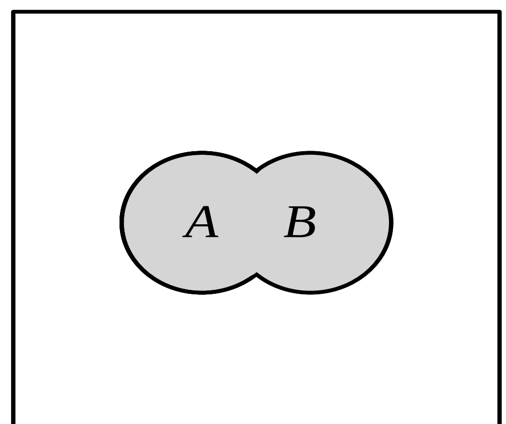
Complement



Containment



Intersection



Union

Complement

- *Crisp Sets*: *Who does not belong to the set?*
- *Fuzzy Sets*: *How much do elements not belong to the set?*
- The complement of a set is an opposite of this set.
  - E.g. if we have the set of *tall men*, its complement is the set of *NOT tall men*. When we remove the tall men set from the universe of discourse, we obtain the complement. If A is the fuzzy set, its complement $\neg A$ can be found as follows:
- $\mu_{\neg A}(x) = 1 - \mu_A(x)$

Containment

- *Crisp Sets*: *Which sets belong to which other sets?*
- *Fuzzy Sets*: *Which sets belong to other sets?*
- Similar to a Chinese box, a set can contain other sets.

  - E.g. the set of *tall men* contains all tall men;
  - *very tall men* is a subset of *tall men*. However, the *tall men* set is just a subset of the set of *men*.
    - In crisp sets, all elements of a subset entirely belong to a larger set.
    - In fuzzy sets, each element can belong less to the subset than to the larger set.

Intersection

- *Crisp Sets: Which element belongs to both sets?*
- *Fuzzy Sets: How much of the element is in both sets?*

- In crisp sets, An intersection between two sets contains the elements shared by these sets. E.g, the intersection of the set of *tall men* and the set of *fat men* is the area where these sets overlap.

- In fuzzy sets, an element may partly belong to both sets with different memberships. A fuzzy intersection is the lower membership in both sets of each element. The fuzzy intersection of two fuzzy sets *A* and *B* on universe of discourse *X*:
- $\mu_{A \cap B}(x) = min\,[\mu_A(x), \mu_B(x)] = \mu_A(x) \cap \mu_B(x),$

Union

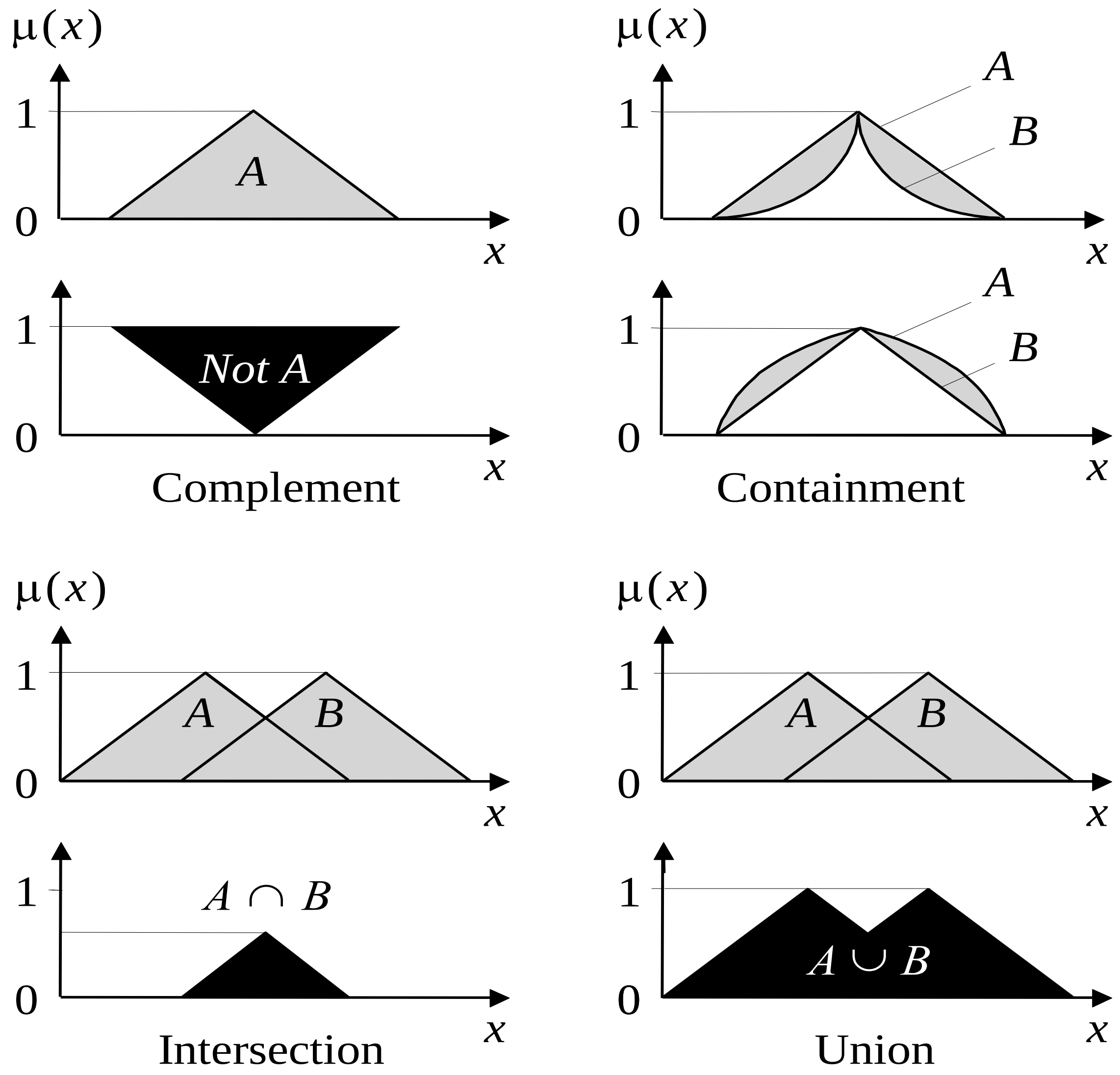
- *Crisp Sets: Which element belongs to either set?*
- *Fuzzy Sets: How much of the element is in either set?*

- In crisp sets: The union of two crisp sets consists of every element that falls into either set.
  - –E.g, the union of *tall men* and *fat men* contains all men who are tall **OR** fat.

- In fuzzy sets: The union is the largest membership value of the element in either set. The fuzzy operation for forming the union of two fuzzy sets A and B on universe X can be given as:

$$\mu_{A \cup B}(x) = max\,[\mu_A(x), \mu_B(x)] = \mu_A(x) \cup \mu_B(x), \quad \text{where } x \in X$$

Operations of fuzzy sets



Linguistic variables and hedges

- At the root of fuzzy set theory lies the idea of linguistic variables.

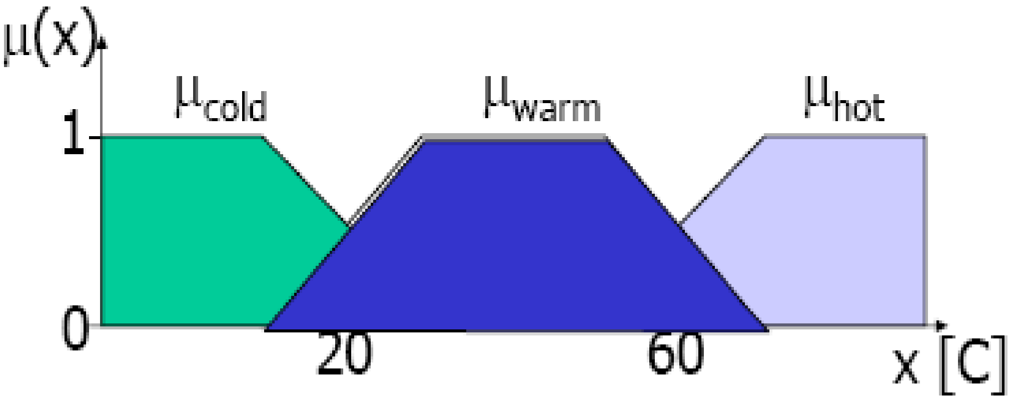
- **A linguistic variable is a fuzzy variable.**
  - For example, the statement "John is tall" implies that the linguistic variable *John* takes the linguistic value *tall*.

Linguistic Variables

- A *linguistic variable* *has a set of linguistic terms/values*. Each linguistic value corresponds to a fuzzy set and explained with its membership function.

linguistic variable : temperature

linguistics terms (fuzzy sets) : { cold, warm, hot }



In fuzzy expert systems, linguistic variables are used in fuzzy rules.

For example:

IF wind is strong
THEN sailing is good

IF project_duration is long
THEN completion_risk is high

IF speed is slow
THEN stopping_distance is short

What is a fuzzy rule?

- A fuzzy rule can be defined as a conditional statement in the form:

IF $\quad$ ***x*** **is** ***A***

THEN $\quad$ ***y*** **is** ***B***

- where *x* and *y* are linguistic variables; and *A* and *B* are linguistic values determined by fuzzy sets on the universe of discourses *X* and *Y*, respectively.

What is the difference between classical and fuzzy rules?

- A classical IF-THEN rule uses binary logic,

Rule: 1
IF speed is > 100
THEN stopping_distance is long

Rule: 2
IF speed is < 40
THEN stopping_distance is short

- The variable *speed* can have any numerical value between 0 and 220 km/h,

- The linguistic variable *stopping_distance* can take either value *long* or *short*.

  – classical rules are expressed in the black-and-white language of Boolean logic.

- We can also represent the stopping distance rules in a fuzzy form:

Rule: 1
IF speed is fast
THEN stopping_distance is long

Rule: 2
IF speed is slow
THEN stopping_distance is short

- In fuzzy rules: the linguistic variable *speed* also has the range (the universe of discourse) between 0 and 220 km/h,
  - This range includes fuzzy sets, such as *slow*, *medium* and *fast*.
- The universe of discourse of the linguistic variable *stopping_distance* can be between 0 and 300 m
- This range may include such fuzzy sets as *short*, *medium* and *long*.

Fuzzy Rules

A fuzzy rule is a linguistic expression of causal dependencies between linguistic variables in form of if-then statements

General form:

If <antecedent> then <consequence>

Example:

if temperature is cold and oil price is cheap then heating is high

linguistic variables

linguistic values/terms (fuzzy sets)

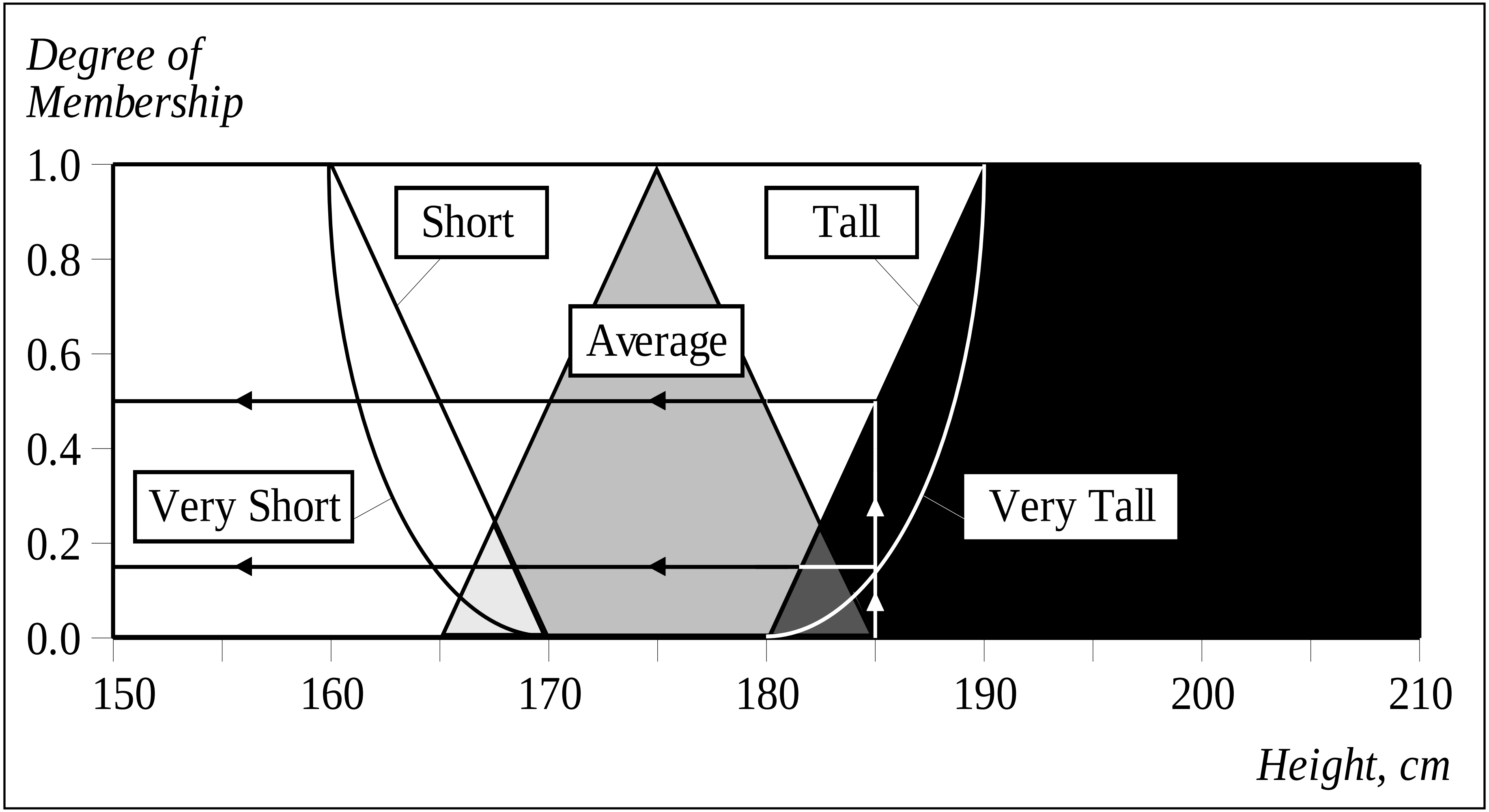
- The range of possible values of a linguistic variable represents the universe of discourse of that variable.

  – For example, the universe of discourse of the linguistic variable *speed* might have the range between 0 and 220 km/h and may include such fuzzy subsets as *very slow*, *slow*, *medium*, *fast*, and *very fast*.

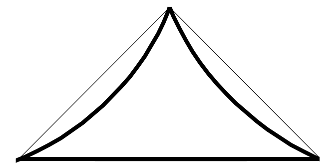
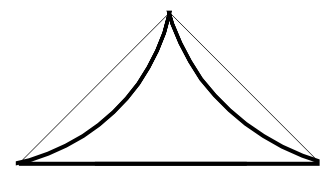
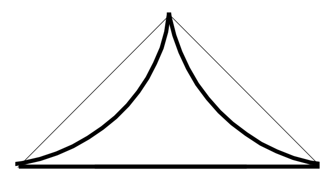
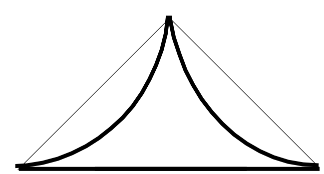
- A linguistic variable carries with it the concept of fuzzy set qualifiers, called *hedges*.

- **Hedges are terms that modify the shape of fuzzy sets. They include adverbs such as *very*, *somewhat*, *quite*, *more or less* and *slightly*.**

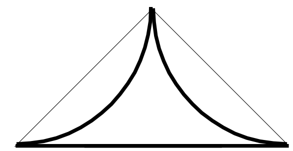
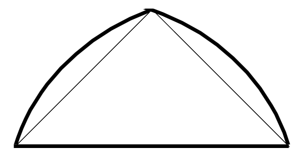
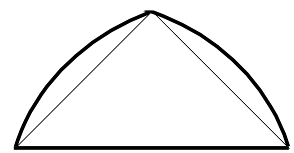
Fuzzy sets with the hedge *very*



Representation of hedges in fuzzy logic

Hedge	*Mathematical Expression*	*Graphical Representation*
A little	$[\mu_A(x)]^{1.3}$	
Slightly	$[\mu_A(x)]^{1.7}$	
Very	$[\mu_A(x)]^{2}$	
Extremely	$[\mu_A(x)]^{3}$	

Representation of hedges in fuzzy logic

Hedge	*Mathematical Expression*	*Graphical Representation*
Very very	$\left[\mu_A(x)\right]^4$	
More or less	$\sqrt{\mu_A(x)}$	
Somewhat	$\sqrt{\mu_A(x)}$	
Indeed	$2\left[\mu_A(x)\right]^2$ if $0 \le \mu_A \le 0.5$ $1 - 2\left[1 - \mu_A(x)\right]^2$ if $0.5 < \mu_A \le 1$