Intelligent systems

Lecture 2:
Approximate Reasoning

Fuzzy Expert Systems
Fuzzy Inference

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Fuzzy inference

- The most commonly used fuzzy inference technique is the so-called Mamdani method.

  – In 1975, Professor of London University built one of the first fuzzy expert **Ebrahim Mamdani** systems to control a steam engine and boiler combination.

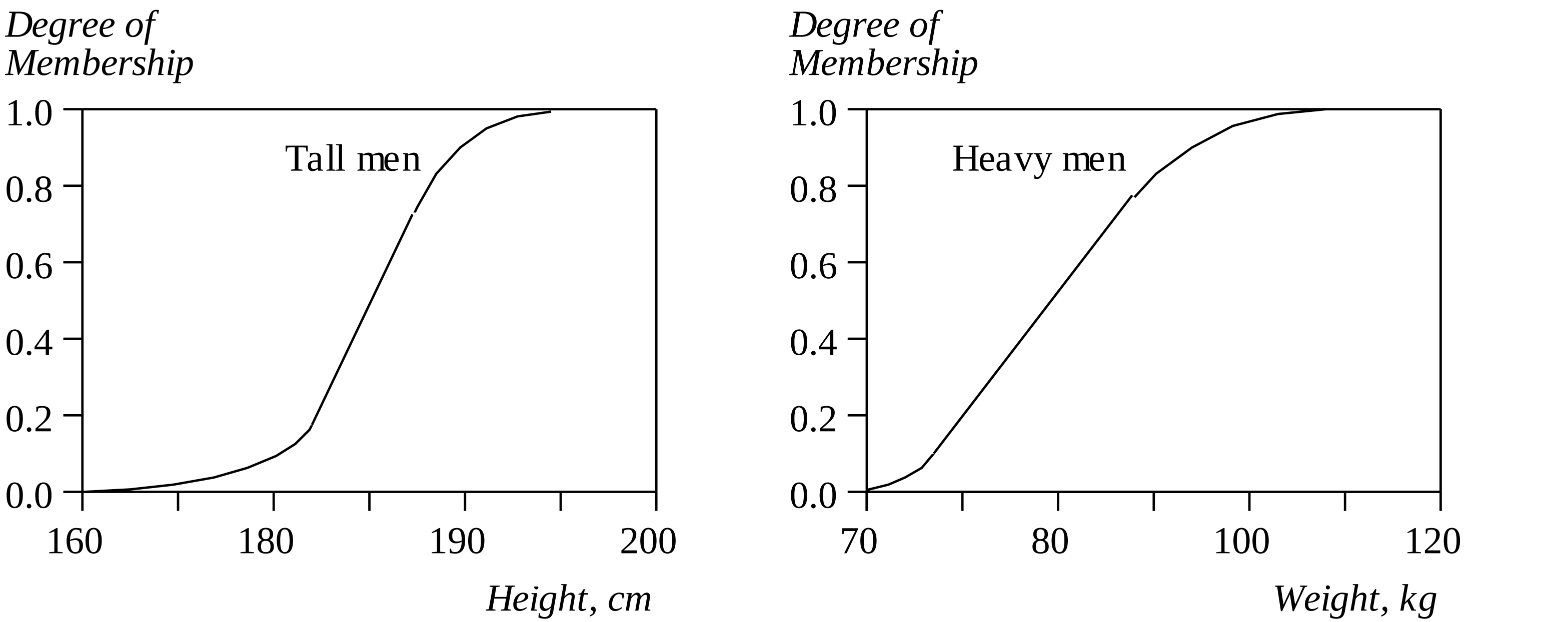
  – He applied a set of fuzzy rules supplied by experienced human operators.

Mamdani fuzzy inference

- The Mamdani-style fuzzy inference process is performed in four steps:

  - fuzzification of the input variables,
  - Rule evaluation;
  - Aggregation of the rule outputs.
  - defuzzification.

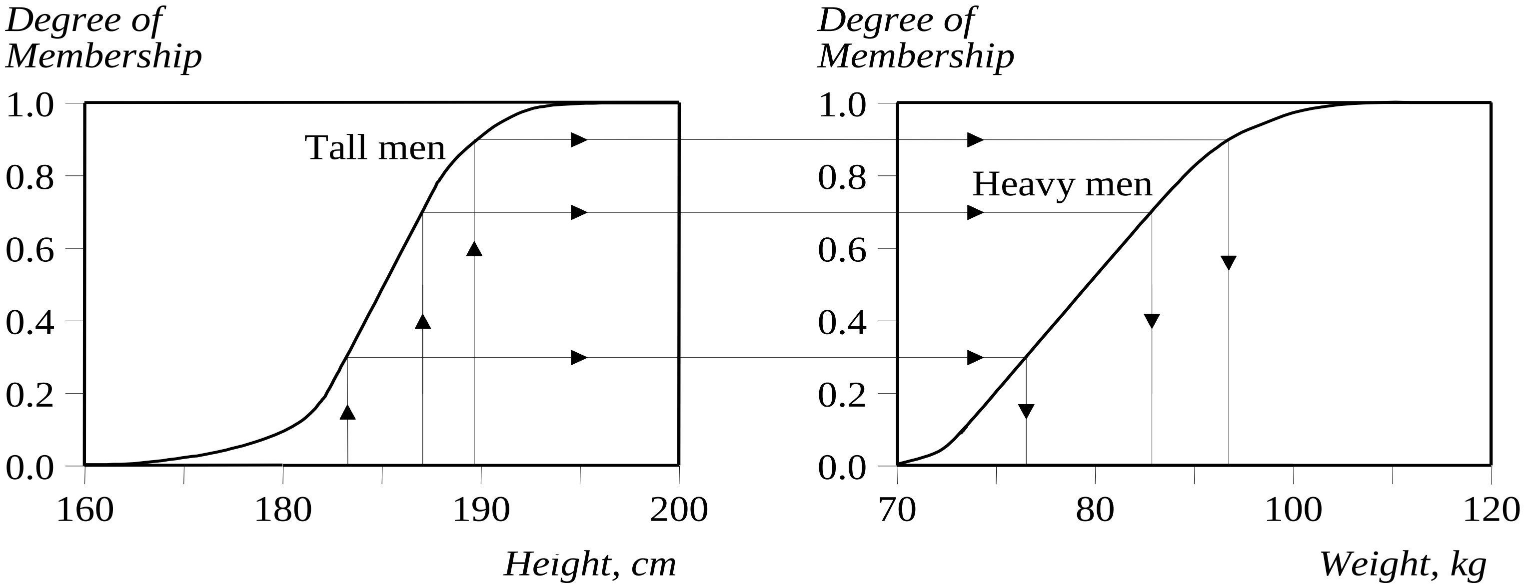
Fuzzy sets of *tall* and *heavy* men



These fuzzy sets provide the basis for a weight estimation model. The model is based on a relationship between a man's height and his weight:

IF height is *tall*
THEN weight is *heavy*

The value of the output or a truth membership grade of the rule consequent can be estimated directly from a corresponding truth membership grade.



- A fuzzy rule can have multiple antecedents, for example:

IF project_duration is long
AND project_staffing is large
AND project_funding is inadequate
THEN risk is high

IF service is excellent
OR food is delicious
THEN tip is generous

Fuzzy Decision Making Procedure

- Fuzzy Decision Making Procedure

- **Fuzzification**: compute the membership degrees for each input variable respect to its linguistic terms.

- **Fuzzy reasoning**: yield out the output fuzzy set using computed membership degrees and the fuzzy rules. It further consists of rule matching, fuzzy inference, and fuzzy aggregation stages

- **Defuzzification**: determine a crisp value from the output membership function as the final result or solution.

Mamdani fuzzy inference Example

- We examine a simple two-input one-output problem that includes three rules:

Rule: 1
IF *x* **is** *A*3
OR *y* **is** *B*1
THEN *z* **is** *C*1

Rule: 2
IF *x* **is** *A*2
AND *y* **is** *B*2
THEN *z* **is** *C*2

Rule: 3
IF *x* **is** *A*1
THEN *z* **is** *C*3

Rule: 1
IF ***project_funding*** **is** ***adequate***
OR ***project_staffing*** **is** ***small***
THEN ***risk*** **is** ***low***

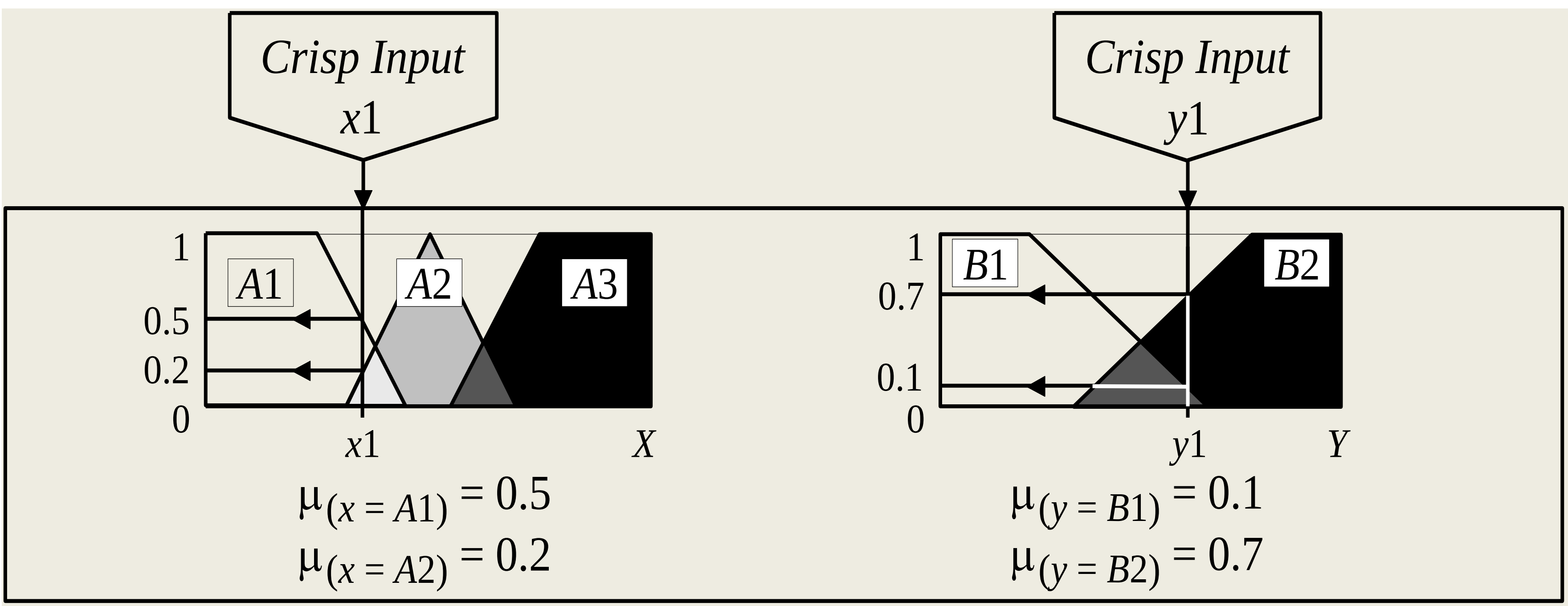
Rule: 2
IF ***project_funding*** **is** ***marginal***
AND ***project_staffing*** **is** ***large***
THEN ***risk*** **is** ***normal***

Rule: 3
IF ***project_funding*** **is** ***inadequate***
THEN ***risk*** **is** ***high***

Step 1: Fuzzification

The first step is to take the crisp inputs, x1 and y1 (project funding and project staffing), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.

x1,y1 are determined by an expert, x1 is rated by an expert as 35% corresponds to the membership functions A1 and A2 (inadequate and marginal) to the degrees of 0.5 and 0.2; and y1 as 60% corresponds to the membership functions B1 and B2 (small and large) to the degrees of 0.1 and 0.7 respectively.



Step 2: Rule Evaluation

- The second step is to take the fuzzified inputs:

$\mu_{(x=A1)} = 0.5$, $\mu_{(x=A2)} = 0.2$, $\mu_{(y=B1)} = 0.1$, $\mu_{(y=B2)} = 0.7$

- And apply them to the antecedents of the fuzzy rules.

- If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation.

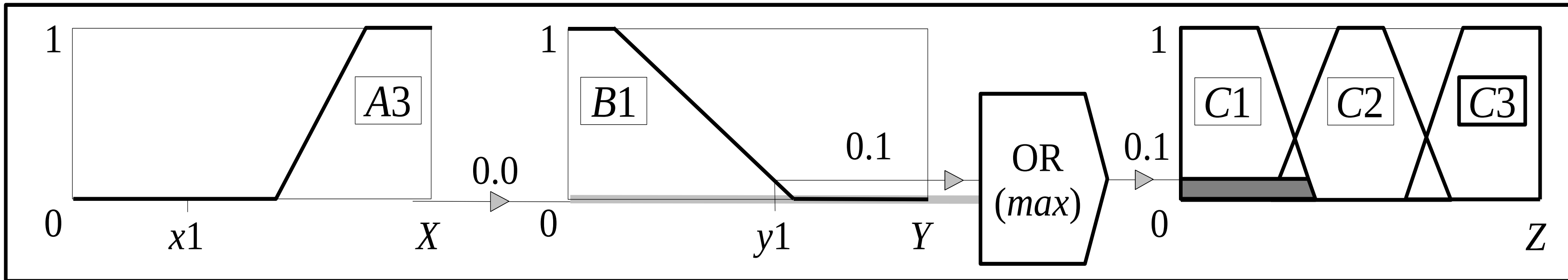
- This number (the truth value) is then applied to the consequent membership function.

- To evaluate the disjunction of the rule antecedents, we use the **OR fuzzy operation**. Typically, fuzzy expert systems make use of the classical fuzzy operation **union**:

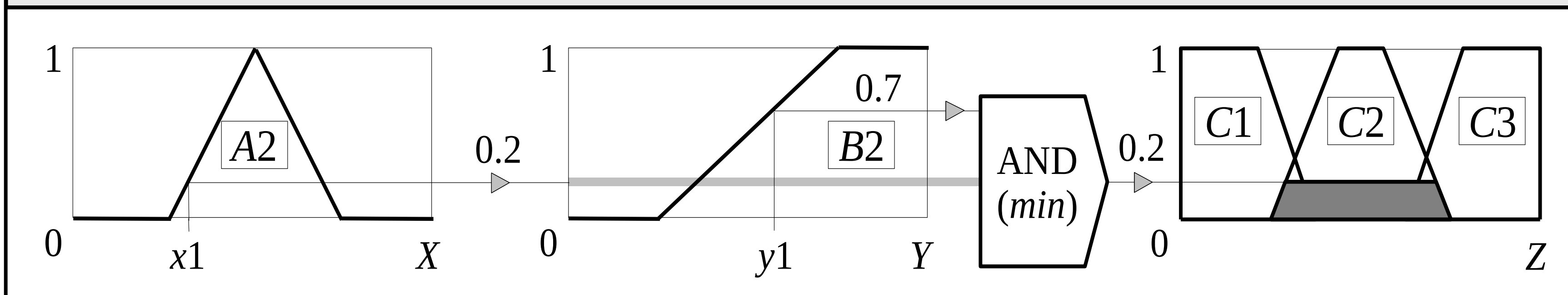
$$\mu_{A \cup B}(x) = \mathbf{\mathit{max}}\ [\mu_A(x),\ \mu_B(x)]$$

- Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the **AND fuzzy operation intersection**:

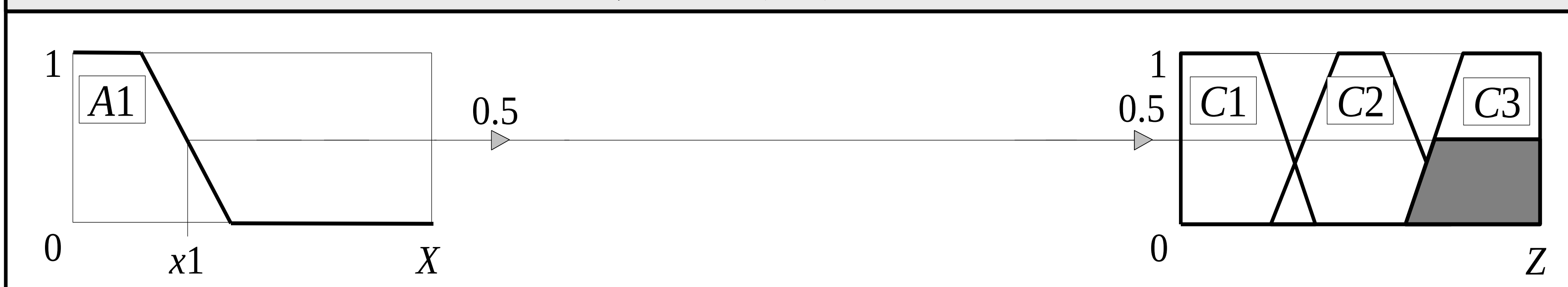
$$\mu_{A \cap B}(x) = \mathbf{\mathit{min}}\ [\mu_A(x),\ \mu_B(x)]$$



Rule 1: IF *x* is *A*3 (0.0) OR *y* is *B*1 (0.1) THEN *z* is *C*1 (0.1)



Rule 2: IF *x* is *A*2 (0.2) AND *y* is *B*2 (0.7) THEN *z* is *C*2 (0.2)



Rule 3: IF *x* is *A*1 (0.5) THEN *z* is *C*3 (0.5)

- Now the result of the antecedent evaluation can be applied to the membership function of the consequent.

- The most common method of correlating the rule consequent with the truth value of the rule antecedent is to cut the consequent membership function at the level of the antecedent truth.
- This method is called **clipping**.

- Since the top of the membership function is sliced, the clipped fuzzy set loses some information.

- Clipping is still often preferred because:
  - It involves less complex and faster mathematics.
  - It generates an aggregated output surface that is easier to defuzzify.

- **Scaling** method:
  - offers a better approach for preserving the original shape of the fuzzy set.

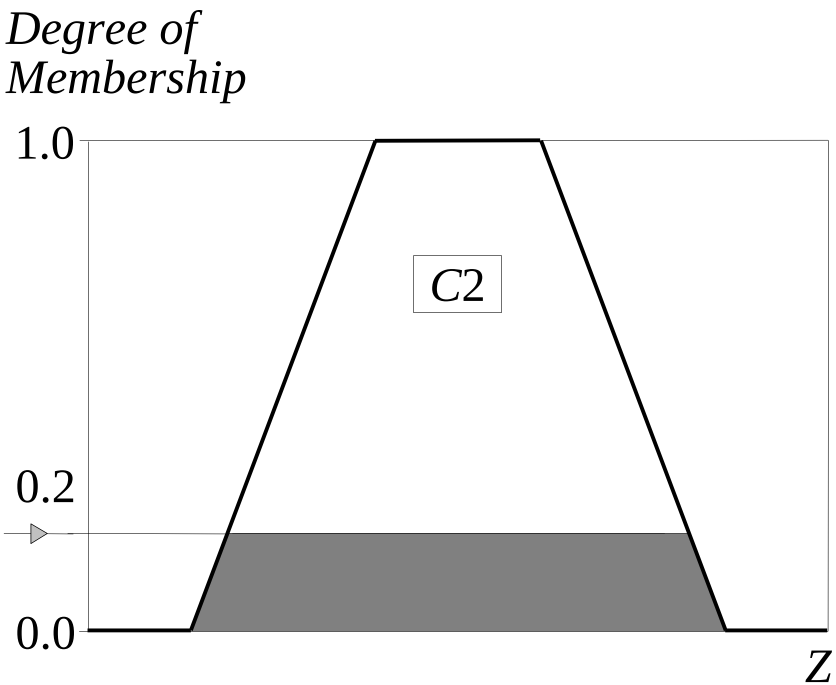
  - The original membership function of the rule consequent is adjusted by multiplying all its membership degrees by the truth value of the rule antecedent.

$$\mu_{A \cup B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) * \mu_B(x)$$

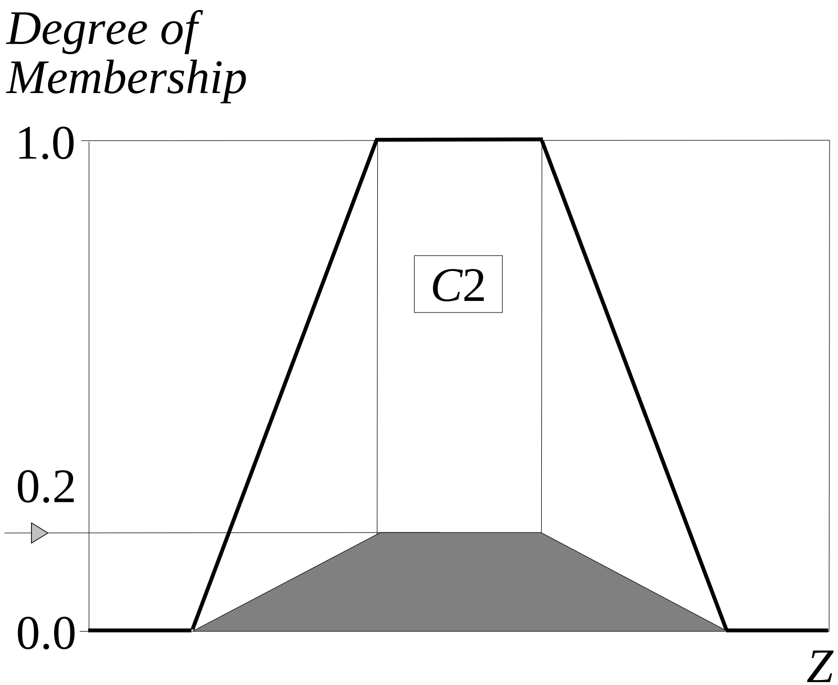
$$\mu_{A \cap B}(x) = \mu_A(x) * \mu_B(x)$$

  - This method, which generally loses less information, can be very useful in fuzzy expert systems.

Clipped and scaled membership functions



Clipped membership function



Scaled membership function

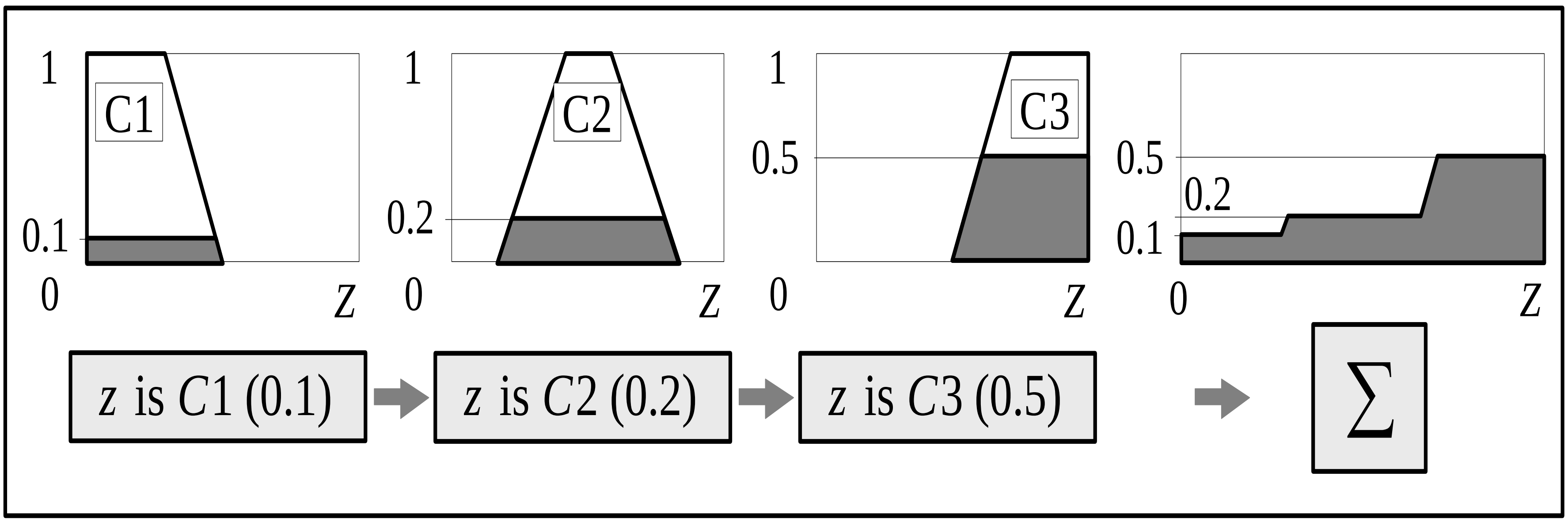
Step 3: Aggregation of the rule outputs

- Aggregation is the process of unification of the outputs of all rules.
- The process includes:
  - Taking the membership functions of all rule consequents previously clipped or scaled.
  - combine them into a single fuzzy set.

- The input of the aggregation process is the list of clipped or scaled consequent membership functions.

- The output is one fuzzy set for each output variable.

Aggregation of the rule outputs



Step 4: Defuzzification

- The last step in the fuzzy inference process is defuzzification.

- The input for the defuzzification process is the aggregated output fuzzy set and the output is a single number.

- The final output of a fuzzy system has to be a crisp number.

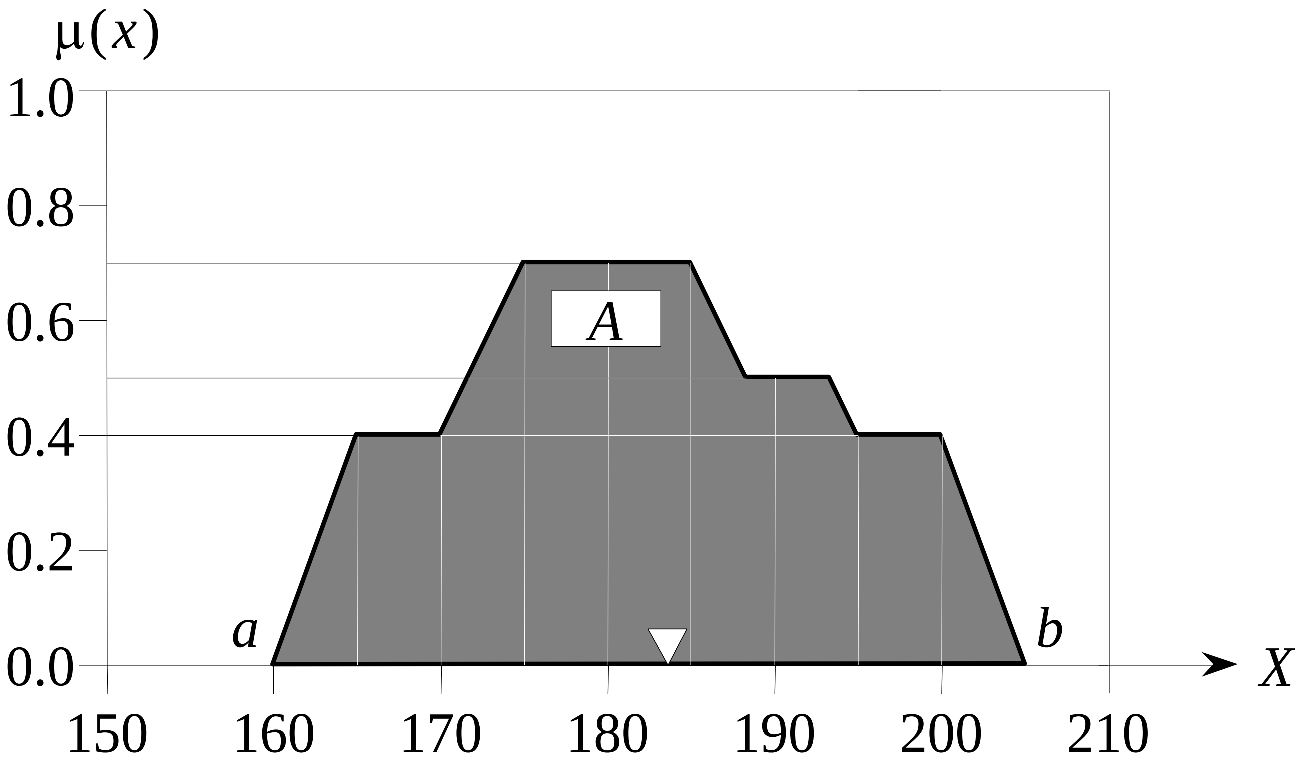
There are several defuzzification methods:

- the most popular one is the **centroid technique**.
- It finds the point where a vertical line would slice the aggregate set into two equal masses.
- Mathematically this **centre of gravity (COG)** can be expressed as:

$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx} = \frac{\sum_{x=a}^{b} \mu_A(x)\, x}{\sum_{x=a}^{b} \mu_A(x)}$$

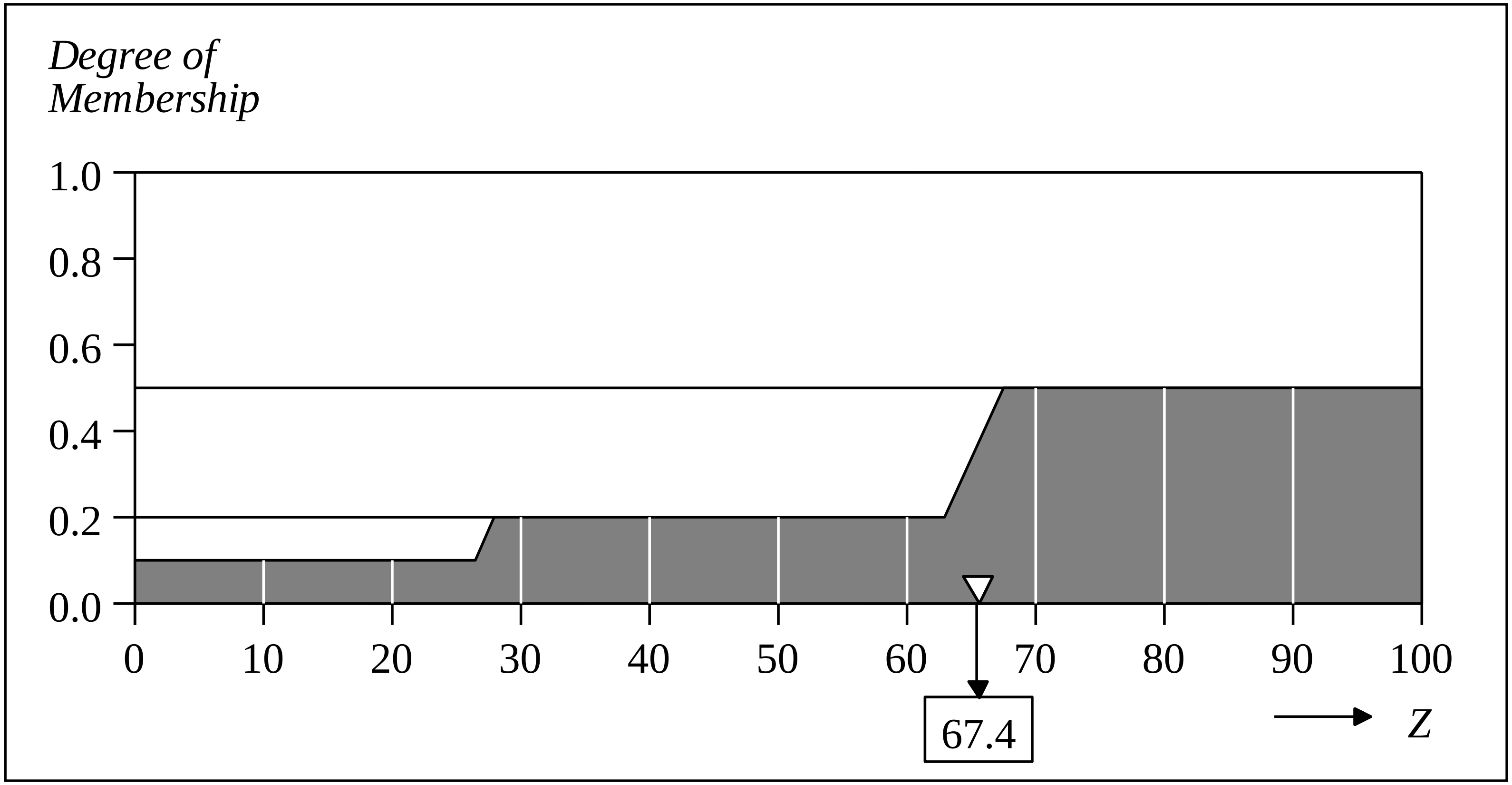
- Centroid defuzzification method finds a point representing the centre of gravity of the fuzzy set, *A*, on the interval, *ab*.

- A reasonable estimate can be obtained by calculating it over a sample of points.



$$COG = \frac{(0+10+20)\times 0.1+(30+40+50+60)\times 0.2+(70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$

This value means that the risk involved in our 'fuzzy' project is 67.4 per cent



Sugeno fuzzy inference

- **Mamdani-style inference:**
  - Requires to find the centroid of a two-dimensional shape by integrating across a continuously varying function.
  - This process is not computationally efficient.

- **Sugeno** (**Sugeno** 1985 Japan) suggested to use a single spike, a *singleton*, as the membership function of the rule consequent.

  – A **singleton**, or more precisely a **fuzzy singleton** is a fuzzy set with a membership function:
    - It is unity at a single particular point on the universe of discourse
    - zero everywhere else.

Sugeno-style fuzzy inference:

- Is very similar to the Mamdani method.
- Sugeno changed only a rule consequent.
- Instead of a fuzzy set, he used a mathematical function of the input variable.

- The format of the Sugeno-style fuzzy rule is

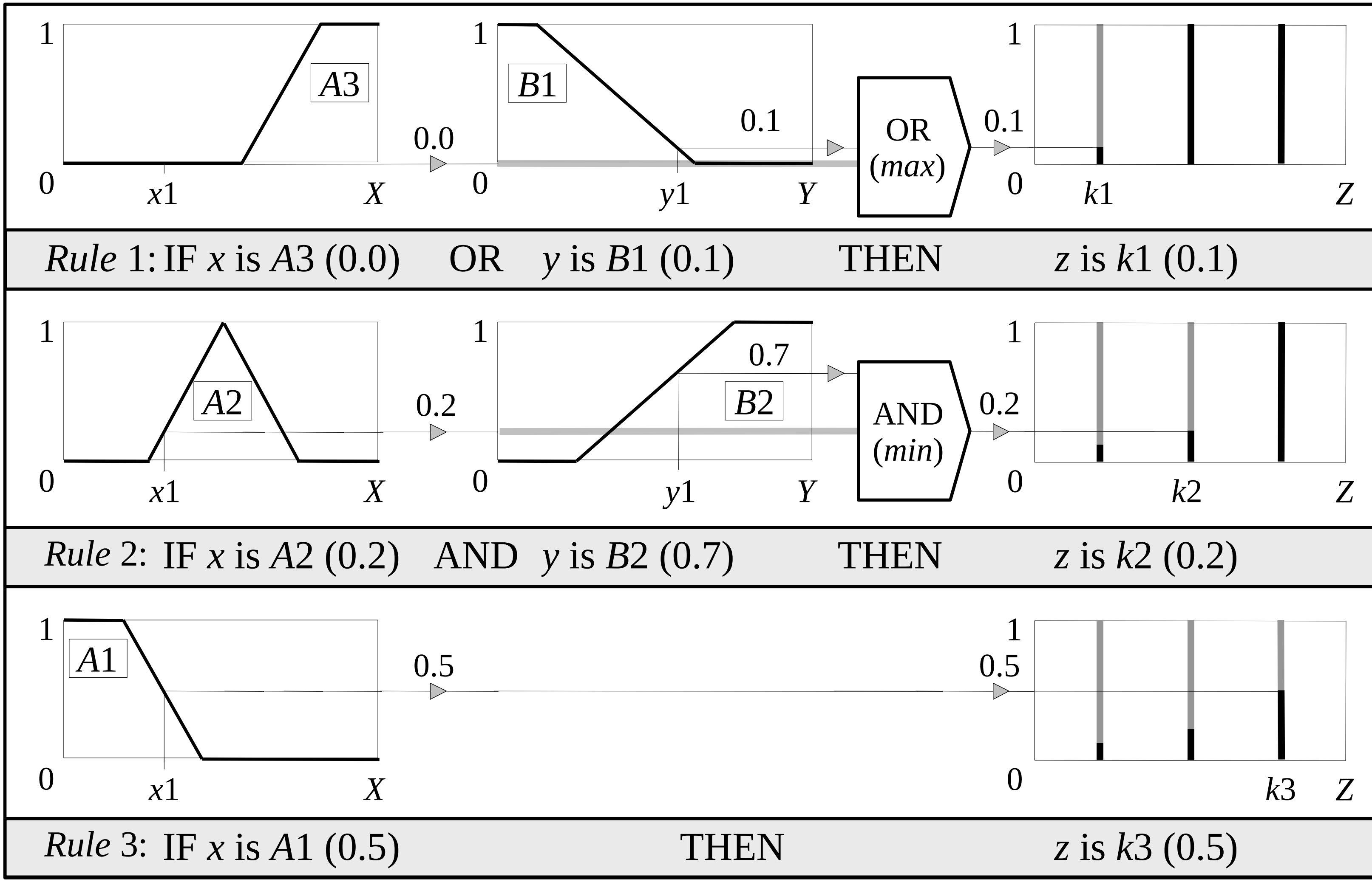
IF x is A

AND y is B

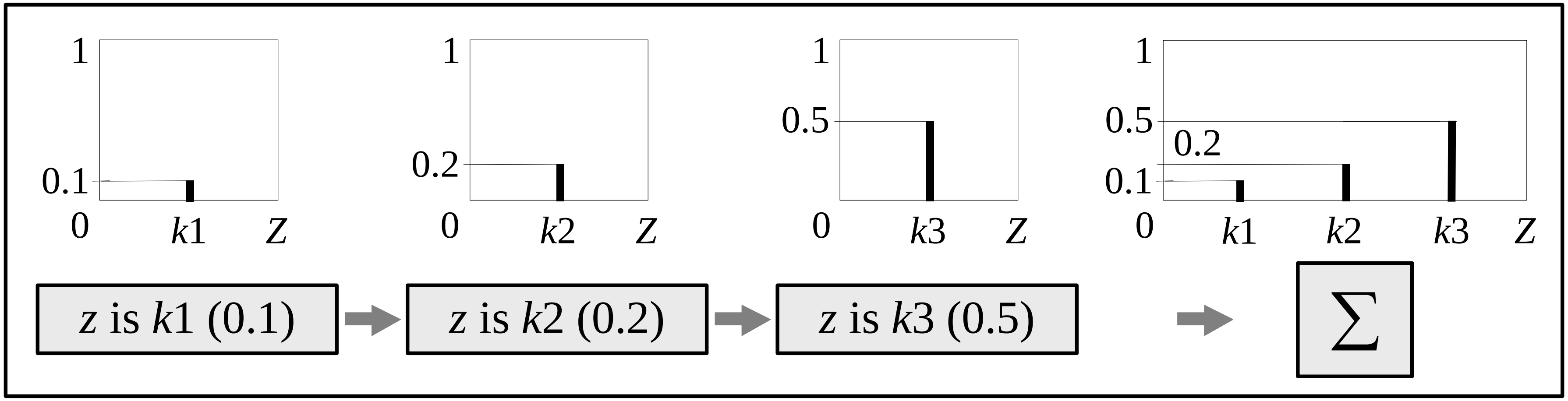
THEN z is $f(x, y)$

- where x, y and z are linguistic variables; A and B are fuzzy sets on universe of discourses X and Y, respectively; and f (x,y) is a mathematical function.

Sugeno-style rule evaluation
Step 2



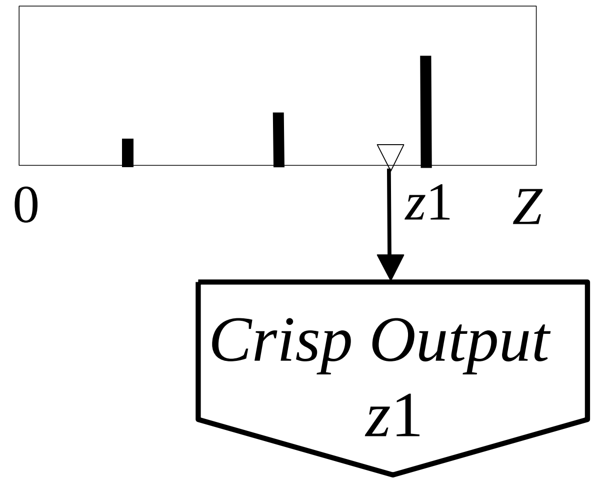
Sugeno-style aggregation of the rule outputs – Step 3



Weighted Average (WA):

$$WA = \frac{\mu(k1)\times k1 + \mu(k2)\times k2 + \mu(k3)\times k3}{\mu(k1)+\mu(k2)+\mu(k3)} = \frac{0.1\times 20 + 0.2\times 50 + 0.5\times 80}{0.1+0.2+0.5} = 65$$

Sugeno-style defuzzification



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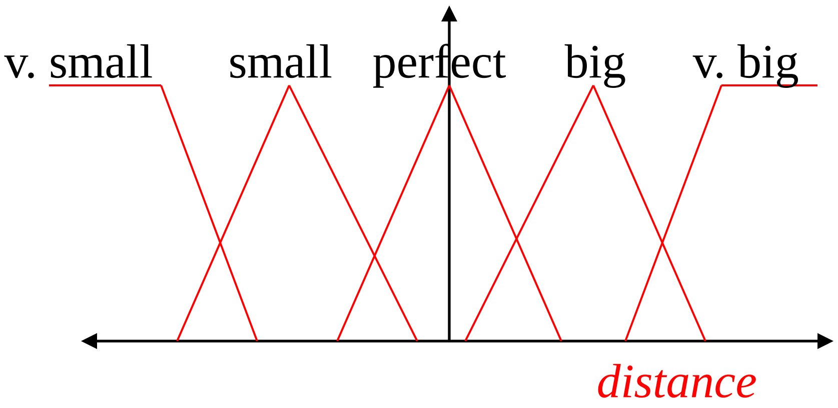
How to make a decision on which method to apply – Mamdani or Sugeno?

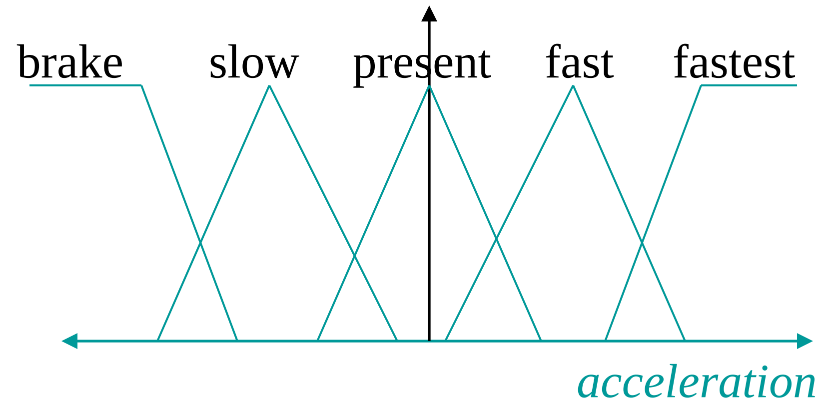
- Mamdani method:
  - widely accepted for capturing expert knowledge.
  - It allows us to describe the expertise in more intuitive, more human-like manner.
  - Mamdani-type fuzzy inference entails a substantial computational burden.
- Sugeno method
  - computationally effective
  - works well with optimisation and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.

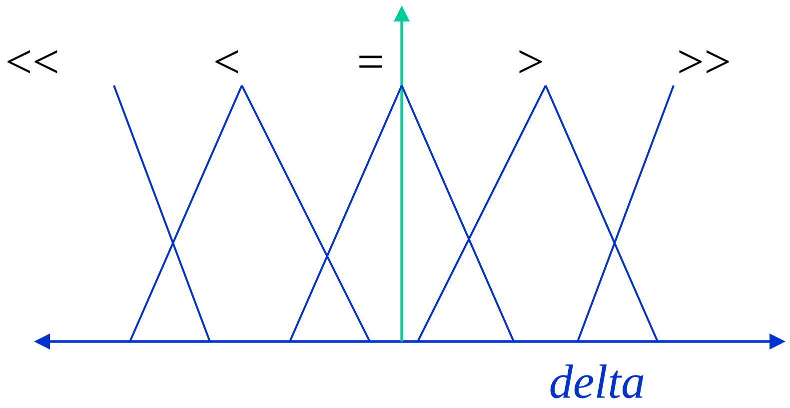
Example

- Rules for controlling a car:
  - Variables are *distance* to car in front and how fast it is changing, *delta*, and *acceleration* to apply
  - Sets are:
    - Very small, small, perfect, big, very big - for distance
    - Shrinking fast, shrinking, stable, growing, growing fast for delta
    - Brake hard, slow down, none, speed up, floor it for acceleration
  - Rules for every combination of distance and delta sets, defining an acceleration set
  - Extension: Allow fuzzy values for input variables (degree to which we believe the value is correct)

Set Definitions







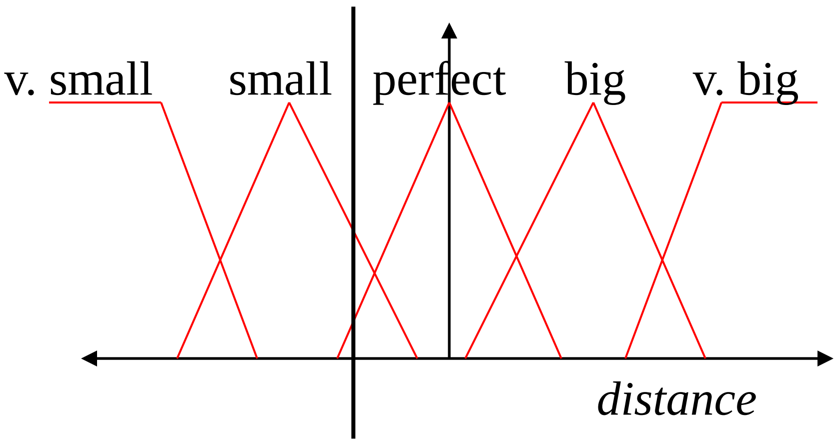
Fuzzy Rules

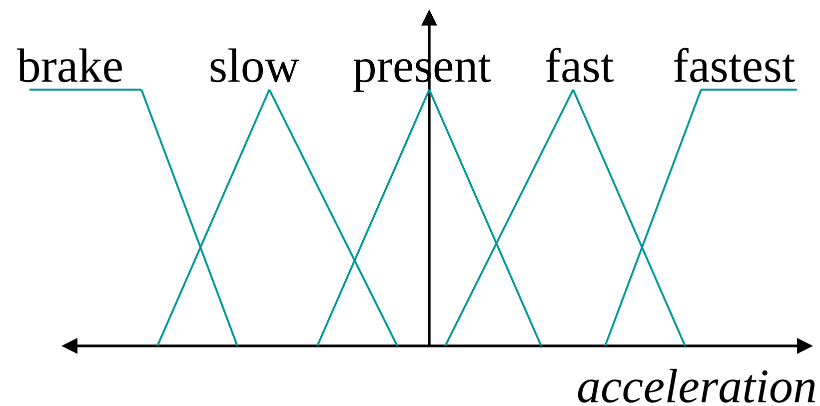
- "If our distance to the car in front is small, and the distance is decreasing slowly, then decelerate quite hard"
  - Fuzzy variables in blue
  - Fuzzy sets in red
  - Conditions are on membership in fuzzy sets
  - Actions place an output variable (decelerate) in a fuzzy set (the quite hard deceleration set)

- We have a certain belief in the truth of the condition, and hence a certain strength of desire for the outcome

- Multiple rules may match to some degree, so we require a means to arbitrate and choose a particular goal - *defuzzification*

Instance





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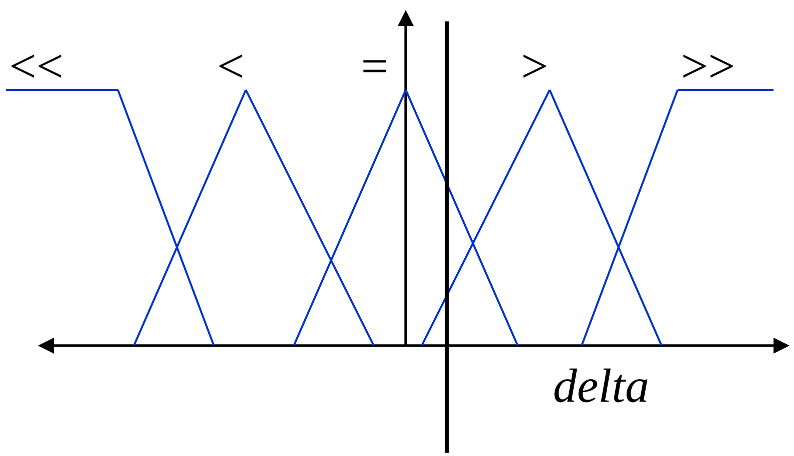
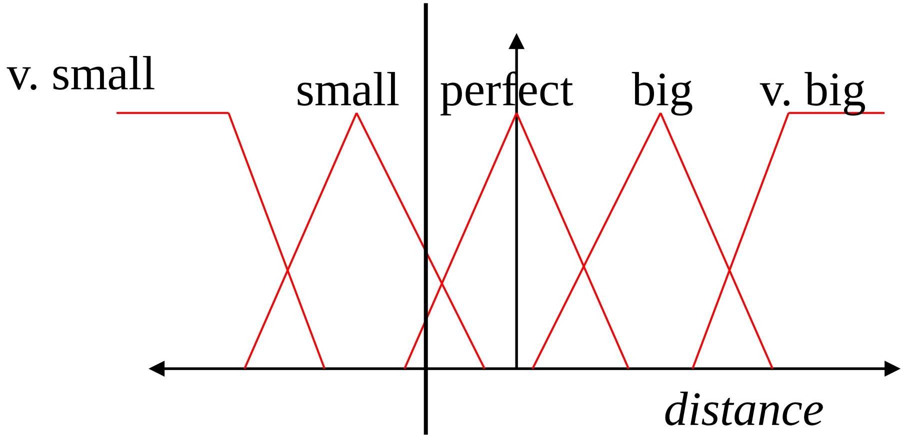


- Distance could be considered small or perfect
- Delta could be stable or growing
- What acceleration?

Fuzzification

- Determine the degree of membership for each term of an input variable :
  - If distance is small and delta is growing, maintain speed

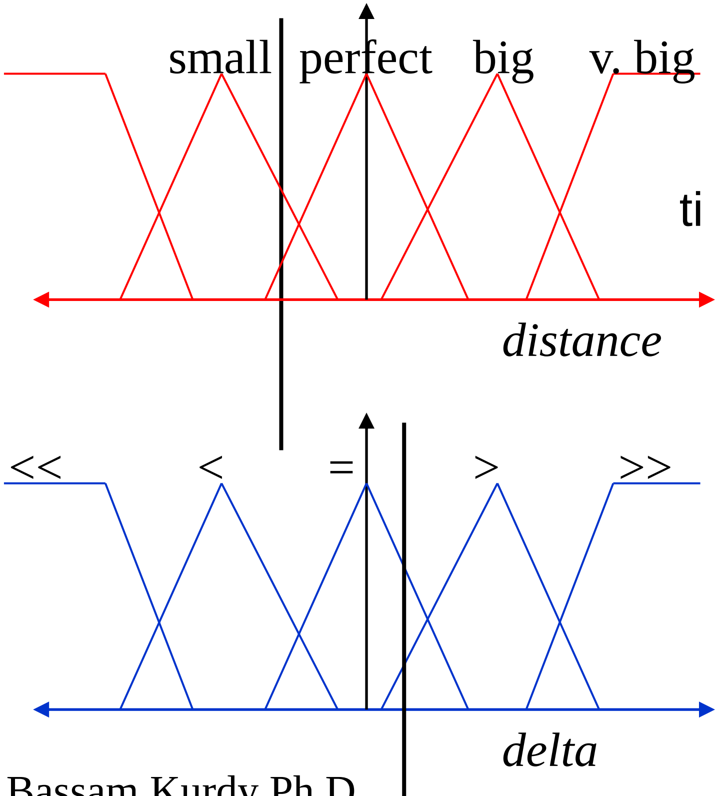


μ small(t)=0.75,

Rule Matching

- Calculate the firing strength of every rule by combining the individual membership degrees for all terms involved in the condition part of the rule through fuzzy AND: min-operator.

If distance is small and delta is growing, maintain speed



ti = min{µsmall(t), µgrowing(p)} = min{0.75,0.3} = 0.3

Matching for Example

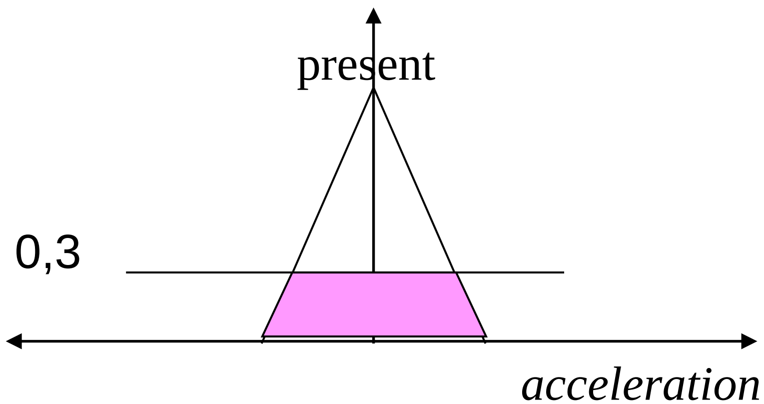
- Relevant rules are:
  - If distance is small and delta is growing, maintain speed
  - If distance is small and delta is stable, slow down
  - If distance is perfect and delta is growing, speed up
  - If distance is perfect and delta is stable, maintain speed

- For first rule, distance is small has 0.75 truth, and delta is growing has 0.3 truth
  - So the truth of the **and** is 0.3

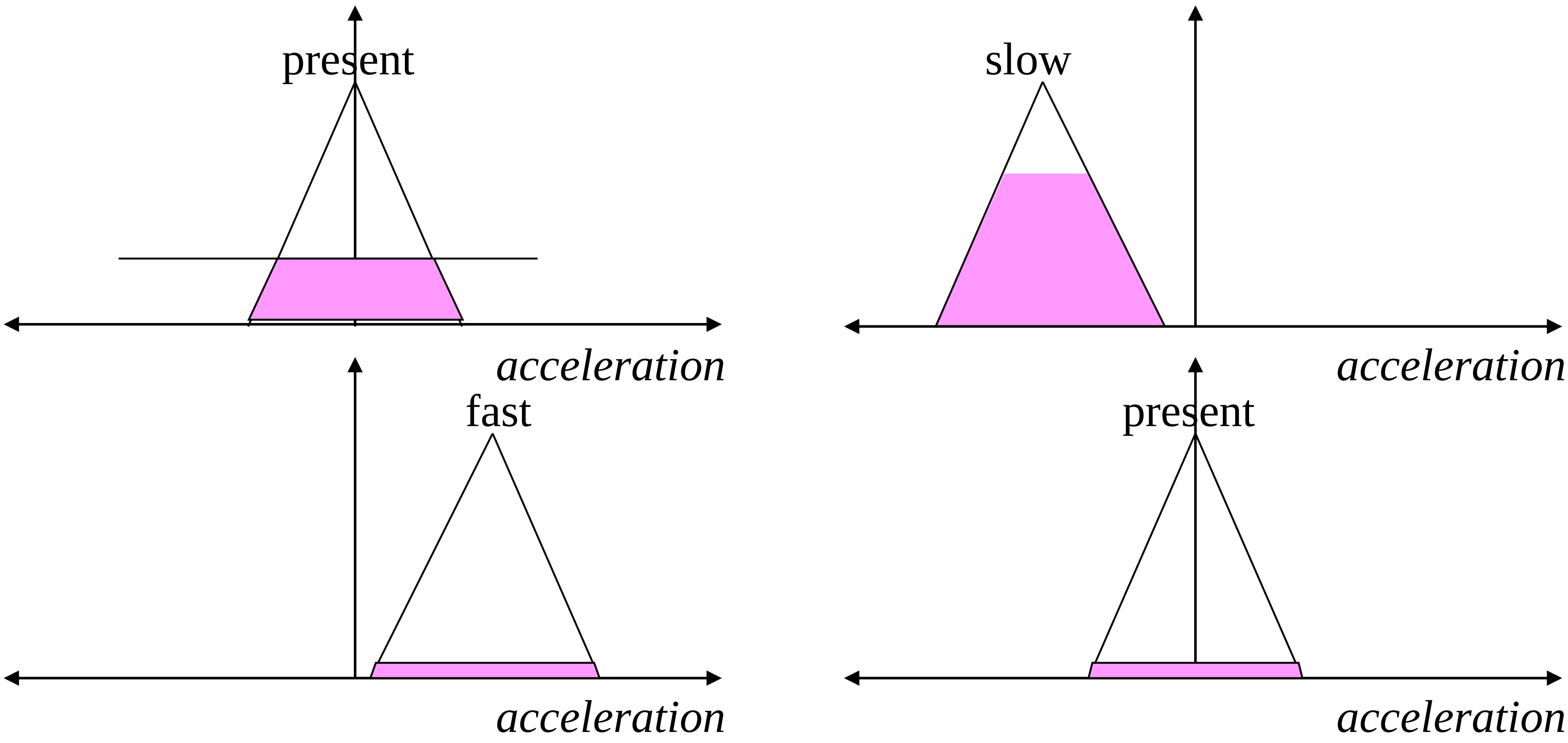
- Other rule strengths are 0.6, 0.1 and 0.1

Fuzzy Inference

- Inference step: For each rule apply the firing strength to modify its consequent fuzzy set, resulting in a new fuzzy set as the response of the rule.

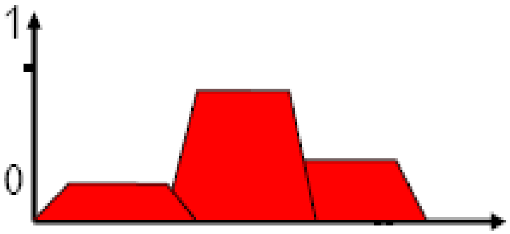


Fuzzy Inference



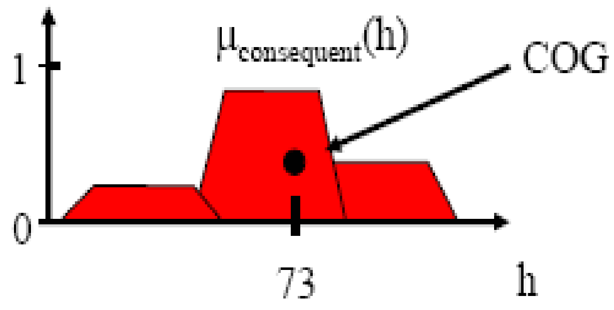
Fuzzy Aggregation

- Aggregation: combine responses of individual rules to yield an overall output fuzzy set using the max-operator for union. The overall fuzzy set is an aggregated effect from all rules.



Defuzzification

- Determine crisp value from output membership function.Commonly used is the "Center of Gravity" method:



Defuzzification

- We have three things sets we have reason to believe we are in, and each set covers a range of values
- Two options in going from current state to a single value:
  - Mean of Max: Take the rule we believe most strongly, and take the (weighted) average of its possible values
  - Center of Mass: Take all the rules we partially believe, and take their weighted average
- In this example, we slow down either way, but we slow down more with Mean of Max
  - Mean of max is cheaper, but center of mass exploits more information

Conclusion

- Advantages
  - Allows use of numbers while still writing “crisp” rules
  - Allows use of “fuzzy” concepts such as medium
  - Biggest impact is for control problems
    - Help avoid discontinuities in behavior
    - In example problem strict rules would give discontinuous acceleration

- Disadvantages
  - Sometimes results are unexpected and hard to debug
  - Additional computational overhead

Test your knowledge

- Given the following rules of Inference:

if	X is A1	or	Y is B1	then Z is C1
if	X is A2	and	Y is B2	then Z is C2
if	X is A3			then Z is C3

And the following Fuzzy Sets where:

- (A1, A2, A3) represent (Light-Weight, Medium-Weight, Heavy-Weight) and
- (B1, B2) represent (Average Height, Tall-Height) and
- (C1, C2, C3) represent (Low-Risk, Medium-Risk, High- Risk) to health.

- First: Draw the Fuzzy Sets given.
  - Light-Weight (60/1, 65/0.75, 70/0.5, 75/0.25, 80/0)
  - Medium-Weight (70/0, 75/0.5 80/1, 85/0.5, 90/0)
  - Heavy-Weight (85/0, 90/0.25, 95/0.5, 100/0.75, 105/1)
  - Average-Height (160/1, 165/0.75, 170/0.5, 180/0)
  - Tall- Height (170/0, 180/0.5, 190/1)
- Second: Using fuzzy inference and the rules provided, provide a % health risk for someone that has a height (Y) of 178cm and a weight (X) of 95Kg? A fuzzy set for risk (C) is provided: