COMP353 Databases

Schema Refinement:

Minimal Bases

(Canonical Covers)

Minimal Basis (Canonical Cover)

- Recall that the number of iterations to compute the closure of a set of attributes depends on the number of attributes
  - The complexity of some other algorithms which we will study (eg, decomposition algorithms) depend on the number of FD's
- To ease the situation, can we "minimize" **F**?

Covers/bases

- Note that FD's on a relation may be represented in different but equivalent ways.
- Recall that, given two sets of FD's **F** and **G** on **R,** we say that: "**G** *follows from* **F** (**F** ⊨ **G)**", provided for every instance **r** of **R,** if **r** satisfies **F**, then **r** satisfies **G**. In this case, we may also say: "**F** *implies* **G**", or "**F** covers **G**", *or* "**G** *is implied by* **F**".
- If **F** ⊨ **G** and **G** ⊨ **F** both hold, then we say that **G** and **F** are equivalent and denote this by **F ≡ G.** In this case, **we may** also say that **F** and **G** are covers for each other.
- Note that **F ≡ G** iff $F^+ \equiv G^+$

Canonical Cover (minimal basis)

- Let **F** be a set of FD's. A canonical cover of **F** is a set **G** of FD's that satisfies the following conditions:
  1. **G** is equivalent to **F**, that is, **G ≡ F**
  2. Every FD in **G** is has a single attribute on the right hand side.
  3. **G** is minimal, that is, if we obtain a set **H** of FD's from **G** by deleting some FD's in **G** or by reducing the left hand side of some FD's, then **H** won't be equivalent to **F** (that is, **H ≢ F)**

Canonical Cover

➔ A canonical cover **G** is minimal in two respects:

1. Every FD in **G** is "required" in order for **G** to be equivalent to **F**
2. Every FD in **G** is as "small" as possible, that is,
   - each attribute on the left hand side **X** is necessary.
   - Recall: the RHS of every FD in **G** is a single attribute

Computing Canonical Cover

Given a set **F** of FD's, how to compute ***a*** canonical cover **G** of **F**?

- **Step 1:** Put the FD's in the **simple** form (i.e., one attribute on the RHS)
  - Initialize **G := F**
  - Replace each FD $X \longrightarrow A_1A_2...A_k$ in **G** with $X \longrightarrow A_1$, $X \longrightarrow A2$, ..., $X \longrightarrow A_k$
- **Step 2: Minimize** the left hand side **X** of every FD
  - E.g., if AB⟶C is in **G**, check if A or B on the LHS is redundant **,** i.e.,

    $(G - \{AB \longrightarrow C\} \cup \{A \longrightarrow C\})^+ \equiv F^+$ **or**

    $(G - \{AB \rightarrow C\} \cup \{B \rightarrow C\})^+ \equiv F^+$?
- **Step 3: Delete** redundant FD's, if any
  - For each FD $X \longrightarrow A$ in **G**, check if $(G - \{X \longrightarrow A\})^+ \equiv F^+$?

Computing Canonical Cover

- R = { A, B, C, D, E, H}
- F = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
- Step 1 – put FD's in the simple form
  - All present FD's are simple

  ➔ G = {A→B, DE→A, BC → E, AC → E, BCD → A, AED → B}

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Computing Canonical Cover

- R = { A, B, C, D, E, H }
- F = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
- Step 2 – Check every FD to see if it is left reduced
  - For every FD **X → A** in **G,** check if the closure of a subset of **X** determines **A**. If so, remove the redundant attribute(s) from **X**

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Computing Canonical Cover

- R = { A, B, C, D, E, H }
- F = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
- G = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
  - A → B

    ➔ obviously OK (no left redundancy)
  - DE → A
    - $D^+ = D$
    - $E^+ = E$

    ➔ OK (no left redundancy)

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Computing Canonical Cover

- R = { A, B, C, D, E, H }
- F = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
- G = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
  - BC → E
    - $B^+ = B$
    - $C^+ = C$

    ➔ OK (no left redundancy)

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Computing Canonical Cover

- R = { A, B, C, D, E, H }
- F = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
- G = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
  - AC → E
    - $A^+ = AB$
    - $C^+ = C$

    ➔ OK (no left redundancy)

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Computing Canonical Cover

- R = { A, B, C, D, E, H }
- F = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
- G = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
  - BCD → A
    - $B^+ = B$
    - $C^+ = C$
    - $D^+ = D$
    - $\{BC\}^+ = BCE$
    - $\{CD\}^+ = CD$
    - $\{BD\}^+ = BD$

    ➔OK (no left redundancy)

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Computing Canonical Cover

- R = { A, B, C, D, E, H }
- F = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
- G = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
  - AED → B
    - A^+ = AB
    - E & D are redundant ➔ we can remove them from AED → B
- G = { $\cancel{A \to}$ B, DE → A, BC → E, AC → E, BCD → A, A → B }

➔ G = { DE → A, BC → E, AC → E, BCD → A, A → B }

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Computing Canonical Cover

- R = { A, B, C, D, E, H}
- F = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
- Step 3 – Find and remove redundant FDs
  - For every FD **X → A** in **G**
    - Remove **X → A** from **G**; call the result **G'**
    - Compute X^+ under **G'**
    - If **A** ∈ X^+, then **X → A** is redundant and hence we remove the FD **X → A** from **G** (that is, we rename **G'** to **G**)

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Computing Canonical Cover

- R = { A, B, C, D, E, H }
- F = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
- G = { DE → A, BC → E, AC → E, BCD → A, A → B }
  - Remove **DE → A** from **G**
    - G' = { BC → E, AC → E, BCD → A, A → B }
  - Compute DE^+ under **G'**
    - $\{DE\}^+$ = DE (computed under G')
  - Since A ∉ DE, the FD DE → A is not redundant
    - G = { DE → A, BC → E, AC → E, BCD → A, A → B }

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Computing Canonical Cover

- R = { A, B, C, D, E, H }
- F = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
- G = { DE → A, BC → E, AC → E, BCD → A, A → B }
  - Remove **BC → E** from **G**
    - G' = { DE → A, AC → E, BCD → A, A → B }
  - Compute BC^+ under **G'**
    - $\{BC\}^+$ = BC

      ➔ BC → E is not redundant
    - G = { DE → A, BC → E, AC → E, BCD → A, A → B }

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Computing Canonical Cover

- R = { A, B, C, D, E, H }
- F = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
- G = { DE → A, BC → E, AC → E, BCD → A, A → B }
  - Remove **AC → E** from **G**
    - G' = { DE → A, BC → E, BCD → A, A → B }
  - Compute $\{AC\}^+$ under **G'**
    - $\{AC\}^+$ = ACBE

    Since E∈ ACBE, AC → E is redundant ➔ remove it from **G**
    - G = { DE → A, BC → E, BCD → A, A → B }

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Computing Canonical Cover

- R = { A, B, C, D, E, H }
- F = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
- G = { DE → A, BC → E, BCD → A, A → B }
  - Remove **BCD → A** from **G**
    - G' = { DE → A, BC → E, A → B }
  - Compute BCD^+ under **G'**
    - $\{BCD\}^+$ = BCDEA
  - This FD is redundant ➔ remove it from **G**
    - G = { DE → A, BC → E, A → B }

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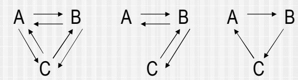
Computing Canonical Cover

- R = { A, B, C, D, E, F }
- F = { A → B, DE → A, BC → E, AC → E, BCD → A, AED → B }
- G = { DE → A, BC → E, A → B }
  - Remove A → B from G
    - G' = { DE → A, BC → E }
  - Compute A^+ under G'
    - $A^+ = A$
  - This FD is not redundant (Another reason why need A → B ?)
    - G = { DE → A, BC → E, A → B }

      ➔ G is a minimal cover for F

Several Canonical Covers Possible?

- Relation R={A,B,C} with F = {A → B, A → C, B → A, B → C, C → B, C → A}
- Several canonical covers exist
  - G = {A → B, B → A, B → C, C → B}
  - G = {A → B, B → C, C → A}



Can you find more ?

Computing a Canonical Cover

This example shows the order of steps 2 and 3 is important!

R= {A,B,C,D} with F = { ABC → D, AB → C, D → C }

1. (step 3; step 2): Doing step 3 first, no FD is redundant (Why?)
   In step 2, ABC → D is left reduced to AB → D. No more changes.
   We thus obtain G = { AB → D, AB → C, D → C } which is equivalent to F but is not minimal! (The red FD is redundant!).
2. (step 2; step 3): Following our algorithm, in step 2 we get G above. In step 3, we remove the redundant FD in G.
   This yields { AB → D, D → C} which is equivalent to F and minimal.

How to Deal with Redundancy?

Relation Schema:

Star (name, address, representingFirm, spokesPerson)
F = { name → address, representingFirm, spokePerson,
representingFirm → spokesPerson }

Relation Instance:

Name	Address	RepresentingFirm	SpokesPerson
Carrie Fisher	123 Maple	Star One	Joe Smith
Harrison Ford	789 Palm dr.	Star One	Joe Smith
Mark Hamill	456 Oak rd.	Movies & Co	Mary Johns

- We can **decompose** this relation into two smaller relations

How to Deal with Redundancy?

Given the relation schema below:

Star (name, address, representingFirm, spokesperson) with

F = {name → address, representingFirm, spokePerson
representingFirm → spokesPerson }

Decompose Star into the following 2 relations:

Star (name, address, representingFirm)
with **F1**={ name → address,representingFirm }

and

Firm (representingFirm, spokesPerson)
with **F2**= { representingFirm → spokesPerson }

How to Deal with Redundancy?

Instance of Star before decomposition:

Name	Address	RepresentingFirm	Spokesperson
Carrie Fisher	123 Maple	Star One	Joe Smith
Harrison Ford	789 Palm dr.	Star One	Joe Smith
Mark Hamill	456 Oak rd.	Movies & Co	Mary Johns

The instance after the decomposition:

Name	Address	RepresentingFirm
Carrie Fisher	123 Maple	Star One
Harrison Ford	789 Palm dr.	Star One
Mark Hamill	456 Oak rd.	Movies & Co

RepresentingFirm	Spokesperson
Star One	Joe Smith
Movies & Co	Mary Johns

Decomposition

- A ***decomposition*** of a relation schema **R** is obtained by splitting **R** into two or more relations, denoted as **R** = {R_1,…,R_m}. Formally, **R** is a decomposition of **R** if the following two conditions hold:
  1. No attribute of **R** is lost or introduced (i.e., $R_1 \cup \ldots \cup R_m = R$)
  2. No schema R_i is a subset or equal to any relation R_j (for $i \neq j$)
  - When $m = 2$, the decomposition **R** = { R_1, R_2 } is called ***binary***
- Not every decomposition of R is "desirable". Why?
- Properties of a decomposition?
  - (1) Lossless-join – this is a **must**
  - (2) Dependency-preserving – this is **desirable**

Explanation follows …

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Example

Relation Instance:

A	B	C
1	2	3
4	2	5

Decomposed into:

A	B
1	2
4	2

B	C
2	3
2	5

To "recover" information, we join the relations:

A	B	C
1	2	3
4	2	5
4	2	3
1	2	5

Why do we got new tuples?

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Lossless-Join Decomposition

- Suppose **R** is a relation and **F** is a set of FD's over **R**.

  A binary decomposition of **R** into relation schemas R_1 and R_2 with attribute sets **X** and **Y** is said to be a **lossless-join decomposition with respect to** F, if for every instance *r* of **R** that satisfies **F**, it holds that $\pi_X(r) \bowtie \pi_Y(r) = r$
- **Thm**: Let **R** be a relation schema and **F** a set of FD's on **R**. A binary decomposition of **R** into R_1 and R_2 with attribute sets **X** and **Y** is lossless **if** $X \cap Y \rightarrow X$ or $X \cap Y \rightarrow Y$, i.e., this binary decomposition is lossless if the common attributes of **X** and **Y** form a key of R_1 or R_2

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Example: Lossless-join

Relation Instance:

A	B	C
1	2	3
4	2	3

$F = \{ B \rightarrow C \}$

Decomposed into:

A	B
1	2
4	2

B	C
2	3

To recover the original relation r, we join the two relations:

A	B	C
1	2	3
4	2	3

No new tuples !

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Example: Dependency Preservation

Relation Instance:

A	B	C	D
1	2	5	7
4	3	6	8

$F = \{ B \rightarrow C, B \rightarrow D, A \rightarrow D \}$

Decomposed into:

A	B
1	2
4	3

B	C	D
2	5	7
3	6	8

Can we enforce $A \rightarrow D$? How ?

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Dependency-Preserving Decomposition

- A dependency-preserving decomposition allows us to enforce every FD (on each insertion of a tuple or when modifying a tuple) by examining just one single relation instance
- Let **R** be a relation schema that is decomposed into two schemas with attribute sets **X** and **Y**, and let **F** be a set of FD's over **R**. The **projection of F on X** (denoted by F_X) is the set of FD's in F^+ that follow from **F** and involve only attributes in **X**
  - Recall that a FD $U \rightarrow V$ in F^+ is in F_X if all the attributes in **U** and **V** are in **X**; In this case, we say this FD is "relevant" to **X**
- The decomposition of < **R, F** > into two schemas with attribute sets **X** and **Y** is **dependency-preserving** if $(F_X \cup F_Y)^+ \equiv F^+$

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Normal Forms

- Given a relation schema **R**, we must be able to determine whether it is "good" or we need to decompose it into smaller relations, and if so, how?
- To address these issues, we need to study **normal forms**
- If a relation schema is in one of these normal forms, we know that it is in some "good" shape in the sense that *certain kinds of problems (related to redundancy) cannot arise*

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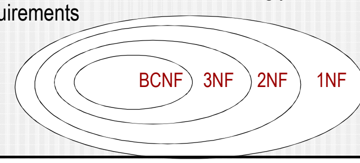
Normal Forms

- The normal forms based on FD's are
  - First normal form (1NF)
  - Second normal form (2NF)
  - Third normal form (3NF)
  - Boyce-Codd normal form (BCNF)
- These normal forms have increasingly restrictive requirements



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Third Normal Form (3NF)

Let **R** be a relation schema, **F** a set of FD's on **R**, $\mathbf{X} \subseteq \mathbf{R}$, and $\mathbf{A} \in \mathbf{R}$.

- We say **R** w.r.t. **F** is in 3NF (**third normal form**), if for every FD $\mathbf{X} \rightarrow \mathbf{A}$ in **F**, at least one of the following conditions holds:
  - $\mathbf{A} \in \mathbf{X}$, that is, $\mathbf{X} \rightarrow \mathbf{A}$ is a trivial FD, or
  - **X** is a superkey, or
  - If **X** is not a key, then **A** is part of some key of **R**
- To determine if a relation <**R, F**> is in 3NF:
  - We check whether the LHS of each nontrivial FD in **F** is a superkey
  - If not, we check whether its RHS is part of any key of **R**

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Boyce-Codd Normal Form

Let **R** be a relation schema, **F** a set of FD's on **R**, $\mathbf{X} \subseteq \mathbf{R}$, and $\mathbf{A} \in \mathbf{R}$.

- We say **R** w.r.t. **F** is in **Boyce-Codd normal form**, if for every FD $\mathbf{X} \rightarrow \mathbf{A}$ in **F**, at least one of the following conditions holds:
  - $\mathbf{A} \in \mathbf{X}$, that is, $\mathbf{X} \rightarrow \mathbf{A}$ is a trivial FD, or
  - **X** is a superkey
- To determine whether **R** with a given set of FD's **F** is in BCNF
  - Check whether the LHS **X** of each nontrivial FD in **F** is a superkey
    - How? Simply compute $\mathbf{X}^+$ (w.r.t. **F**) and check if $\mathbf{X}^+ = \mathbf{R}$

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