

Matrix games

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1 Introduction

Introduction

Given

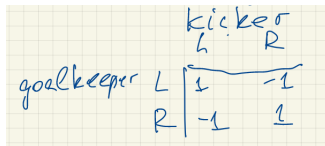
- Player I (rows, Alice)
- Player II (columns, Bob)
- a *payoff* matrix $A \in \mathbb{R}^{m \times n}$

Every round

- 1 Alice picks (row) strategy $i \in [m] := \{1, \dots, m\}$ Bob picks (col) strategy $j \in [n]$
- 2 Bob pays Alice the amount $a_{i,j}$

zero-sum game

Example: penalty game



A handwritten payoff matrix for a penalty game on a grid background. The matrix is written in blue ink. The columns are labeled 'kicker' with sub-labels 'L' and 'R'. The rows are labeled 'goalkeeper' with sub-labels 'L' and 'R'. The payoffs are as follows:

		kicker	
		L	R
goalkeeper	L	1	-1
	R	-1	1

Figure: penalty game

Example: prisoners dilemma

	Confess A	Stay quiet A
Confess B	6 6	10 0
Stay quiet B	0 10	2 2

Worst case

- Alice gets $\min_{j \in [n]} a_{i,j}$

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- Alice gets $\max_{j \in [m]} \min_{i \in [n]} a_{i,j}$
- Bob gets $\min_{i \in [n]} \max_{j \in [m]} a_{i,j}$

We claim:

$$\max_i \min_j a_{i,j} \leq \min_j \max_i a_{i,j}$$

"Tallest dwarf is not as tall as the smallest giant."

But equality does not hold in general!

Proof of the min-max theorem

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$$\min_j a_{ij} \leq \min_j \max_i a_{ij} \quad \forall i$$