Definition 0.1. Given a vector space V a **norm** $\|\cdot\|$ is mapping from V to $[0, +\infty)$ such that

1. **positive definiteness:** $\|x\| = 0$ if and only if $x = 0$.
2. **absolute homogeneity:** $\|\lambda x\| = |\lambda|\|x\|$ for all $\lambda \in \mathbb{R}$.
3. **triangle inequality:** $\|x + y\| \leq \|x\| + \|y\|$ for all $x, y \in V$.

1 Problem set

1. Show that all norms are *convex* functions.
2. Let $g : \mathbb{R}^m \to \mathbb{R}$ be a convex function and $A \in \mathbb{R}^{m\times d}$ a linear operator (matrix). Show that $f(\cdot) = g(A\cdot)$ is also convex.
3. Let x^* be local minimum of a convex function $f : \mathbb{R}^d \to \mathbb{R}$. Show that x^* is a global minimum.
4. If $\bar{x}$ is a stationary point of the **convex** function f, then $\bar{x}$ is a global minimizer of f (give a geometric intuition).