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Modeling a Maunder Minimum

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Abstract. We introduce intermittency into a mean field dynamo model by imposing stochastic fluctuations either in an additive electromotive force or the alpha effect. Sufficiently strong small scale fluctuations with time scales of the order of 0.3-3 years can produce long term variations in the system on time scales of the order of hundreds of years. The variation of the magnetic field does not resemble that of the sunspot number, but is more reminiscent of the ^{10}Be record. The interpretation of our results focuses attention on the connection between the level of magnetic activity and the sunspot number, an issue that must be understood if long term solar effects are to be well understood.

1. Introduction

The sun shows variability on a broad range of time scales, from milliseconds to millenia and on to much longer scales. Here we are interested in the time scales associated with the solar cycle and its intermissions. A salient manifestation of this cyclic behavior is seen in the sunspot number, whose annual mean oscillates on a scale of eleven years (or twenty-two years, if one goes by magnetic polarity variations). The term "cycle" is used in the sense that the sunspot number performs the same kind of oscillation approximately every eleven years, with a magnitude that varies strongly in a way that is reminiscent of a chaotic oscillator. Although we do not have sufficient data to conclude that the solar oscillation is chaotic, in the sense that it is describable by a low order deterministic dynamical system (Spiegel & Wolf 1987), this possibility offers a natural way to model various irregularities of the cycle (Spiegel 1977; Tavakol 1978; Ruzmaikin 1981).

Like the sun, simple chaotic systems repeat themselves over and over again, but never the same way twice, much as the sun does in executing its cycle. On the other hand, the simplest chaotic dynamos (Allan 1962; Robbins 1978) do not provide the kind of intermittency evinced in grand minima like the Maunder minimum (Eddy 1978) when the amplitude of the oscillation in sunspot numbers went nearly to zero for seventy-five years in the time of Newton. This behavior is more suggestive of the possibility that a strongly intermittent chaotic oscillator needs to be invoked in trying to understand the solar oscillation. Oscillators that behave this way are known (Fautrelle & Childress 1982; Spiegel 1981) but when they are in the active phase, their variations do not resemble those of the sunspot number.

As we shall discuss at the end of this paper, it is not obvious how to connect the output of a model with the sunspot number. It may therefore not be damning if the variations produced by a model do not reproduce qualitatively the variations seen in the sunspot number. Nevertheless, in terms of lumped models, whose behavior is purely temporal, it has been possible to produce variations that resemble those of the sun in various qualitative ways (Weiss et al. 1984; Platt et al. 1993b). But the solar variability is manifestly spatio-temporal and such lumped models, though possibly interesting in their own ways, can at best provide a guide to the actual processes involved in the solar cycle. In this paper, we describe on attempt to go beyond such simple models.

2. Spatio-temporal variability

The spatio-temporal dynamics of the solar cycle as seen in space-time diagrams like the Maunder butterfly diagram suggest that waves play an active part in the solar activity cycle. A nonlinear version of Parker's (1955) dynamo waves or waves arising from overstable convection could be the cause of the drift of the center of activity through the cycle. The finite width of the activity zone at any time suggests that the waves in question are themselves confined or guided by a layer of some corresponding thickness. Such a layer could lie deep in the convection zone (DeLuca 1986) or just below it (Spiegel & Weiss 1980). It should have only the standard ingredients of a dynamo — differential rotation and cyclonic convection — from which we can make a model. Here we use mean field theory (Krause & Rädler 1980) with the effects of small scale motions subsumed into turbulent diffusivity and the α-effect. Though mean field dynamos cannot tell the whole story of the solar magnetic fluctuations (e.g. Hoyng 1987; 1988), they will adequately suit our purpose of trying to model the intermittency signaled by the Maunder minimum.

We have tried for some time to see whether the mechanisms that produce grand minima in the lumped models can be used equally effectively in the case of spatio-temporal models. Our starting point in this effort was a working model of the solar cycle (Rüdiger & Brandenburg 1995) that had not previously produced grand minima. It is not at all obvious that the mechanisms of intermittency could in fact produce periods of inactivity globally in models with large spatial extent, any more than one can do this in a turbulent medium. In fact, we failed in our attempt and this paper is about the meaning of this failure.

The situation is complicated by the existence of several forms of intermittency that have been isolated in dynamical systems theory. However, there is a particular one that seems most likely to be at work in the solar cycle called *on/off intermittency* (Platt et al. 1993a; see also Spiegel 1994). This is the form of intermittency detected in the output of a probe in a turbulent fluid registering abrupt changes in the output between excited and quiescent states. One may think of this as a series of bursts or a chaotic relaxation oscillations. What characterizes models that have been made of this process (e.g. Spiegel 1981, Ott & Chen 1990, Pikovsky & Grassberger 1991) is that the (potentially) unstable oscillator performing the cycle is driven to instability through coupling to an aperiodic driver that is continuously chaotic or stochastic. The role of the driver is to move the system into and out of the unstable state. Observations of the bursts cannot readily distinguish the chaotic driver from a stochastic alternative which works equally well (Hardenberg et al. 1996), and the number of degrees of freedom involved in such an object in the solar case will be hard to determine (but see Heagy et al. 1994). Fortunately, the qualitative features of the process do not depend sensitively on this difference, and we shall use a stochastic driver here.

The effects of stochastic noise on mean field $\alpha\Omega$ dynamos have been studied previously (Choudhuri 1992, Moss et al. 1992, Hoyng et al. 1994), mainly to model irregularities of the solar cycle on time scales comparable to the solar cycle itself or shorter. Moss et al. (1992) suggested that a modulation of the solar cycle on longer timescales of the order of centuries should also be possible. More recently, work along those lines (Schmitt et al. 1996) has introduced the on/off intermittency mechanism into a mean-field style of dynamo, as had been used already in a lumped model of the solar cycle (Platt et al. 1993b). The present paper is similarly based on a relatively realistic model of the solar dynamo, in which we embed an on/off intermittency mechanism.

Apart from technical details, the main difference between our work and that of Schmitt et al. is that we consider basically a standard α-effect while they introduced a lower cutoff excluding field generation below a field strength of 1 kG at the base of the convection zone. This kind of filtering is reasonable, and it is analogous to Durney's (1995) introduction of a critical field value in the tachocline above which a magnetic tube is ejected. This deus ex machina seems to be needed to produce grand minima in spatio-temporal models and this issue is what this paper is really about, as we shall discuss at the end. Another difference with the calculation of Schmitt et al. is that we consider the full sun whereas they studied only a single hemisphere and produced one-winged butterflies. This turns out to be another aspect of the difficulty that this paper is designed to discuss.

Making a spatio-temporal system demonstrate on/off intermittency has something in common with laying a rug. Once the rug is nailed down, if there is a bump somewhere, there seems to be no way to squeeze it out of existence. We have had the same trouble with the dynamo model. If we produce a grand mimimum locally somewhere in space, we find invariably that there is excitation elsewhere. Furthermore, while we can get a whole hemisphere to be quiet at one time, the other one may still be active. The output of the dynamo model is therefore not like the sunspot number, except that it does have cycles. As we have learned at the Fermi School on the Interaction of the Solar Cycle with Terrestrial Activity in June of 1996, certain terrestrial data have much more in common with the output of the dynamo models than the sunspot number. Moreover, some of these terrestrial data are very convincing proxy data for the solar activity (Beer et al. 1994), while others may mimic the sunspot number. We conclude that the heart of the issue is one of knowing how to interpret the output of dynamo models, as we shall explain at the end of this paper. Before coming to this, however, we need to sketch some of the model results.

We describe some details about the model itself in Sect. 3, including the manner in which the fluctuations are introduced. Readers who are not interested in such detailed information should skip directly to Sect. 4 where we outline the main results. In these, we focus on models with a rather high noise level exceeding the electromotive force from the α-effect by an order of magnitude.

3. The Model

We use a mean-field model of the solar dynamo (Rüdiger & Brandenburg 1995) with anisotropic α-effect and turbulent magnetic diffusivity and that includes magnetic buoyancy. The prescribed angular velocity is taken from helioseismology (Christensen-Dalsgaard & Schou 1988). To obtain a 22 yr magnetic cycle period, we introduce a scaling factor in the magnetic diffusivity of 0.5 and, to get a butterfly diagram with sufficient activity at low latitudes, we introduce a suitable latitude dependence in α.

In the original model calculations, the value of α was typically set at a value approximately twenty times the critical value for instability. To produce on/off intermittency (Platt et al. 1993a), in the highly supercritical case, we would need very large fluctuations at that value. In this exploratory study, we prefer to operate at more modest parameter values and to avoid the large fluctuations they entail. Therefore we have set the system nearer to criticality so that the driver can move it into and out of the unstable state easily. So we choose a slightly subcritical value of α and introduce only modest fluctuations in its magnitude. For this purpose we introduce a scaling factor c_α in front of certain components of the α-tensor (those components which result from the interaction of rotation and stratification). For details see the original discussion of the model (Rüdiger & Brandenburg 1995) where, with $c_\alpha = 1$, the resulting toroidal magnetic field is a few kgauss and the poloidal field at the surface about 10 gauss. In the cases studied here, however, where the dynamo is just marginally excited, the generated magnetic field is weak, and could not explain the field strength observed in the sun. This weakness of the modeling can be avoided, as the work of Schmitt et al. (1996) shows.

We consider separately two kinds of fluctuation in the model. In addition to fluctuations in α, we introduce a fluctuating electromotive force $\boldsymbol{\mathcal{E}}$. The latter is a more elementary process that does not require much dynamo theory. The picture here is that in the bulk of the convection zone a small scale dynamo operates (e.g. Meneguzzi & Pouquet 1989, Nordlund et al. 1992) producing a highly intermittent magnetic field in space and time and this is represented by $\tilde{\boldsymbol{\mathcal{E}}}$. In the bulk of the convection zone the fluctuations are immense, hence even spatial and temporal averages (such as $\boldsymbol{\mathcal{E}} = \langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle$) remain fluctuating, albeit on longer scales. We adopt white noise with vanishing mean value and a root mean square value of unity. The temporal power spectrum of $N = N(t, r, \theta)$ is flat for frequencies smaller than $2\pi/\tau$. The value of τ determines the time span over which long term variability of the resulting mean magnetic field is possible. The amplitude of the noise is measured by $n_{\mathcal{E}}$ in terms of the rms-velocity of the turbulent motions and the local equipartition field strength. If the cycle-producing dynamo is in a stably stratified shear layer just below the photosphere (a tachocline), the fluctuating emf should really act on the boundary only, but we have not attempted to implement this version of the forcing.

We can include either these emf fluctuations or, as mentioned, we can put in fluctuations in the α effect. To do this, we add a fraction n_α of the noisy component to the original α.

In normal mean-field theory, the angular brackets refer to ensemble averages; in practice they are approximated by spatial and temporal course graining averages. The error resulting from this approximation is sometimes interpreted as a source of stochastic noise (Hoyng 1987; 1988; Moss et al. 1992). But, as we have seen, there is ample reason for expecting fluctuations to appear in any realistic model.

4. Results

4.1. Fluctuating α-effect

To regulate the strength of the effect, we introduce a factor c_α in front of the familiar α term, as described in Sect. 3. With everything else fixed, instability occurs when c_α exceeds the critical value $c_\alpha^{(\mathrm{crit})} \approx 0.03$. We begin by adopting the subcritical value $c_\alpha = 0.02$. We adjust the fluctuations in α to have a time scale $\tau = 3\,\mathrm{yr}$.

If the noise level, n_α, is too low, the magnetic field decays to zero, as it does when $n_\alpha = 0.02$. Short noisy boosts in α are insufficient to bring the magnetic field to appreciable strength. On the other hand, if n_α is rather larger than this, the magnetic field is almost entirely dominated by the noise and cyclic behavior does not occur, except for irregular reversals on the timescale of centuries. Such a case is depicted in Fig. 1, where we show the butterfly diagram of the toroidal magnetic field at the bottom of the convection zone. A very similar result, with reversals on a long time scale, is found even when the non-random component of α is absent altogether provided there is global shear as in accretion disks or the tachocline (Vishniac & Brandenburg 1996).

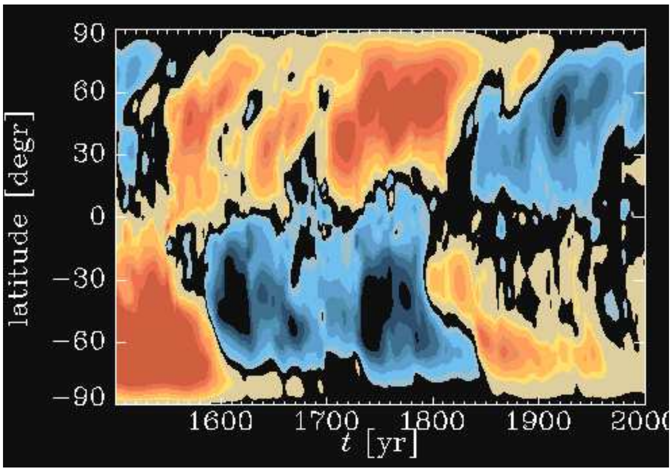

Fig. 1. Butterfly diagrams of the toroidal field at the base of the convection zone for $c_\alpha = 0.02$, $n_\alpha = 5$ and $\tau = 3\,\mathrm{yr}$

To get sufficiently large magnetic field strengths, we focus attention on models with $c_\alpha = 1$. With $\tau = 3\,\mathrm{yr}$ we find long term variability on a time scale of $200-500\,\mathrm{yr}$; see Fig. 2. This variability, though not really global, may be associated with grand minima and maxima, as discussed in Sect. 5.

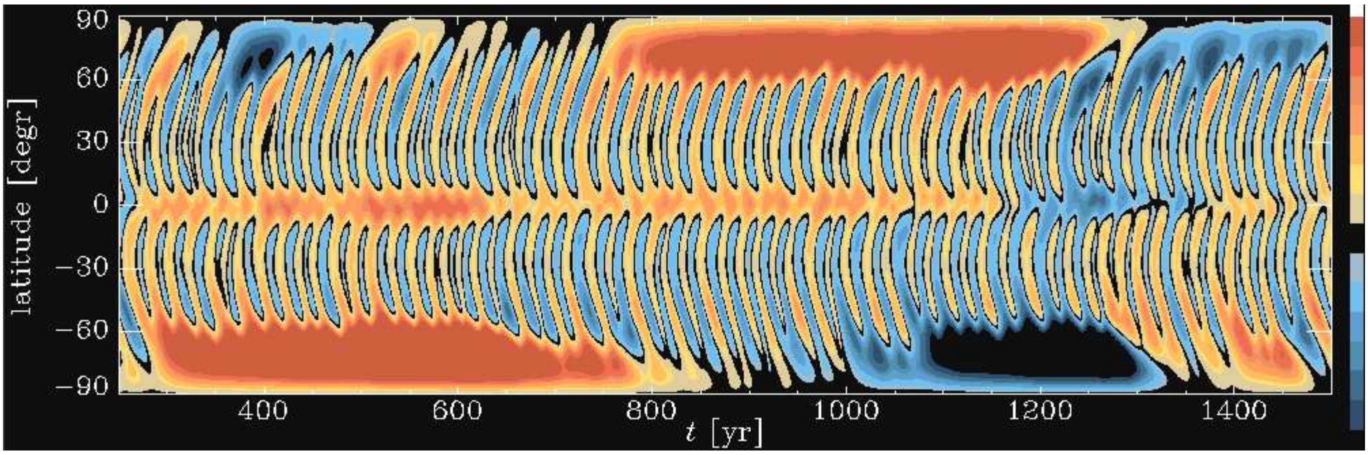

Fig. 2. Butterfly diagrams of the toroidal field at the base of the convection zone for $c_\alpha = 1$, $n_\alpha = 0.2$ and $\tau = 3\,\mathrm{yr}$

Even if τ is decreased, to a value of $\tau = 0.3\,\mathrm{yr}$ say, long term variability still occurs, but individual cycles may vary significantly in amplitude see Fig. 3. Such behavior is found only if the fluctuations of α are sufficiently larger than the average value; here a factor of five is needed. Individual fluctuations of α can still be much larger because we assumed an exponential distribution for the probability density of α fluctuations.

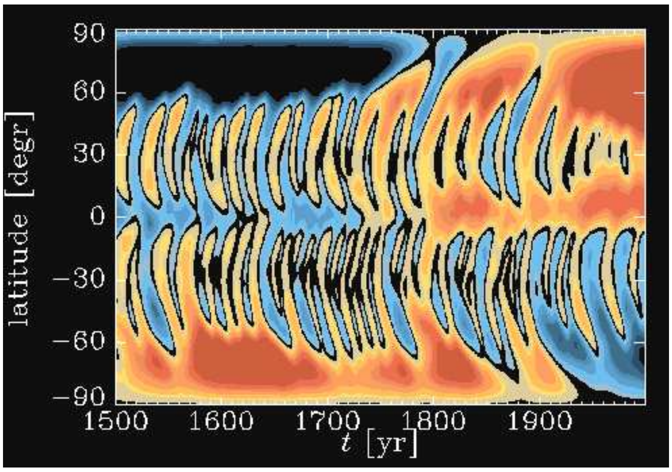

Fig. 3. Butterfly diagrams of the toroidal field at the base of the convection zone for $c_\alpha = 1$, $n_\alpha = 5$ and $\tau = 0.3\,\mathrm{yr}$

4.2. Magnetic noise

If we introduce an external stochastic emf, as described in Sect. 3, we find a qualitatively similar behavior to that with fluctuating α; see Fig. 4. Fluctuations of various kinds can evidently produce long-time intermittency.

The model shows two distinct activity waves, one migrating equatorwards and the other polewards. The two waves seem to be modulated independently, in each hemisphere. So do the modulations in the two hemispheres seem to be only weakly coupled, with little tendency to form a dipole structure. It may be that the approximate antisymmetry of the toroidal magnetic field of the sun, suggested by Hale's polarity law, may not be a very stable feature, and that other types of symmetry might have occurred in the past. Other, more regular parity variations of the magnetic field have previously been seen in nonlinear models (Brandenburg et al. 1989a,b, 1990, Jennings & Weiss 1991, Sokoloff & Nesme-Ribes 1994).

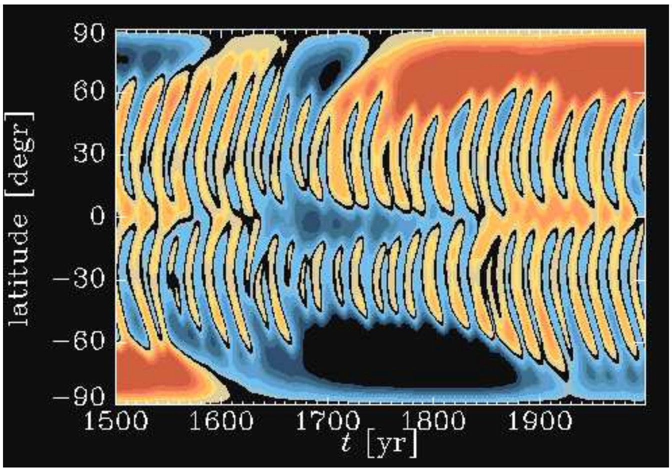

Fig. 4. Butterfly diagrams of the toroidal field at the base of the convection zone for $c_\alpha = 1$, $n_{\mathcal{E}} = 1$ and $\tau = 3\,\mathrm{yr}$

5. Interpretation

We have made an intermittent dynamo but not a grand mimimum, as has been done for lumped models (Platt et al. 1993b). Simple dynamo models, both with fluctuating α-effect and with magnetic noise injected in the bulk of the convection zone, can produce intermittency on sufficiently long time scales, but they do not switch off globally over the whole sun, in both hemispheres at once. We suggest that the problem is inherent to all models with propagative behavior and that it cannot be expected that the cycle-producing processes of the sun switch off everywhere for several cycles.

To produce grand minima in a dynamo we need then to introduce an additional feature that relates to the production of sunspots, or more specifically, strong flux tubes of sufficiently large cross-section and that is the message of the procedures of Durney (1995) and Schmitt et al. (1996). A simple way of formulating this problem is to say that the sunspot number, which is a global parameter, is a functional of the various fields produced in the model. To get that functional, we need to operate with an explicit sunspot production mechanism.

The rationale of this argument is rooted in the notion that there are two (at least) distinct dynamos in the sun. One is the convection zone that continually produces rapidly fluctuating fields of moderate strength. Down below, in the tachocline (say), another dynamo process operates with longer time scales and this gives rise to more intense field concentrations. We have seen a similar kind of symbiosis in the simulations of the geodynamo by Glatzmaier & Roberts (1995). In this view, when the local field produced by the internal dynamo is too weak, the grouping of flux ropes into a sunspot field is not achieved, even though much of the normal activity continues in the convection zone. It is indeed possible that the spot-producing mechanism is not intrinsic to the dynamo process but, for the present purposes, we illustrate the problem with a particular functional of the dynamo field.

Let $R(t)$ be the average magnetic field strength between $\pm 10°$ and $\pm 30°$ latitude, but allow only those areas where the field exceeds the root mean square value by 30 percent to contribute to this average. This quantity is plotted in Fig. 5. Note that $R(t)$ shows periods with almost vanishing magnetic activity as in a Maunder minimum. The grand minima on this interpretation of the output of the models result from what may be regarded as lean cycles magnetically. If there are magnetic droughts, they are only local, not global, though the magnetic means may be low. In that sense, the sunspot number, though valuable for having focused our attention on an interesting feature of the magnetodynamics, may in some ways be a misleading indicator of what is happening overall.

This view is supported by studies of other indicators of solar activity than the sunspot number. The most complete records of variations that may result from solar activity fluctuations are found in the ^{10}Be records from ice cores (Beer et al. 1994, 1996). The ^{10}Be variations are plausibly attributed to modulation of the cosmic ray flux by the magnetic field in the solar wind (Beer et al. 1996). In Fig. 6 we reproduce data kindly provided by Dr. Jürg Beer comparing the ^{10}Be records with the sunspot number for nearly four centuries. We see that a cyclic modulation

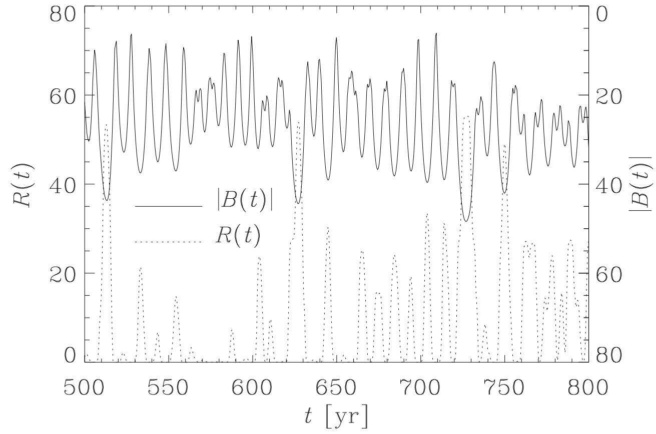

Fig. 5. Time series of $|B|$ (dotted line) and the activity parameter R (solid line) for the same run as in Fig. 2. Note that the scale for $|B|$ increases downwards, in order to mimic the approximate anticorrelation between the ^{10}Be data and the sunspot number.

is very much in evidence during the Maunder minimum, with a rather modest reduction in its amplitude compared to that of the sunspot number.

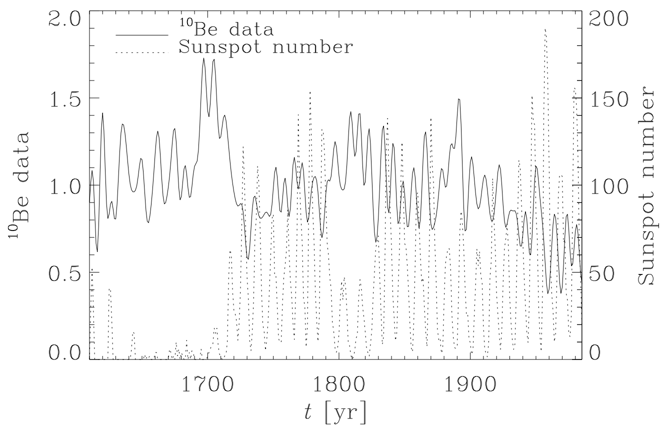

Fig. 6. ^{10}Be data (solid line) together with the sunspot number (dotted line), as provided by Dr. Jürg Beer. The data are from a shallow core (300 m) drilled at Dye 3, Greenland, in 1986. The data were filtered using a spectral filter with a cut-off of 6 years and interpolated using a cubic spline. The younger part (1783-1985) is published in (Beer et al. 1990), and the whole record appeared in Beer et al. (1994).

The situation as brought out in the cited papers of Beer et al. is that various measures of solar activity are not perfectly correlated. If we were to think of the output of a model solar dynamo as we would one of these other measures, we would not be surprised that it does not necessarily represent all of them faithfully. Unfortunately, there is as yet there no clear theoretical indication which one of them a model should most closely represent. If, in a grand minimum, there is high magnetic activity in only one hemisphere, it is not unreasonable that the modulations of ^{10}Be should continue with reasonable strength. The question for the theory then is what is the relationship between the variations produced by the models and (say) the sunspot number. This is particularly difficult since the sunspots seem to be produced well within the sun and not at the surface, whereas some of the other activity measures are no doubt superficially produced. Even if we do have a good model of the cyclic mechanism, we also need to understand how sunspots form and surface before we can predict their number.

Nor is it unimportant to try to predict the sunspot number, or something akin to it, for it is this quantity that seems to be connected to some climatological variations. The most striking evidence of this is the discovery in tree ring data (Douglass 1927) that "the sunspot curve flattens out in a striking manner ... from 1670 or 16780 to 1727." This discovery was made by A.E. Douglas before he had "received a letter from Professor E. Maunder ... calling attention to the prolonged dearth of sunspots between 1645 and 1715." As Douglass and others have argued, variations in tree ring thickness in turn are connected to rainfall, so it is not an idle project to try to understand how to go from the workings of a solar dynamo to the manufacture of sunspots.

In summary, solar activity waves at high and low latitudes and in the two hemisphere, lead somewhat independent, weakly correlated lives and fluctuate separately under the influence of noise. Spatial variations of the solar cycle during the Maunder minimum are not an immediate indicator of the sunspot number and if the sunspots have their origin well beneath the solar surface we must go another step in the discussion before we obtain results that are fully consistent with historic records of sunspots (Ribes & Nesme-Ribes 1993). It is not at all obvious how the effects of solar subconvective activity will appear from outside the sun.

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