# Defining $\chi$ with $c_p$ or $c_v$

`> Here Ed Spiegel's email of the end of July.`

That was basically clear and in principle one can make either choice: we can write

$$\frac{DT}{Dt} = \frac{\mathcal{H}-\mathcal{C}}{\varrho c_v} + \frac{\nabla\cdot(K\nabla T)}{\varrho c_v} + (\gamma-1)T\frac{D\ln\varrho}{Dt}\ , \tag{1}$$

which naturally leads to the definition $\chi_v \equiv K/(\varrho c_v)$ — or we write

$$\frac{DT}{Dt} = \frac{\mathcal{H}-\mathcal{C}}{\varrho c_p} + \frac{\nabla\cdot(K\nabla T)}{\varrho c_p} + \frac{\gamma-1}{\gamma}T\frac{D\ln p}{Dt}\ , \tag{2}$$

which naturally leads to the definition $\chi_p \equiv K/(\varrho c_p)$.

However, I see two points that make one variant preferrable over the other:

1. Eq. (1) is more convenient if $\ln\varrho$ varies less than $\ln p$, while Eq. (2) is more useful if $\ln p$ varies less. Now for subsonic convection local pressure balance is enforced by sound waves, while strong temperature fluctuations can cause relativily strong density fluctuations. Thus, Eq. (1) and correspondingly $\chi_p$ may be more useful.
2. As was shown by Ed Spiegel himself, for weakly compressible convection one obtains the same threshold value for Ra as in the incompressible case if one defines

$$\mathrm{Ra} \equiv \frac{(\Theta_{\mathrm{top}}-\Theta_{\mathrm{bot}})gH^3}{\Theta_{\mathrm{mid}}\chi_p\,\nu}\ , \tag{3}$$

where $\Theta = T - \beta_{\mathrm{ad}}z$ denotes potential temperature. Thus, $\chi_p$ is more convenient.