Master-Thesis

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Blah blah blah

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Abstract

Things about this thesis. why and what question should be answered. and what are the answers. (zusammenfassung)

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wie ich das buch schreibe, R markodwn bookdown und so und welche versionen ich nutze

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zb plotly_save

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Chapter 4

Mathematical Fundations

This chapter provides an overview of the mathematical calculations and conventions used in this Thesis. It's important to note that most of the time mathematical formulas are written in matrix notation. In the majority of cases, this will result in a direct translation into R-code. All necessary assumptions needed for the modeled return structure are provided in this chapter to enable each reader to make sense of the stated formulas. It is crucial to note that reality is too complicated and can only be partially modeled. Simplistic, basic models are employed that don't hold up in real-world situations, but these models or variations on them are frequently used in finance and have proven to be helpful. The complexity of solving advanced and basic models do not differ for the PSO, because the dimension of the objective function is based on the number of selectable elements, see chapter 6.

4.1 Basic Operators

A compendium that compares commonly used mathematical symbols to R-code and its meanings can be found in the table below:

Latex/Displayed	R-Code	Meaning
×	%*%	Matrix or vector multiplication and cross-product
A^T	t(A)	transpose of Matrix or vector A
·	*	Scalar or elementwise vector multiplication

4.2 Return calculation

Any portfolio optimization strategy based on historical data must start with returns. These returns are calculated using adjusted closing prices, which show the percentage change over time. Adjusted closing prices are reflecting dividends and are cleaned of by stock splits and rights offerings. These Returns are essential for comparing assets and for analyzing dependencies.

4.2.1 daily returns

The default timeframe for all raw data in this thesis is one workday, and we only use simple returns. The simple returns can be calculated as follows if we know the adjusted closing price P of one asset on workdays t_i and t_{i+1}:

$$R_{i+1} = \frac{P_{t_{i+1}}}{P_{t_i}} - 1$$

4.2.2 annualized returns

4.3 Markowitz Modern Portfolio Theory (MPT)

In 1952, Harry Markowitz published his first ground-breaking work, which had a significant influence on modern finance, primarily by outlining the effects of diversification and efficient portfolio. The definition of an efficient portfolio is one that has either the maximum expected return for a given risk target or the minimum risk for the given expected return target. A simple quote to define diversification could be: “A portfolio has the same return but less variance than the sum of its parts”. This is true if the assets are not perfectly correlated because bad and good movers can make up for each other, reducing the likelihood of extreme events. You can find more specific information at (Maringer, 2005).

4.3.1 Assumptions of Markowitz Portfolio Theory

The following are the Markowitz assumptions that can be combined, according to (Maringer, 2005):

- Perfect market without taxes or transaction costs.
- Short sales are disallowed.
- Assets are infinitely divisible.
- Expected Returns, Variances and Covariances contain all information.
- Investors are risk-adverse, they will only accept greater risk if they are compensated with a higher expected return.

The assumption that the returns are normally distributed is not required, but it will be assumed in this case to make the problem simpler. (Maringer, 2005) has further details regarding the requirements for utilizing other distributions. It is obvious that these assumptions are unrealistic in real-life.

4.4 Portfolio Math

Proofs for the fundamental calculations required for portfolio optimization as shown in (Zivot, 2021) will be provided in this section. The returns are presented differently than in most sources, because its the most common data-format used in practice. Suppose there are N assets that are described by a return vector R of random variables and a portfolio weight vector w, respectively:

$$R = \begin{bmatrix} R_1 & R_2 & \cdots & R_N \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ w_2 \\ \cdots \\ w_N \end{bmatrix}$$

Each return is additionally simplified in this thesis so that it is normally distributed with $R_i = \mathcal{N}(\mu_i, \sigma_i^2)$. As a result, linear combinations of normally distributed random variables are jointly normal distributed and have a mean, variance, and covariance that can be used to fully describe them.

4.4.1 expected returns

The following formula can be used to get the expected returns of a vector with normally distributed random variables $R \in \mathbb{R}^N$:

$$\begin{aligned} E[R] &= \begin{bmatrix} E[R_1] & E[R_2] & \cdots & E[R_N] \end{bmatrix} \\ &= \begin{bmatrix} \mu_1 & \mu_2 & \cdots & \mu_N \end{bmatrix} = \mu \end{aligned}$$

and μ_i can be estimated in R with the base-function `mean()` and historical data.

4.4.2 expected portfolio return

The following equation can be used to get the linear combination of expected returns *mu* and a weighting vector w (for example, portfolio weights):

$$\mu \times w = \begin{bmatrix} E[\mu_1] & E[\mu_2] & \cdots & E[\mu_N] \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ \cdots \\ w_N \end{bmatrix}$$
$$= E[\mu_1] \cdot w_1 + E[\mu_2] \cdot w_2 + \cdots + E[\mu_N] \cdot w_N = \mu_P$$

4.4.3 portfolio returns

Let $R \in \mathbb{R}^{T \times N}$ denote a realized return Matrix of N assets and T days in the past. The portfolio return on each day can be calculated with the formula of the expected portfolio return. This is possible on all days T by:

$$R \times w = \begin{bmatrix} R_{1,1} & R_{1,2} & \cdots & R_{1,N} \\ R_{2,1} & R_{2,2} & \cdots & R_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ R_{T,1} & R_{T,2} & \cdots & R_{T,N} \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} = \begin{bmatrix} R^P_1 \\ R^P_2 \\ \vdots \\ R^P_N \end{bmatrix} = R_P$$

4.4.4 Covariance

The general formula of the covariance matrix $\sum$ of a random vector R with N normally distributed elements and $\sigma_{i,j}$ as correlation of two unique assets is described as:

$$Cov(R) = E[(R - \mu)^T \times (R - \mu)]$$
$$= \begin{bmatrix} \sigma_1^2 & \sigma_{1,2} & \cdots & \sigma_{1,N} \\ \sigma_{2,1} & \sigma_2^2 & \cdots & \sigma_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N,1} & \sigma_{N,2} & \cdots & \sigma_N^2 \end{bmatrix}$$
$$= \sum$$

and can be estimated in R with the base-function `cov()` and historical data.

4.4.5 Portfolio Variance

Let R be a random vector with N normally distributed elements and w a weighting vector. Suppose the covariance matrix $\sum$ of R is known, then the variance of the linear combination of R can be calculated as:

$$Var(R \times w) = E[(R \times w - \mu \times w)^2]$$
$$= E[((R - \mu) \times w)^2]$$

Since $(R-\mu)\times w$ is a scalar we can transform $((R-\mu)\times w)^2$ to $((R-\mu)\times w)^T\times((R-\mu)\times w)$ which results in:

$$\begin{aligned}
Var(R\times w) &= E[((R-\mu)\times w)^T\times((R-\mu)\times w)] \\
&= E[(w^T\times(R-\mu)^T)\times((R-\mu)\times w)] \\
&= w^T\times E[(R-\mu)^T\times(R-\mu)]\times w \\
&= w^T\times\sum\times w
\end{aligned}$$

The same hold for a estimation of $\sum$.

Chapter 5

Activ vs Passiv Investing

The fundation of Asset Management

passiv vs activ studie `https://www.scirp.org/journal/paperinformation.aspx?paperid=92983`

gut gut file:///C:/Users/Axel/Desktop/Master-Thesis-All/Ziel%20was%20bea ntwortet%20werden%20soll/Quellen%20nur%20wichtige/Rasmussen2003_Bo ok_QuantitativePortfolioOptimisat.pdf

Chapter 6

Challenges of Passiv Investing

This Chapter will analyse two common challenges of Passiv-Investing and create simple examples to test the PSO. The first one is the mean-variance portfolio (MVP) from the modern portfolio theory of Markowitz which is simply said an optimal allocation of assets regarding risk and return. The second challenge is the index-tracking-problem which tries to construct a portfolio which has a minimal tracking error to a given benchmark.

6.1 Mean-variance portfolio (MVP)

Markowitz has shown that diversifying the risk on multiple assets will reduce the overall risk of the portfolio. This result was the beginning of the widely used modern portfolio theorie which uses mathematical models to archive portfolios with minimal variance for a given return target. All these optimal portfolios for a given return target are called efficient and create the efficient frontier.

6.1.1 MVP

Let there be N assets and its returns on T different days which creates a return matrix $R \in \mathbb{R}^{T \times N}$. Each element $R_{t,i}$ contains the return of the i-th asset on day t. The covariance matrix of the returns is $\sum \in \mathbb{R}^{N \times N}$ and the expected returns are $\mu \in \mathbb{R}^N$. The MVP with risk aversion parameter $\lambda \in [0, 1]$ like shown in (Maringer, 2005) can be formalized as follows:

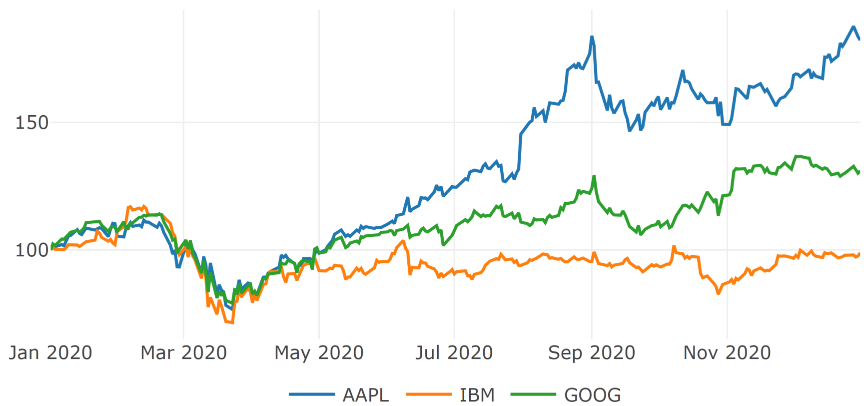
$$\underset{w}{minimize} \quad \lambda \; w^T \sum w - (1 - \lambda) \; \mu^T w \tag{6.1}$$

The risk aversion parameter λ defines the trade-off between risk and return. With $\lambda = 1$, the minimization problem only contains the the variance term and so on results in a minimum variance portfolio and $\lambda = 0$ transforms the problem to a minimization of the negative expected returns so on results in a maximum return portfolio. All possible $\lambda \in [0, 1]$ represent the efficient frontier.

6.1.2 MVP example

We will analyze a small example to understand the meaning of the efficient frontier without going into detail how it was solved. First of all we are loading the daily returns of IBM, Google and Apple from the year 2020.

The cumulated daily returns are:



The calculated expected daily returns and the covariance matrix for the 3 assets are:

```
mu <- as.vector((last(ret_to_cumret(returns))/100)^(1/nrow(returns))-1) %>%
  setNames(., colnames(returns))
p0("expected daily return estimation")
```

```
## [1] "expected daily return estimation"
```

```
mu
```

```
##           AAPL            IBM           GOOG
##  0.00237641721 -0.00004622149  0.00106873786
```

```
cov <- as.matrix(nearPD(cov(returns))$mat)
p0("covariance estimation")
```

```
## [1] "covariance estimation"
```

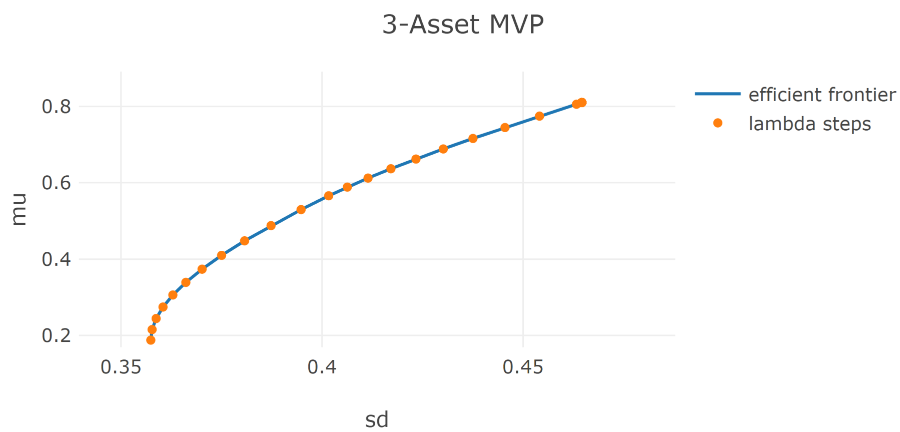
```
cov
```

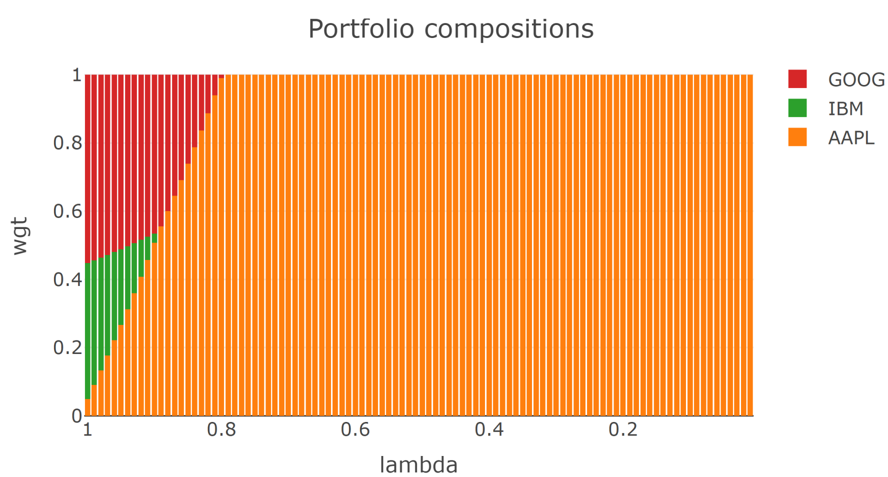
```
##              AAPL         IBM         GOOG
## AAPL 0.0008635696 0.0004356282 0.0005337719
```

```
## IBM  0.0004356282 0.0006626219 0.0004086728
## GOOG 0.0005337719 0.0004086728 0.0005827306
```

We now have all the necessary data to solve the MVP (6.1) with $\lambda \in \{0.01, 0.02, ..., 0.99, 1\}$. We calculate all 100 portfolios by solving the quadratic minimization problem for each λ.

Following that, we convert the daily returns and standard deviation to yearly returns and standard deviation before plotting the efficient frontier.





6.2 Index-tracking portfolio (ITP)

Indices are asset baskets that are used to track the performance of a specific asset group. The well-known Standard and Poor's 500 index (short: S&P 500), for example, tracks the top 500 stocks in the United States. All indices are not investible and only serve to visualize the performance of these asset groups without incurring transaction costs. Asset managers use such indices as benchmarks to compare the performance of their funds. Each fund has its own benchmark, which contains roughly the same assets that the manager could purchase. If the fund underperforms its benchmark, it may be an indication that the fund manager made a poor decision. That is why all fund managers strive to outperform their benchmarks through carefully chosen investments. The past has proven that this is rearly achived with activ managemnt after costs (Desmond Pace and Grima, 2016). This is the reason why passiv managed funds with the goal to track there benchmarks are becoming more frequent. This is why passively managed funds with the purpose of tracking their benchmarks are becoming more common. This can be accomplished through either full or sparse replication. In most circumstances, a full replication that achieves the exact performance we seek is not achievable because not all assets in an index are investable. And, if so, it would be unwise because benchmarks with numerous indexes can contain over ten thousand separate assets, resulting in a massive amount of transaction costs. A sparse replication of the performance is the most prevalent approach. To do so, the portfolio manager must define his benchmark, which should overlap with his fund's investing universe. Following that, he will reduce this universe using investor principles such as liquidity and availability. Now he can begin to optimize a portfolio, taking into account the investor constraints, in order to match the benchmark performance. Typically, this is accomplished by lowering the variance between the ITP's daily returns and the benchmark:

$$minimize \;\; Var(r_p - r_{bm})$$

To obtain the portfolio weights w, we must first substitute r_p as shown below:

$$r_p = R * w$$

The Variance is then solved up until a quadratic problem dependent on the portfolio weights w is represented:

$$Var(r_p - r_{bm}) = Var(R*w - r_{bm}) = Var(R*w) + Var(r_{bm}) - 2 \cdot Cov(R*w, r_{bm})$$

We must now solve each of the three terms, beginning with the easiest.

$$Var(r_{bm}) = \sigma_{bm}^2 = constant$$

The variance of the portfolio can be solved by looking at the Portfolio math Using Matrix Algebra section in (Zivot, 2021):

$$Var(R * w) = w^T * Cov(R) * w$$

And the last term can be solved in general as (https://bookdown.org/compfinezbook/introcompfinr/Multivariate-Probability-Distributions-Using-Matrix-Algebra.html 3.6.5):

$$Cov(A*a,b) = Cov(b,A*a) = E[(b-\mu_b)(A*a-\mu_A*a)] = E[(b-\mu_b)(A-\mu_A)*a] = E[(b-\mu_b)(A-\mu_A)]*a = Cov(A$$

```r
A = matrix(c(1,4,2,4,6,3,8,4,4,10), ncol=2)
a = c(0.2, 0.8)
b = c(4,4,5,5,7)

cov(A %*% a, b)
```

```
##      [,1]
## [1,] 2.15
```

```r
t(a) %*% cov(A, b)
```

```
##      [,1]
## [1,] 2.15
```

```r
t(cov(A, b)) %*% a # das hier wird gebraucht
```

```
##      [,1]
## [1,] 2.15
```

This results in the final formula of the ITP:

$$\begin{aligned} Var(r_p - r_{bm}) &= Var(R \times w - r_{bm}) \\ &= Var(R \times w) - 2 \cdot Cov(R \times w, r_{bm}) + Var(r_{bm}) \\ &= w^T \times Cov(R) \times w - 2 \cdot Cov(r_{bm}, R)^T \times w + \sigma_{bm}^2 \end{aligned} \tag{6.2}$$

The minimization problem of the ITP in the general stricture which all optimizers need is:

$$min(\frac{1}{2} \cdot b^T \times D \times b - d^T \times b)$$

Because it is a minimization we can ignore constant terms and stretching coefficients and still find the same minimum. This results in the general substitution of the ITP as follows:

$$D = Cov(R)$$

and

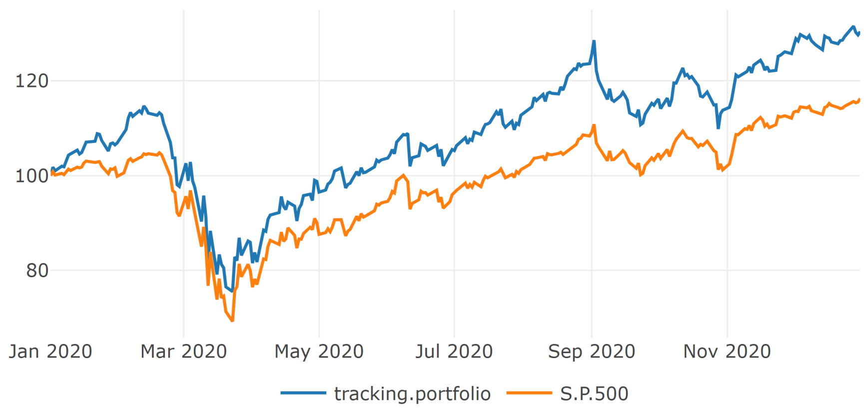
$$d = Cov(r_{bm}, R)$$

Now we need to add some basic constraints like in the MVP, to sum up the weights to 1 and being long only.

6.2.1 Example ITP

We will show the results of tracking the S&P 500 with a tracking portfolio that can only invest in IBM, Apple and Google without going into details:

```
##      AAPL       IBM      GOOG
## 0.2677928 0.4041880 0.3280192
```



Chapter 7

Analytical Solver for Quadratic Problems

The benefits and drawbacks of analytical solvers for quadratic problems will be discussed in this chapter after chapter 5's discussion of some common problems and their solutions. Because it would go beyond the scope of this thesis to explain the mathematical underlying principles of how a solver addresses quadratic problems, only the application and analysis are discussed here.

7.1 Quadratic Programs (QP)

A quadratic program is specifically a minimization problem of some function that returns a scalar and consists of an quadratic term and an linear term dependent on the variable of interest. Additionally can the problem be constrained by some linear inequalities. The formal formulation is to find x that minimizes the following problem:

$$min\ \frac{1}{2} \cdot x^T \times D \times x - d^T \times x$$

and holds under the following linear constraints:

$$A^T \times x >= b_0$$

with a symmetric positive definite matrix D. \

Some other sources notate the problems with different signs or coefficients that are all exchangeable with the above stated problem. Additionally has the problem above the same notation that is used in the R-Package `quadprog` which will reduce substitution efforts. All modern programming languages do have many solvers for quadratic problem. They differ mostly on the computational time on different problems and the requirements to use it. Some commercial QP-solvers do additionally accept more complex constraints like absolute (e.g. $|A^T \times x| >= a_0$) or mixed-integer (e.g. $x \in \mathbb{N}$). But specially the mixed-integer constraint will result in a enormously increase of memory.

7.1.1 QP Solver from quadprog

All modern programming languages do have solvers for quadratic programs. Most of them

$$a \ddddot{b}$$

Chapter 8

Simple_Particle_Swarm_Optimization

first pso examples and explainations

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