Sensors and Actuators A 92 (2001) 37–42
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Tracking system with five degrees of freedom using a 2D-array of Hall sensors and a permanent magnet

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Accepted 28 December 2000

Abstract

Based on a 2D-array of 16 cylindrical Hall sensors and a permanent magnet, a tracking system with five degrees of freedom is analysed in this paper. The system accuracy is studied, including offset drifts, sensitivity mismatches and the number of sensors. A detection distance as large as 14 cm (during 1 h without calibration) is achieved using a magnet of 0.2 cm^3. The position and orientation of the marker is displayed in real time with a sampling frequency up to 50 Hz. The sensing system is small enough to be hand-held and can be used in a normal environment.

Keywords: Tracking; Hall; Sensors array; Magnetic detection; Non-destructive evaluation

1. Introduction

Magnetic field sensors are often used for positional sensing system with the advantage of non-contact operation. The number of degrees of freedom (d.f.) of the magnetic marker can vary from 1 to 5 [1–3]. The marker is a magnetic dipole, either a permanent magnet [4] or a coil emitting high frequency electromagnetic signal [5]. The main advantage of the latest is the elimination of external noise, while the advantage of the permanent magnet is the simplicity of the system since no external excitation of the marker is required. For a number of d.f. >3, only expensive and cumbersome systems have been used up to now.

In this paper, we demonstrate the feasibility of using highly sensitive silicon Hall sensors for tracking the position of a magnet, free in space, at a rather large distance of 14 cm for a magnet volume of 0.2 cm^3. The accuracy of the positioning is evaluated with a set of simulations. These simulations implement a model of the sensors, where intrinsic noise (offset fluctuations) and sensitivity variation are taken into account.

An array with a large number of devices can be built to obtain redundant information, since our sensors are integrated, with flux-concentrators, in a SMD package. A summary of the sensors characteristics is also presented.

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2. Description of the tracking system

Our tracking system (Fig. 1) consists of four components, as shown in the block diagram of Fig. 2. The marker — not shown in Fig. 2 — is a rare earth cylindrical magnet, $\varnothing$ 6 mm $\times$ 7 mm, with a magnetic moment $m = 0.2$ A m^2. The sensing system itself is a 4×4-array of 16 cylindrical Hall sensors with integrated flux-concentrators [6]; the distance between two sensors is 3 cm. The specific geometry of these sensors has been optimised to take maximal benefit of the flux concentration produced by two pieces of amorphous ferromagnetic material (Fig. 3a). This magnification of the magnetic field permits to increase the signal to noise ratio as well as the signal to offset ratio (see Fig. 3b and Table 1). Each sensor measures one component of the magnetic field in the-plane of the matrix, the *x*- for half of them and the *y*-component for the others. The signal of each sensor is then amplified with a low-noise electronic circuitry designed for this specific application. A conventional acquisition electronics converts the analogue amplified signals to digital and sends the result to a computer for further processing. The computer implements an iterative algorithm to recalculate the position of the magnet with respect to the sensors' array.

The Levenberg-Marquardt optimisation algorithm [7] is based on the equation of an ideal dipole

$$\vec{B}(\vec{m},\vec{r}) = \frac{\mu_0}{4\pi}\frac{r^2\vec{m} - 3\vec{r}(\vec{r}\times\vec{m})}{r^5} + \vec{B}_{\text{Earth}}, \quad (1)$$

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PII: S0924-4247(01)00537-4

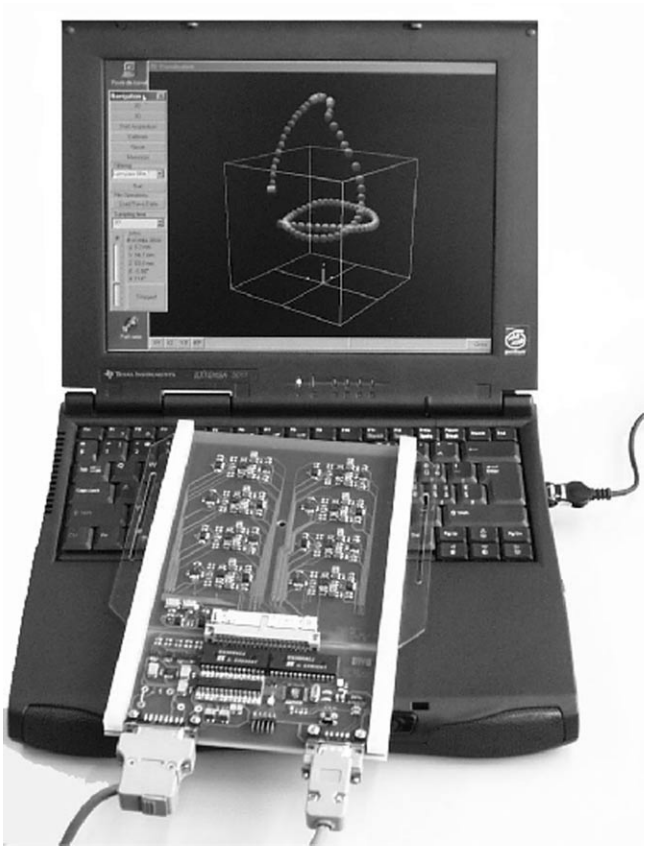

Fig. 1. Photograph of the tracking system. On the screen is shown the 3D display of a recorded trajectory of the marker (side of the cube = 10 cm; sampling = 50 Hz).

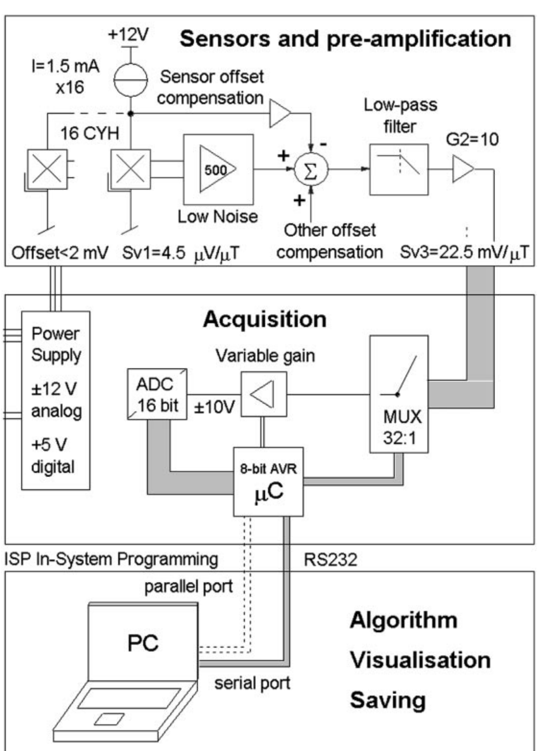

Fig. 2. Block diagram of the whole system.

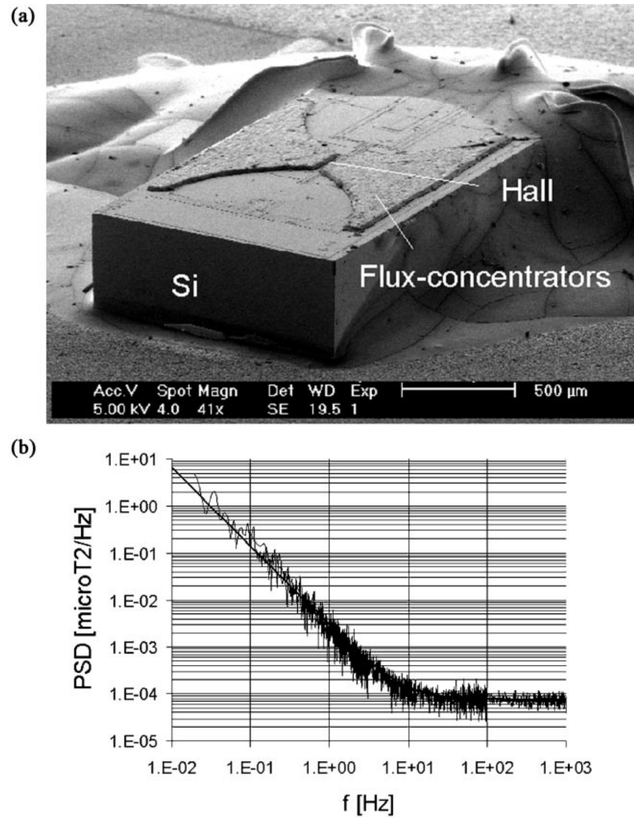

Fig. 3. (a) Cylindrical Hall sensor with flux-concentrators [6]. (b) Noise power spectrum density. The $1/f$ noise is the predominant noise in the frequency range of interest. The slope of the $1/f^{\alpha}$ is $\alpha = 1.5$, the corner frequency (where, $1/f$ noise becomes equal to the white noise) $f_c = 10$ Hz, and the integration of the noise from 0.1 s to 1 h gives about 6 μT.

where the magnetic dipole m produce a magnetic field B at a distance r. The algorithm also takes a homogenous earth magnetic field into account. The Levenberg-Marquardt algorithm is an iterative algorithm using a trust region method: smooth transition from steepest descent direction to the Newton direction (quadratic model) in a way that gives the global convergence properties of steepest descent and the fast local convergence of Newton's method. The calculated position, five co-ordinates (x, y, z, θ, φ) as defined in Fig. 4, is then visualised in 2D versus time or in 3D in real time up to 50 Hz.

Table 1
Main characteristics of the cylindrical Hall sensors with flux-concentrators, with 1 mA bias current[a]

Item	Symbol	Value	Unit
Gap between concentrators	d	30	μm
Sensitivity	S	3	V/T
Input resistance	R_{in}	4	kΩ
Output resistance	R_{out}	14	kΩ
Offset voltage	V_{off}	<2	mV
Temperature coefficient of sensitivity	αS	0.1	%/°C
Temperature coefficient of the offset voltage	αV_{off}	1	%/°C

[a] Refers to sentron@swissplace.ch.

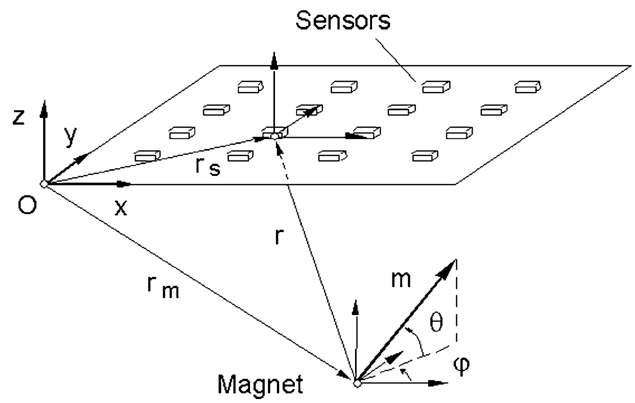

Fig. 4. Definition of the co-ordinates system. The position of the magnet is defined by five co-ordinates: x, y, z and two angles θ and φ.

3. Examples of recorded traces

We attached the magnet at the end of the spring and use it as a pendulum. With this system, all five degrees of freedom can be tested. Some recordings of tracking are shown in Figs. 1 and 5. The system has been tested with success up to a distance $z = 140$ mm; at this distance $B < 15$ μT. The precision of the calculated position is a few millimetres and depends strongly of the orientation of the marker and its position (see Section 4).

4. Limits of the accuracy

The limits of the system are imposed by the validity of the model of the ideal dipole, the extrinsic noise, the intrinsic noise of the sensing system, and the matching of the sensitivities of different sensors.

The approximation of an ideal dipole is reasonable if the distance r is large compared to the magnet dimensions. In our case, if $r > 30$ mm the error is <1.5%. Note that one can choose an optimum ratio length/diameter of the magnet to reduce this error (0.02% with an ideal ratio of 0.87).

As long as we are interested in low frequencies applications, that is frequencies <20 Hz, the main extrinsic noise is the inhomogeneity of the environmental magnetic field, including the earth magnetic field. If, in a particular application, the matrix is not moved, we do not need to take care of the environmental field inhomogeneities: the initial calibration is enough. If, on the contrary, the matrix is moved, the accuracy of the measurement will decrease, because of two reasons. In the first place, the inhomogeneities of the environmental field (due to large metallic objects for instance) will be superimposed on the field of the magnet. In the second place, even with a uniform environmental field, a rotation of the matrix adds noise or offset if the sensitivity of the sensors is not perfectly calibrated or presents a drift. Remind that the earth field is about 50 μT.

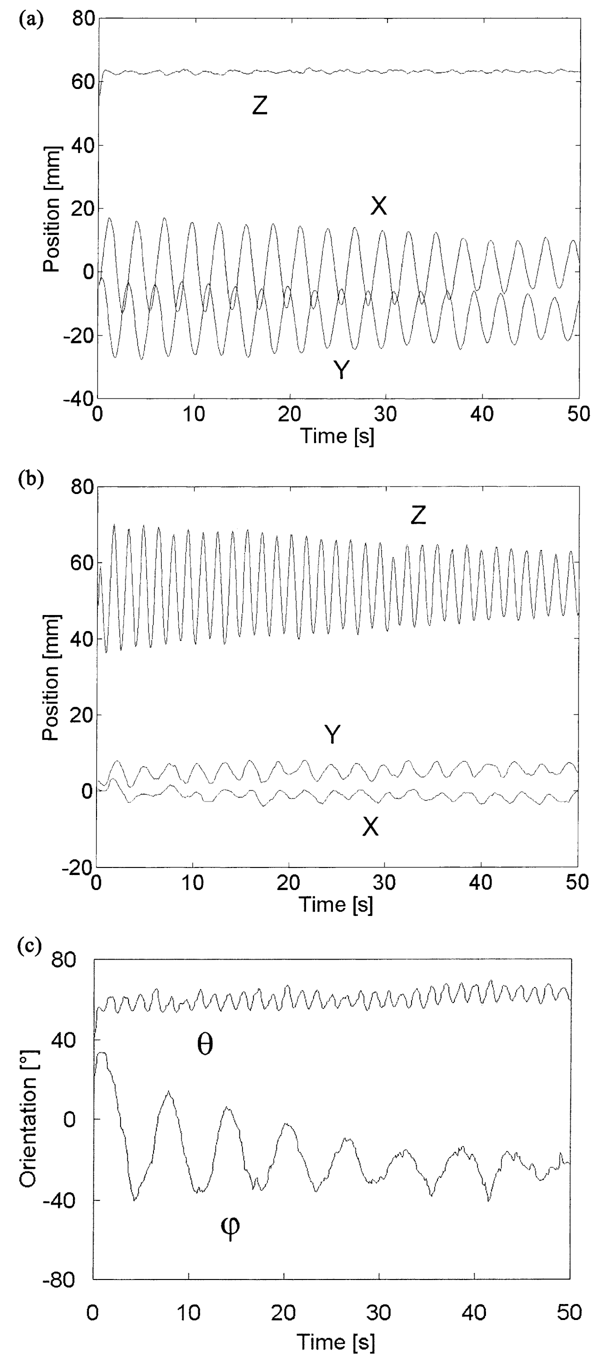

Fig. 5. (a) Harmonic movements of the marker attached to a pendulum. (b) The pendulum is combined with a spring to obtain another periodic movement, but with a different frequency, in the z-direction. (c) In the third example, the orientation of the magnet is shown; here the oscillations are rapidly damped.

The intrinsic noise is the real limitation and is mainly due to the $1/f$ noise of the sensors. The detectivity of the applied Hall sensors is about 5 μT for a d.c. field during one hour (Fig. 3b). Their sensitivity is 3 V/T (1 mA bias current), with variation from one sensor to another of ±5%. The limitation imposed by the intrinsic noise will be studied in details in Section 5.

5. Simulation and positioning accuracy

It is not trivial to characterise directly the whole system. Indeed, at a given position (x, y, z), the positioning accuracy depends on the orientation of the marker. Since all combinations of the pair (θ, φ) must be tested to find the worst case, a characterisation with the real system is virtually impossible. Moreover, even for testing only a specific orientation, it would not be easy to control accurately all the 5 d.f. of the real marker. On the other hand, once we have modelled a single sensor (low frequency noise, sensitivity variations), simulating the whole system is straightforward and becomes a very powerful tool.

We present here, a few simulations illustrating interesting characteristics of matrices, such as the one we have built. The figures are calculated only for one special case — the trajectory of the magnet is a straight line between two points (0, −75 mm, z) and (0, 75 mm, z), passing over the centre of the matrix — but, are representative of many trajectories. Random offsets or sensitivities are first introduced in every Hall sensors of the matrix. The magnet is moved along the line defined above. At each position, the results are calculated for many different angular position of the magnet. The positioning errors are defined as follows:

error in the xy plane : $\sqrt{(x_s - x_r)^2 + (y_s - y_r)^2}$

error in the z-direction : $\sqrt{(z_s - z_r)^2}$

and error in the orientation : angle between the vectors

$$\begin{pmatrix} \cos\theta_s \times \cos\varphi_s \\ \cos\theta_s \times \sin\varphi_s \\ \sin\theta_s \end{pmatrix} \text{ and } \begin{pmatrix} \cos\theta_r \times \cos\varphi_r \\ \cos\theta_r \times \sin\varphi_r \\ \sin\theta_r \end{pmatrix} \tag{2}$$

where index 's' stand for simulated and 'r' for real. This procedure is repeated introducing each time new random variations of offsets or sensitivities. After hundred such virtual experiments, the mean value of the positioning error is analysed.

Firstly, random offsets are used, keeping equal sensitivities for all sensors. Fig. 6 shows that the accuracy of the recalculated position decreases rapidly whenever the magnet goes outside of the sensors' array (that is for values of y superior to ±50 mm). Fig. 7 shows the dependence of the accuracy on the orientation of the magnet. We observe an error more important, when the dipole is vertical compared to any horizontal orientation. This is due to the fact that we measure only the component of the field in the xy-plane. Fig. 8, shows the strong degradation of the result with the distance z. This was predictable by looking at the Eq. (1) of an ideal dipole, where we can see that the field B decreases with z^{-3}. And what is more, as shown by the dashed lines in the same figure, the error can be quite well approximated by a constant multiplied by z^3. Actually, this is valid only if the magnet is in the centre of the surface delimited by the array ($-40 < y < 40$ mm). For the same reason, and with the same limitation, the accuracy in the z-direction is about four times better than in the xy-plane.

So far, unknown offsets (1/f noise) have been considered and their influence on the positioning error has been simulated. For a more detailed simulation, we should also evaluate the importance of inaccurate values of the sensitivity of the sensors (the variation between two sensors is mainly due to the magneto-concentrators). Fig. 9 shows that this parameter is less important, and, above all, leads to inaccuracies linearly related to z (not to z^3).

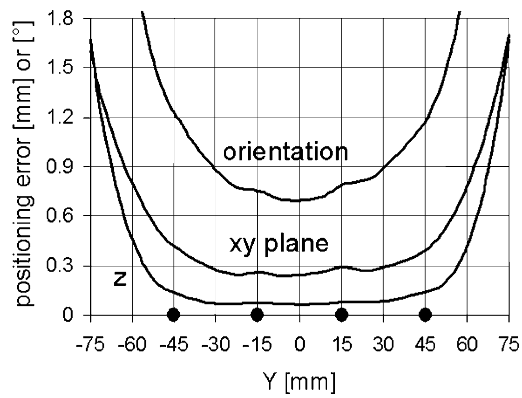

Fig. 6. Simulated positioning error. Three errors are considered: the error in the xy-plane, in the z-direction and the error in the orientation of the marker. In this example, a random offset between +10 and −10 μT is added to the theoretical value of the field seen by each sensor. The result of the simulation is given for every position of the magnet between $y = -75$ and $y = 75$ mm ($x = 0$), at distance $z = 30$ mm. At each point, all possible orientations (θ, φ) of the marker are taken into account: a mean value is calculated (see also Fig. 7). The position of the sensors (dots with 30 mm gaps, on the co-ordinate y-axis) is also represented.

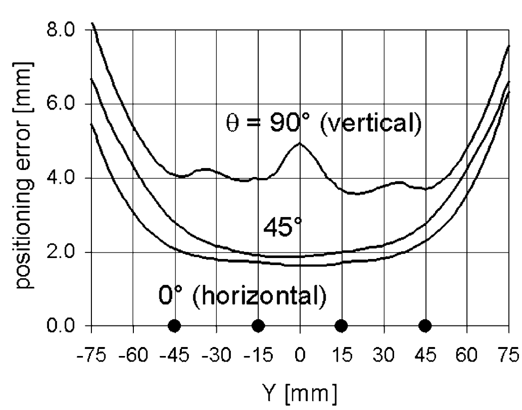

Fig. 7. Positioning error in the xy-plane for various orientations of the marker (under the same conditions as in the simulation of Fig. 6, but at distance $z = 60$ mm). We observe an error twice more important when the marker is vertical ($\theta = 90°$) compared to the horizontal position ($\theta = 0°$); at least, when the magnet is centred above the sensors' array (between $y = -50$ and $y = 50$ mm).

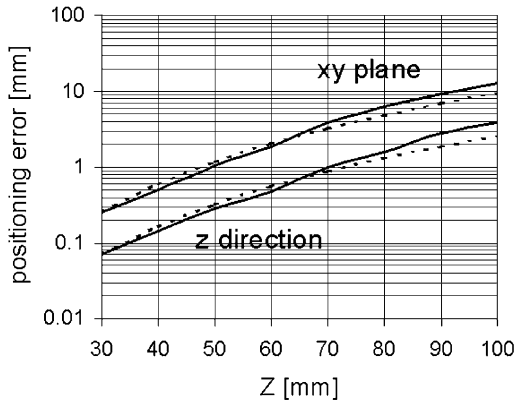

Fig. 8. Positioning error in the xy-plane and in the z-direction, as a function of the distance z (under the same conditions as in the simulation of Figs. 6 and 7). The mean error between $y = -20$ and $y = 20$ mm is used. The dashed lines are good approximations of the result, where the error is assumed to be proportional to z^3.

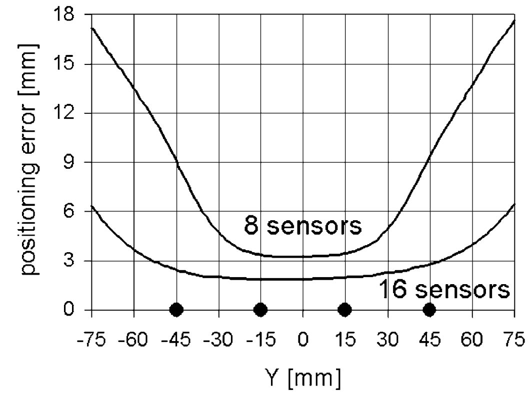

Fig. 10. Improvement of the accuracy with the number of sensors. Positioning error in the xy-plane, at distance $z = 60$ mm (under the same conditions as in Figs. 6–8).

The improvement of the signal to noise ratio with the redundancy of information available thanks to a large number of sensors is well known. Fig. 10 shows that we can extrapolate this result to our case, where the field of the magnet is highly inhomogenous over the matrix. An improvement of a factor between 1.75 and 3 is obtained, when increasing the number of sensors from 8 to 16; which is very profitable.

In this section, we discussed the mean accuracy. The worst accuracy, with the most disadvantageous orientation and noise (different for each position (x, y, z) of the magnet), has also been calculated, but is not illustrated here. We can retain that the maximum error can be a few times worst than the mean value (up to four times).

Simulations may help to foresee future improvements. For instance, we can already predict that the position accuracy will be proportional to the level of noise (what we modelled with unknown offsets). The same goes for the accuracy with which the sensitivity of the sensors is known. Once again, this is valid only if the magnet is centred ($-40 < y < 40$ mm).

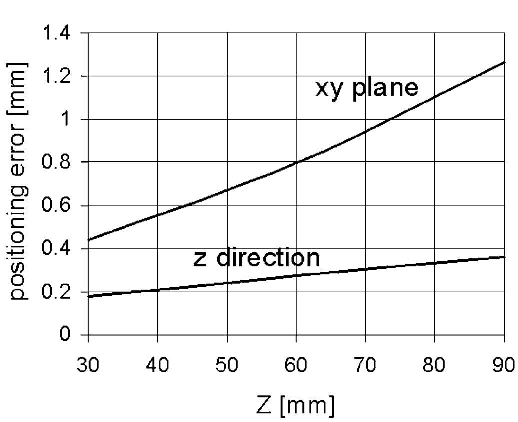

Fig. 9. Positioning error in the xy-plane and in the z-direction, without offset on the measure of the magnetic field (as previously in Figs. 6–8), but taking into consideration an error in the sensitivity of the sensors: a random value up to ±5%.

We discussed here, the absolute position accuracy. However, in many applications only the relative position is relevant. In this case, the accuracy is much better. Note that the values in Fig. 6 and next ones, represent the accuracy at least 1 h after the calibration of the array. For some applications, a regular re-calibration is conceivable, leading to a better accuracy. In some other applications, fast movements or the frequency of periodic trajectories is of main interest, and not the absolute position.

6. Conclusions

A tracking system with 5 d.f. based on a 2D-array of Hall devices has been realised and analysed. This system is novel in the sense that we use new SMD-packaged sensors, allowing to combine many sensors to improve the signal to noise ratio with the redundancy of information while keeping the system low cost.

The use of highly sensitive cylindrical Hall sensors, integrated with flux-concentrators allows to increase the signal to noise ratio, leading to a detection distance as large as 14 cm. The different sources of error have been studied together with the positioning accuracy. The influence of the number of sensors in the matrix is highlighted, showing the importance of redundant information. The cylindrical Hall sensors are optimised to be combined with concentrators in the plane of the silicon chip. In the future, the size and shape of these concentrators will be improved. It will increase the signal to noise ratio, and therefore, the accuracy and detection distance of the tracking system.

Promising applications are 3D computer mouse, medical instrumentation positioning and magnetic pen.

Acknowledgements

The authors wish to thank professor M. Unser and S. Jonic, at the Institute of Applied Optics (EPFL, Switzerland), for their help with the improvement of the algorithm.

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Biographies

Vincent Schlageter was born in Lausanne, Switzerland, on May 20, 1972. He received his MSc degree in Micro-engineering from the Swiss Federal Institute of Technology in Lausanne (EPFL), in March 1996. During the summer 1994, he was at the Belgian Nuclear Research Centre (SCK/CEN, Mol), in the Teleoperation Group. He carried out his diploma research (MSc degree) at University College Dublin, Ireland. Then, he received a MSc degree in Biomedical Engineering from the University and the Swiss Federal Institute of Technology in Lausanne, in October 1997. He is now a research assistant in the Institute of Micro-systems (EPFL) and in the Institute of Physiology (UNIL). His current research topic is magnetic sensors' arrays for medical applications.

Pierre-A. Besse was born in Sion, Switzerland, in 1961. He received the diploma in Physics and his PhD degree on semiconductor optical amplifiers from the Swiss Federal Institute of Technology, ETH Zurich in 1986 and 1992, respectively. In 1986, he joined the group of micro- and opto-electronics of the institute of Quantum Electronics at ETH Zurich, where he is engaged in research in optical telecommunication science. He worked on theory, modeling, characterisation and fabrication of compound semiconductor devices. In August 1994, he joined the Institute of Microsystems at the Swiss Federal Institute of Technology at Lausanne (EPFL) as senior assistant, where he is starting activities on sensors and actuators micro-systems. Actually, his major fields of interest are: physical principals and new phenomena for optical, magnetic, inductive and strain sensors. He has been involved in international projects. He has written and co-authored over 80 scientific papers and conference contributions. He holds eight patent applications.

Rade S. Popovic was born in Yugoslavia (Serbia) in 1945. He obtained the Dip. Eng. degree in Applied Physics from the University of Beograd, Yugoslavia in 1969 and the MSc and DSc degrees in Electronics from the University of Nis, Yugoslavia in 1974 and 1978, respectively. Since 1969 to 1981, he was with Elektronska Industrija Corp., Nis, Yugoslavia, where he worked on research and development of semiconductor devices and later became head of the company's CMOS department. Since 1982 to 1993, he worked for Landis & Gyr Corp., Central R&D, Zug, Switzerland, in the field of semiconductor sensors, interface electronic, and micro-systems. There, he was responsible for research in semiconductor device physics (1983–85), for micro-technology R&D (1986–90) and was appointed vice president (Central R&D) in 1991. In 1994, he joined the Swiss Federal Institute of Technology at Lausanne (EPFL) as professor for micro-technology systems. He teaches conceptual products and system design and micro-electronics at the Department of Micro-engineering of the EPFL. His current research interests include sensors for magnetic, optical, and mechanical signals, the corresponding micro-systems, physic of sub-micron devices, and noise phenomena.

Pavel Kucera born 1940 in Czechoslovakia. He obtained PhD degree in 1963 at the Medical Faculty of the Charles' University in Plzen, Czechoslovakia, where he continued research and teaching in pathophysiology until 1968. Then, he moved to the Institute of Physiology, Medical Faculty of the University of Lausanne, where he became ordinary professor in 1982. His expertise is in cell and organ physiology and his research, although mainly fundamental, is closely linked to pathophysiological and clinical problems. His projects aim to answer concrete questions concerning mechanisms, diagnosis and treatment of some diseases in the domains of urology, gastroenterology, cardiology, and neurology. He participates in several multicentric collaborations with clinicians, biologists, physicists, chemists, engineers and with industry.