Interpolación y Aproximación

Ejercicio 1

$$\sum L_i(x) f(x_i)$$

$3 = x$

		x = 3	x = 7	x = 3
$i=0$	$\frac{(x-2)(x-4)}{(1-2)(1-4)} \cdot 1 = \frac{(x-2)(x-4)}{3}$	$-1/3$	5	29
$i=1$	$\frac{(x-1)(x-4)}{(2-1)(2-4)} \cdot 8 = \frac{(x-1)(x-4)(-4)}{}$	8	72	x = 7
$i=2$	$\frac{(x-1)(x-2)}{(4-1)(4-2} \cdot 64 = (x-1)(x-2)\frac{32}{3}$	$\frac{64}{3}$	320	253

Ejercicio 3

x	0	1/6	1/2
y	0	1/2	1

$$L_0(x) = \frac{(x-1/6)(x-1/2)}{(0-1/6)(0-1/2)} = \frac{(x-1/6)(x-1/2)}{12} \quad f(x_0) = \frac{(\,)(\,)}{12} \cdot 0 \checkmark$$

$$L_1(x) = \frac{(x-0)(x-1/2)}{(1/6-0)(1/6-1/2)} = \frac{-(x)(x-1/2)}{18} \quad f(x_1) = \frac{-(\,)(\,)}{18}\,\frac{1}{2} \Big/ 36$$

$$L_2(x) = \frac{(x-0)(x-1/6)}{(\frac{1}{2}-0)(\frac{1}{2}-1/6)} = \frac{x(x-1/6)}{6} \quad f(x_2) = \frac{x(x-1/6)}{6}\,1$$

$$P(x) = \frac{-x(x-1/2)}{36} + \frac{x(x-1/6)}{6} = \frac{-x(x-1/2)+6x(x-1/6)}{36} = \frac{-x^2+1/2x+6x^2-x}{36}$$

$$\frac{-\frac{1}{2}x + 5x^2}{36} = -\frac{1}{72}x + \frac{5}{36}x^2$$

Ejercicio 5

(a)

x	f(x)	Δf	$\Delta^2 f$	$\Delta^3 f$	$\Delta^4 f$
1	0				
2	0,301	0,301			
3	0,477	0,176	-0,125		
4	0,602	0,125	-0,051	0,074	
5	0,699	0,097	-0,028	0,023	-0,051

Uso el polinomio de

Se utiliza el regresivo debido a que el punto a interpolar está cerca del principio del intervalo

$P(x) = \frac{0,301}{1!}(x-1) - \frac{0,125}{2!}(x-1)(x-2) + \frac{0,074}{3!}(x-1)(x-2)(x-3)$

$- \frac{0,051}{4!}(x-1)(x-2)(x-3)(x-4)$ $\quad 0,301(\,) - \frac{1}{16}(\,)(\,) + \frac{37}{3000}(\,)(\,)(\,) - \frac{17}{8000}$

$(x-1)(x-2) = x^2 - 2x - 1x + 2 = x^2 - 3x + 2$

$(x^2-3x+2)(x-3) = x^3 - 3x^2 + 2x - 3x^2 + 9x - 6 = x^3 - 6x^2 + 11x - 6$

$(x-4)(x^3-6x^2+11x-6) = x^4 - 6x^3 + 11x^2 - 6x - 4x^3 + 24x^2 - 44x + 24$

$= x^4 - 10x^3 + 35x^2 - 50x + 24$

$= 0,301x - 0,301 - \frac{1}{16}(x^2 - 3x + 2) + \frac{37}{3000}(x^3 - 6x^2 + 11x - 6) - \frac{17}{8000}(x^4 - 10x^3 + 35x^2 - 50x + 24)$

$= 0,301x - 0,301 - \frac{x^2}{16} + \frac{3x}{16} - \frac{2}{16} + \frac{37}{3000}x^3 - \frac{37}{500}x^2 + \frac{407}{3000}x - \frac{37}{500} - \frac{17}{8000}x^4 + \frac{17}{800}x^3 - \frac{119}{1600}x^2 + \frac{17}{160}x - \frac{31}{1000}$

$P(x) = -2,125 \times 10^{-3} x^4 + 0,03358\,33\, x^3 - 0,2108375x^2 + 0,7304166\, x - 0,551$

Uso progresivo

Estan equiespaciados

Ejercicio 6

$a_0 = 0 \quad a_1 = 5 \quad a_2 = -6 \quad a_3 = 6$

$P(x) = 0 + \frac{5}{1!1^1}(x+1) - \frac{6}{2!1^2}(x+1)(x-0) + \frac{+6}{3!1^3}(x+1)(x-0)(x-1)$

$(x+1)(x) = x^2 + x$

$(x+1)(x-1)x = (x^2-1)x = x^3 - x$

$P(x) \; 0 + 5(x+1) - 3(x+1)(x) + (x+1)(x-0)(x-1)$

$P(x) = 0 + 5x + 5 - 3x^2 - 3x + x^3 - x \Rightarrow$

$P(x) = 5 + x - 3x^2 + x^3$

0					
5	5				
4	-1	-6			
3	-1	0	6	0	0
8	5	6	6	0	0
25	17	12	6		
	x^1	x^2	x^3		

Esercizio 7

1,000						
1,02470	0,0247					
1,04881	0,02411	$-5,9\,10^{-4}$				
1,07238	0,02357	$-5,4\,10^{-4}$	$0,5\,10^{-4}$			
1,09544	0,02306	$-5,1\,10^{-4}$	$0,3\,10^{-4}$	$-0,2\,10^{-4}$		
1,11803	0,02259	$-4,7\,10^{-4}$	$0,4\,10^{-4}$	$0,1\,10^{-4}$	$0,3\,10^{-4}$	
1,14017	0,02214	$-4,5\,10^{-4}$	$+0,2\,10^{-4}$	$-0,2\,10^{-4}$	$-0,3\,10^{-4}$	$-0,6\,10^{-4}$

$$P(x) = 1. + \frac{0,0247}{1!}(x-1) - \frac{5,9\,10^{-4}}{2!}(x-1)(x-1,05) + \frac{0,5\,10^{-4}}{3!}(x-1)(x-1,05)(x-1,10)$$

$$+\frac{0,2\,10^{-4}}{4!}(x-1)(x-1,05)(x-1,10)(x-1,15) + \frac{0,3\,10^{-4}}{5!}(x-1)(x-1,05)(x-1,10)(x-1,15)(x-1,20)$$

$$-\frac{0,6\,10^{-4}}{6!}(x-1)\dots(x-1,25)$$

$$P(1,01) = 1 + 2,47\,10^{-4} + 1,18\,10^{-7} + 3\,10^{-10} - 4,2\,10^{-12} + 2,47381\,0^{-13} + 1,9152\,10^{-14}$$

$$P(1,01) = 1,00024\ldots \quad \text{(uso NR.P)}$$

con Lagrange

$$L_0(1,12).1 = \frac{(1,12-1,05)(1,12-1,10)(1,12-1,15)(1,12-1,20)(1,12-1,30)(1,12-1,25)}{(1-1,05)(1-1,10)(1,13-1,15)(1-1,20)(1-1,30)(1-1,25)}$$

$$L_1(1,12).1,02470 = \frac{(1,12-1)(1,12-1,10)(1,12-1,15)\dots(1,12-1,30)}{(1,05-1)(1,05-1,10)\dots}\,1,02470$$

$$L_2(1,12).1,04881 = \frac{(1,12-1)(1,12-1,05)(1,12-1,15)\dots(1,12-1,30)}{(1,10-1)(1,10-1,05)(1,10-1,15)\dots}\,1,04881$$

$$L_3(1,12).1,07238 = \frac{(1,12-1)(1,12-1,05)(1,12-1,10)(1,12-1,20)\dots(1,07238)}{(1,15-1)(1,15-1,05)(1,15-1,10)(1,15-1,20)\dots}$$

$$L_4(1,12).1,09544 = \frac{(1,12-1)(1,12-1,05)(1,12-1,10)(1,12-1,15)(1,12-1,25)(1,09544)}{(1,20-1)\dots}$$

$$L_5(1,12).1,11803 = \frac{(1,12-1)(1,12-1,05)(1,12-1,10)(1,12-1,15)(1,12-1,20)(1,12-1,30).1,11803}{(1,12-1,25)1,14017}$$

$$L_6(1,12)1,14017\,(1,12-1)$$

$$P(1,12) = L_0(1,12) + L_1(1,12)1,02470 + L_2(1,12)1,04881 + L_3(1,12)1,07238 + L_4(1,12).1,09544 + L_5(1,12).1,11803 + L_6(1,12)1,14017$$

Ejercicio (12)

x	f	Δf	$\Delta^2 f$	
-1	10			
		-7		
0	3		2	
		-1		0
2	1		K/3	
		K-1		0
3	K		1/4K-15/2	
		$\frac{28-K}{2}$		
5	28			

¡Así es de grado (2)

$$\frac{\frac{K}{3}-2}{3-(-1)} = \frac{\frac{K-6}{3}}{4} \Rightarrow \frac{K-6}{12} = 0$$

$K-6=0 \rightarrow K=6$

$$\frac{K-1}{3-2} = \frac{K+1}{1} = \qquad \frac{K-1+1}{3-0} = \frac{K}{3} \qquad \frac{K-1-\frac{28+K}{2}}{5-2} = \frac{K-1-14-K/2}{2} = \frac{1}{4}K - \frac{15}{2}$$

Ejercicio (13)

x	f	Δ	$\Delta^2 f$	$\Delta^3 f$	
0	4				
		-1			
1	3		0		
		-1		1/20K+1/20	
4	0		1/4K+1/4		0
		K		(*)2	
5	K		K/2+41/2		
		(*)			
7	123		$\frac{71}{6}+\frac{K}{6}$		
		97			
8	220				

$$\frac{K}{5-4} = \frac{K}{1} \qquad \frac{K+1}{5-1} = \frac{K+1}{4} \qquad \frac{97-\frac{123}{2}+\frac{K}{2}}{3} = \frac{71}{6}+\frac{K}{6}$$

$$\frac{123-K}{7-5} = \frac{123-K}{2} \ (*)$$

$$\frac{\frac{1}{4}K+\frac{1}{4}-0}{5-0} = \frac{1}{20}K+\frac{1}{20}$$

$$\frac{\frac{123}{2}-\frac{K}{2}-K}{7-4} = \frac{-\frac{3K}{2}+\frac{123}{2}}{3} = -\frac{K}{2}+\frac{41}{2} \qquad \frac{-\frac{K}{2}+\frac{41}{2}-\frac{1}{4}K-\frac{1}{4}}{7-1} = \frac{-\frac{3}{4}K+\frac{81}{4}}{6} = \frac{27}{8}-\frac{1}{8}K \ (*)2$$

$$\frac{\frac{27}{8}-\frac{1}{8}K-\frac{1}{20}K-\frac{1}{20}}{7} = \frac{\frac{133}{40}-\frac{7}{40}K}{7} = -\frac{1}{40}K+\frac{19}{40} \Rightarrow \frac{1}{40}K = \frac{19}{40} \Rightarrow \frac{19 \cdot 40}{40} = K \Rightarrow K=19$$

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14) A_ Como serán equiespaciados

3,5
5,75 9/4
9 3,25 1
13,25 4,25 1 (6)

(F) porque es cero

(B) hacer tabla, y ver si $\Delta^4 f = 0$

c) Δ Δ^2 Δ^3 Δ^4 Δ^5 (F) n puntos, n-1 grado

d) equiespaciado $ax^3+bx^2+cx+D=$

-1	Δ	Δ^2	Δ^3
3	4		
0	-3	-7	3
-7	-7	-4	

$(0,3)$ $D=3$

$\rightarrow -a+b-c=-4$
$\rightarrow a+b+c=-3$
$8a+4b+2c=-10$
$\rightarrow 4a+2b+c=-5$

a	b	c	
-1	1	-1	-4
1	1	1	-3
4	2	1	-5
0 6.2	0		-7
0 2.6	-3		-21
0	0	6	0

$c=0$
$2b=-7 \Rightarrow -7/2 \Rightarrow b=-3,5$
~~2·2~~ 5
$a+b+c=-3$
$a=-3+3,5 \rightarrow 0,5=a$

$0,5x^3-3,5x^2+3=0$

e) y F)

8) 1 3,5
5 1,5
6,5 1,5 0
k k-6,5 k-8

VERD.
ojo si k = 8 no existe pq es de 1
pero si k ≠ 8 el pol es de 3

A) Es de grado 2 por que la diferencia finita de orden 3 es nula y b) orden "n" no es nulo

15)

	Δ	Δ^2	Δ^3
a			
b	b-a		
0	-b	-2b+a	a+3b+c
c	c	c+b	

-b - b+a
-2b+a
c+b+2b+a = a+3b+c

18) si x = 3 ⟹ equiespaciados
5 1
7 2 y-9
y y-7
si y = 9 es de 2°
si y ≠ 9 es de 3.
y-7-2 = -9+y
si x ≠ 3
∴ depende de (x,y)

1 5
2 7 2
x y y-7 / x-2

$$\frac{\frac{y-7}{x-2} - 2}{x-1} = \frac{\frac{y-7+2x+4}{x-2}}{x-1}$$
$$\frac{y-2x-3}{(x-2)(x-1)}$$

19) si k = 1,5
(1,5 ; 7,5)

	Δ	Δ^2	Δ^3
3			
5	2		
7	2	0	
7,5	0,5	-1,5	-1,5

equi s/equi

		Δf	Δ^2
0	3	4	
0,5	5	4	0
1	7	5k-7 / k-1	k-3 / k-0,5
k	5k		

k = 3 , Δ^2 nula
g 1 (c)

$$\frac{\frac{5k-7}{k-1} - 4}{k-0,5} = \frac{5k-7-4k+4}{(k-0,5)}$$
$$\left(\frac{k-3}{k-0,5}\right)$$

17)

0			
7	7		
24	17	10	
a	a-24	a-41	0

si a = 51 pol grado 1 (unico)

0			
7	7		
24	17	10	
51	27	10	0
b	b-51	b-78	b-88

b-88 = 0
b ≠ 88 (no unico

21)

20			
20	0		
k	k-20	k-20	
15	15-k	-2k+35	-3k+55
12	-3	k-18	3k-53
10	-2	1	19-k
10	0	2	1

3k-53+3k-55 = 0
6k = 108
k = 18

15-k-k+20 | -3-15+k | -2k+35-k+20
-2k+35 | -18+k | -3k+55

k-18+2k-35 | 1-k+18
3k-53 | 19-k

Eje 22

x	y	x^2	x.y
2	1	4	2
3	6	9	18
4	8	16	32
5	12	25	60
6	18	36	108
Σ 20	45	90	220

$$\begin{cases} a\Sigma x_i^2 + b\Sigma x_i = \Sigma x_i y_i \\ a\Sigma x_i + bn = \Sigma y_i \end{cases}$$

$$\begin{cases} a\cdot 90 + b\,20 = 220 \\ a\,20 + b5 = 45 \end{cases} \rightarrow 4a + b = 9 \rightarrow b = 9 - 4a$$

$$a90 + 20(9-4a) = 220$$

$$90a + 20\cdot 9 - 20\cdot 4a = 220$$

$$90a - 80a = 220 - 180 \Rightarrow 10a = 40 \rightarrow a = 4$$

$$b = 9 - 4\cdot 4 = 9 - 16 \Rightarrow b = -7$$

$P(x) = 4x - 7$

$Error = \Sigma (\bar{y} - y)^2$

$\bar{y} = 4x - 7$

$\bar{Y}$	1	5	9	13	17
$\bar{Y} - y$	0	−1	1	1	−1
$(\,)^2$	0	1	1	1	1 = 4

Eje 24

x	y	x^2	x.y
10.00	1	100	10
10.1	1,20	102,01	12,12
10.2	1,25	104,04	12,75
10.3	1,267	106,09	13,0501
10.4	1,268	108,16	13,1872
10.5	1,276	110,25	13,398
Σ 61,5	7,261	630,55	74,5053

$$\begin{cases} a\Sigma x_i^2 + b\Sigma x_i = \Sigma x_i y_i \\ a\Sigma x_i + bn = \Sigma y_i \end{cases} \quad \begin{cases} a\,630,55 + b\,61,5 = 74,5053 \\ a\,61,5 + b6 = 7,261 \end{cases}$$

$$b = \frac{7,261}{6} - a\frac{61,5}{6}$$

$$a\,630,55 + \left(\frac{7,261}{6} - a\frac{61,5}{6}\right)61,5 = 74,5053$$

$$630,55a + 74,4253 - 630,375a = 74,5053$$

$$0,175a = 0,08 \Rightarrow a = 0,4571$$

$$b = \frac{7,261}{6} - 0,4571\frac{61,5}{6} = -3,4751$$

Eje 25

x	y_i	x^2	x^3	x^4	$x_i \cdot y_i$	$x_i^2 \cdot y_i$
1850	232					
1860						
1870						
1880						
1890						
1900						
1910						
1920						

Eje 25

$\Sigma x_i = 20900 \quad \Sigma y = 886,8 \quad \Sigma x^2 y = 3256601300$

$\Sigma x_i y_i = 1699218 \quad \Sigma x^3 = 7,55117\,10^{10}$

$\Sigma x_i^2 = 39721000 \quad \Sigma x^4 = 1,435913789610^{14}$

$$\begin{cases} a\Sigma x_i^4 + b\Sigma x^3 + c\Sigma x^2 = \Sigma x^2 y \\ a\Sigma x^3 + b\Sigma x^2 + c\Sigma x = \Sigma x\, y \\ a\Sigma x^2 + b\Sigma x + cn = \Sigma y \end{cases}$$

$$\begin{cases} 1,435913789610^{14}\, a + b\,7,55117\,10^{10} + c\,39721000 = 3256601300 \\ 7,55117\,10^{10} a + 39721000 b + c\,20900 = 1699218 \\ a\,39721000 + b\,20900 + c\cdot 11 = 886,8 \end{cases}$$

	$1,43591\,10^{14}$	$7,5512\,10^{10}$	39721000	3256601300
3,	$7,5512\,10^{10}$	39721000	20900	1699218
7	39721000	20900	11	886,8
	0	$-1,4762,10^{18}$	$1,61888\,10^{15}$	$1,9218\,10^{18}$
	0	$-4,42959\,10^{11}$	-463100000	$5,3060\,10^{11}$

$$-1,476210^{18} b + 1,61888\,10^{15} c = 1,9218\,10^{18}$$

$$1,61888\,10^{15} c = 1,9218\,10^{18} + 1,476210^{18} b$$

$$c = 1187,1757 + 911,9101 b$$

$$-4,42961\,0^{11} b - 463100000\,(1187,1757 + 911,9101 b) = 5,30601\,0^{11}$$

$$-4,42961\,0^{11} b - 5,49781\,0^{11} - 4,2231 b\,10^{11} = 5,30601\,0^{11}$$

$$-8,65271\,0^{11} b = 1,08041\,0^{12}$$

$$b = -1,24861\,0^{22}$$

Ejercicio 26

$\Sigma x = 2,31 \quad \Sigma x^2 = 1,2911 \quad \Sigma xy = 5,0790$

$\Sigma y = 9,797 \quad \Sigma x^2 y = 3,3286$

Pol grad 1

$$\begin{cases} \Sigma x^2 \cdot a + b\Sigma x = \Sigma xy \\ \Sigma x \cdot a + b\,n = \Sigma y \end{cases} \quad \begin{cases} 1,2911a + b\,2,31 = 5,0790 \\ a\,2,31 + 6b = 9,797 \end{cases}$$

$b = \frac{9,797}{6} - \frac{2,31}{6}a \Rightarrow 1,2911a + 2,31\left(\frac{9,797}{6} - \frac{2,31}{6}a\right) = 5,0790$

$1,2911a + 3,7718^{45} - 0,8899^{85}a = 5,0790$

$0,4017a = 1,3072 \Rightarrow a = 3,2542$

s/redondeo $\rightarrow$ $0,40175 = 1,3071852a \Rightarrow a = 3,2536$

$b = 0,3801770$

Grado 1: $P(x) = 3,2536x + 0,38017$

c	b	a	
n	Σx	Σx^2	$= \Sigma y$
Σx	Σx^2	Σx^3	$= \Sigma xy$
Σx^2	Σx^3	Σx^4	$= \Sigma x^2 y$

$\Sigma x^3 = 0,796041$

$\Sigma x^4 = 0,518247711$

1,2 6	2,31	1,2911	= 9,797
2,31	1,2911	0,7960	= 5,0790
6 1,2911	0,7960	0,5182	= 3,3286
0	+2,4105	+1,7936	+7,84293
0	+2,004	+1,55988	+8,2152
0	0	-0,1657	-4,0855

$-2,4105b + 1,7936a = -7,84293$

$a = \frac{-7,84293}{-1,7936} + \frac{2,4105b}{-1,7936}$

$a = 4,3727 - 1,3439b$

$a + 0,1657 = -4,0855$

$a = 24,6560$

Ejercicio 28

x	y	x^2	Ln(y)	x ln(y)
1	7	1	Ln(7)	Ln(7)
2	11	4	Ln(11)	2 ln(11)
3	17	9	Ln(17)	3 Ln(17)
4	27	16	Ln(27)	4 Ln(27)
Σ 10	62	30	10,47285563	28,42476881

$$\begin{cases} a\Sigma x^2 + B\Sigma x = \Sigma x \ln(y) \\ a\Sigma x + Bn = \Sigma \ln y \end{cases}$$

$$\begin{cases} a\,30 + 10B = 28,4247 \\ 10a + 4B = 10,4729 \end{cases} \quad \begin{matrix} \cdot 10 \\ \cdot 30 \end{matrix}$$

$$+20B = +29,93179\,8077$$

$$B = 1,496899038$$

$$b = e^B \Rightarrow b = e^{1,496899038}$$

$$b = 4,4678$$

$$y = b \cdot e^{ax} \Rightarrow y = 4,4678\, e^{0,4485}$$

$$a.30 + 10\,1,496899038 = 28,4247 \Rightarrow a = 0,4485$$

Ejercicio 27

Σx	ΣY	Σln(y)	Σln(x)	$\Sigma \ln(x)^2$	Σ ln(x) ln(y)
15	19,7	4,929954	4,787491	6,199504	7,55031

$$\begin{cases} a\Sigma \ln^2(x) + B\Sigma \ln(x) = \Sigma \ln(x)\ln(y) \\ a\Sigma \ln(x) + Bn = \Sigma \ln(y) \end{cases}$$

$$\begin{cases} a.6,199504 + B(4,787491) = 7,55031 \\ a\,4,787491 + 5B = 4,929954 \end{cases}$$

$$a = \frac{7,55031}{6,199504} - \frac{4,787491b}{6,199504} = 1,2179 - 0,7722b$$

$$4,787491(1,2179 - 0,7722b) + 5b = 4,929954$$

$$5,8307 - 3,69691b + 5b = 4,929954$$

$$1,3031b = -0,900746$$

$$b = -0,6912 \qquad a = 1,7516$$

$$y = b\,x^a$$

$$e^B = b \Rightarrow e^{-0,6912} = 0,5009$$

uso R s y 3 dig

$$\begin{cases} b = 0,501 \\ a = 1,752 \end{cases}$$

$$y = 0,501\, x^{1,752}$$