Sequential Cognition Processes: A Framework For Reasoning with Non-Monotonic Logics

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Abstract

Approaches to cognitive modelling with non-monotonic logics have thus far been largely *ad hoc* and poorly standardised, making inter-model comparisons difficult. As an attempt to systematically represent non-monotonic logics in a framework that standardises cognitive modelling under these logics without sacrificing their expressiveness, we introduce the Sequential Cognition Process (SCP). Under the assumption that human reasoning can be represented as a sequence of distinct cognitive operations on an initial knowledge base, and using an adapted interpretation of the Weak Completion Semantics (WCS), SCPs are able to accurately model several classical experiments in cognitive modelling such as the Wason Selection Task and the Suppression Task. Further, SCPs are shown to be suitable for modelling common reasoner deviations from the general participant responses in both of these experiments.

1 Introduction

The human mind is complex. So complex that thousands of approaches from dozens of fields have failed to capture their complexity. The sheer size of the brain – containing over 5000 times as many neurons as the largest practical neural networks (**?**) – and our limited understanding of the fundamental learning processes it employs mean that researchers can neither completely describe nor predict human actions nor model human thought processes. Instead, much of the current state of the art in cognitive modelling relies on one of two general approaches: creating systems that structurally approximate the human brain, and creating systems which approximate a more abstract intuition of cognition. The first type of system encompasses many algorithms related to machine learning and deep learning (@TODOref); the second type, with which this paper is concerned, has existed in some form for far longer than humans have been thinking creatures. Every shark stalking its prey, every tiny proto-mammal hiding from a hungry dinosaur, and every man driving to work in the morning, has applied this type of reasoning when trying to make predictions about the actions of other agents in their world. By applying case-sepecific reasoning to a known world state we are able to make imperfect, but quick, predictions about the mental state of other agents in our world.

Due in part to the difficulty of cloning dinosaurs to hunt participants in cognitive research experiments, and partly due to concerns about the ease with which they could answer questionnaires on the experience later, most cognitive tasks used by researchers tend to be more dull than the examples above.

Non-monotonic logics have proven able to adequately model a large number of standard cognitive reasoning tasks such as The Wason Selection Task(**?**), Suppression Task (**?**), and @TODOmore. These approaches, though seemingly effective and well-founded in isolation, are often unable to integrate or be compared to data models of other tasks, even when they rely on the same underlying logic. Although the non-monotonic logics themselves are generally carefully described, procedures ranging from best practise in deciding appropriate knowledge bases to the mechanisms by which abnormalities should be created tend to be re-imagined on a case-by-case basis.

$\rightarrow$	$\top$	u	$\bot$
$\top$	$\top$	u	$\bot$
u	$\top$	$\top$	u
$\bot$	$\top$	$\top$	$\top$

Table 1: A table showing the implication operator in 3-valued Łukasiewicz logic.

Further, cognitive frameworks using non-monotonic logics are almost always designed to describe the most common (general) conclusion drawn by participants in the experiment. Modelling other individual reasoners or classes of reasoners who differ from the norm is often a non-trivial process. It has been shown the Weak Completion Semantics is able model the four most general cases of the Wason Selection Task under the assumption that reasoners who differ from the general case (*deviant reasoners*) follow a sequence of mental processes that is still highly similar to that of the general reasoner (**?**).

This paper introduces the Sequential Cognition Process (SCP) which generalises the assumption of sequential cognitive operations, each of which uses an epistemic state as input and produces as epistemic state as output. Using SCPs and well-founded set of cognitive operations it is possible to apply traditional search techniques to problems in cognitive modelling with non-monotonic logics that have previously required expert-made models. The SCP framework introduces a number of desirable properties: they introduce a partially standardised (though extensible) set of allowable cognitive operations, they standardise the structure of what constitutes an epistemic state, they are easily and modified to accommodate deviant reasoners when a well-founded general model already exists, and their sequential structure makes them well-suited to scoring algorithms that allow intra and inter-experimental modelling and comparisons.

In the field of non-monotonic logics, reasoning is represented as a collection of defeasible inferences (@TODOref). Unlike in classical logic, conclusions need not hold in perpetuity, or even in the same model and revision is always possible. @TODOref showed that monotonic logics are not capable of describing human reasoning in the Suppression Task (@TODOref).

2 Mathematical Preliminaries

2.1 Propositional Logic

2.2 Non-monotonic Logics

2.3 The Weak Completion Semantics

The Weak Completion Semantics is a non-monotonic logic which procedurally encodes several well-known cognitive phenomenon. The WCS makes use of 3-valued Łukasiewicz logic (Table 2). It adds abnormalities to non-ground inferences, and replaces the classical inference ($\leftarrow$), with a bijective ($\leftrightarrow$).

The Weak Completion of a program P is defined as follows:

1. Replace all clauses of the form $A \leftarrow body_1, ..., A \leftarrow body_n$ with $A \leftarrow body_1 \vee ... \vee body_n$.

2. Replace all occurrences of $\leftarrow$ with $\leftrightarrow$.

Applying this procedure to P results in wcP which is the weak completion of P.

The next requirement to apply the WCS framework is the introduction of a semantic operator ϕ_{SvL} (**?**). Let J be the result of applying the semantics operator to an interpretation I and logic program P. The J is defined as follows:

$$J^{\top} = \{A| \text{ there exists a clause } A \leftarrow Body \in P \text{ with } I(Body) = \top\}$$

$$J^{\bot} = \{A| \text{ there exists a clause } A \leftarrow Body \in P$$
$$\text{and for all clauses } A \leftarrow Body \in P \text{ we find } I(Body) = \bot\}$$

Using $I = <\emptyset, \emptyset>$, the least model of P ($\text{lm}_{\text{Ł}}wcP$) can be calculated by iterating $\phi_{SvL,P}$.

3 Experiments

Two classical experiments in modelling human cognitions are the Wason Selection Task (WST) and the Suppression task. Both of these logics illustrate that human reasoners can draw inferences which differ from the conclusions of classical logic.

3.1 The Suppression Task

The Suppression Task refers to an experiment conducted by **?** and is a classical example of the inadequacy of monotonic logics for modelling human reasoning. In classical logic, if our knowledge base kb is such that $kb \models \phi$, then it must be the case that $kb \cup \psi \models \phi$. However, in the suppression task participants no longer draw classically valid inferences when new information is added. The task is often formulated as follows:

- $e \rightarrow l$: If she has an essay to write (e), she will study late in the library (l).

- $\top \rightarrow e$: She has an essay to write (e).

- $o \rightarrow l$: If the library is open (o), she will study late in the library (l).

Given only the rules ($e \rightarrow l$) and ($\top \rightarrow e$), the participants consistently concluded that she would study late in the library, seemingly drawing the classical logic inference $\frac{e \rightarrow l, e}{l}$ with *modus ponens*. But when given the additional rule $o \rightarrow l$, participants no longer believe that they have enough information to judge whether she will study late in the library, and a significant portion of them no longer draw the classical conclusion. This effect, called Suppression, demonstrates the need for something more than classical logic for modelling human reasoning.

Work by **?** has shown that the WCS is an adequate non-monotonic logic for modelling the Suppression Task. Under the WCS and Łukasiewicz 3-valued logic the task is usually modelled as follows for the case without suppression:

1. Initial logic program: $P = \{e \rightarrow l, \top \rightarrow e\}$. This program represents the task without information about what happens if the library is open.

2. Addition of Abnormality: $P = \{e \wedge \neg ab_1 \rightarrow l, \top \rightarrow e, \bot \rightarrow ab_1\}$. The program now reflects the possibility that some abnormal event may prevent her from going to the library, but because we have no information about the nature of this event, it is set to false by default.

3. Weak Completion: $wc(P) = \{e \wedge \neg ab_1 \leftrightarrow l, \top \leftrightarrow e, \bot \leftrightarrow ab_1\}$. Weak completion is applied to the logic program and, in this case, only implications need to be changed to bijections.

4. Semantic Operator:

   - Execution 1: $\top = \{e\}, \bot = \{ab_1\}$
   - Execution 2: $\top = \{e, l\}, \bot = \{ab_1\}$

After application of the semantic operator l is true in the least model, and so participants conclude that she will study late in the library (as when P is evaluated classically). However, in the case where Suppression is observed, the same process yields a different result because of the presence of the extra conditional ($o \rightarrow l$).

1. Initial logic program: $P = \{e \rightarrow l, \top \rightarrow e, o \rightarrow l\}$. The initial program now includes information about the extra (suppressing) conditional.

2. Addition of Abnormality: $P = \{e \wedge \neg ab_1 \rightarrow l, o \wedge \neg ab_2 \rightarrow l, \top \rightarrow e, \neg o \rightarrow ab_1, \neg e \rightarrow ab_2\}$. Adding abnormalities is a poorly described process and often relies on intuitionist views of what actually constitutes an abnormal situation. In this case we adopt the practise of identifying abnormalities as described in Appendix 13.2.

3. Weak Completion: $wc(P) = \{((e \wedge \neg ab_1) \vee (o \wedge \neg ab_2)) \leftrightarrow l, \top \leftrightarrow e, \neg o \leftrightarrow ab_1, \neg e \leftrightarrow ab_2\}$. Weak completion is applied to the logic program, combining rules with shared heads and replacing implications with bijections.

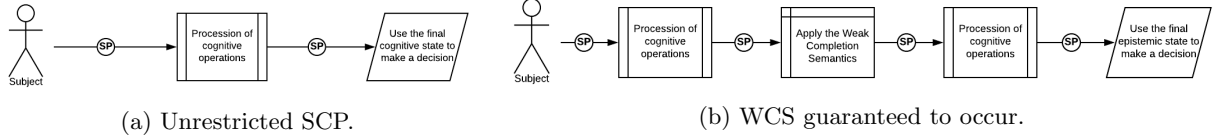(a) Unrestricted SCP. (b) WCS guaranteed to occur.

Figure 1: The most general description on an SCP with and without guaranteeing the WCS is applied at least once. An agent transitions from one epistemic state to another and then uses it to make a decision. SP nodes indicate state points.

4. Semantic Operator:

- Execution 1: $\top = \{e\}, \bot = \{ab_2\}$

Now suppression has been displayed in the logic program and the variable l remains unknown in the least model.

4 Sequential Cognition Process

4.1 SCPs: an Intuitive Description

Although there is evidence that the brain can perform several simultaneous operations when considering a task (such as when considering an image (**?**), the SCP framework assumes that at some points in reasoning about a given task, the mental processes of the agent converge to a set of epistemic states, called a *state point*. Whatever happens between these points of convergence can contain any number of parallel processes. The processes that occur between any two state points in a reasoning task is called a *cognitive operation*. It follows that any cognitive operation is valid as long it takes a set of epistemic states as input and produces a set of epistemic states as output. Figure 1a describes an SCP that is powerful enough to model any cognitive task that involves an epistemic state transition. However, it does not provide any useful information; the nature of the processes followed is completely undescribed. Suppose, instead, that some cognitive task is being modelled, and that researchers have reason to believe that The Weak Completion Semantics should play a part in their model. Under this new restriction, and assuming a sufficiently expressive epistemic state, Figure 1a is still an accurate model of the process, but now so is Figure 1. By sacrificing some of the ambiguity – and, thus, expressiveness – of the model, the information content of the model description has increased. This trade-off is a feature of the SCP framework and finding the right depth of complexity to model the task accurately and still provide meaningful information is more art than science at present.

4.2 SCP: a Formal Framework

An SCP Task $\Pi = (s_i, \gamma, M)$ consists of an initial epistemic state, and known goal state γ, and a known set of cognitive operations M. An epistemic state s_k describes all the information available to the agent at state point k. The precise contents of an epistemic state should be chosen so that at least some $m \in M$ are able to accept that state as an input. In the case of a system containing only the one complex operation which applies the WCS, one possible epistemic state is $s = (KB, V)$, where KB describes a knowledge base consisting only of rules of the ($head \leftarrow body$) and V is a mapping of atom names appearing in KB to a truth value in Łukasiewicz logic ($\top, u, \bot$). In principle this definition will serve for the rest of the paper, but it is extended slightly so that $s_k = (KB, V, R)$ where R is a set of labelled categorization criteria sets (LCS). An $LCS \in R$ consists of a category name and list of rules and atoms which fit into that category.

A base point is a single epistemic state. A state point p is defined recursively by $p = \{\bar{p} \oplus Q\}$ where $\bar{p}$ is a base point, and Q is a set of state points, and $\oplus$ represents the exclusive-or operation[1]. It is never the case that $p \in_s p$. State point containment $\in_s$ for state points p and q is defined recursively as follows:

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[1] $(X \oplus Y) = ((X \cup Y) - (X \cap Y))$ for sets X and Y.

$$p \in_s q = \begin{pmatrix} p \in q & \text{True} \\ p \notin q \text{ and } q \text{ is a base point} & \text{False} \\ \exists_{r \in p} p \in_s r = \text{True} & \text{True} \\ \text{Otherwise} & \text{False} \end{pmatrix}$$

A cognitive operation $m = (\chi, e), m \in M$ consists of a precondition χ and a process e, s.t. for an input state point p, every base point $\bar{p} \in_s p$ is either either accepted as input $\bar{p} \models m[\chi]$ under whatever definition of $\models$ is used for the complex operation m, or else rejected. Every base point is evaluated by the complex operation in isolation (no other base point $\bar{q}$ can affect the output of m on base point $\bar{p}$). To capture the fact that cognitive operation may utilize non-monotonic logic, Applying m to an input base point $\bar{p} \in p$ always yields a state point p' (written $J[\bar{p}, m] = p'$) which describes all possible resulting epistemic states that can be generated.

Applying m to an input state point p is done by replacing every base point $\bar{p} \in p$ with $J[\bar{p}, m]$. In a cognitive operation is called monotonic if it always yields a base point as an output given a base point input $(J[\bar{p}, m] = \bar{p'})$. It follows that the depth of a state point is directly related to the number of complex operations which have been performed on it. If a base point does not meet the precondition, it is either ignored complete and not processed (cruel), or passed exactly as is to the next complex operation (lenient). It is worth noting that the type of cognitive states produced as output my m may not be the same as those of the input[2].

An SCP $\pi = (s_i, m_1, ..., m_n)$ describes an initial epistemic state (or state point if the input is uncertain) and state point p_k is defined recursively by $p_k = J[p_{k-1}, m_k]$. An SCP is called *credulously valid* if $p_n \models \gamma$ for at least one epistemic state in the final state p_n. An SCP is called *sceptically valid* if $p_n \models \gamma$ for every epistemic state in the final state. In cases where all operations are monotonic, sceptical validity is the same as credulous validity.

4.3   Choosing an Epistemic State

The choice of epistemic state is dependent on the properties that are known or suspected to be true for the cognitive task as a whole. For example, a researcher working on drawing inferences using Propositional Logic can be certain that any SCP they create should be expressive enough to pass a knowledge base to a cognitive operation. Thus, it might suffice to simply define $s_i = (KB)$ where KB is a set of propositional rules. By contrast, a researcher using the WCS requires a system capable of both communicating a set of rule to the next complex operation, but also of describing the results of repeated applications of the semantic operator. It might seem intuitive to simply keep the epistemic state used for the propositional case and to use a non-complex operation function on the weakly completed program to find the least model. And, though this approach works for many examples, it makes it impossible to do something else *after* applying the semantic operator. What if the conclusion drawn was meant to form part of the background knowledge of another process? In practice using $s_k = (KB, V, R)$, where V is a set of (variable name, value pairs) R is a categorization variable, is able to model all aspects of the WCS, including information related the least model.

The final property of a cognitive operation that needs to be discussed is how it is able to interact with the categorization variable R. Imagine a case drawn from @TODOref where the difference between creating abnormalities for obligate and factual conditionals is discussed. The intuition behind the authors' work can be summarised by saying that there are two different types of conditional statement, those that *have to be* true, and those that are *usually* true. If it rains r, I will normally take my umbrella u (in the absence of something abnormal happening, like a plague of umbrella-stealing gnomes), this a statement that is usually true. But some things are definitely true. For example, when it rains, water has to fall out of the sky s. This is an essential property of rain, not subject to abnormalities. Thus one useful set of categorizations for researchers seeking cosmic knowledge of weather patterns may be: $R = \{obligate : \{u \leftarrow r\}, factual : \{s \leftarrow r\}\}$. With these labels we might then expect that the process followed by the operation $m \in M$ which creates

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[2]This paper will focus on cases where the type of cognitive state remains constant, but there is no reason in principle that a base point input $s_k = (KB, V, R)$ for the WCS could not be returned as a base point $s_k = (KB, V, R, D)$ where D is a set of default rules. Future models of human cognition may well rely on background knowledge which draws inferences from multiple types of non-monotonic logics.
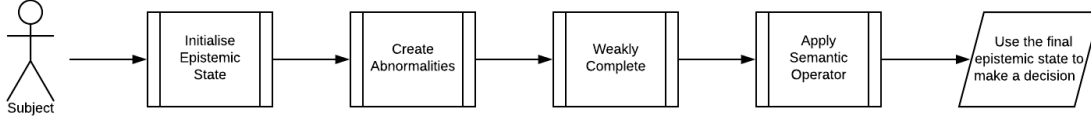
Figure 2: A generalised illustration of the WCS in an SCP.

abnormalities would treat the two conditionals in KB differently, because of their assignments in R. R then is a way of expressing meta information to the cognitive operations, and it is completely possible that some operation m_k might change R is such a way that future operation m_{k+l} produces different output. This is a technique we will exploit in several examples in this paper.

As yet, there are no definite rules for creating an epistemic state, but Albert Einstein's advice from 1950 still rings true: "Everything should be made as simple as possible, but no simpler." The ideal epistemic state is one that enables every reasonable cognitive operation in M that might help model the problem, without adding superfluous functionality that might render searching the SCP space infeasible.

4.4 Choosing a set of Complex Operations

The set of possible complex operations M determines many attributes of the achievable final state point p_n. If every $m \in M$ is monotonic, then p_n will be monotonic. If some cognitive operation is computationally complex or produces a very large number of output state points, then search using that base point becomes less efficient. If some complex operation m' (such as weakly completing) is known or believed to occur in the SCP, then a restriction on the cognitive states exists s.t. the either the initial state is of a format suitable as input for m', or there exists another cognitive operation which is able to output a state point which contains base points of a suitable format.

More abstractly, the set of cognitive operations should be well-founded in the literature. The set of possible complex operations is infinite and an SCP only meaningfully describes human cognition when it contains cognitive operations that have been justified empirically (*modus ponens-modus tolens* assymetry, suppression, denial of the antecedent, etc.).

5 Modelling with SCPs

5.1 SCPs Under the Weak Completion Semantics

5.1.1 The Wason Selection Task

5.1.2 The Suppression Task

Figure 2 illustrates a generalised SCP to describe the Suppression Task as a series of sequential steps directly mirroring the discrete steps outlined in Section 3.1, each cognitive operation passing information to the next process[3]. To model the SCP the implicit sequence of operations in the Suppression Task is systematized and refined into a set of complex operations. Further we introduce an initial epistemic state $s_i = (KB, V, R)$. One interpretation of the requirements of the suppression task $\pi = (s_i, \gamma, M)$ using SCPs and the WCS is as follows:

- $s_i = \{KB_i, V_i, R_i\}$

- $KB_i = \{e \to l, \top \to e, o \to l\}$, $V_i = \{e : u, l : u, o : u\}$, $R_i = \{\}$

- $\gamma = (l \models \top)$ or $(l \models \bot)$ (that is, l is no longer mapped to unknown in the final epistemic state.)

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[3]It is important to note that a diagram like this is valid for *any* cognitive modelling task because any process may be arbitrarily complex and non-sequential. and so the overall linear process of (actor, complex decision, observed results) is always valid for retroactive modelling, and at least as powerful as the non-monotonic logic framework it uses for modelling.
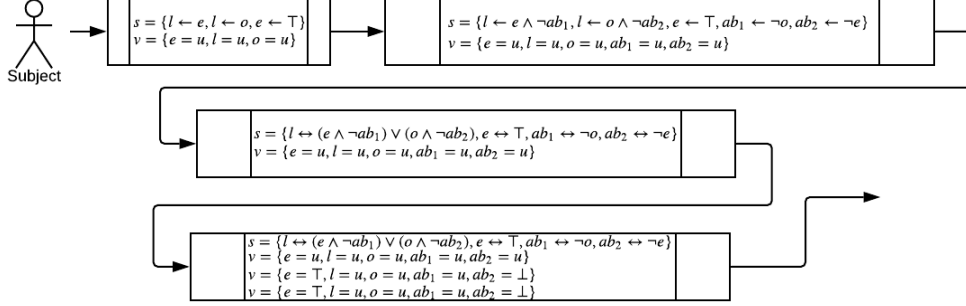
Figure 3: The standard case of the Suppression Task, demonstrating the suppression effect. Where the epistemic state in the boxes represents the output of that cognitive operation.

- $M = \{\texttt{init}, \texttt{addAB}, \texttt{WeaklyComplete}, \texttt{semanticOperator}\}$

where all cognitive operations require as in input and produce as output a state point p where every ground point $\bar{p} \in_s p$ is of format $\bar{p} = (KB, V, R)$, $\texttt{init}$ is always the first cognitive operation and adds the initial variables and rules to epistemic state; $\texttt{addAB}$ adds abnormalities to the current epistemic state using the procedure described in Algorithm 1 (but now also adds those abnormalities to the variable list of the epistemic state; $\texttt{WeaklyComplete}$ weakly completes the knowledge base of the current epistemic state; and $\texttt{semanticOperator}$ returns an epistemic state that leaves the knowledge base unchanged but updates the variables of that state to return the least model of the epistemic state.

Treating Figure 2 as an SCP, we observe the sequence of output states seen in Figure 3. Note that in the final state l remains unknown and the suppression effect is demonstrated.

5.1.3 Extending the Suppression Task with the SCPs

The previous example merely showed that SCP are suitable for modelling the suppression task. In this example we consider one of the most powerful characteristics of SCPs, the ability to model unusual results as deviations from general reasoning. A significant portion of people still believe that she will study late in the library when presented with the suppression task. Several possible explanations are intuitive, the first and simplest, is the assumption that the reasoner is using classical logic and drawing the classical conclusion. However, what if that is not the case? What if they do reason in exactly the same way as the other reasoners, except for one or two small deviations?

In this example we consider two possible deviations that could explain the classical result of the Suppression Task: variable deletion, and variable fixation. Both of these operations will be discussed in a way that may seem overly prosaic to many readers, but the reason for the examples given is to reinforce how reasonably we might expect these cognitive operations occur in reasoning tasks.

Variable Deletion

Consider the sequence of numbers (1, 44 ,27 ,8 ,0 , -4, 6, 7, 346, 7, 74, 7, 234, -55, 2.4, 18). Now without looking back at the numbers, ask yourself some questions: how many numbers were there? Were any of them prime? How many numbers were repeated? In all probability you are not entirely sure. This simple thought experiment provides support for our first extension, the idea that variables can be "forgotten", that is, that information that existed in the knowledge base at one point in time might no longer exist at a later timepoint.

This is not the only imaginable case where a variable might be removed from the knowledge base of the person being modelled. The size of the knowledge base used for cognitive modelling is always implicitly restricted to relevant variables, only those variables whose values might reasonably be expected to affect the final conclusions drawn with regard to the research question. Finding which variables and rules are relevant
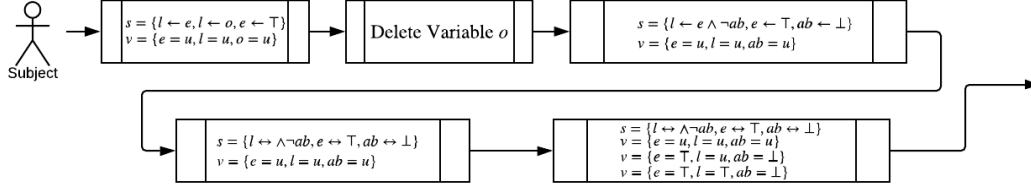
Figure 4: The Suppression Task in which the additional operation of deleting the variable o occurs.
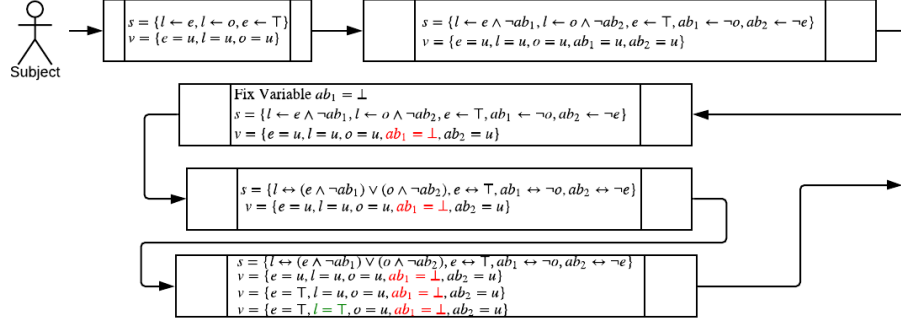

Figure 5: The Suppression Task in which the additional operation of fixing the variable ab_1 to false occurs.

is, however, non-trivial. For another real-life example, imagine a mystery novel. Three hundred pages of plot descriptions, character actions, and dialogues. In a good murder mystery novel every piece of information that reveals the killer's identity is hidden in the story itself, yet we do not hold every fact and interaction in the book in our epistemic model of the book, so discerning the identity of the killer remains a mystery until the last page. But when the mystery is solved, many details that we internalised while reading (and recall in retrospect) suddenly make the conclusion seem obvious. We have not forgotten this information, we had merely incorrectly deemed it irrelevant at the time and ignored it in our cognitive processing.

The exact details of how to delete a variable from a knowledge base is non-trivial, and there is no best practice for doing so. But in simple cases the process can be intuitive. Given $kb = \{a \leftarrow \top, b \leftarrow \top\}$ deleting variable a might be as simple as removing all rules that mention it from the knowledge base (though an argument could also be made that it should be set to unknown).

In the case of the Suppression Task we argue that one cognitively valid reason for drawing the classical conclusion to the task may be forgetting (or disregarding) the variable o. Figure 4 illustrates this case, and shows how the insertion of a complex operation can completely change the final epistemic state.

Variable Fixing

The second case of a potential complex operation to add to the search space of our SCPs is the idea of Variable Fixing. The idea that some conclusions can be fixed *a priori*. Consider a person who strongly doubts the effectiveness of vaccines, we will call her Karen. Karen started her day convinced that giving her child the MMR vaccine is more dangerous than the disease itself. Later that day Karen spoke to her doctor who strongly advised that she vaccinate her child. He offered her a variety of peer-reviewed papers and studies that showed the relative safety of the vaccination. Karen listened carefully to the trained medical professional, and then went home. After some thought Karen decided that he was wrong, and her opinion on vaccines didn't change.

In this example Karen shows a very powerful type of cognitive bias, the unwillingness to change change her opinions, despite powerful evidence to the contrary. This phenomenon has observed across a great many

fields of study, from medical psychology (**?**) (**?**) to political sciences(**?**). In the context of cognitive modelling with logics, it indicates that a mental rules or variables are immutable, regardless of new evidence or valid beliefs that would logically contradict them. Non-monotonic logics, as a class, are already capable of dealing with bias effect, as non-monotonic logics are built on the basis of a preference operation. However, for the case of the Weak Completion Semantics and SCPs, it serves to introduce a complex operation $fixX$ which can fix the value of a rule or variable at every point after it occurs in the SCP.

In practise this necessitates a way of denoting fixed variables so that future operations do not change them, it also strongly suggests the plausibility of some operation $unfixX$ which would allow a variable to be fixed to a new value again after it occurs. For the purposes of the Suppression Task it suffices to show that the $fixX$ operation can change the conclusions of the general reasoner to those of the deviant reasoner.

Figure 5 shows the effect of adding a complex operation which fixes the value of the abnormality to false in v so that, no matter what rules are present in kb when the semantic operator is applied, ab_1 will remain false[4].

5.2 SCPs Under other Logics

6 Conclusions and Future Work

7 Overview

For my Master's thesis I intend to describe, implement, and test a novel framework for modelling human reasoners called the Sequential Cognitive Process (SCP). SCPs retain most of the expressiveness of the hitherto *ad hoc* approaches to modelling reasoners with non-monotonic logics, but also introduce a number of desirable properties:

1. They enable modelling of individual reasoners through addition or removal of well-described mental operations. This is in contrast to most non-monotonic logic approaches which can model the general reasoner across a number of cognitive tasks but do not extend well to explaining reasoning that deviates from the norm.

2. They standardise a broad class of non-monotonic logical frameworks so that cognitive comparisons, even across different frameworks are possible, in particular I have given some thought to the use of string-matching algorithms borrowed from the field of bioinformatics (such an extended version of the Needleman Wunsch Aglorithm(**?**)) to comparing SCPs as operation sequences.

3. Because they are sequential (essentially linked lists of mental operations) they lend themselves well to traditional search techniques, and their utility in finding – though not yet justifying – models of the Suppression Task has already been demonstrated for both general and individual reasoners.

4. They standardise a set of well-founded cognitive processes (such as, the Suppression Effect, Affirmation of the Antecedent, Conjunction Fallacy bias, and the introduction of abnormalities) as reusable epistemic operations which can be constrained or expanded according the needs of the researcher.

5. Because they are sequences of actions with standardised input and output formats, there is certainly potential for traditional heuristic approaches to problem solving with SCPs as well as purpose-built heuristic adapted to the cognitive task being considered.

A limited Python version of this system has already been implemented[5] for the Weak Completion Semantics (**?**). This implementation has proved effective at modelling both the Suppression Task (**?**) and the Wason Selection Task (**?**). Work to expand the implementation is ongoing.

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[4]The question of which solution best models deviant reasoners in the suppression task is one of significant interest and I intend to go into some detail about possible scoring algorithms and heuristics for determining the most likely SCP for a cognitive task when multiple candidate SCPs come to the desired conclusions.

[5]Available on Github: https://github.com/AxelInd/SequentialCognitionPrcoesses. For access, please contact me with your details.

8 What is an SCP

An SCP is a model of the thought process of a reasoner, whether individuals or a group. The SCP assumes that some epistemic state (of whatever form the nonmonotonic logic requires) is modified by mental operations in the SCP to reach a final epistemic state that satisfies some goal properties. An SCP Problem $\pi = (s_i, \gamma, M)$ describes the requirements and limitations of the SCP before it is created[6]. s_i contains the initial state of the SCP, and γ is the goal of the SCP (e.g. to mimic the classical case of the Suppression Task), M is the set of allowable cognitive operations.

The SCP framework makes several core assumptions about the nature of cognition:

1. Evolving Epistemic States: SCPs are built on the idea that some mental model of the world exists before the problem-solving begins (even if the model is empty), and is updated by each mental process that acts on it, yielding a new epistemic state as output from mental process m_n to be used as input for mental process m_{n-1}.

2. Sequential Reasoning: the assumption that a reasoner will perform only one mental operation at a time before moving on to the next one. Although it initially seems very limitting, the complexity of mental operations is unconstrained, and could itself involve parallel processes provided that a unified mental state is achieved before the next operation begins.

3. Reusable Operations: in order to be more than just a container for some other non-monotonic logic, cognitive tasks must share at least some complex operations. If this were not the case, standardising a set of cognitive operations to model unseen tasks would be impossible. Fortunately, there is ample evidence of consistent mental approaches to reasoning (**?**).

4. Limited mental processing power: SCPs assume a sequential and finite number of mental operations to reach a goal. Without this property, it would be impossible to accurately model human cognition. Further, there would be no reasonable scoring system for determining the best explanation when multiple SCPs all model the same thought process (the simplest scoring system intuitively is to compare the length of the two candidate SCPs and to prefer the smaller).

9 State of the Art

9.1 More Questions Than Answers

The example of the Suppression Task (**?**) illustrates one of the glaring problems with cognitive modelling in non-monotonic reasoning as it stands at present: it is unstructured. What rules should be followed when adding abnormalities? How do we choose the order to apply these operations? Is this the only reasonable sequence in which to apply these operations? All these questions arise from a lack of structure in the frameworks being used. For example, work by **?** discusses the idea of obligate and factual conditionals, which are treated differently in terms of how abnormalities are added. From this it immediately follows that extensions to the WCS are possible, but where can they occur? Which extensions and changes are well founded in the psychological literature? What are the properties of the search spaces of these extensions? Is it even possible to compare different cognitive models to one other if they produce identical output? SCPs are an attempt to create a framework for non-monotonic logics that is suitable to answer these questions.

One of the most important questions to answer, is if it possible for these non-monotonic logics to model individual reasoners as well as general reasoners. Work by (**?**) showed that simple extensions to the Weak Completion Semantics allowed researchers to model the four most common decisions of participants in the Wason Selection Task. There is significant potential for research into modelling the unusual choices of participants who do not conform to the normally observed results. Part of my thesis will be dedicated to justifying a view of these unusual reasoners as deviations from the normal cognitive process who still follow a very similar reasoning approach but with small and well-founded differences when compared to general reasoners. In the following sections we briefly discuss this idea in terms of the Suppression Task.

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[6]This formulation is inspired by the field of AI Planning

9.2 Example: The Suppression Task

9.2.1 What is the Suppression Task?

The Suppression Task refers to an experiment conducted by **?** and is a classical example of the inadequacy of monotonic logics for modelling human reasoning. In classical logic, if our knowledge base kb is such that $kb \models \phi$, then it must be the case that $kb \cup \psi \models \phi$. However, in the suppression task participants no longer draw classically valid inferences when new information is added. The task is often formulated as follows:

- $e \rightarrow l$: If she has an essay to write (e), she will study late in the library (l).

- $\top \rightarrow e$: She has an essay to write (e).

- $o \rightarrow l$: If the library is open (o), she will study late in the library (l).

Given only the rules ($e \rightarrow l$) and ($\top \rightarrow e$), the participants consistently concluded that she would study late in the library, seemingly drawing the classical logic inference $\frac{e \rightarrow l, e}{l}$ with *modus ponens*. But when given the additional rule $o \rightarrow l$, participants no longer believe that they have enough information to judge whether she will study late in the library, and a significant portion of them no longer draw the classical conclusion. This effect, called Suppression, demonstrates the need for something more than classical logic for modelling human reasoning.

9.2.2 Modelling the Suppression Task with the WCS

Work by **?** has shown that the Weak Completion Semantics is an adequate non-monotonic logic for modelling the Suppression Task. The reader is assumed to be familiar with the Weak Completion Semantics and so it is only briefly discussed in Appendix 13.1.

Under the Weak Completion Semantics and Łukasiewicz 3-valued logic the task is usually modelled as follows for the case without suppression:

1. Initial logic program: $P = \{e \rightarrow l, \top \rightarrow e\}$. This program represents the task without information about what happens if the library is open.

2. Addition of Abnormality: $P = \{e \wedge \neg ab_1 \rightarrow l, \top \rightarrow e, \bot \rightarrow ab_1\}$. The program now reflects the possibility that some abnormal event may prevent her from going to the library, but because we have no information about the nature of this event, it is set to false by default.

3. Weak Completion: $wc(P) = \{e \wedge \neg ab_1 \leftrightarrow l, \top \leftrightarrow e, \bot \leftrightarrow ab_1\}$. Weak completion is applied to the logic program and, in this case, only implications need to be changed to bijections.

4. Semantic Operator:

   - Execution 1: $\top = \{e\}, \bot = \{ab_1\}$
   - Execution 2: $\top = \{e, l\}, \bot = \{ab_1\}$

After application of the semantic operator l is true in the least model, and so participants conclude that she will study late in the library (as when P is evaluated classically). However, in the case where Suppression is observed, the same process yields a different result because of the presence of the extra conditional ($o \rightarrow l$).

1. Initial logic program: $P = \{e \rightarrow l, \top \rightarrow e, o \rightarrow l\}$. The initial program now includes information about the extra (suppressing) conditional.

2. Addition of Abnormality: $P = \{e \wedge \neg ab_1 \rightarrow l, o \wedge \neg ab_2 \rightarrow l, \top \rightarrow e, \neg o \rightarrow ab_1, \neg e \rightarrow ab_2\}$. Adding abnormalities is a poorly described process and often relies on intuitionist views of what actually constitutes an abnormal situation. In this case we adopt the practise of identifying abnormalities as described in Appendix 13.2.

3. Weak Completion: $wc(P) = \{((e \wedge \neg ab_1) \vee (o \wedge \neg ab_2)) \leftrightarrow l, \top \leftrightarrow e, \neg o \leftrightarrow ab_1, \neg e \leftrightarrow ab_2\}$. Weak completion is applied to the logic program, combining rules with shared heads and replacing implications with bijections.

4. Semantic Operator:

- Execution 1: $\top = \{e\}, \bot = \{ab_2\}$

Now suppression has been displayed in the logic program and the variable l remains unknown in the least model.

10 Modelling the Suppression Task with the SCPs

11 Implementation

A simple implementation of the SCP framework has already been implemented for the WCS, it has been shown able to model both the Suppression Task and the Wason Selection Task in terms of both general and deviant reasoning. The overall structure of the SCP implementation is as a pipeline, a linked list of operations each of which relies on input from the previous operation in the list.

11.1 Class Diagram

Figure 6 shows a simplified class diagram of the implementation of the SCP program. The scp class makes use of a basic implementation of 3-valued logic and a sequence of `complexOperation` objects in order to evaluate the knowledge base and variables of the SCP after each operation.

Each `complexOperation` is a pipeline, it contains no data other than pointers to its successor and predecessor and passes the results of its `evaluatev()` and `evaluatekb()` functions to the next operation as input. The code itself is heavily commented and contains files which implement and execute examples of the Suppression Task and Wason Selection task under WCS.

11.1.1 Additional Features

Several other features will be added to the implementation soon:

- DeNovo Searching: finding an SCP that satisfies a goal condition form scratch, iterating over the set off cognitive operation in M. This search is important for coming up with an explanation for the general (most common) results of a cognitive task.

- Insertion Searching: finding which additional complex operations can be added to an existing SCP to produce results consistent with the deviant reasoner. This search is important for describing unusual results as deviations from the general reasoner. It is also important for scoring the likelihood of these the SCP.

- Scoring: a modified version of Needleman-Wusch algorithm for string matching has already been theoretically described and must now be implemented in order determine how similar different models are to each other.

Figure 6: A simplified illustration of class interactions in the SCP implementation.
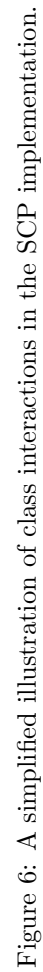
12   Milestones

A portion of the work to create and formalise SCPs has already been completed. A literature review, time spent programming, and time spent formalising the idea of SCPs has already devoted to the problem. however there is still a significant amount of work to be done:

Timeline

- Standardise implementation interface: 3 Feb - 9 Feb

- Extend Interface to Conjunction Fallacy under WCS: 10 Feb - 16 Feb

- Complete paper on SCPs under the WCS: 17 Feb - 23 Feb

- Conceptually extend SCPs to Reiter's Default Logic: 24 Feb - 1 March

- Extend the existing implementation to handle Default Logic: 2 March - 8 March

- Explore scoring algorithms for SCps: 3 March - 9 March

- Examine application of heuristics to SCPs: 10 March - 15 March

- Complete paper on SCPs under Default Logic: 16 March - 23 March

- Write Thesis: 23 March - 20 April

- Correction, updates, and new ideas: 21 April - ...

Deliverables

- Conference ready paper on SCPs and the WCS: 23 Feb

- Conference ready paper on SCPs and the default logic: 23 March

- Complete, documented, extensible implementation of SCPs containing various examples, search functionality, and best-practice design: 1 April

$\rightarrow$	$\top$	u	$\bot$
$\top$	$\top$	u	$\bot$
u	$\top$	$\top$	u
$\bot$	$\top$	$\top$	$\top$

Table 2: A table showing the implication operator in 3-valued Łukasiewicz logic.

13 Appendix

13.1 Weak Completion

The Weak Completion Semantics is a non-monotonic logic which procedurally encodes several well-known cognitive phenomenon. The WCS makes use of 3-valued Łukasiewicz logic (Table 2). It adds abnormalities to non-ground inferences, and replaces the classical inference ($\leftarrow$), with a bijective ($\leftrightarrow$).

The Weak Completion of a program P is defined as follows:

1. Replace all clauses of the form $A \leftarrow body_1$, ..., $A \leftarrow body_n$ with $A \leftarrow body_1 \vee ... \vee body_n$.

2. Replace all occurrences of $\leftarrow$ with $\leftrightarrow$.

Applying this procedure to P results in wcP which is the weak completion of P.

The next requirement to apply the WCS framework is the introduction of a semantic operator ϕ_{SvL} (**?**). Let J be the result of applying the semantics operator to an interpretation I and logic program P. The J is defined as follows:

$$J^{\top} = \{A|\text{ there exists a clause } A \leftarrow Body \in P \text{ with } I(Body) = \top\}$$

$$J^{\bot} = \{A|\text{ there exists a clause } A \leftarrow Body \in P$$
$$\text{and for all clauses } A \leftarrow Body \in P \text{ we find } I(Body) = \bot\}$$

Using $I = <\emptyset, \emptyset>$, the least model ($\text{lm}_{Ł}wcP$) can be calculated by iterating $\phi_{SvL,P}$.

13.2 Systematically Adding Abnormalities

Adding abnormalities is a poorly described process and often relies on intuitionist views of what actually constitutes an abnormal situation. For this reason I have provided one concrete method for adding abnormalities to a logic program (though many others exist and can be extended at will). This particular one is based on the principle that if a variable v_1 has the power to make another variable v_2 true, then it may be the case that participants believe that any other variables V' satisfying v_2 may only do so when v_1 holds. This intuition is counter to classical logic, but follows intuitively in cases like the Suppression Task where participants believe that the library not being open may be the reason that the she doesn't study late in the library, even when she has an essay to write. The precise procedure is as follows:

In the case of SCPs, the procedure for creating abnormalities is handled by a single complex epistemic operation and so may be substituted for any competing method with ease without disrupting the flow of the SCP.

Algorithm 1 One way of adding abnormalities to a logic program prior to applying the Weak Completion Semantics.

1: **function** ADDABNORMALITIES(P)
2:     $P' := []$
3:     **for** Unique $head$ with $(body \rightarrow head) \in P$ **do**
4:         $headDependencies :=$ [all non-ground bodies s.t. $(body \rightarrow head) \in P$]
5:         **for** $body_i$ with $(body \rightarrow head) \in P$ **do**
6:             $ab_x :=$ new abnormality   ▷ Where x is a unique identifier
7:             create new rule $head \leftarrow body_i \wedge \neg ab_x$
8:             $abDependencies = headDependencies \backslash body_i$ ▷ $\bot$ if it would be empty
9:             $rule =$ create new rule $(ab_x \leftarrow dep_1 \wedge ... \wedge dep_n)$ for $dep \in abDependencies$
10:            $P' + = rule$
11:        **end for**
12:    **end for**
13:    **return** P'
14: **end function**