Sequential Cognition Processes

A FRAMEWORK FOR REASONING WITH NON-MONOTONIC LOGICS

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Research Questions

Is there a way to systematically represent some (or all) non-monotonic logics in a framework that standardizes cognitive modelling under these logics without sacrificing their expressiveness?

- Applications in human reasoning.
- Searchability.

Can the reasoning of individuals be explained with small deviations from models for general reasoners?

How can cognitive modelling explain why humans do not follow classical logic rules?

My Achievements

A systematic, searchable, and extensible framework for modelling human reasoning.

- General.
- Individual.

Creation of a scoring algorithm for comparing these models

- Within the same task.
- Across tasks.

Novel techniques for modelling individual cases of the Suppression Task and the Wason Selection Task.

Experiments: The Suppression Task

In classical logic $\mathrm{th}(KB) \subseteq \mathrm{th}(KB \cup \theta)$

Experiment by (Byrne, 1989) showed that this does not always hold in human reasoning.

Heavily studied.

Demonstrable in some non-monotonic logics.

Suppression Task Formulation

1. If she has an essay to write, she will study late in the library $(l|e)$.
2. If the library is open, she will study late in the library $(l|o)$.
3. She has an essay to write $(e \leftarrow \top)$.

``Will she study late in the library?”

Told only (1) and (3) (the *el* case)
- ``she will study late in the library.”
- Classical inference.

Told (1), (2), and (3) (the *elo* case)
- We are not sure if she will study late in the library.
- Suppression of the inference has occurred.

Weak Completion Semantics

Powerful non-monotonic approach.

Using 3-valued Łukasiewicz logic and the WCS guarantees a minimal least model.

Weak completion of a program:

1) Replace all clauses of the form $A \leftarrow body_1 \ldots A \leftarrow body_n$ with $A \leftarrow body_1 \vee \cdots \vee body_n$.
2) Replace all occurrences of $\leftarrow$ with $\leftrightarrow$.

Semantic Operator:

- $J^{\top} = \{A | \text{there exists a clause } A \leftarrow body \in P \text{ with } I(body) = \top\}$
- $J^{\perp} = \{A | \text{there exists a clause } A \leftarrow body \in P \text{ and for all clauses}$
  $A \leftarrow body \in P \text{ we find } I(body) = \perp\}$

Applying the WCS to the Suppression Task

el case:

$\mathrm{KB}_{noSup} = \{e \leftarrow \top, l \leftarrow e \ \wedge \neg ab_1, ab_1 \leftarrow \bot\}$

$wc\ \mathrm{KB}_{noSup} = \{e \leftrightarrow \top, l \leftrightarrow e \ \wedge \neg ab_1, ab_1 \leftrightarrow \bot\}$

least model $\top = \{e, l\}, \bot = \{ab_1\}$

elo case:

$\mathrm{KB}_{\mathrm{sup}} = \{e \leftarrow \top, l \leftarrow e \ \wedge \neg ab_1, l \leftarrow o \ \wedge \neg ab_2, ab_1 \leftarrow \neg o, ab_2 \leftarrow \neg e\}$

$\mathrm{wc}\ \mathrm{KB}_{sup} = \{e \leftrightarrow \top, l \leftrightarrow (e \ \wedge \neg ab_1) \ \vee (o \ \wedge \neg ab_2), ab_1 \leftrightarrow \neg o, ab_2 \leftrightarrow \neg e\}$

least model $\top = \{e\}, \bot = \{ab_2\}$

<u>$\therefore$Suppression observed.</u>

Sequential Cognitive Processes

Model human reasoning as a *sequence* cognitive operations.

Can be used to encode:

- Uncertainty in background knowledge
- Non-monotonic final world states

Epistemic States

Set of **structural variables.**

Structural variables encode the beliefs of the reasoner at a given moment.

No fixed set of structural variables, more can be added as required.

For the Weak Completion Semantics:

Propositional Knowledge Base

Conditionals $(\psi|\varphi)$

Epistemic state

$$s = \{S, \Delta, V, R\}$$

(variable, value) pairs

(label:{instruction})

Cognitive Operation

Takes a **state point** as *input*, produces a state point as *output*.

- State point: list of epistemic states.

Monotonic cognitive operations: #inputs = #outputs.

Non-monotonic cognitive operations: #inputs != #outputs.

$$(\overset{\text{Pre-condition}}{\downarrow \atop \chi}, \underset{\uparrow \atop \text{effect}}{e})$$

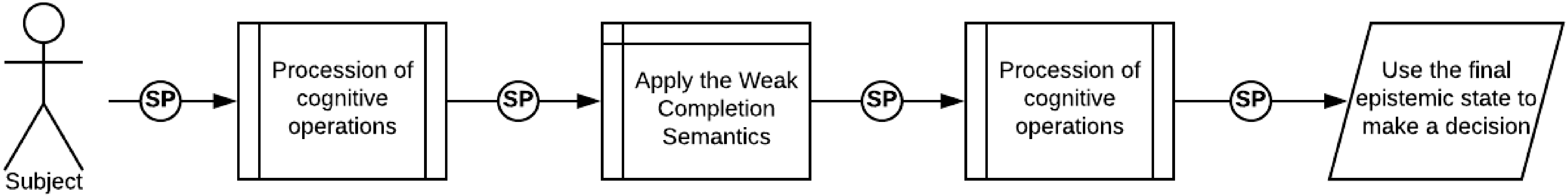
Well founded:

- In psychology, logic, physiology, etc.

Computational limitations of the human mind:

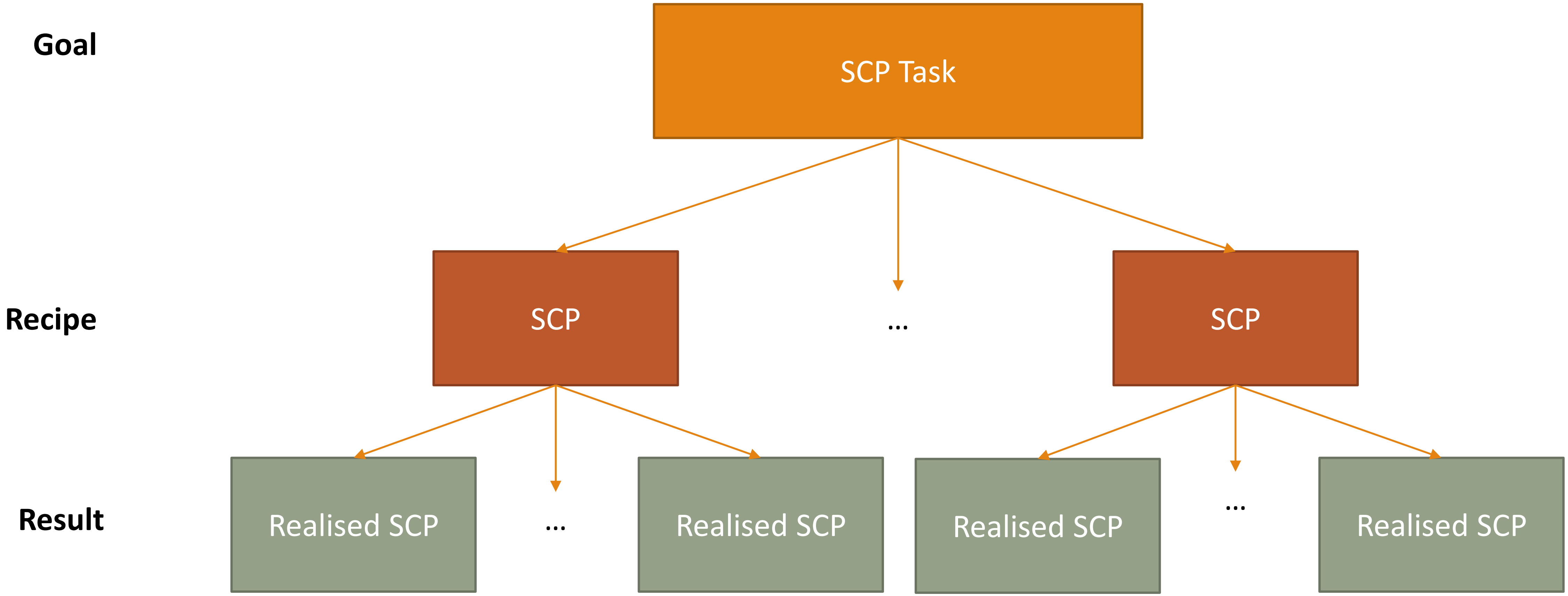
- Time complexity.
- Space complexity.

Sequential Cognitive Processes

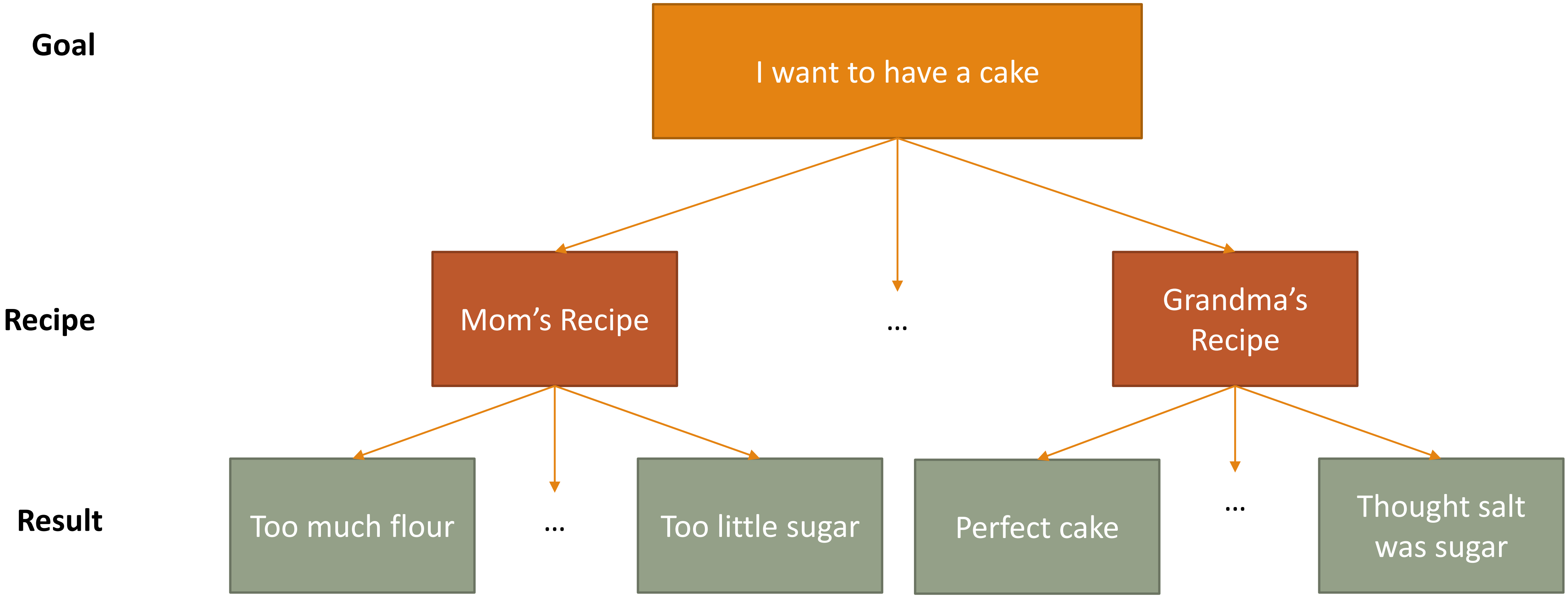


- A sequence of transformations on an initial state point.
- Can be used to model existing non-monotonic logics.
- Easily extended to model individual reasoners.
- Allows classical search techniques.

The SCP Framework



SCP Framework: Cake Example



SCP Tasks

Models a final goal using a known initial state point and set of operations.

SCP Task

Initial State Point

External Evaluation function

$$\Pi = (s_i, M, f(), \gamma)$$

Set of Cognitive Operations

Goal

SCPs

Gives a sequence of instructions which **could** result in the desired goal.

SCP   Transition Model

$$\mu = (\pi, f())$$

External Evaluation Function

$$\pi = (s_i \longmapsto m_1 \longmapsto \cdots \longmapsto m_n)$$

Initial State   Cognitive Operations

Realised SCPs

A single path through an SCP.

Results in a single epistemic state output from every cognitive operation.

Single Path

$$r = (k, f())$$

Realised SCP

External Evaluation Function

Modelling the Suppression Task

The *el* Case

$s_i^{el} = \{S, \Delta_{\text{noSup}}, V\}$

$M = \{addAB, semantic, wc\}$

$$f() = \begin{cases} will\ study\ late & l \in p_n[V] \vDash \top \\ will\ not\ study\ late & l \in p_n[V] \vDash \bot \\ we\ do\ not\ know & otherwise \end{cases}$$

$\gamma_{el} = \ will\ study\ late$

$$\Pi_{el} = (s_i^{el}, M, f(), \gamma_{el})$$

The *elo* Case

- $s_i^{elo} = \{S, \Delta_{\text{sup}}, V\}$
- $\gamma_{elo} = we\ do\ not\ know$

$$\Pi_{elo} = (s_i^{elo}, M, f(), \gamma_{elo})$$

A possible SCP

	el	elo
init	$S = \{e \leftarrow \top\}$ $\Delta = \{(l \| e)\}$ $V = \{(e : u), (l : u)\}$	$S = \{e \leftarrow \top\}$ $\Delta = \{(l \| e), (l \| o)\}$ $V = \{(e : u), (l : u), (o : u)\}$
addAB	$S = \{e \leftarrow \top, l \leftarrow e \wedge \neg \mathrm{ab}_1, ab_1 \leftarrow \bot\}$ $V = \{(e : u), (l : u), (\mathrm{ab}_1 : u)\}$	$S = \{e \leftarrow \top, l \leftarrow e \wedge \neg \mathrm{ab}_1, l \leftarrow o \wedge \neg \mathrm{ab}_2, \mathrm{ab}_1 \leftarrow \neg o, \mathrm{ab}_2 \leftarrow \neg e\}$ $V = \{(e : u), (l : u), (o : u), (\mathrm{ab}_1 : u), (\mathrm{ab}_2 : u)\}$
wc	$S = \{e \leftrightarrow \top, l \leftrightarrow e \wedge \neg \mathrm{ab}_1, ab_1 \leftrightarrow \bot\}$ $V = \{(e : u), (l : u), (\mathrm{ab}_1 : u)\}$	$S = \{e \leftrightarrow \top, l \leftrightarrow (e \wedge \neg \mathrm{ab}_1) \vee (o \wedge \neg \mathrm{ab}_2), \mathrm{ab}_1 \leftrightarrow \neg o, \mathrm{ab}_2 \leftrightarrow \neg e\}$ $V = \{(e : u), (l : u), (o : u), (\mathrm{ab}_1 : u), (\mathrm{ab}_2 : u)\}$
Semantic	$S = \{e \leftrightarrow \top, l \leftrightarrow e \wedge \neg \mathrm{ab}_1, ab_1 \leftrightarrow \bot\}$ $V = \{(e : \top), (l : \top), (\mathrm{ab}_1 : \bot)\}$	$S = \{e \leftrightarrow \top, l \leftrightarrow (e \wedge \neg \mathrm{ab}_1) \vee (o \wedge \neg \mathrm{ab}_2), \mathrm{ab}_1 \leftrightarrow \neg o, \mathrm{ab}_2 \leftrightarrow \neg e\}$ $V = \{(e : \top), (l : u), (o : u), (\mathrm{ab}_1 : u), (\mathrm{ab}_2 : \bot)\}$
	$f(\pi_{\mathrm{el}})$ = 'She will study late'	$f(\pi_{\mathrm{elo}})$ = 'we do not know'

Modelling Individual Cases

Let's go a step further.

Given the *elo* case, some participants still believe that she will study late in the library.

This is the *classical* inference.

- $\mu_{clas} = (\pi_{clas}, f())$ is a valid SCP for this problem.
- Where $\pi_{clas} = \left(s_i^{elo} \longmapsto th\right)$.
- And th adds all classically valid inferences to S.

But it can be modelled with non-monotonic logics.

	el	elo
init	$S = \{e \leftarrow \top\}$ $\Delta = \{(l\|e)\}$ $R = \{\text{delete} : \{o\}\}$ $V = \{(e : u), (l : u), (o : u)\}$	$S = \{e \leftarrow \top\}$ $\Delta = \{(l\|e), (l\|o)\}$ $R = \{\text{delete} : \{o\}\}$ $V = \{(e : u), (l : u), (o : u)\}$
$\texttt{remove}_\bot$ (for case $\{o\}$ only)	$S = \{e \leftarrow \top\}$ $\Delta = \{(l\|e)\}$ $V = \{(e : u), (l : u)\}$	$S = \{e \leftarrow \top\}$ $\Delta = \{(l\|e), (l\|\bot)\}$ $V = \{(e : u), (l : u)\}$
addAB	$S = \{e \leftarrow \top, l \leftarrow e \wedge \neg \mathrm{ab}_1, ab_1 \leftarrow \bot\}$ $V = \{(e : u), (l : u), (\mathrm{ab}_1 : u)\}$	$S = \{e \leftarrow \top, l \leftarrow e \wedge \neg \mathrm{ab}_1, l \leftarrow \bot \wedge \neg \mathrm{ab}_2, \mathrm{ab}_1 \leftarrow \bot, \mathrm{ab}_2 \leftarrow \neg e\}$ $V = \{(e : u), (l : u), (\mathrm{ab}_1 : u), (\mathrm{ab}_2 : u)\}$
wc	$S = \{e \leftrightarrow \top, l \leftrightarrow e \wedge \neg \mathrm{ab}_1, ab_1 \leftrightarrow \bot\}$ $V = \{(e : u), (l : u), (\mathrm{ab}_1 : u)\}$	$S = \{e \leftrightarrow \top, l \leftrightarrow (e \wedge \neg \mathrm{ab}_1) \vee (\bot \wedge \neg \mathrm{ab}_2), \mathrm{ab}_1 \leftrightarrow \bot, \mathrm{ab}_2 \leftrightarrow \neg e\}$ $V = \{(e : u), (l : u), (\mathrm{ab}_1 : u), (\mathrm{ab}_2 : u)\}$
Semantic	$S = \{e \leftrightarrow \top, l \leftrightarrow e \wedge \neg \mathrm{ab}_1, ab_1 \leftrightarrow \bot\}$ $V = \{(e : \top), (l : \top), (\mathrm{ab}_1 : \bot)\}$	$S = \{e \leftrightarrow \top, l \leftrightarrow (e \wedge \neg \mathrm{ab}_1) \vee (\bot \wedge \neg \mathrm{ab}_2), \mathrm{ab}_1 \leftrightarrow \bot, \mathrm{ab}_2 \leftrightarrow \neg e\}$ $V = \{(e : \top), (l : \top), (\mathrm{ab}_1 : \bot), (\mathrm{ab}_2 : \bot)\}$
	$f(\pi_{\mathrm{el}})$ = 'She will study late'	$f(\pi_{\mathrm{elo}})$ = 'She will study late'

Comparing SCPs

It is possible to compare SCPs.

Assumption of common origin.

Needleman Wunsch Algorithm.

- Simple string-matching.
- Application in bioinformatics.

$$D_{0,j} = \text{insertion cost} \times j$$

$$D_{i,0} = \text{deletion cost} \times i$$

$$D_{i,j} = \min \begin{pmatrix} D_{i-1,j-1} & + & s(a_i, b_j) \\ D_{i-1,j} & + & s(a_i, -) \\ D_{i,j-1} & + & s(-, b_j) \end{pmatrix}$$

Extending Needleman Wunsch

We can apply the NW algorithm to SCPs.

- Cognitive operations which occur across tasks are *identical*.

Match/Mismatch on a per operation basis.

$$D_{0,0} = 0$$

$$D_{0,j} = D_{0,j-1} + s_{\text{ext}}(a_{j-1}, \texttt{insert})$$

$$D_{i,0} = D_{i-1,0} + s_{\text{ext}}(b_{i-1}, \texttt{insert})$$

$$D_{i,j} = \min \begin{pmatrix} D_{i-1,j-1} & + & s_{\text{ext}}(a_i, b_j) \\ D_{i-1,j} & + & s_{\text{ext}}(a_i, \texttt{insert}) \\ D_{i,j-1} & + & s_{\text{ext}}(\texttt{insert}, b_j) \end{pmatrix}$$

Cognitive Op	Match	Mismatch	Insertion
s_i	3	$\frac{s_{\text{ext}}(x,\texttt{insert})+s_{\text{ext}}(y,\texttt{insert})}{2}$	-1
`addAB`	1	$\frac{s_{\text{ext}}(x,\texttt{insert})+s_{\text{ext}}(y,\texttt{insert})}{2}$	-1
delete	1	$\frac{s_{\text{ext}}(x,\texttt{insert})+s_{\text{ext}}(y,\texttt{insert})}{2}$	-1
`semantic`	1	$\frac{s_{\text{ext}}(x,\texttt{insert})+s_{\text{ext}}(y,\texttt{insert})}{2}$	-1
`th`	5	$\frac{s_{\text{ext}}(x,\texttt{insert})+s_{\text{ext}}(y,\texttt{insert})}{2}$	-5
`wc`	1	$\frac{s_{\text{ext}}(x,\texttt{insert})+s_{\text{ext}}(y,\texttt{insert})}{2}$	-1

Alignment

		s_{sup}	th
	0	-1	-6
s_{sup}	-1	3	-2
addAB	-2	2	0.0
wc	-3	1	-1.0
semantic	-4	0	$\boxed{-2.0}$

Low similarity

		s_{sup}	addAB	delete	wc	semantic
	0	-1	-2	-3	-4	-5
s_{sup}	-1	3	2	1	0	-1
addAB	-2	2	4	3	2	1
wc	-3	1	3	3.0	4	3
semantic	-4	0	2	2.0	3	$\boxed{5}$

Preferred

SCP Strengths and Weaknesses

Strengths	Weaknesses
Simplicity.	No infinite loops.
Adaptability.	Potentially high branching factor.
Can encode other logical frameworks.	Justifiability of chained cognitive operations.
Experimental evidence.	
Searchability.	
Extendability.	
Can find preferred models.	

Applications

Experimental

- Model existing empirical data.
- Predict new results.

Practical

- Extensible set of tools for researchers.
- Inter-model comparisons to find common motifs in human defeasible reasoning.
- Existing implementation can be used and adapted to suit researchers.

Conclusion

SCPs are a powerful tool for modelling human reasoning.

- Able to simulate and extend existing non-monotonic logics.
- Allow plausibility comparisons.
- Allow for problem-solving through search.

My thesis answered the question:

> “Is there a way to systematically represent some (or all) non-monotonic logics in a framework that standardizes cognitive modelling under these logics without sacrificing their expressiveness?”

And the answer seems to be **yes**.

Thank You.

Appendix

		A	T	T	A	C	A
	0	-1	-2	-3	-4	-5	-6
A	-1	1	0	-1	-2	-3	-4
T	-2	0	2	1	0	-1	-2
G	-3	-1	1	1	0	-1	-2
C	-4	-2	0	0	0	1	0
T	-5	-3	-1	1	0	0	0

Optimum Alignment: $\begin{pmatrix} A & T & T & A & C & A \\ A & - & T & G & C & T \end{pmatrix}$