Lesson 8: Functions and Their Graphs

Introduction

The third step of the Quantitative Reasoning Process is *Apply Quantitative Tools*. This lesson will expand your knowledge of functions gained in the previous lesson to see how they can help you solve questions in everyday life when used as tools in the Quantitative Reasoning Process.

Mathematical functions and the Quantitative Reasoning Process help solve many different types of practical problems encountered in everyday life. Here are some examples:

- How long will it take me to pay off this loan?
- How much money should I save each month if I want to buy a car?
- How much money should I save each month in order to retire at age 62?
- How many calories should I eat each day if I want to compete in a triathlon?
- Should I buy a car with better gas mileage?
- Should I accept a job offer that pays a commission rather than a salary?
- Which health insurance plan is best for my situation?
- How much money do I need to start a community charity?

Here is a real-life example that shows how the Quantitative Reasoning Process is used to fight the poaching of elephants in Asia and Africa. Poachers kill elephants for the ivory in their tusks. Poaching deaths have led to declining elephant populations over the past century. In an effort to protect elephants, international agreements banned the trade of raw ivory from Asia and Africa in the 1970s and 1980s. However, 30,000 elephants are still killed every year by poachers who illegally take their tusks. Detecting poached ivory has previously been too expensive and time-consuming for governments to enforce these trade laws.

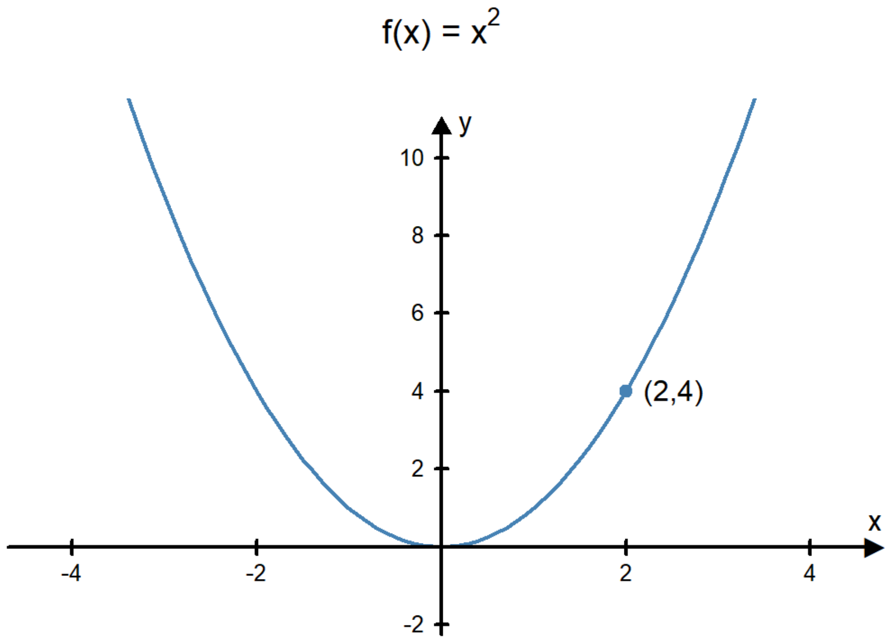
Recently, researchers at the University of Utah developed methods to use carbon-14 dating to determine the age of ivory. The new method is less expensive than previous methods, so it is more accessible to government agencies. The carbon-14 dating method is used to determine if the sample of ivory is old and comes from an elephant killed before the ivory trade ban, or if the ivory is recent and comes from a poached elephant. Governments can now identify illegally sourced ivory and fight poaching of elephants, all because of the wise use of mathematical functions and the Quantitative Reasoning Process (Age and legality of ivory revealed by carbon-14 dating can fight poachers; July 1, 2013; Phys.org↩).

The Coordinate Plane

When using functions, it is helpful to create a graph of the function. Functions are graphed on the **coordinate plane**. The **horizontal axis** (also called the x-axis) represents the inputs of a function and the **vertical axis** (also called the y-axis) represents the outputs of the function.

Every point on the graph represents an input and output of the function. Each point is represented as an **ordered pair**, (x, y), where the first number is the input and the second number is the output. The point $(0, 0)$ is referred to as the **origin**.

When a function is graphed, the graph is a visual representation of all the inputs and corresponding outputs of the function. For example, the following graph represents the function $f(x) = x^2$.



Every point on the curve represents an input and an output of the function. The point $(2, 4)$ is a point on the curve. That is because if you use 2 as an input in the function $f(x) = x^2$ then you get an output of 4. In other words, $f(2) = 4$. Some of the other ordered pairs related to this function are $(-1, 1)$, $(3, 9)$, $(-2, 4)$, and $(0, 0)$. If you use the first number in each pair as an input of the function $f(x) = x^2$, you get the second number as the output.

Three Types of Functions

Functions are divided into *families* of functions. Each family represents a group of functions with similar properties. Three of the most common families of functions are **linear functions**, **quadratic functions**, and **exponential functions**. There are many more families of functions, but these three function families are helpful for solving everyday problems and making informed decisions.

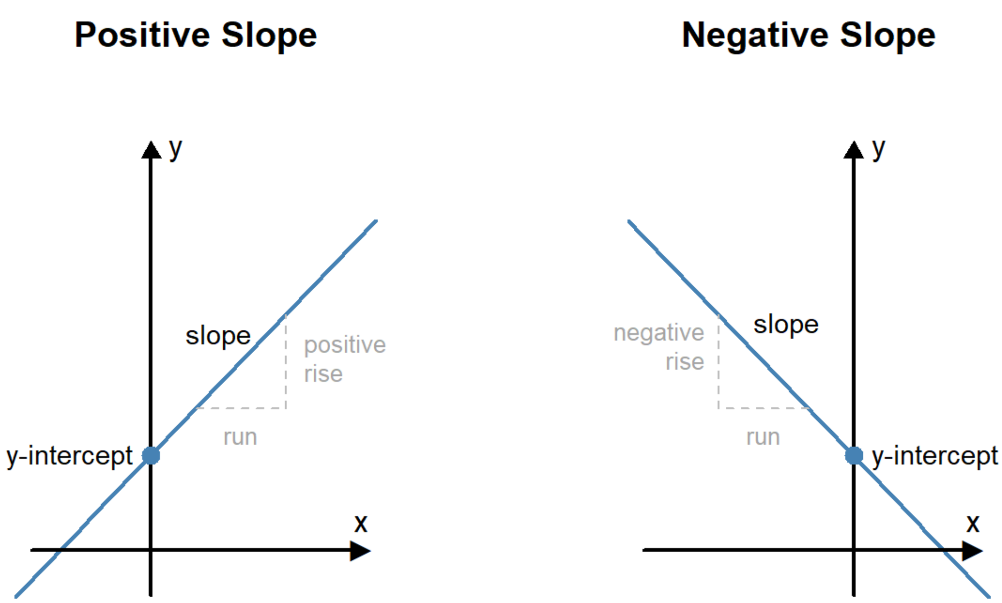
Linear Functions

Linear functions help you describe real-world situations with math symbols.

> A **linear function** is a function of the form $f(x) = mx + b$ where m and b are constants.

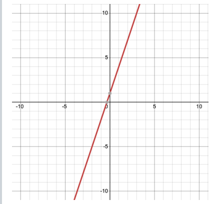
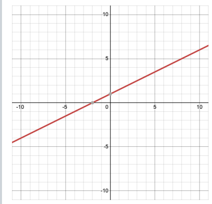
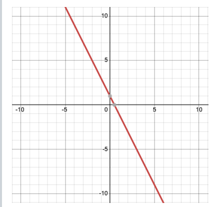
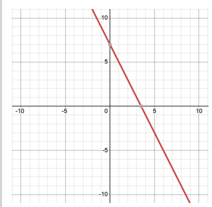
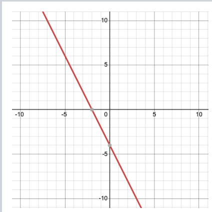
Two features that help graph a linear function are the slope and the *y-intercept*. The equation $f(x) = mx + b$ is said to be in slope-intercept form.

- The y-intercept is the point on the graph where the line crosses the vertical axis (y-axis).
- The slope of a line is a number that tells how steep the line is. If the slope is ⅔, starting from any point on the line, go up 2 units for every 3 units you go to the right. If the slope is −4, you would start at any point on the line and then go down 4 units for each 1 unit you go to the right.



The graph of a linear function, $f(x) = mx + b$ is a straight line that crosses the y-axis at the point $(0, b)$ and has a slope of m.

Here are examles of linear equations with different slopes and y-intercepts:

<u>Linear Function</u>	<u>Slope</u>	<u>Y-Intercept</u>	<u>Graph</u>
$f(x) = 3x + 1$	3	$(0, 1)$	
$f(x) = \frac{1}{2}x + 1$	$\frac{1}{2}$	$(0, 1)$	
$f(x) = -2x + 1$	-2	$(0, 1)$	
$f(x) = -2x + 7$	-2	$(0, 7)$	
$f(x) = -2x - 4$	-2	$(0, -4)$	

Go to desmos.com/calculator and try graphing $f(x) = 3x + 2$ then see what happens to the graph as you change the slope and the y-intercept.

Example 7

Determine which of the following are linear functions:

a. $f(x) = 3x - 2$

b. $3x + 4y = -7$

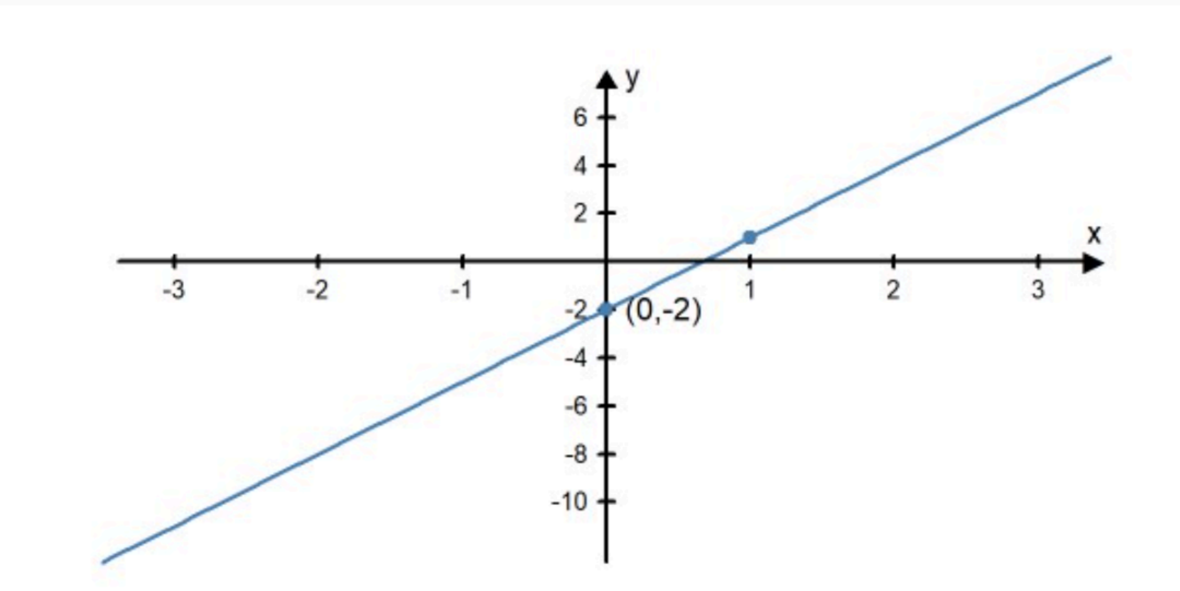
c. $y = 4x^2 - \sqrt{3}$

Solution

Part A

The function $f(x) = 3x - 2$ is a linear function because its form matches the general form of a linear function: $f(x) = mx + b$. In this example, $m = 3$ and $b = -2$. We could graph this function by using the fact that it will cross the y-axis at the point $(0, -2)$ and has a slope of 3.

$$f(x) = 3x - 2$$



Example 7 continued

Part B

The function $3x + 4y = -7$ is also a linear function. It is not currently in the form $f(x) = mx + b$, but we could rewrite it in this form using some algebra rules. First, we solve the equation for y:

$$3x + 4y = -7$$

$$4y = -3x - 7$$

$$y = \frac{-3x - 7}{4}$$

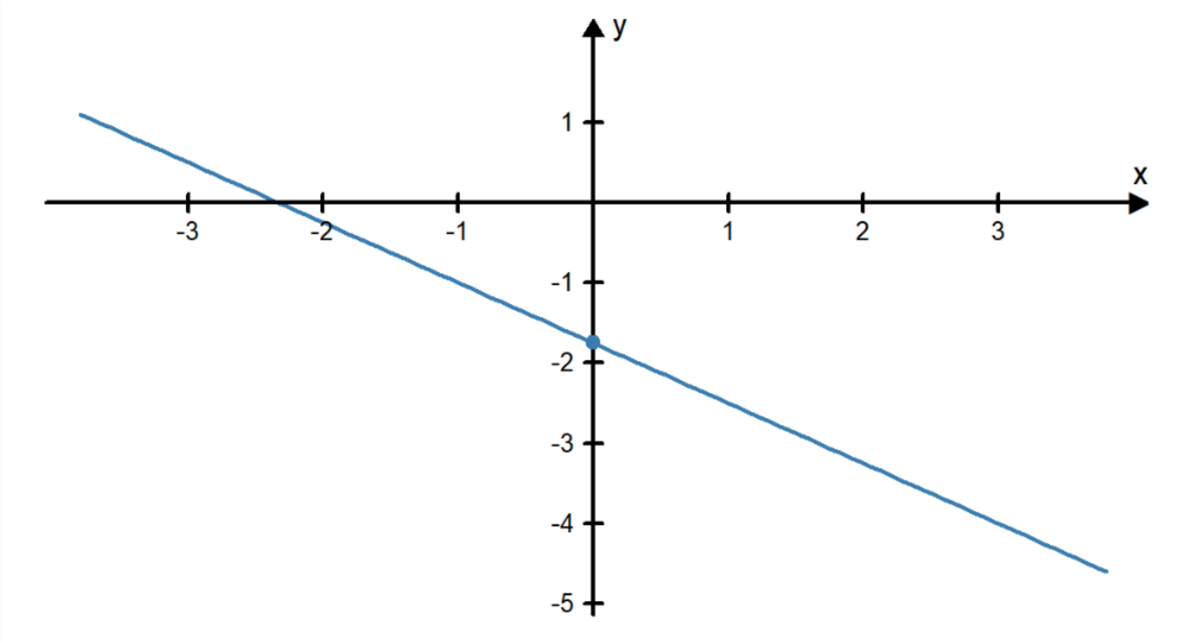
$$y = \frac{-3}{4}x - \frac{7}{4}.$$

Recall from last week's lesson that $y = f(x)$, so we can change the y to an $f(x)$. This makes the equation:

$$f(x) = -\frac{3}{4}x - \frac{7}{4}$$

Now this function is written in the slope-intercept form of a linear function. We could graph it using a y-intercept of $-\frac{7}{4}$ and a slope of $\frac{-3}{4}$.

$$f(x) = \frac{-3}{4}x - \frac{7}{4}$$



Part C

The equation $y = 4x^2 - \sqrt{3}$ is not a linear function. Notice that in a linear function the x variable is not squared. Because equation c has an x^2 term, we know it is not a linear function.

Example 8

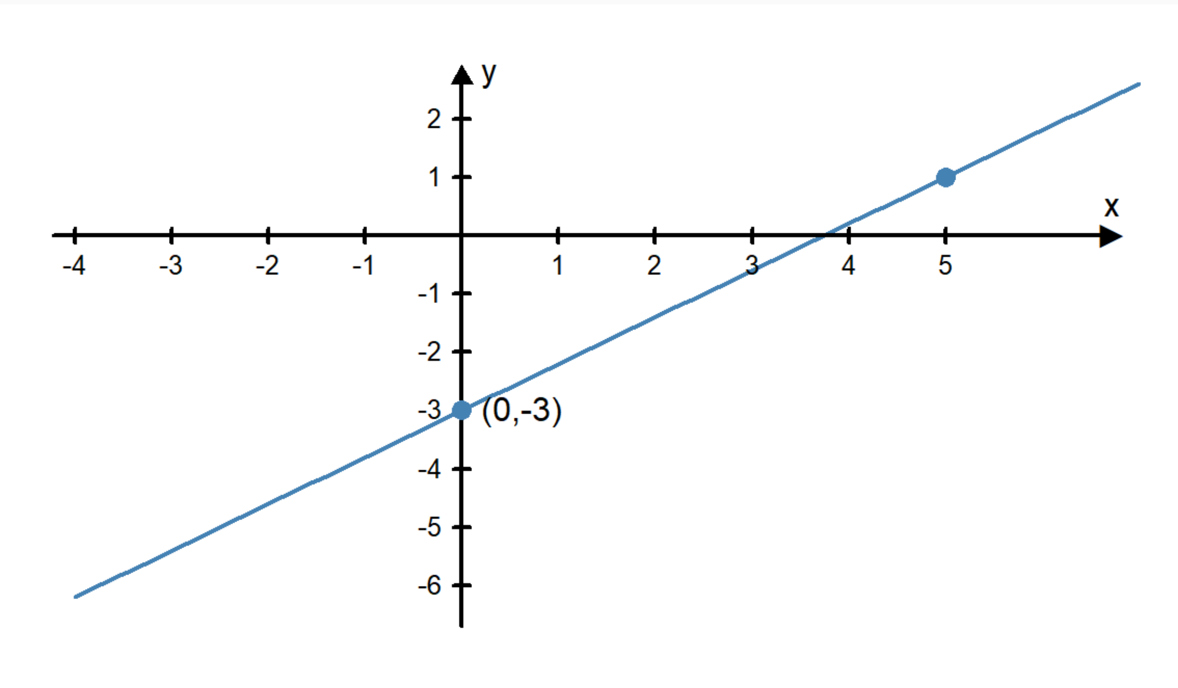
Part A

Graphing Linear Functions $f(x)=\frac{4}{5}x-3$.

Solution

The equation of this linear function is written in slope-intercept form, so we know the y-intercept is $(0,-3)$ and the slope is $\frac{4}{5}$. We can use this information to help us graph the function:

$$f(x)=\frac{4}{5}x-3$$



Part B

Graph the linear function $12x-3y=9$.

Solution

First, we need to rewrite this equation in the slope-intercept form for a linear function. We start by solving for y.

$$12x-3y=9$$

$$-3y=-12x+9$$

$$y = \frac{-12x + 9}{-3}$$

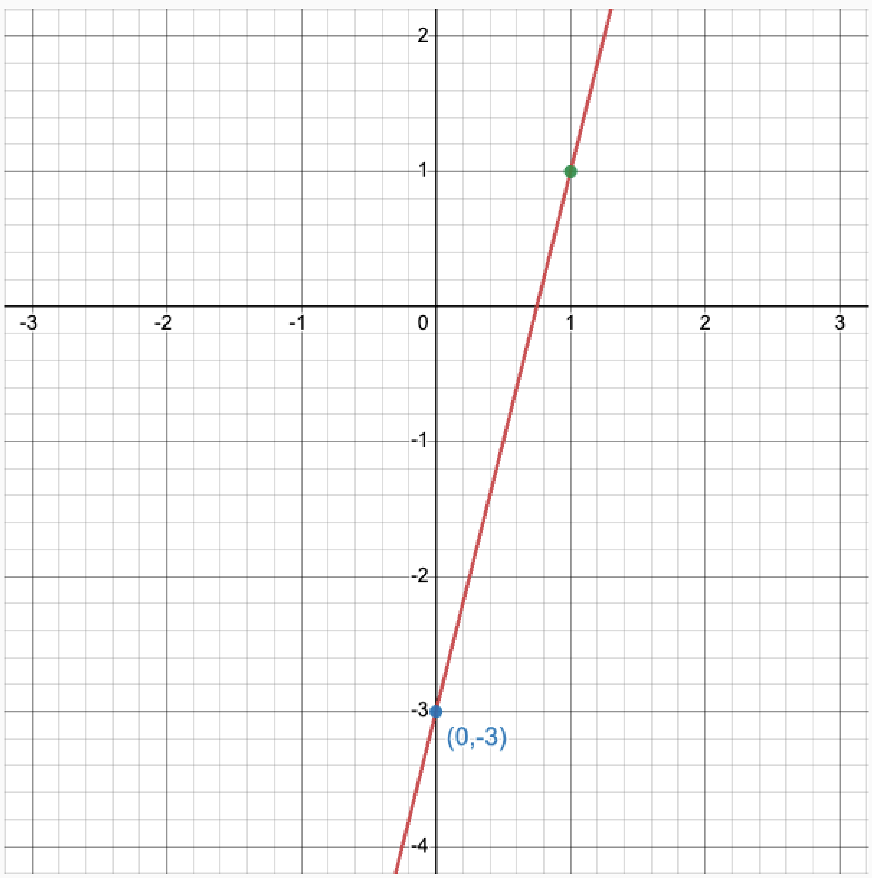
$$y = \frac{-12}{-3}x + \frac{9}{-3}$$

$$y = 4x - 3$$

$$f(x) = 4x - 3$$

We now know that we can graph this linear function by drawing a line that goes through the point (0,-3) with a slope of $m = 4$

$$f(x) = 4x - 3$$



A Linear Application: Calories per Day

A calorie is a unit of measurement that we use to measure energy. Dani is a 22-year-old woman with an average weight and height who does little physical activity. She spends about 1,860 calories of energy each day. This means if Dani eats more than 1,860 calories in a day, her body will store the extra calories and she will gain weight. If she eats less than 1,860 calories in a day, her body will need to use some of the stored calories to make up the difference and she will lose weight.

Dani is trying to improve her health and realizes that by exercising she can increase her caloric consumption. Dani decides to start walking each morning before she goes to work. She has determined that for her weight and the speed (approximately 10 minutes per kilometer), she will burn about six calories for each minute she walks (**Calories Burned Per Minute of Walking**↩).

Her schedule varies every day so she can't always walk for the same amount of time. However, Dani realizes that she can create a linear function that will give her the total amount of calories she can consume that is based on the number of minutes she walks. This will allow her to better monitor her calorie consumption and meet her health goals.

The number of calories she spends on a day she doesn't go for a walk, 1,860 calories, will be the y-intercept of the linear function. The slope of the function will be the number of calories burned per minute, or 6 calories per minute. This gives her the following linear function:

$$f(x) = 6x + 1860$$

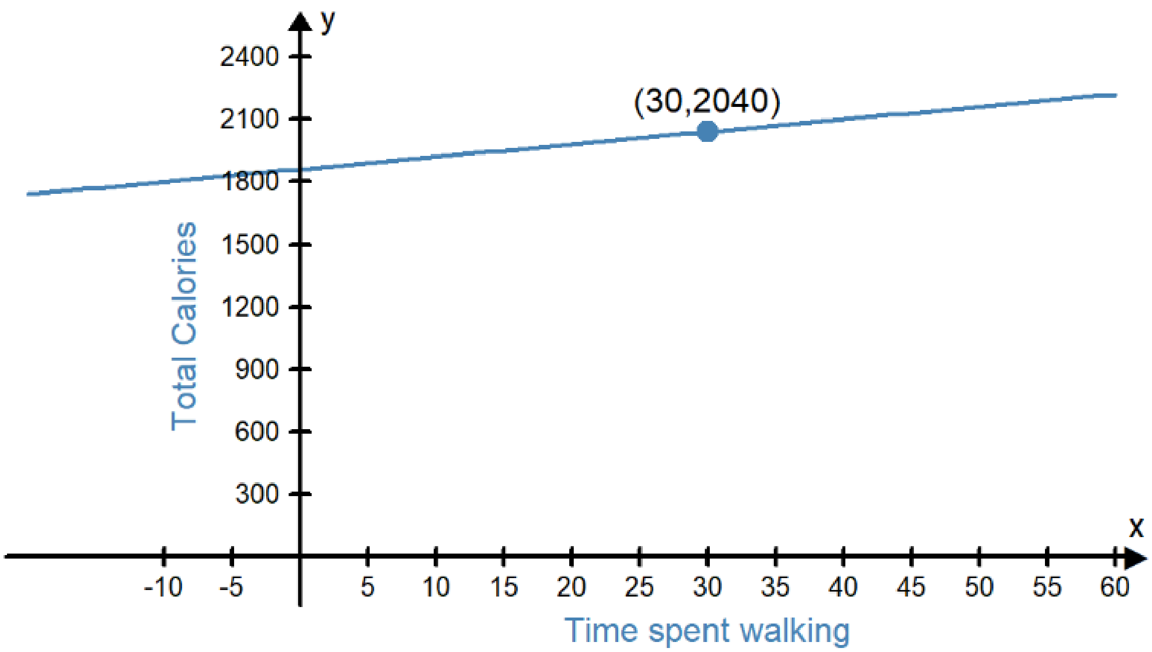
Dani makes the following chart to determine how many total calories she would need each day, depending on how long her walk is:

x = Length of Walk	**f(x)= Total calories**
5 minutes	1890 calories
10 minutes	1920 calories
15 minutes	1950 calories
20 minutes	1980 calories
25 minutes	2010 calories
30 minutes	2040 calories
35 minutes	2070 calories
40 minutes	2100 calories

45 minutes	2130 calories
50 minutes	2160 calories
55 minutes	2190 calories
60 minutes	2220 calories

On a day when Dani walks 30 minutes prior to work, she uses the following graph to help her see how the walk affects the total number of calories she would burn that day. Based on this graph, she knows that her caloric expenditure is 2,040 calories.

$$f(x) = 6x + 1860$$



Quadratic Functions

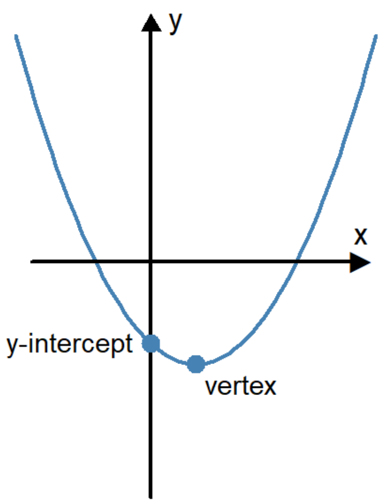
Another commonly used type of graph represents a quadratic function.

A **quadratic function** is a function in the form $f(x) = ax^2 + bx + c$ where a, b, and c are constants.

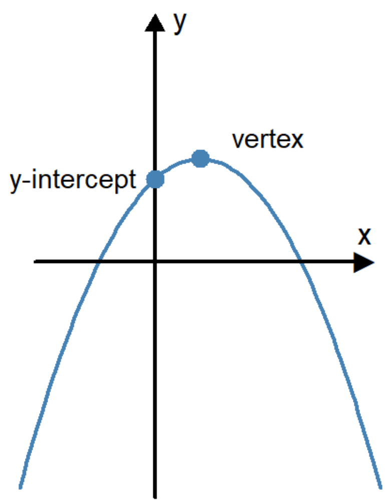
A U-shaped curve called a **parabola**. Three features that help us graph a quadratic function are the **y-intercept**, the **direction** the parabola opens, and the **vertex** of the parabola.

- The *y-intercept* is the point on the graph where the parabola crosses the vertical axis (y-axis).
- The *direction* of the parabola refers to whether the U-shape of the parabola opens up or opens down.
- The *vertex* of the parabola is either the highest or the lowest point on the parabola (depending on the direction of the parabola).

Parabola Opens Up



Parabola Opens Down



Important thing you need to know about quadratic functions, $f(x) = ax^2 + bx + c$:

- The graph of a quadratic function is a **parabola**.
- How to find the **vertex**:
  - The vertex of the parabola has an x-coordinate of $x = \frac{-b}{2a}$.
  - You find the y-coordinate of the vertex by solving for $f(\frac{-b}{2a})$.
- The y-intercept is at $(0, c)$.
- If a is positive, the parabola opens up. If a is negative, the parabola opens down.

Example 9

Determine which if the following equations are quadratic functions.

A. $f(x) = -4x^2 + 2x - 3$

B. $f(x) = \frac{-3}{x^2}$

C. $x^2 - 2y = 3x$

Solution

Both Equations A and C are quadratic functions.

Notice that Equation A is already written in the form $f(x) = ax^2 + bx + c$. In this case, $a = -4$, $b = 2$, and $c = -3$.

In Equation C, the function is not written in standard form, but it can be written as $f(x) = \frac{1}{2}x^2 - \frac{3}{2}x + 0$.

Equation B is not a quadratic function because the x^2 term is in the denominator of a fraction, so it cannot be written in the form $f(x) = ax^2 + bx + c$.

Example 10

Sketch the graph of $f(x) = 2x^2 - 4x + 1$

Solution

First, we find the vertex of the parabola. We know it is at the point where $x = \frac{-b}{2a}$. So in this function, the vertex will be at

$$x = \frac{-b}{2a} = \frac{-(-4)}{2 \times 2} = \frac{4}{4} = 1$$

Since we know the x value of the vertex will be at $x = 1$, we can now solve for the y value by replacing x with 1 in the function. This is the same thing as calculating $f(\frac{-b}{2a})$.

$$f(1) = 2(1)^2 - 4(1) + 1$$

$$f(1) = 2 - 4 + 1$$

$$f(1) = -1$$

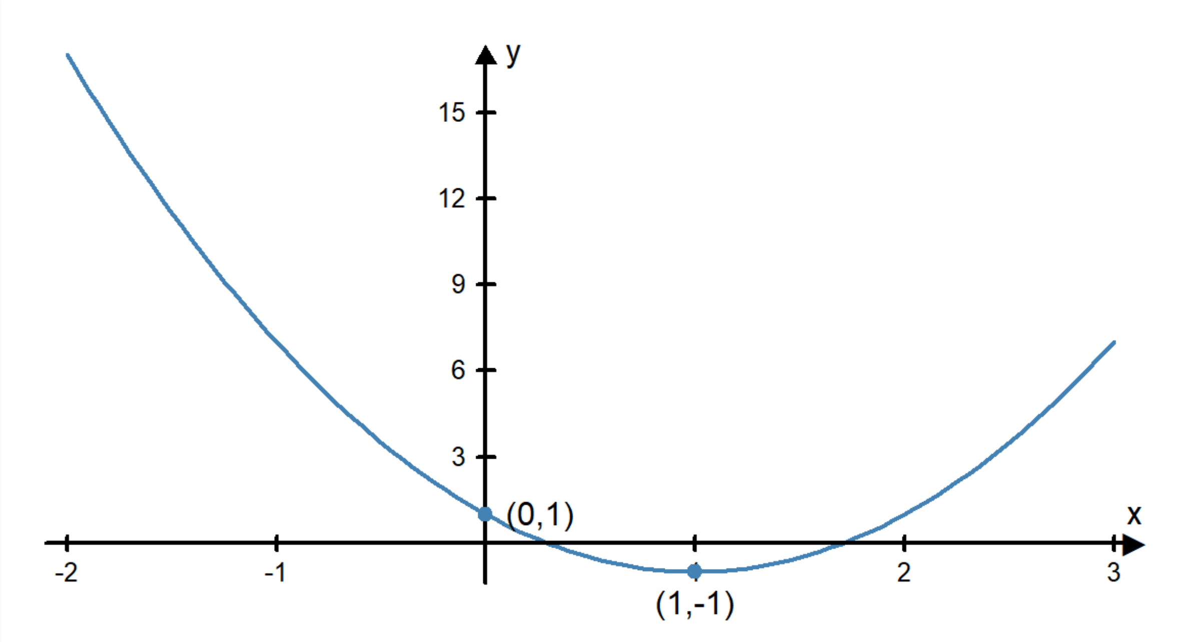
Therefore, the vertex of this parabola is $(1, -1)$.

Next, we find where the parabola crosses the y-axis. We know this happens at the point $(0, c)$. In this example, the parabola will cross the y-axis at the point $(0, 1)$ since $c = 1$.

Finally, we determine whether the parabola opens up or down. In this parabola $a = 2$ because $a > 0$ we know the parabola opens up.

Putting this information together gives you the following graph:

$$f(x) = 2x^2 - 4x + 1$$


y
15
12
9
6
3
(0,1)
x
-2
-1
2
3
(1,-1)

Practice Problem 2

Find the vertex, the y-intercept, and the direction of the following quadratic function.

$$f(x) = 3x^2 - 18x + 16$$

x-coordinate of the vertex:

y-coordinate of the vertex:

y-intercept:

What direction is the parabola?

- [ ] Parabola opens up
- [ ] Parabola opens down

Check your answer.

x-coordinate of vertex: 3

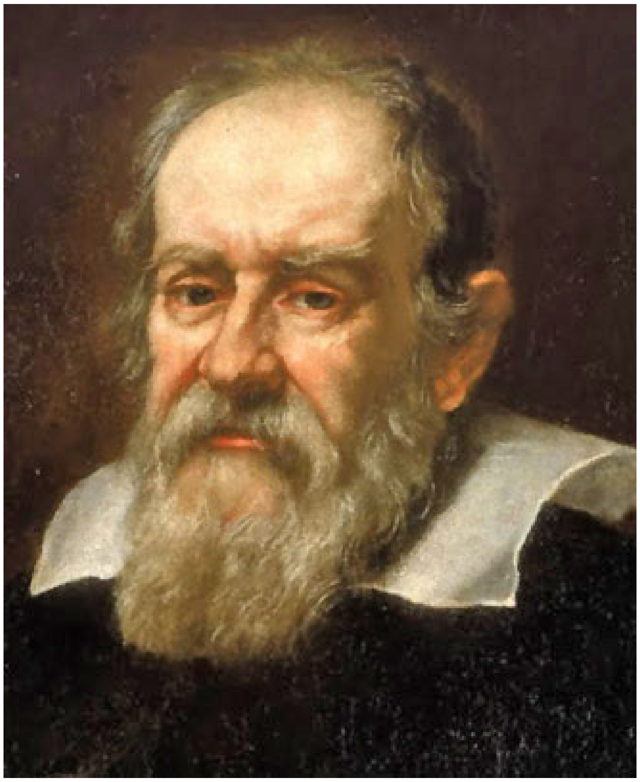
y-coordinate of vertex: -11

y-intercept: (0, 16)

Parabola opens up.

A Quadratic Application: Exploding Fireworks

One famous application of quadratic functions was discovered by the Italian mathematician and physicist Galileo Galilei.



In the 16th century, Galileo developed the following equation to show the path of a free-falling object:

$$f(x) = -4.9x^2 + v_0x + h_0$$

Where x represents time, v_0 represents the initial velocity of the object, and h_0 represents the initial height of the object (in meters). The input of the function represents how long the object has been in the air (time) and the output of the function is the height of the object above the ground.

Andre is designing a fireworks show. He knows that he will fire a rocket from an initial height of 0.61 meters above the ground and it will have an initial velocity of approximately 62 meters/sec.

Andre needs to know how long to make the fuse of the rocket. For safety reasons, he needs to make sure the firework will be more than 152 meters above the ground when it explodes. He wants to have the firework explode when it reaches its highest point.

Using Galileo's free-fall equation, Andre finds this equation for his rocket:

$$f(x) = -4.9x^2 + 62x + 0.61$$

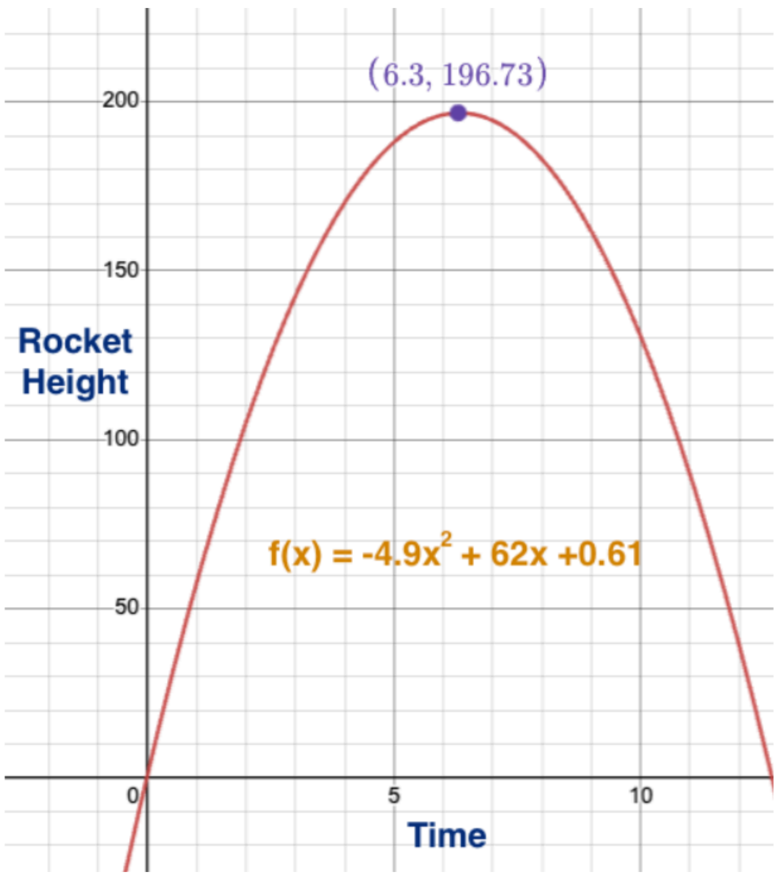
Because of the negative in the equation, he knows the parabola opens down, so the highest point will be at the vertex of the parabola. He finds the vertex of the parabola:

$$x = \frac{-b}{2a} = \frac{-62}{2 \cdot -4.9} = 6.33$$

$$f(6.33) = -4.9(6.33)^2 + 62(6.33) + 0.61 = 196.73$$

Since the vertex of the parabola is 197 meters, Andre knows the rocket will be 197 meters above the ground 6.33 seconds after he fires the rocket. So, he needs to use a fuse that will burn for 6.33 seconds.

Andre's graph verifies his solution:



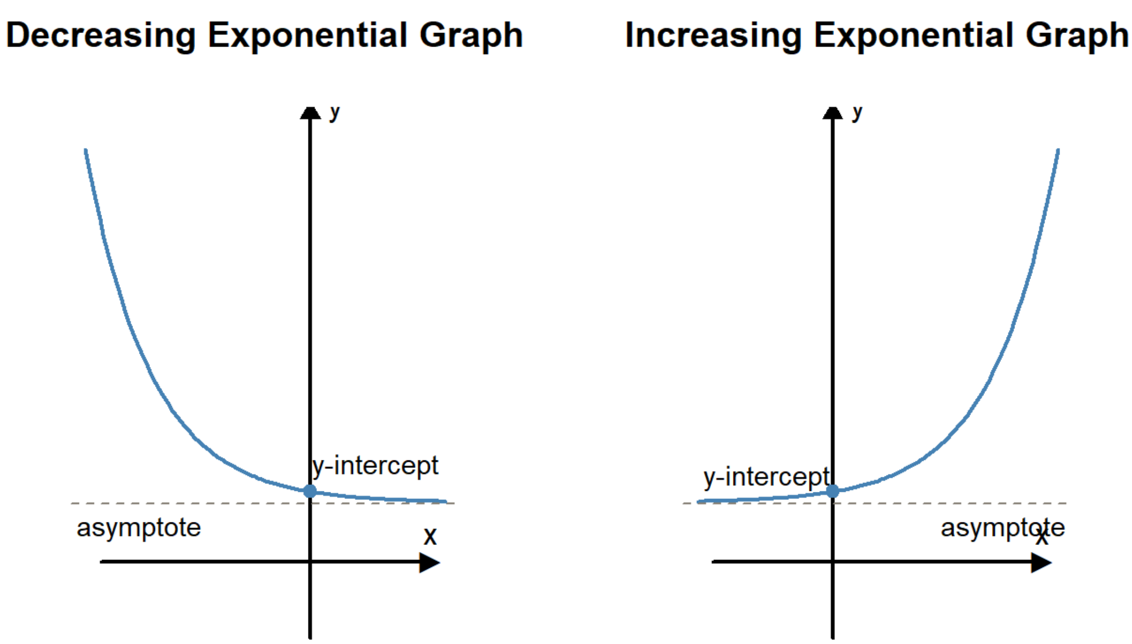
Exponential Functions

Exponential functions are used in many real-world applications such as computing compound interest in bank accounts and modeling the growth of populations.

> **An Exponential Function** is a function in the form $f(x) = a \times b^x + c$ where a, b, and c are constants and $b > 0$ and $b \neq 1$.

There are three features that help graph exponential functions:

- The y-intercept is the point where the graph crosses the vertical axis (y-axis).
- The horizontal asymptote is a line that the function gets closer and closer to, but never crosses.
- The graph can be either increasing or decreasing. An increasing graph goes up as you move along the graph from left to right. A decreasing graph goes down as you move along the graph from left to right.



Important thing you need to know about exponential functions, $f(x) = a \times b^x + c$:

- The graph of an exponential function has a y-intercept at $(0, a + c)$.
- Plotting another point on the graph helps determine the steepness of the graph.
- The graph has a horizontal asymptote at $y = c$.
- When $0 < b < 1$ the graph is decreasing when $b > 1$ it is increasing.

Example 11

Classify each of the following functions as being linear, quadratic, or exponential.

a. $f(x) = 3 \cdot 2^x + 4$

b. $f(x) = 4 - 3x$

c. $f(x) = 3x^2 - 4 + 2$

Solution

a. This function is exponential. We know because the variable x is in the exponent.

b. This function is linear. We know because it is in the form $f(x) = mx + b$ where $m = -3$ and $b = 4$.

c. This function is quadratic. We know because it is in the form $f(x) = ax^2 + bx + c$ where $a = 3$, $b = 4$, and $c = 2$.

Example 12

Part 1

Graph the function: $f(x) = 3 \cdot 5^x + 2$.

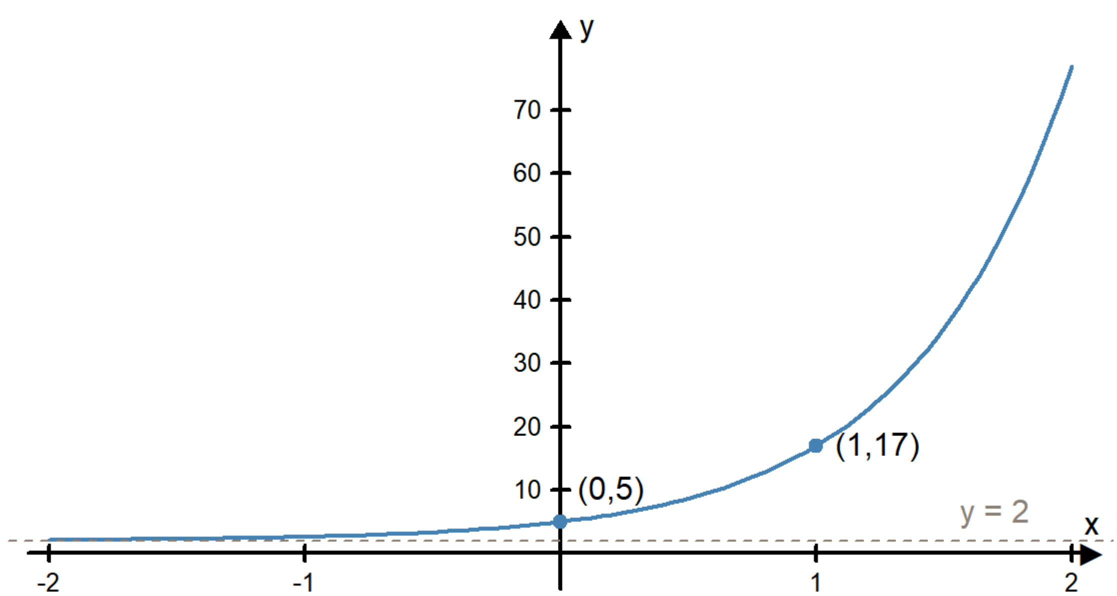
Solution

This is an exponential function of the form $f(x) = a \cdot b^x + c$ where $a = 3$, $b = 5$, and $c = 2$.

- The y= intercept is at $(0, a + c) = (0, 3 + 2) = (0, 5)$.
- When $x = 1$, $f(1) = 3 \cdot 5 + 2 = 17$. The graph goes through the point $(1, 17)$.
- The horizontal asymptote is at $y = 2$.
- Since $b = 5$ is greater than 1, we know the function is increasing.

This information tells us the graph looks like this:

$$f(x) = 3 \cdot 5^x + 2$$



Part 2

Graph the function: $f(x) = 7(\frac{1}{2})^x + 1$.

Solution

This is an exponential function of the form $f(x) = a(b^z) + c$ where $a = 7$, $b = 1/2$, and $c = 1$.

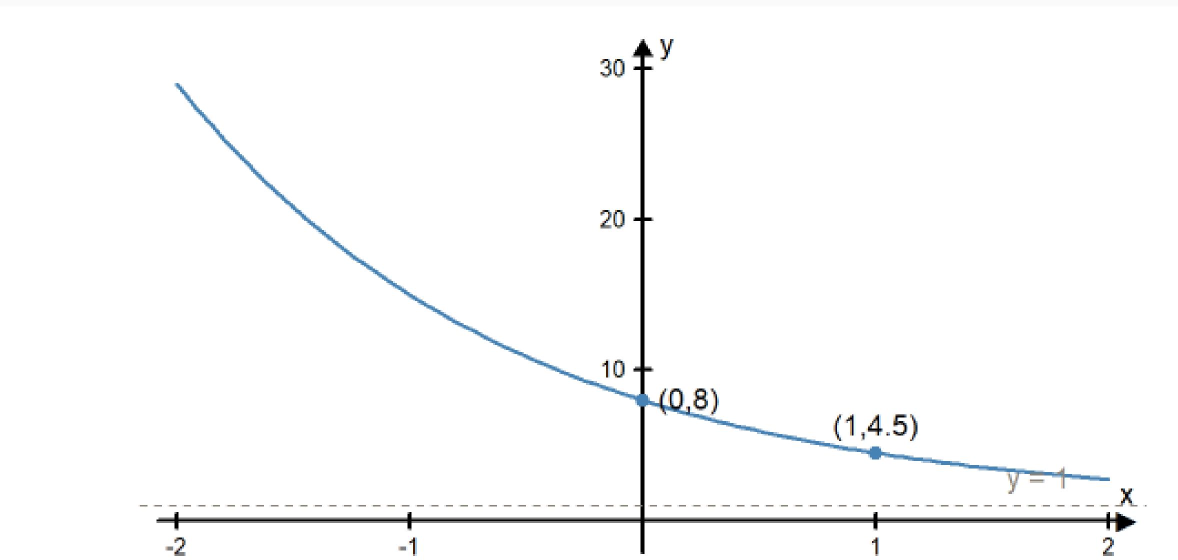
- The y-intercept is at $(0, a + c) = (0, 7 + 1) = (0, 8)$.
- When $x = 1$, $x = 1$, $f(1) = 7(\frac{1}{2}) + 1 = \frac{9}{2} = 4.5$. The graph goes through the point $(1, 4.5)$.

$$f(1) = 7(\frac{1}{2}) + 1 = \frac{9}{2} = 4.5. \text{ The graph goes through the point}(1, 4.5).$$

- The horizontal asymptote is at $y = 1$.
- Since $b = 1/2$ is between 0 and 1, we know the function is decreasing.

This information tells us the graph looks like this:

$$f(x) = 7 \cdot \frac{1}{2}^x + 1$$

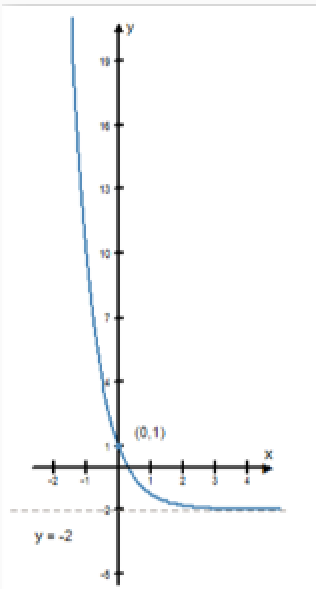


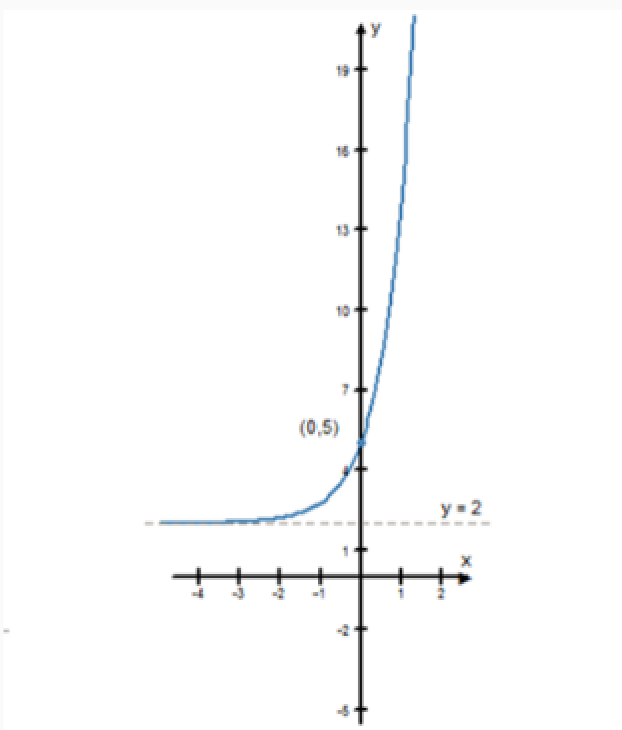
Graphing Practice

Practice Problem 3

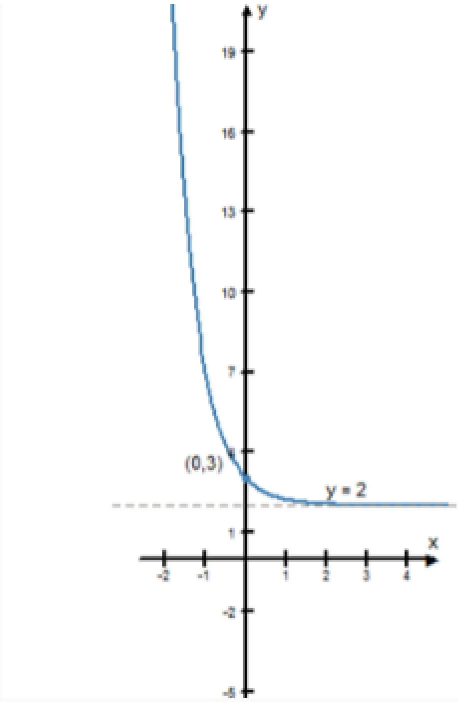
Select the correct graph of the exponential function:

$$f(x) = 3(\frac{1}{4})^x - 2$$

a. 

b. 

c.



Check your answer.

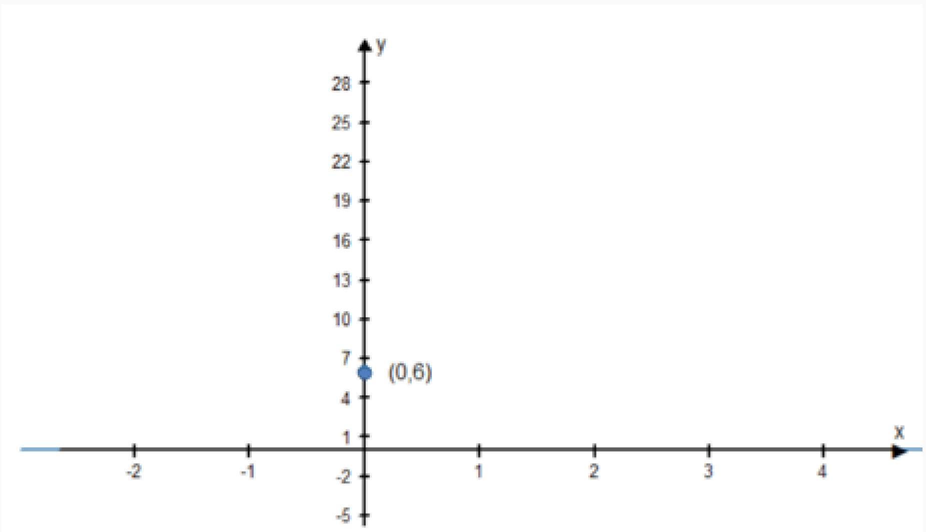
The correct answer is a.

Practice Problem 4

Follow the given steps to graph the exponential function: $f(x) = 2 \cdot 3^x + 4$.

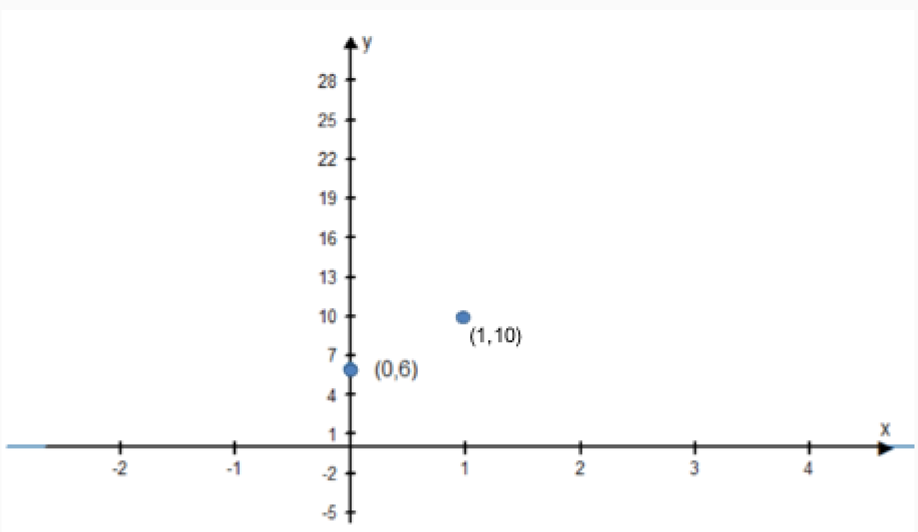
Step 1: Find the y-intercept.

Check your answer.



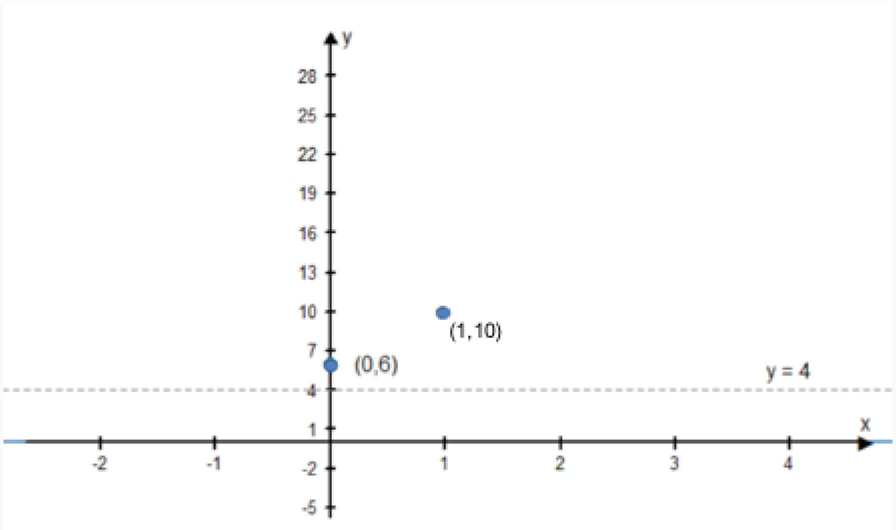
Step 2: Plot another point. Find $f(1)$ on the graph.

Check your answer.



Step 3: Find the horizontal asymptote.

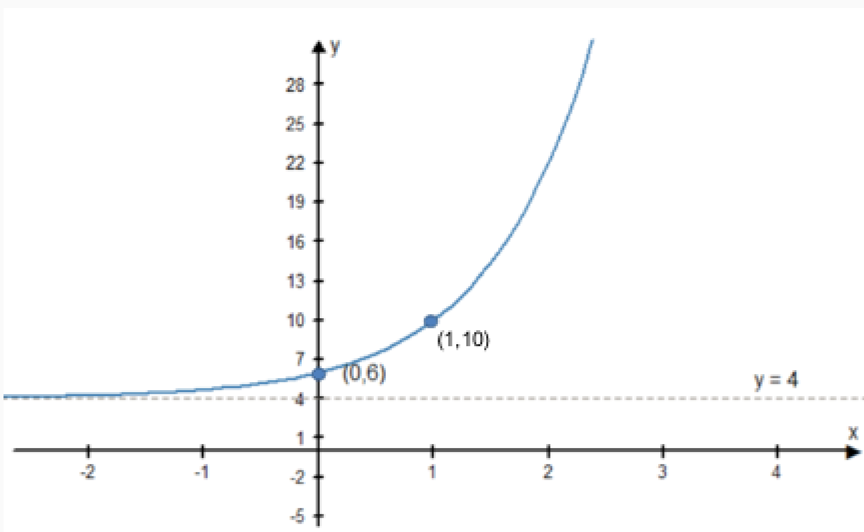
Check your answer.



Step 4: Is the function increasing or decreasing? Use that information to draw the graph.

Check your answer.

Increasing.



An Exponential Application: Carbon-14 Dating

In the introduction, we learned that carbon-14 dating is used to determine the age of organic materials. This is a good example of an application involving exponential functions.

Carbon-14 dating works by comparing the ratio of carbon-14 to carbon-12 in the organic material. Carbon-12 is a stable form of carbon that does not decay over time. Carbon-14, on the other hand, is radioactive and is therefore an unstable form of carbon. Living things have a consistent ratio of carbon-12 to carbon-14. However, when an organism dies, it stops replenishing carbon-14 into its chemical makeup. Without new carbon-14 sources being introduced into the organism, the carbon-14 will decay over time into elements other than carbon. This will cause the carbon-12 to carbon-14 ratio to change with time. By looking at the current carbon-12 to carbon-14 ratio of the dead organism and comparing it to a carbon-12 to carbon-14 ratio in a living organism, scientists can use this difference to calculate how long an organism has been dead.

This is what is used to determine the age of ivory. While the tusk is still intact on a living elephant, the ratio of carbon-12 and carbon-14 will stay consistent. But when the tusk is removed from the elephant, the amount of carbon-14 begins to change.

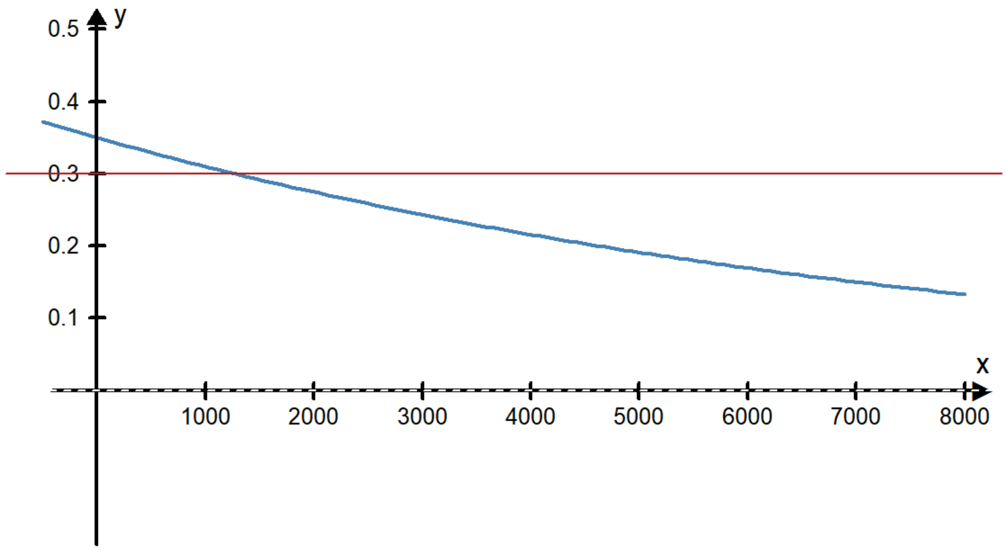
The half-life of carbon-14 is about 5,730 years. This means that half of the carbon-14 decays away every 5,730 years. Knowing this, scientists can create the following exponential equation to describe the amount of carbon-14 that is present over time.

$$f(x) = A(\frac{1}{2})^{x/5730}$$

In this equation, A represents the amount of carbon-14 initially present in the living organism. The input for this function is time (in years) since the organism died and the output is the current amount of carbon-14 present in the organic material.

If a particular ivory tusk from an elephant had 0.35 grams of carbon-14 present when the elephant was living, the equation will become:

$$f(x) = 0.35(\frac{1}{2})^{x/5730}$$



If researchers test a piece of ivory and find there are currently 0.30 grams of carbon-14 present, how old would this piece of ivory be? Looking at the graph, the red line shows that the y-value will be 0.3 after a little more than 1,000 years. So this piece of ivory is over 1,000 years old.

As discussed at the beginning of this lesson, new techniques for carbon-14 dating are being developed by researchers at the University of Utah. Because carbon-14 has such a long half-life, it isn't very helpful at determining the age of organic materials that are less than about 50,000 years old. New methods developed by the University of Utah researchers are particularly helpful in fighting poaching because they found a way to make carbon-14 dating work for specimens that are only 30–50 years old.

Check Your Understanding

Practice Problem 5

What type of function is this?

$$f(x) = -5x + 2$$

- ☐ Linear function
- ☐ Exponential function
- ☐ Quadratic function

Social Media Marketing Example

Viktor is running a small business and wants to improve his marketing. He decides to use the Quantitative Reasoning Process to make some decisions about the type of marketing he wants to do.

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1. Understand the Problem

Viktor's business is an online tractor sales business. Viktor does not have a store and runs the business out of his home. Most of his business is done through his website and over his phone. He believes his sales will increase if more people see his website. He decides to use Facebook, Twitter, and Instagram to market his business. He wants these social media tools to drive traffic to his website.

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2. Identify Variables & Assumptions

Viktor finds a marketing company willing to run a social media marketing campaign for his business for a fixed monthly fee. The company offers three levels of service with different costs. As he thinks about his situation, Viktor identifies the following variables:

- The number of customers visiting his website per week
- The number of customers who purchase a tractor per week
- The percentage of paying customers who visited his website
- His average monthly sales
- The choice of a marketing plan from the marketing company (Plan A, Plan B, or Plan C)
- The cost of hiring a marketing company

He makes the following assumptions:

- The claims of the social media marketing company are accurate. They claim the following:
  - Plan A (highest cost) will increase the traffic to his website by 5% per month.
  - Plan B (medium cost) will increase the traffic to his website by 3.5% per month.
  - Plan C (lowest cost) will increase the traffic to his website by 2% per month.
- If more customers visit his website, he will sell more tractors.

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3. Apply Quantitative Tools

According to the analytics for Viktor's website, he currently gets around 75 unique visitors per month. He hopes to double the visits to 150 unique visitors per month.

Because the marketing plans claim to increase traffic to his website by a fixed percentage every month, Viktor can use an exponential function. He creates the following three exponential functions:

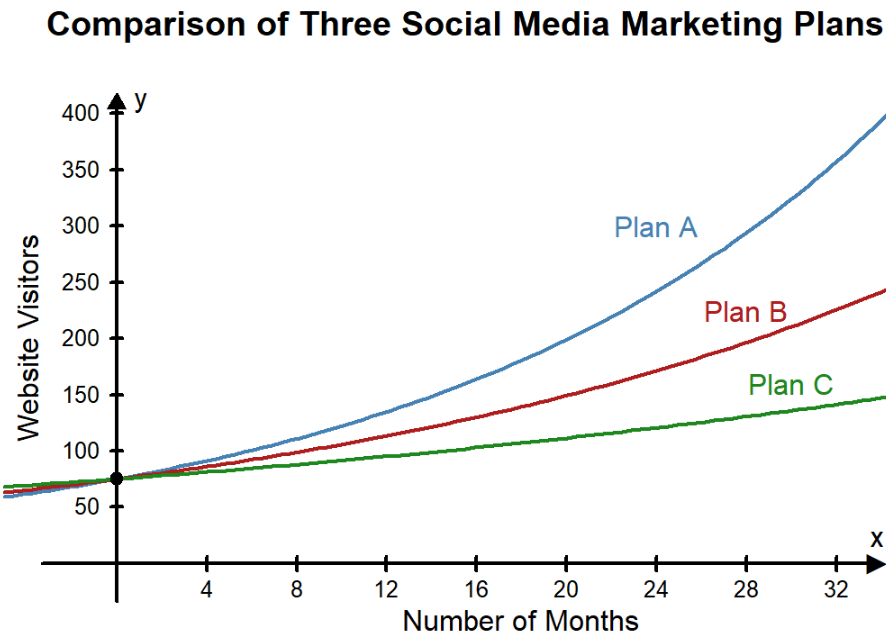
Plan A: $f(x) = 75(1.05)^x$

Plan B: $f(x) = 75(1.035)^x$

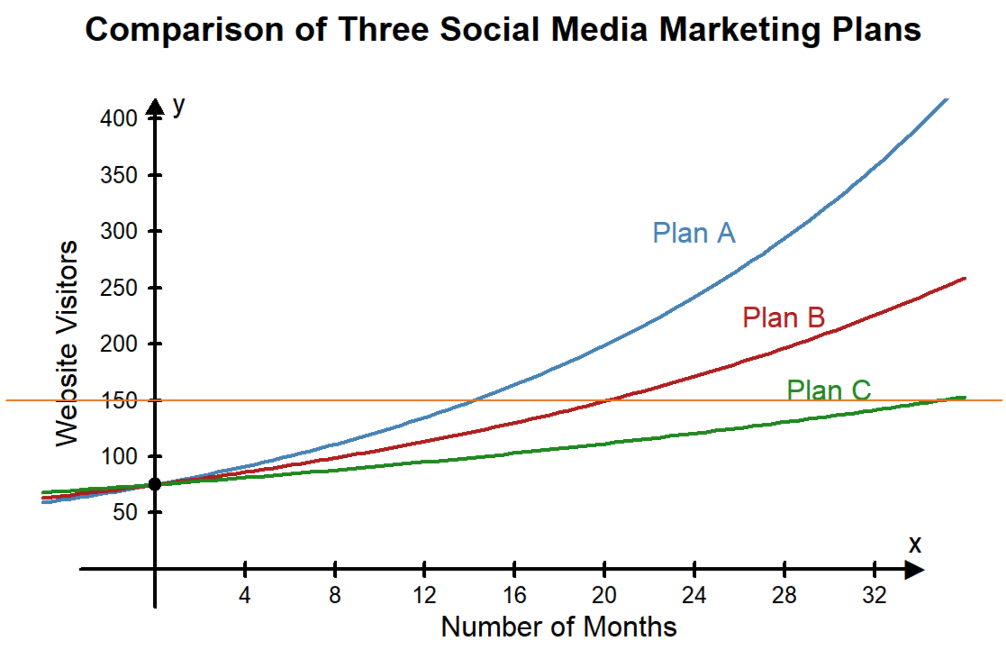
Plan C: $f(x) = 75(1.02)^x$

In these equations, the input x represents the months the social media marketing campaign has been running. The output $f(x)$ represents the number of visitors to the website that month.

To compare the three plans, Viktor creates the following graph with the three exponential functions on the same chart.



Because he is trying to get to 150 website visitors per month, Viktor draws a line on the graph at $y = 150$ to see how long it will take to reach 150 website visitors with each plan.



Looking at the graph, he finds where the line intersects each curve. The x-value at the intersection point will represent the number of months required double the traffic to his website:

Plan	Approximate Number of Months
Plan A	about 14 months
Plan B	about 20 months
Plan C	about 35 months

Plan A is the fastest, Plan B takes about 6 months longer than Plan A, and Plan C takes about 15 months longer than Plan B.

4. Make an Informed Decision

Viktor compares each plan's cost to the time required to double the traffic to his website (see chart above). Plan A is much more expensive and faster than Plan B, but Plan B only takes a little longer to double the traffic to his website. While Plan C is the least costly, it takes much more time to get the result he wants.

Viktor selects Plan B.

5. Evaluate Your Reasoning

After using the marketing company for six months, Viktor reevaluates his assumptions. He reviews his website traffic over the six months to see how things are going. According to the claims of the social media marketing company, he should have seen approximately a 3.5% increase in traffic to his website each month. He decides to compare the actual number of visits to his website to the predicted number of visits from the exponential function he used to make his decision. In the table below, the second column shows the anticipated website visitors if the marketing plan worked as expected. The third column shows the actual number of visitors to the website.

Month	Predicted Visitors	Actual Visitors
1	$f(1) = 75(1.035)^1 = 77.63$	80
2	$f(1) = 75(1.035)^2 = 80.34$	81
3	$f(1) = 75(1.035)^3 = 83.15$	85
4	$f(1) = 75(1.035)^4 = 86.06$	79
5	$f(1) = 75(1.035)^5 = 89.07$	87
6	$f(1) = 75(1.035)^6 = 92.19$	93

He noticed a dip in Month 4, with fewer visitors than expected. He was not concerned because Month 4 was December, which is usually the slowest month for his business, and he found the number of visitors back up to the expected amount by Month 6. He determines that social media marketing is working as expected.

Now that he knows more customers are visiting his website, he decides to examine the accuracy of his assumption that more website visitors will increase tractor sales. He compares his current deals to sales for the same months the previous year and finds on average, his sales have increased by 4% since hiring the social media marketing company.

Viktor is pleased with the results of his decision to use the Quantitative Reasoning Process and plans to keep using the company.

Lesson Checklist

By the end of this lesson, you should be able to do the following:

- Correctly apply the order of operations
- Correctly use the rules of exponents
- Use function notation to evaluate a function (linear, quadratic, exponential) at a point
- Distinguish among linear, exponential, and quadratic graphs
- Identify graphs of linear, quadratic, and exponential functions from their equations
- Interpret functional values or graphs in the context of a given situation