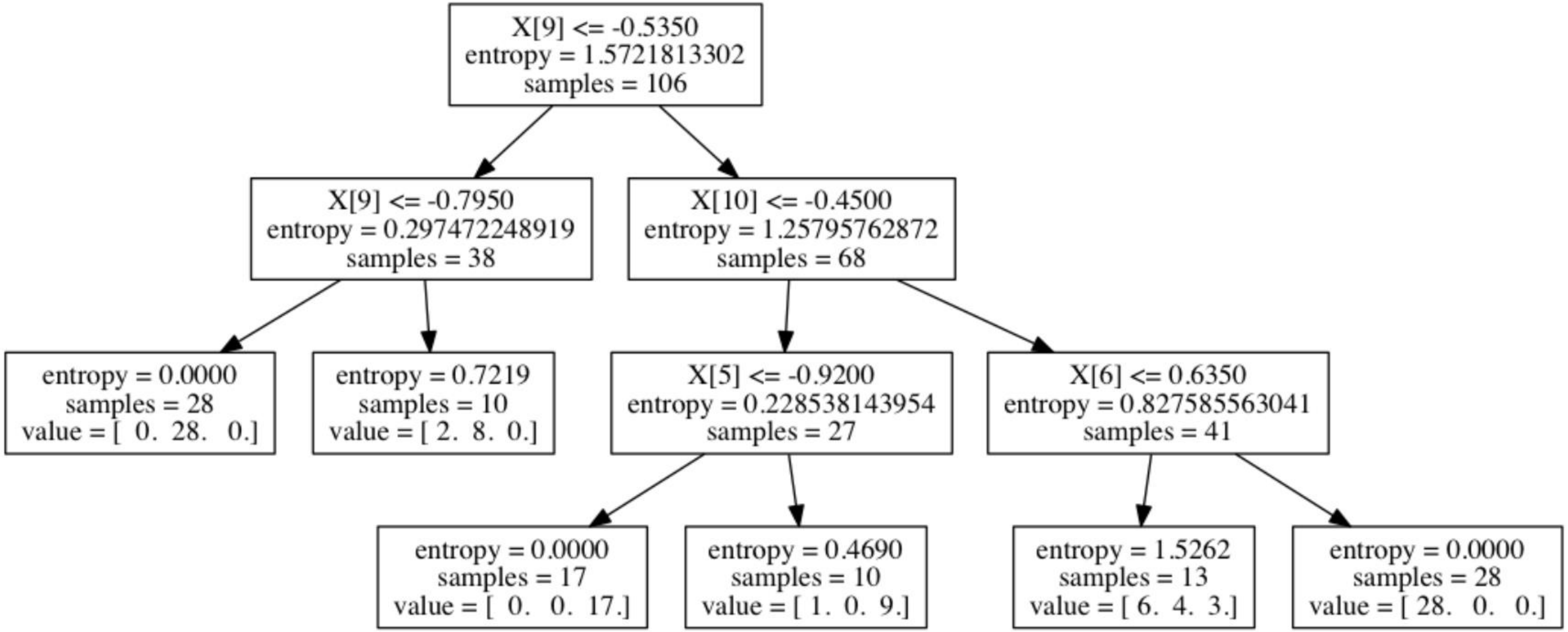
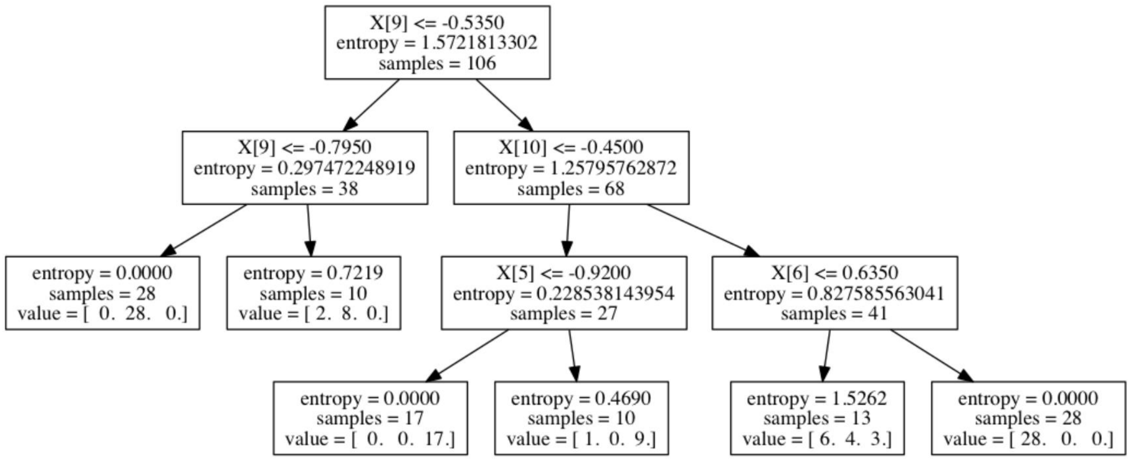
Decision trees

$$G(D, x) = \text{Entropy}(D) - \sum_{v \in x} \frac{|D_v|}{|D|} \text{Entropy}(D_v)$$



Decision trees



advantages:

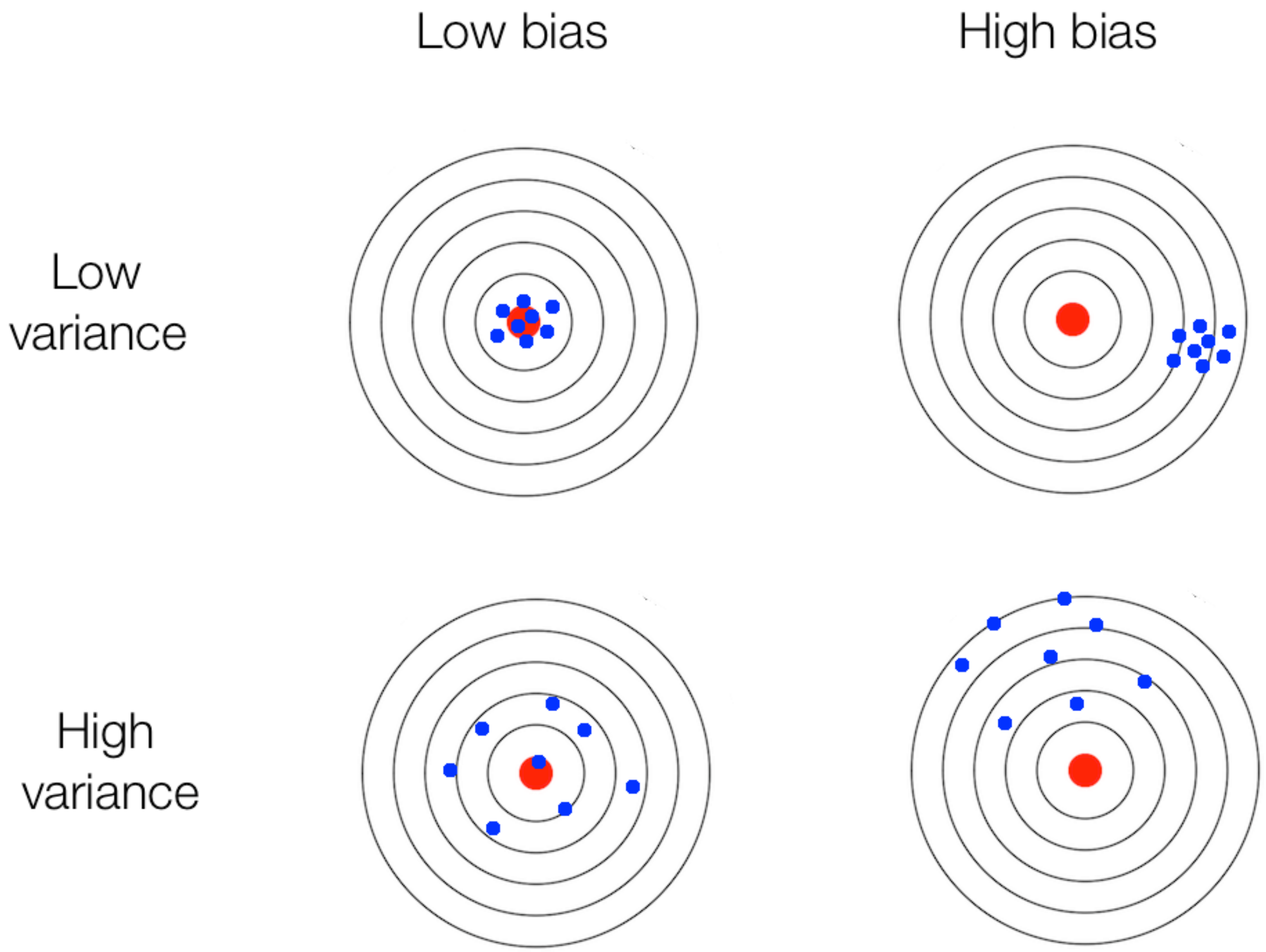
- ease of interpretation
- handles continuous and discrete features
- invariant to monotone transformation of features
- variable selection automated
- **low bias** (deep trees)

disadvantages:

- **high variance**
- overfitting

Bias-variance tradeoff

- Error due to bias
- Error due to variance
- Irreducible error



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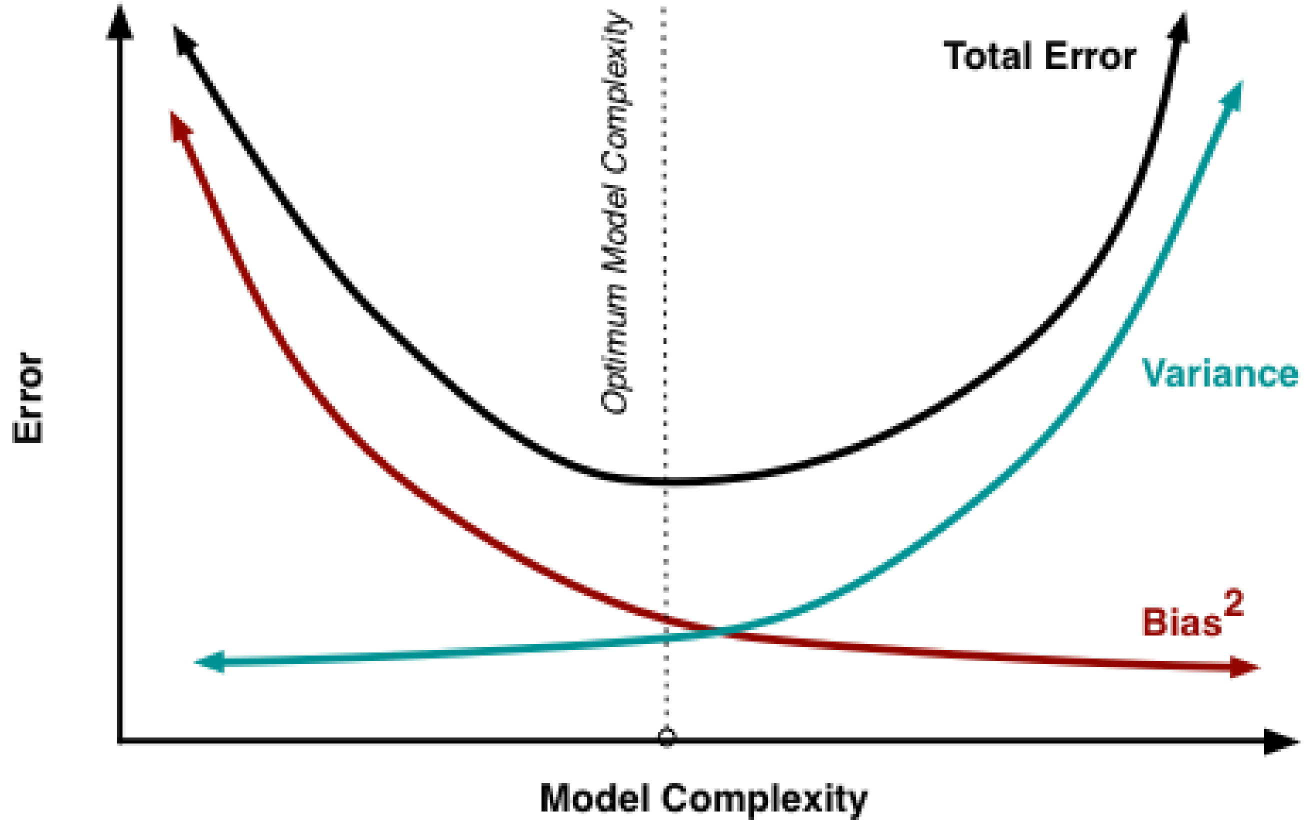
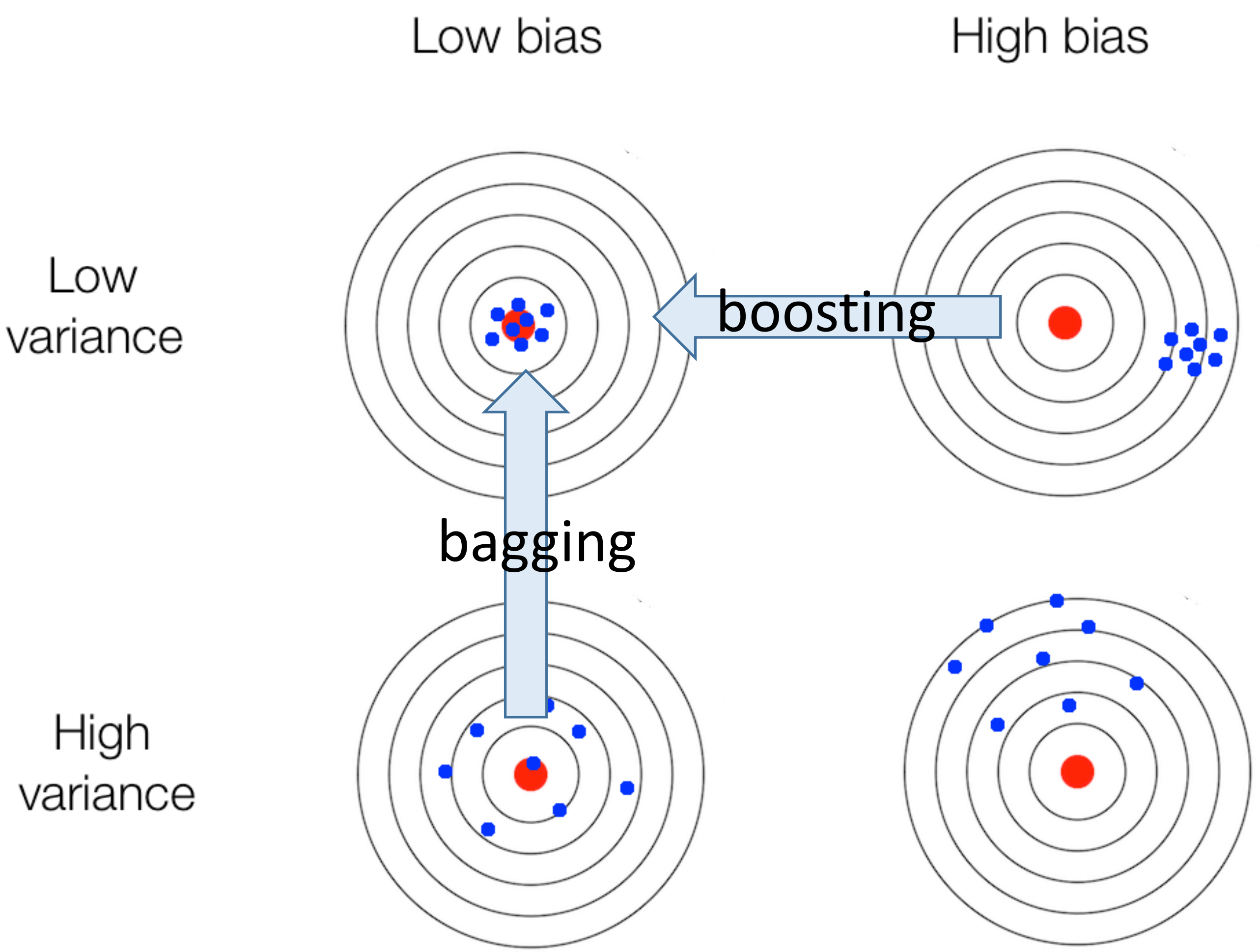


bagging/boosting



Bagging

- Simulate the notion of different train set samples:

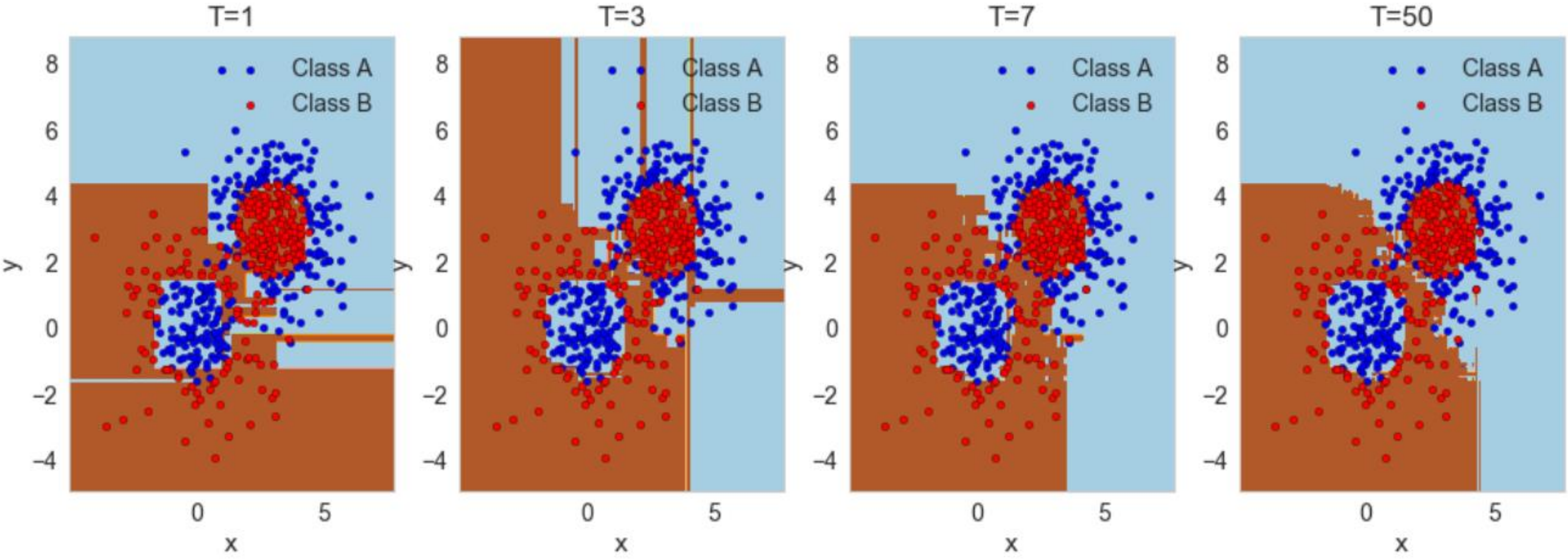
1. sample data points from train set to create new train set
2. fit low bias high variance model on new train set
3. repeat steps (1) and (2) *T* times
4. average the predictions of the *T* models

$$\hat{f}(x,\theta) = \frac{1}{T}\sum_{t=1}^{T} f_t(x,\theta)$$

Bagging: Random Forests

- Train set contains *n* data points with *m* features.
- Construct *T* low bias high variance decision trees by following these steps:
  1. Sample ***n*** data points at random **with replacement** from the train set.
  2. At each node, select $h<<m$ features at random and compute the best split using only these *h* features.
  3. Each tree is grown to the largest extent possible. There is no pruning or early stopping.
- Step 3 ensures that the bagged models are low bias by learning deep complex decision trees.

Bagging: Random Forests



Boosting

- reduce the bias of a high bias low variance model
- turning an ensemble of weak learners into a strong learner
- the meta-model is additive, i.e. adaboost:

$$\hat{f}(x, \theta) = \sum_{t=1}^{T} \alpha_t f_t(x, \theta)$$

Boosting: adaboost

$\boxed{y_i \in \{-1, +1\}}$

Initialize the weights $D_1(i) = 1/n, i = 1, 2, \ldots, n.$

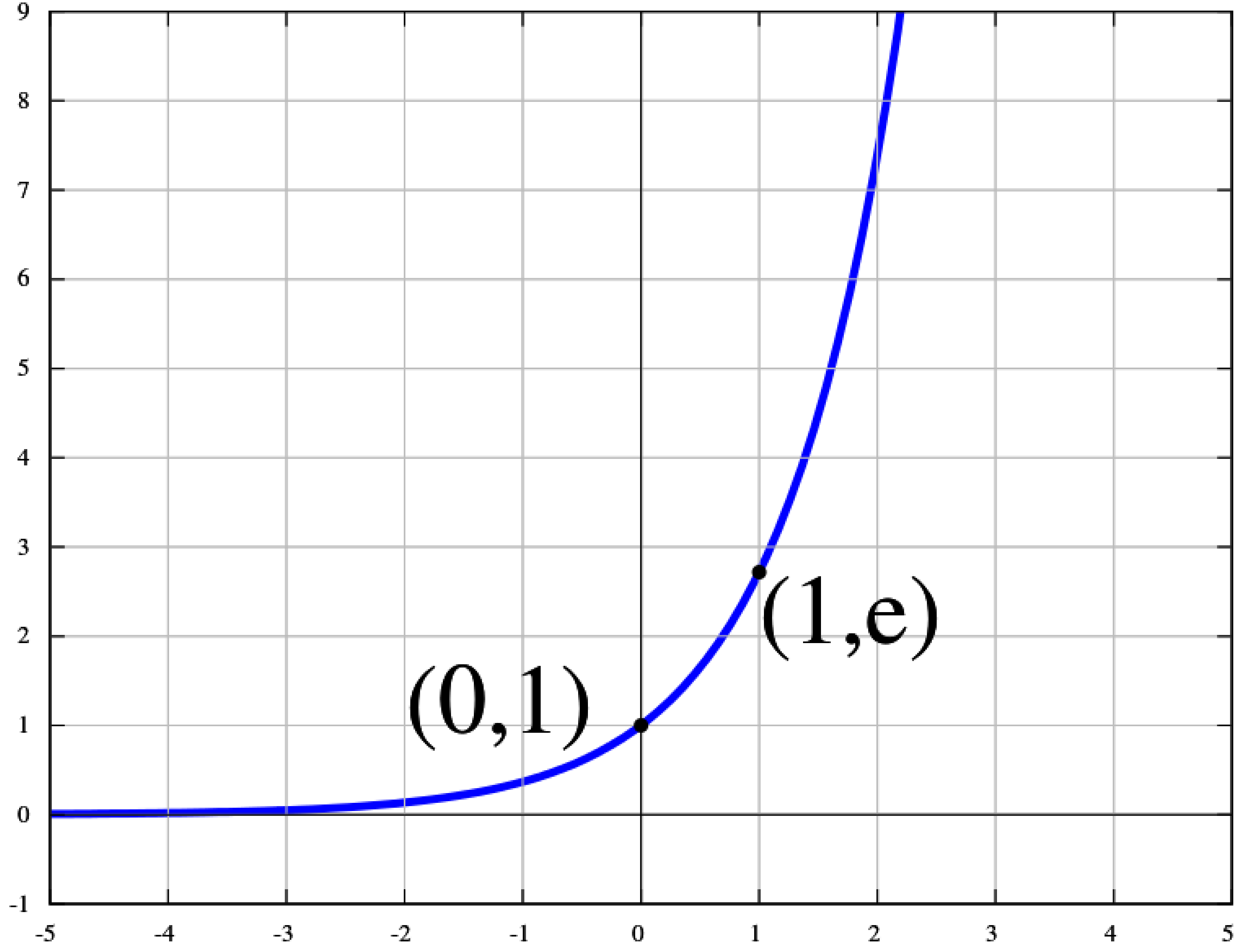
For $t = 1$ to T:

1. Fit a weak classifier $f_t(x, \theta)$ to the trainset data using weights $D_1(i)$.
2. Set $\alpha_t = \frac{1}{2} ln(\frac{1-error}{error})$.
3. Update weights:

$$D_{t+1}(i) = \frac{D_t(i)exp(-\alpha_t y_i f_t(x_i, \theta))}{Z_t},$$

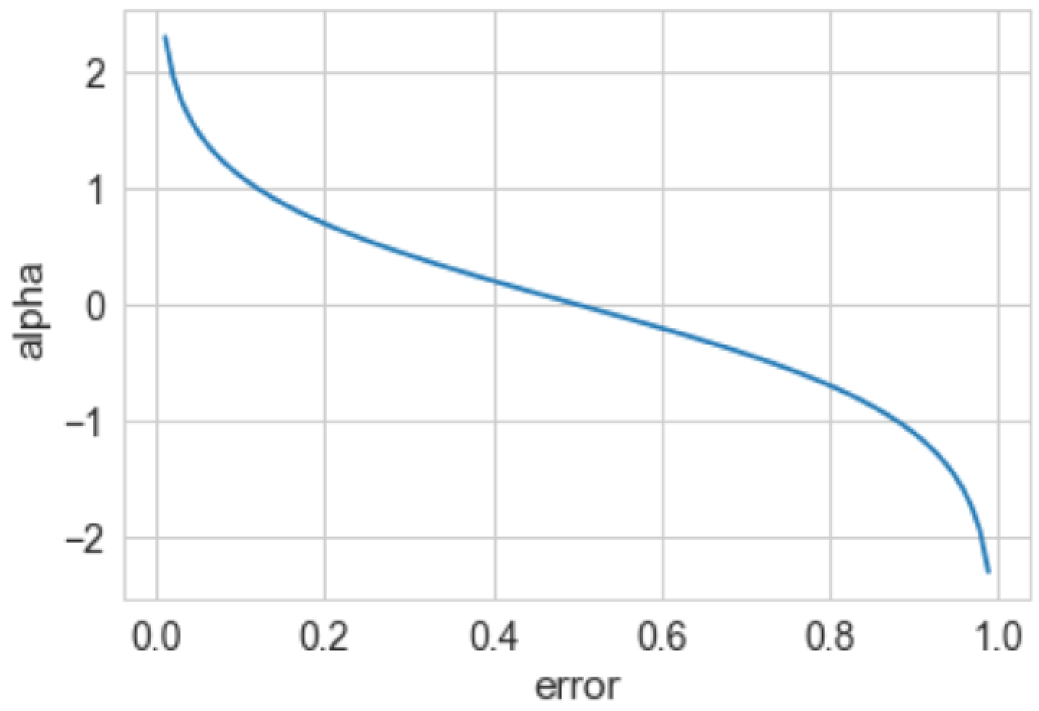
where Z_t is a normalizing factor that makes sure that $\sum D_{t+1}(i) = 1.$

$$\hat{f}(x, \theta) = \sum_{t=1}^{T} \alpha_t f_t(x, \theta)$$



9
8
7
6
5
4
3
2
1
0
-1
-5
-4
-3
-2
-1
0
1
2
3
4
5
(0,1)
(1,e)

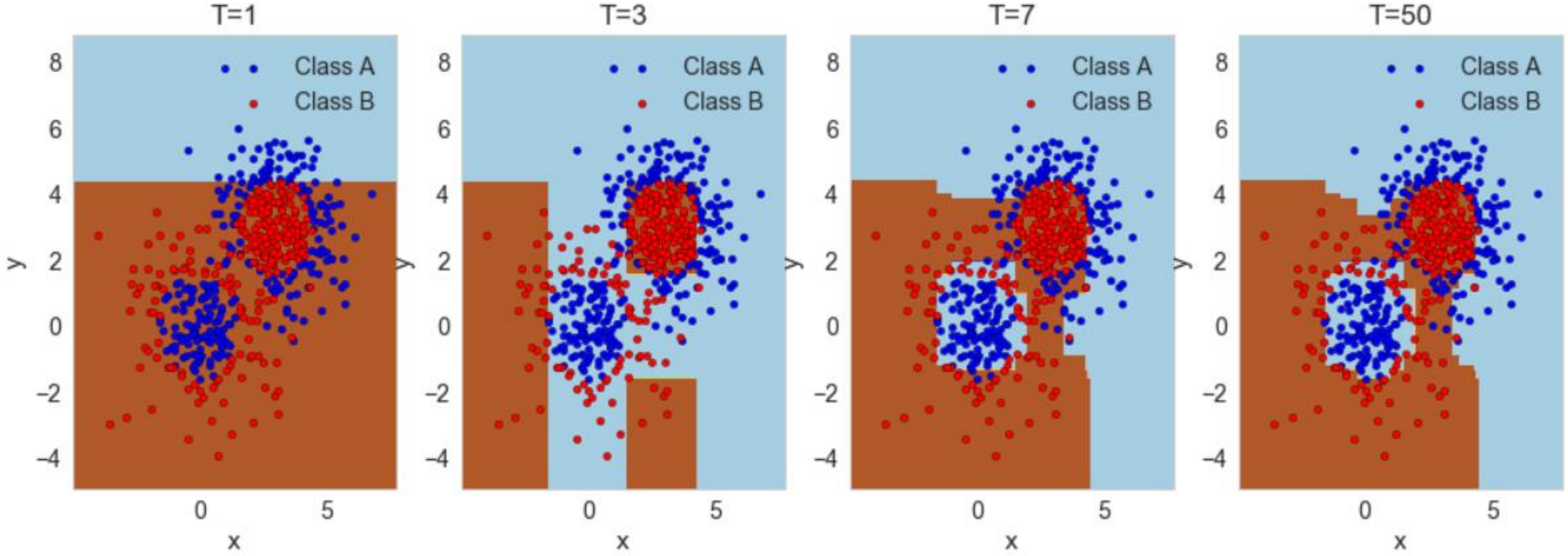
Boosting: adaboost



- The weight of a weak model in the boosted meta-model increases exponentially as the error approaches 0. Better models are given exponentially more weight.
- The weight is zero if the error rate is 0.5. A model with 50% accuracy is no better than random guessing, so it is ignored.
- The weight decreases exponentially as the error approaches 1. A negative weight is given to classifiers with worse than 50% accuracy. “Whatever that classifier says, do the opposite!”.

$$\hat{f}(x,\theta) = \sum_{t=1}^{T} \alpha_t f_t(x,\theta)$$

Boosting: adaboost



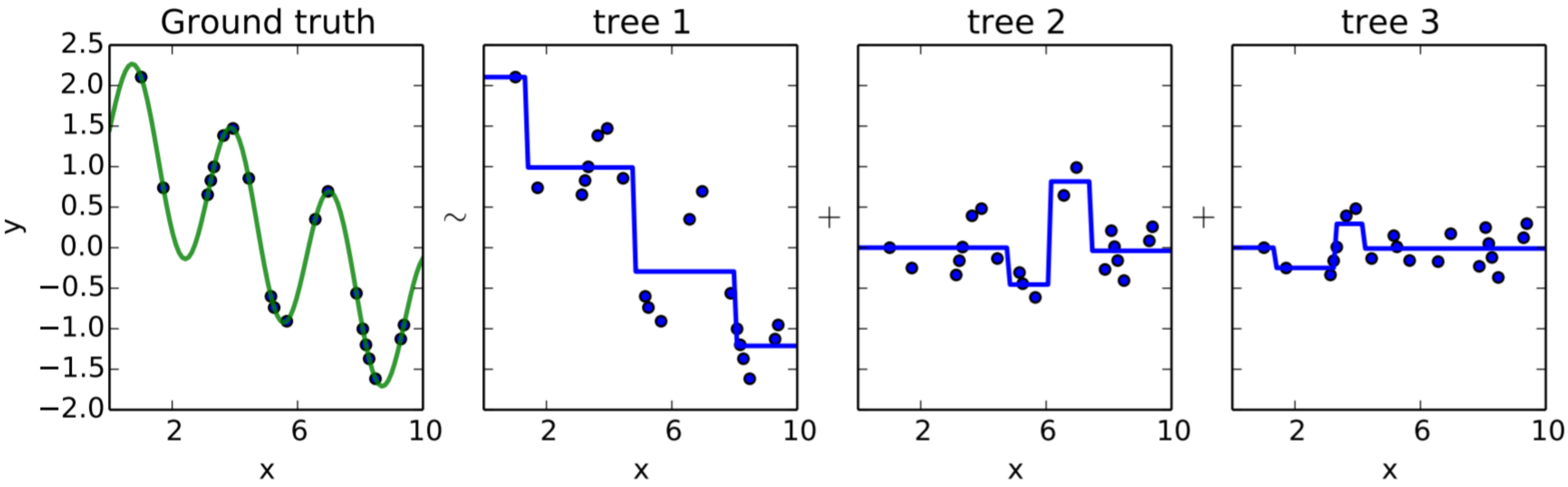
Boosting: Gradient Boosting

$$\hat{f}(x, \theta) = \sum_{t=1}^{T} f_t(x, \theta)$$

1. Fit a model $f_1(x, \theta) = y$
2. Fit a model to the **residuals** $h_1(x) = y - f_1(x, \theta)$
3. Create a new model $f_2(x, \theta) = f_1(x, \theta) + h_1(x)$

$$f_0(x, \theta) = \frac{1}{n} \sum_{i=1}^{n} y_i$$

$$f_{t+1}(x, \theta) = f_t(x, \theta) + h_t(x)$$



Ground truth
tree 1
tree 2
tree 3
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y
x
2
6
10
~
+
+