2.2 e)

In[219]:= **Solve[{y == 0, -Sin[x] - σ * y == 0 }] (*To take the fixed points*)**

Out[219]= $\left\{\left\{y \rightarrow 0, x \rightarrow \boxed{2 \pi c_1 \text{ if } c_1 \in \mathbb{Z}}\right\}, \left\{y \rightarrow 0, x \rightarrow \boxed{\pi + 2 \pi c_1 \text{ if } c_1 \in \mathbb{Z}}\right\}\right\}$

(*From above we have fixed points when y==
  0 and x can take values such as 2π*constant or π +
   2π*constant for instace. We should limit our investigation so -π<x$\leq$
π so we can choose the fixed points (0,0) and (π,0) which is when c1=0 *)

In[220]:=
```
xMin = -π;
xMax = π;
yMin = -π / 2;
yMax = π / 2;
tMin = 0;
tMax = 20;
systemSolver[x0_, y0_] :=
  Table[NDSolve[{D[x[t], t] == y[t], D[y[t], t] == -Sin[x[t]] - σ y[t],
    x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, tMin, tMax}], {σ, {0}}];

table1 = Table[{xMin, y}, {y, yMin, yMax, 0.9}];
table2 = Table[{xMax, y}, {y, yMin, yMax, 0.9}];
table3 = Table[{x, yMin}, {x, xMin, xMax, 0.9}];
table4 = Table[{x, yMax}, {x, xMin, xMax, 0.9}];
initialConditions = Join[table1, table2, table3, table4];
plot =
  Table[ParametricPlot[Evaluate[{x[t], y[t]} /. systemSolver[initialConditions[[i, 1]],
      initialConditions[[i, 2]]]], {t, tMin, tMax}, PlotRange → {{-5, 5}, {-3, 3}},
    PlotLabel → "F.P at (0,0), Stable center, σ=0"] /.
   Line[x_] :> {Arrowheads[{0, 0.045, 0.045, 0.045, 0}], Arrow[x]},
  {i, Length[initialConditions]}];
Show[{plot}]
```

Out[233]=



F.P at (0,0), Stable center, $\sigma=0$
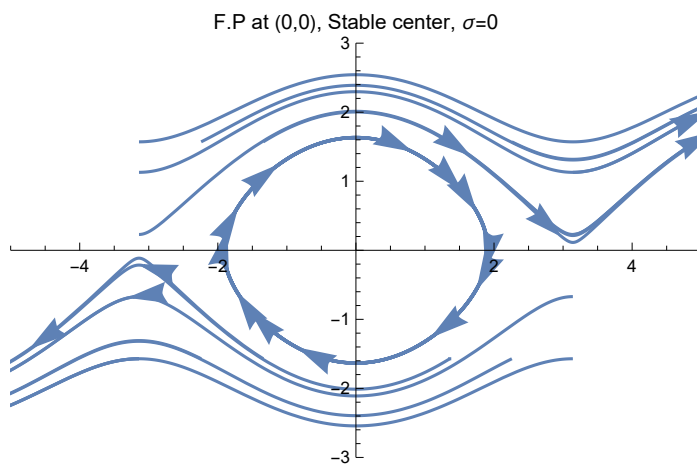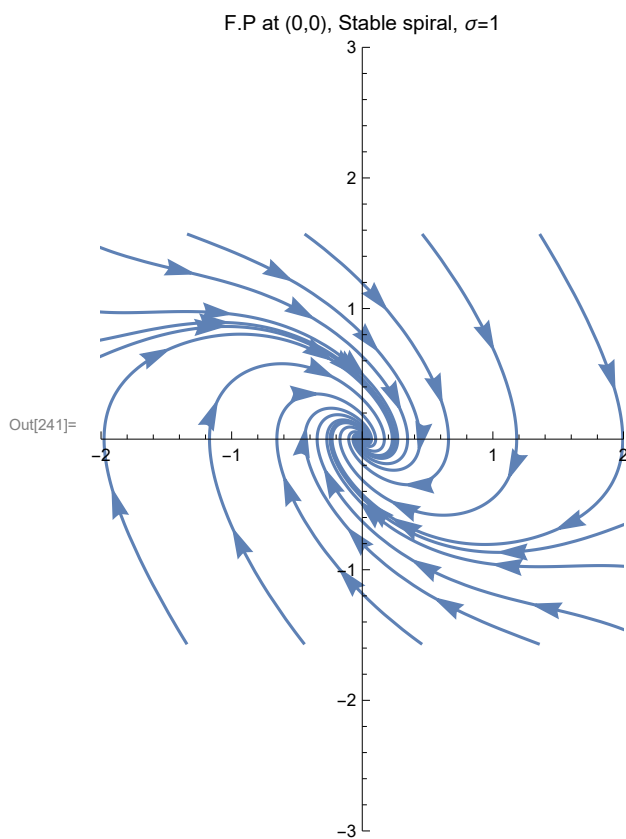
```
In[234]:= systemSolver[x0_, y0_] :=
    Table[NDSolve[{D[x[t], t] == y[t], D[y[t], t] == -Sin[x[t]] - σ y[t],
       x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, tMin, tMax}], {σ, {1}}];

    table1 = Table[{xMin, y}, {y, yMin, yMax, 0.9}];
    table2 = Table[{xMax, y}, {y, yMin, yMax, 0.9}];
    table3 = Table[{x, yMin}, {x, xMin, xMax, 0.9}];
    table4 = Table[{x, yMax}, {x, xMin, xMax, 0.9}];
    initialConditions = Join[table1, table2, table3, table4];
    plot =
     Table[ParametricPlot[Evaluate[{x[t], y[t]} /. systemSolver[initialConditions[[i, 1]],
          initialConditions[[i, 2]]]], {t, tMin, tMax}, PlotRange → {{-2, 2}, {-3, 3}},
        PlotLabel → "F.P at (0,0), Stable spiral, σ=1"] /.
       Line[x_] :> {Arrowheads[{0, 0.045, 0.045, 0.045, 0}], Arrow[x]},
      {i, Length[initialConditions]}];
    Show[{plot}]
```
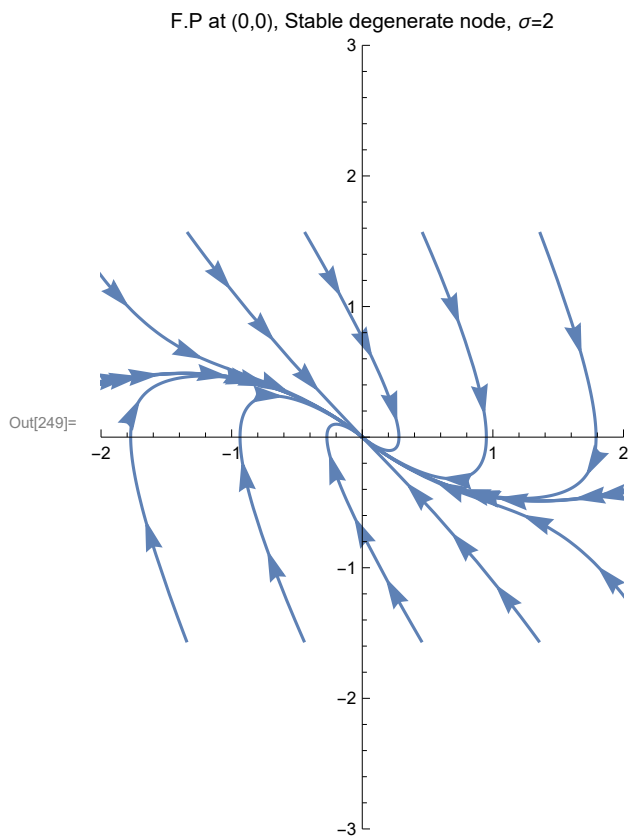
Out[241]=



F.P at (0,0), Stable spiral, $\sigma=1$

In[242]:= **systemSolver[x0_, y0_] :=**
   **Table[NDSolve[{D[x[t], t] == y[t], D[y[t], t] == -Sin[x[t]] - σ y[t],**
     **x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, tMin, tMax}], {σ, {2}}];**

**table1 = Table[{xMin, y}, {y, yMin, yMax, 0.9}];**
**table2 = Table[{xMax, y}, {y, yMin, yMax, 0.9}];**
**table3 = Table[{x, yMin}, {x, xMin, xMax, 0.9}];**
**table4 = Table[{x, yMax}, {x, xMin, xMax, 0.9}];**
**initialConditions = Join[table1, table2, table3, table4];**
**plot =**
  **Table[ParametricPlot[Evaluate[{x[t], y[t]} /. systemSolver[initialConditions〚i, 1〛,**
    **initialConditions〚i, 2〛]], {t, tMin, tMax}, PlotRange → {{-2, 2}, {-3, 3}},**
   **PlotLabel → "F.P at (0,0), Stable degenerate node, σ=2"] /.**
    **Line[x_] :> {Arrowheads[{0, 0.045, 0.045, 0.045, 0}], Arrow[x]},**
   **{i, Length[initialConditions]}];**
**Show[{plot}]**

Out[249]=



F.P at (0,0), Stable degenerate node, $\sigma$=2
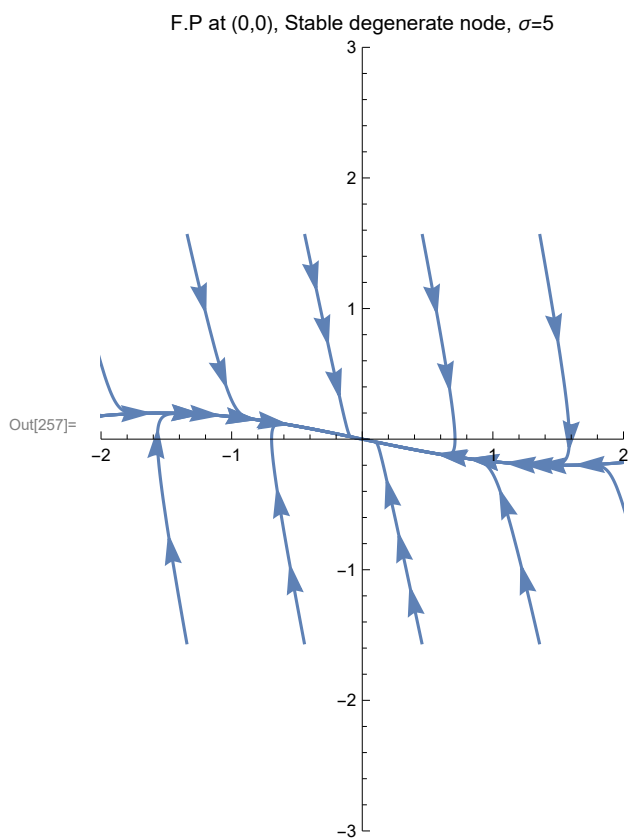
In[250]:= **systemSolver[x0_, y0_] :=**
   **Table[NDSolve[{D[x[t], t] == y[t], D[y[t], t] == -Sin[x[t]] - σ y[t],**
     **x[0] == x0, y[0] == y0}, {x[t], y[t]}, {t, tMin, tMax}], {σ, {5}}];**

**table1 = Table[{xMin, y}, {y, yMin, yMax, 0.9}];**
**table2 = Table[{xMax, y}, {y, yMin, yMax, 0.9}];**
**table3 = Table[{x, yMin}, {x, xMin, xMax, 0.9}];**
**table4 = Table[{x, yMax}, {x, xMin, xMax, 0.9}];**
**initialConditions = Join[table1, table2, table3, table4];**
**plot =**
   **Table[ParametricPlot[Evaluate[{x[t], y[t]} /. systemSolver[initialConditions[[i, 1]],**
       **initialConditions[[i, 2]]]], {t, tMin, tMax}, PlotRange → {{-2, 2}, {-3, 3}},**
     **PlotLabel → "F.P at (0,0), Stable degenerate node, σ=5"] /.**
     **Line[x_] :> {Arrowheads[{0, 0.045, 0.045, 0.045, 0}], Arrow[x]},**
    **{i, Length[initialConditions]}];**
**Show[{plot}]**

Out[257]=



F.P at (0,0), Stable degenerate node, $\sigma$=5

In[258]:= **table = TextGrid[{{"fix point (x,y)", "(0,0)"},**
**{"σ=0", "Stable center"},**
**{"σ=1", "Stable spiral"},**
**{"σ=2", "Stable degenerate node"},**
**{"σ=5", "Stable degenerate node"}}, Frame → All]**

Out[258]=

| fix point (x,y) | (0,0) |
|---|---|
| $\sigma$=0 | Stable center |
| $\sigma$=1 | Stable spiral |
| $\sigma$=2 | Stable degenerate node |
| $\sigma$=5 | Stable degenerate node |