

Get all fixed points  $x^*$  for  $dx/dt = rx + 4x^3 - 9x^5$

`In[ ]:= Solve[r * x + 4 * x^3 - 9 * x^5 == 0, x]`

`Out[ ]:=`  $\left\{ \left\{ x \rightarrow 0 \right\}, \left\{ x \rightarrow -\frac{1}{3} \sqrt{2 - \sqrt{4 + 9 r}} \right\}, \left\{ x \rightarrow \frac{1}{3} \sqrt{2 - \sqrt{4 + 9 r}} \right\}, \right.$   
 $\left. \left\{ x \rightarrow -\frac{1}{3} \sqrt{2 + \sqrt{4 + 9 r}} \right\}, \left\{ x \rightarrow \frac{1}{3} \sqrt{2 + \sqrt{4 + 9 r}} \right\} \right\}$

## Bifurcation diagram of $x^*$ vs $r$

```

In[ ]:= f[x_] := r * x + 4 * x^3 - 9 * x^5;
derivOfx = D[f[x], x];
conplot = ContourPlot[{
  ConditionalExpression[f[x], derivOfx < 0] == 0,
  ConditionalExpression[f[x], derivOfx > 0] == 0},
{r, -1, 1}, {x, -1, 1}, ContourStyle -> {{Black}, {Black, Dashed}},
PlotLegends -> {"stable", "unstable"}];

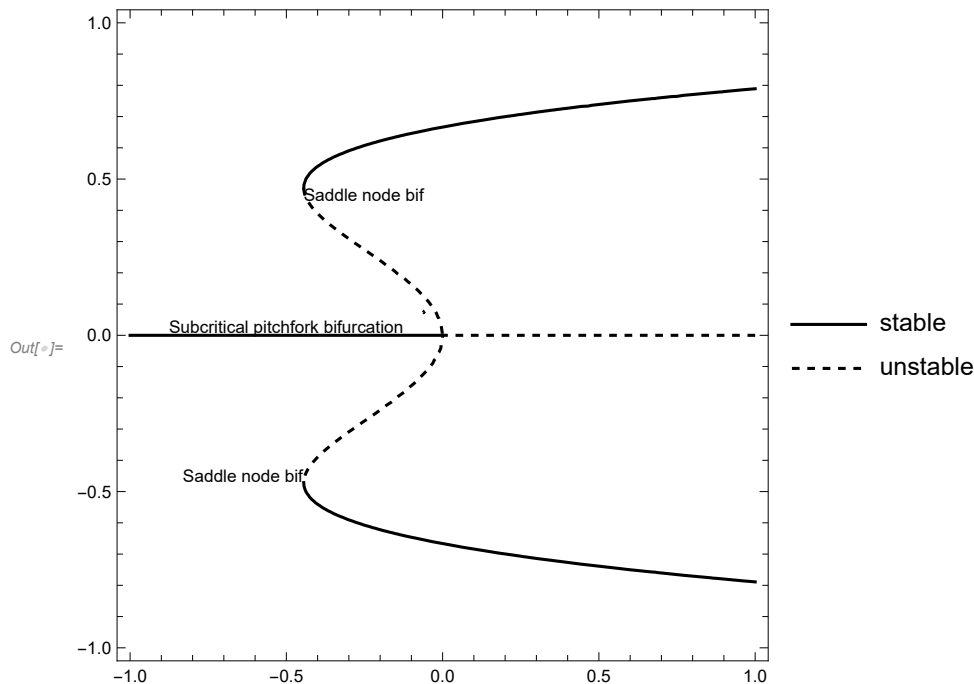
align[Right] = {1, 0};
align[Center] = {0, 0};
align[Left] = {-1, 0};

lText = Text["Saddle node bif", {-4 / 9, 0.45}, align[Left]];
cText = Text["Subcritical pitchfork bifurcation", {-0.5, 0.02}, align[Center]];
rText = Text["Saddle node bif", {-4 / 9, -0.45}, align[Right]];

txt = Graphics[{lText, cText, rText}];

Show[conplot, txt]

```



## Find r critical

```
In[ ]:= Clear[x];
f[x_] := r * x + 4 * x^3 - 9 * x^5;
dx = D[f[x], x];
Solve[dx == 0, x]
```

$$\text{Out[ ]}= \left\{ \left\{ x \rightarrow -\frac{\sqrt{2 - \sqrt{4 + 5r}}}{\sqrt{15}} \right\}, \left\{ x \rightarrow \frac{\sqrt{2 - \sqrt{4 + 5r}}}{\sqrt{15}} \right\}, \left\{ x \rightarrow -\frac{\sqrt{2 + \sqrt{4 + 5r}}}{\sqrt{15}} \right\}, \left\{ x \rightarrow \frac{\sqrt{2 + \sqrt{4 + 5r}}}{\sqrt{15}} \right\} \right\}$$

```
In[ ]:= x1 = -Sqrt[2 - Sqrt[4 + 5 * r]] / Sqrt[15];
x2 = Sqrt[2 - Sqrt[4 + 5 * r]] / Sqrt[15];
x3 = -Sqrt[2 + Sqrt[4 + 5 * r]] / Sqrt[15];
x4 = Sqrt[2 + Sqrt[4 + 5 * r]] / Sqrt[15];
Solve[f[x1] == 0, r]
Solve[f[x2] == 0, r]
Solve[f[x3] == 0, r]
Solve[f[x4] == 0, r]
```

$$\text{Out[ ]}= \left\{ \left\{ r \rightarrow 0 \right\} \right\}$$

$$\text{Out[ ]}= \left\{ \left\{ r \rightarrow 0 \right\} \right\}$$

$$\text{Out[ ]}= \left\{ \left\{ r \rightarrow -\frac{4}{9} \right\} \right\}$$

$$\text{Out[ ]}= \left\{ \left\{ r \rightarrow -\frac{4}{9} \right\} \right\}$$