a)

In[27]:=
```
Mq = {{sigma + 1, 3}, {-2, sigma - 1}};
Eigenvalues[Mq]
```

Out[28]= $\left\{-i\sqrt{5}+\text{sigma},\ i\sqrt{5}+\text{sigma}\right\}$

b)

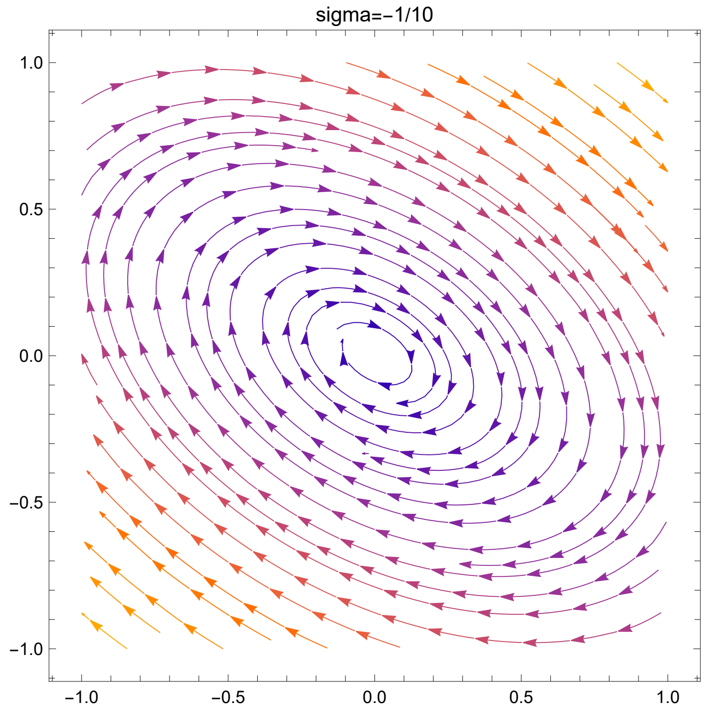
In[29]:=
```
systemsolver[x0_, y0_] := DSolve[{x'[t] == (sigma + 1) * x[t] + 3 * y[t],
     y'[t] == -2 * x[t] + (sigma - 1) * y[t], x[0] == x0, y[0] == y0},
    {x, y}, {t}];
sol = systemsolver[u, v];
X[t_, x0_, y0_, sigma_] = {x[t], y[t]} /. sol
```

Out[31]= $\left\{\left\{\frac{1}{5}e^{\text{sigma}\,t}\left(5\,u\,\text{Cos}\left[\sqrt{5}\,t\right]+\sqrt{5}\,u\,\text{Sin}\left[\sqrt{5}\,t\right]+3\sqrt{5}\,v\,\text{Sin}\left[\sqrt{5}\,t\right]\right),\right.\right.$
$\left.\left.-\frac{1}{5}e^{\text{sigma}\,t}\left(-5\,v\,\text{Cos}\left[\sqrt{5}\,t\right]+2\sqrt{5}\,u\,\text{Sin}\left[\sqrt{5}\,t\right]+\sqrt{5}\,v\,\text{Sin}\left[\sqrt{5}\,t\right]\right)\right\}\right\}$

c)

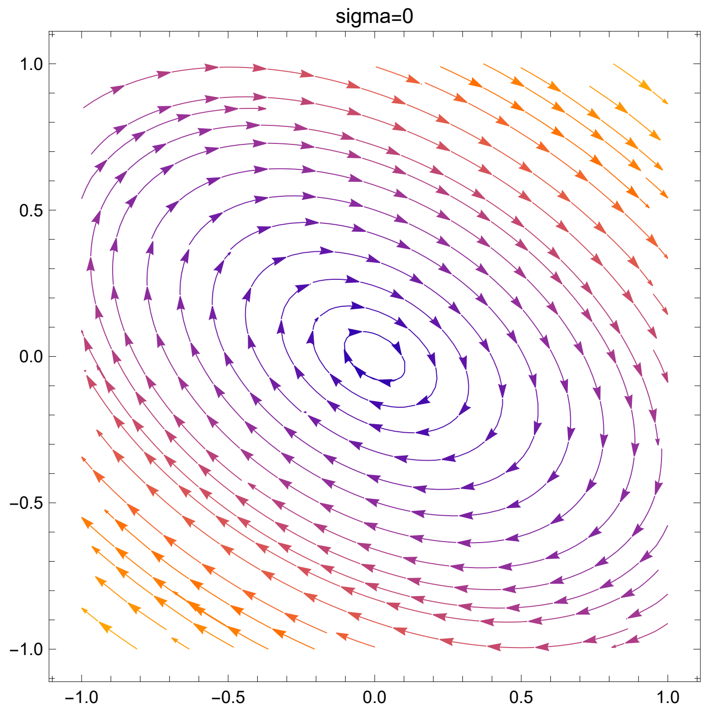
In[14]:=
```
dotx[x_, y_, sigma_] := (sigma + 1) * x + 3 * y;
doty[x_, y_, sigma_] := -2 * x + (sigma - 1) * y;
plot1 = StreamPlot[{dotx[x, y, -1 / 10], doty[x, y, -1 / 10]},
   {x, -1, 1}, {y, -1, 1}, PlotLabel → "sigma=-1/10"];

plot2 = StreamPlot[{dotx[x, y, 0], doty[x, y, 0]},
   {x, -1, 1}, {y, -1, 1}, PlotLabel → "sigma=0"];

plot3 =  StreamPlot[{dotx[x, y, 1 / 10], doty[x, y, 1 / 10]},
   {x, -1, 1}, {y, -1, 1}, PlotLabel → "sigma=1/10"];

Show[plot1]
Show[plot2]
Show[plot3]
```

Out[19]=

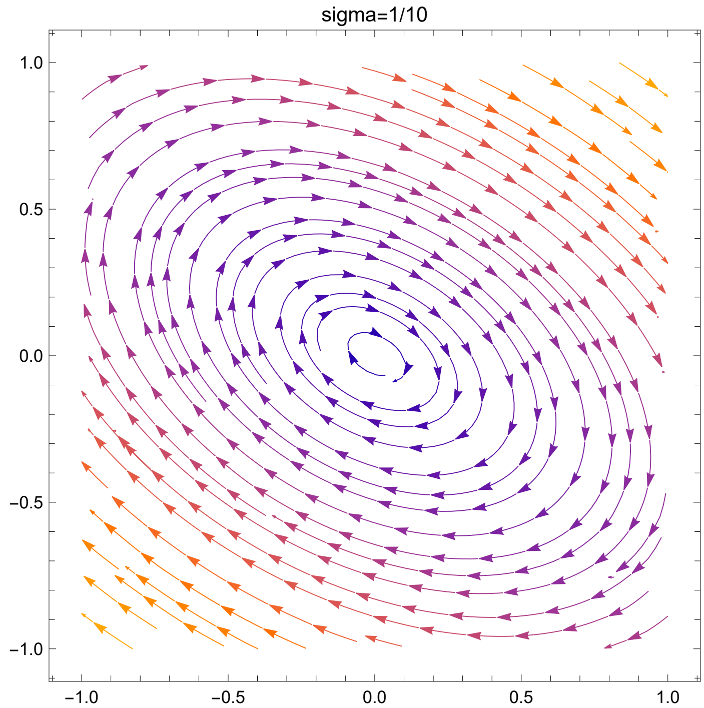


sigma=-1/10
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0

Out[20]=



sigma=0
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
0.0
0.5
1.0



Out[21]=

d)

```
(*Make the system again but in another way, good way to practice*)
```

In[32]:=
```
X[t_] = {x[t], y[t]};
dynsystem = X'[t] == Mq.X[t];
sol = DSolve[dynsystem, {x, y}, t];
Xdot[t_, sigma_] = {x[t], y[t]} /. sol /. {C[1] → u, C[2] → v};
Xdot[t_, sigma_] // Simplify
```

Out[36]= $\left\{\left\{\frac{1}{5}\, e^{\text{sigma\_ t\_}} \left(5\, u \,\text{Cos}\left[\sqrt{5}\, \text{t\_}\right] + \sqrt{5}\, (u + 3 v)\, \text{Sin}\left[\sqrt{5}\, \text{t\_}\right]\right),\ \frac{1}{5}\, e^{\text{sigma\_ t\_}} \left(5\, v \,\text{Cos}\left[\sqrt{5}\, \text{t\_}\right] - \sqrt{5}\, (2 u + v)\, \text{Sin}\left[\sqrt{5}\, \text{t\_}\right]\right)\right\}\right\}$

In[37]:=
```
sol = Solve [Xdot[t, 0] == Xdot[0, 0], t];
T = t /. sol (*calculate the period of the ellipse*)
T = Simplify[T /. C[1] → 1]
```

Out[38]= $\left\{ \frac{2 \pi c_1}{\sqrt{5}} \text{ if } c_1 \in \mathbb{Z} \right\}$

Out[39]= $\left\{ \frac{2 \pi}{\sqrt{5}} \right\}$

e)

In[40]:=
```
sol = Xdot[t, 0];
a = FindMaximum[Norm[sol] /. {u → 1, v → 2}, t][[1]];
b = FindMinimum[Norm[sol] /. {u → 1, v → 2}, t][[1]];
ratio = a / b;
(*Calculate the ratio for the minor and major axes of the ellipse*)
NumberForm[ratio, 10]
```

Out[43]//NumberForm=
```
1.618033989
```

d) Analytically calculate the direction of the major ellipse axis in the (x, y) plane. For definiteness, normalise the vector to one and choose the direction so that the first component is positive.

In[44]:=
```
timemajoraxis = FindMaximum[Norm[sol] /. {u → 1, v → 2}, t][[2]][[1]]
 (*Calculate the time where the major axis is placed*)
```

Out[44]= t → 0.727378

In[45]:=
```
direction = sol /. {u → 1, v → 2, timemajoraxis}
```

Out[45]= {{3.07, -1.89737}}

```
Normalize[direction[[1]]] (*Normalizing the direction which is the answer*)
```

Out[46]= {0.850651, -0.525731}