Convert from polar to cartesian

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2+y^2}$$

$$r' = \left(\sqrt{x^2+y^2}\right)' = \left(\left(x^2+y^2\right)^{1/2}\right)' =$$

$$= \frac{(x^2+y^2)'}{2\sqrt{x^2+y^2}} = \frac{\cancel{2}xx' + \cancel{2}yy'}{\cancel{2}\sqrt{x^2+y^2}} =$$

$$= \frac{xx' + yy'}{\sqrt{x^2+y^2}} = \frac{xx' + yy'}{r}$$

$$\Rightarrow \quad rr' = xx' + yy'$$

$$\Rightarrow \quad x' = \frac{(rr' - yy')}{x} \quad ①$$

$$\theta' = \left(\arctan\left(\frac{y}{x}\right)\right)' = \frac{(y/x)'}{1+(y/x)^2} = \frac{y'x - x'y}{r^2}$$

$$\Rightarrow \quad \theta' r^2 = y'x - x'y \quad \Rightarrow \quad x' = \frac{y'x - \theta' r^2}{y} \quad ②$$

$$① = ② \Rightarrow$$

$$\frac{rr' - yy'}{x} = \frac{y'x - \theta' r^2}{y}$$

$$rr'y - y^2y' = y'x^2 - \theta' r^2 x$$

$$rr'y + \theta' r^2 x = y^2 y' + y' x^2$$

$$y' = \frac{rr'y + \theta' r^2 x}{x^2+y^2} = \frac{r(r'y + \theta' r x)}{r^2}$$

$$y' = \frac{r'y + \theta' r x}{r} = \underline{\frac{y}{r} r' + x \cdot \theta'} \quad (3)$$

(3) in (2) gives:

$$x' = \frac{y'x - \theta' r^2}{y} = \frac{\left(\frac{y}{r} r' + x\theta'\right) x}{y} - \frac{\theta' r^2}{y} =$$

$$= \frac{y r' x}{y r} + \frac{x^2 \theta'}{y} - \frac{\theta' r^2}{y} =$$

$$= \frac{r' x}{r} + \theta' \frac{(x^2 - r^2)}{y} = \frac{\theta' (x^2 - x^2 - y^2)}{y} =$$

$$= \frac{x}{r} r' - \frac{\theta' y^2}{y} = \underline{\frac{x}{r} r' - \theta' y} \quad (4)$$

so we have

(3): $y' = \frac{y}{r} r' + x \cdot \theta'$

(4): $x' = \frac{x}{r} r' - \theta' y$

where x' correspond to $\dot{x}_1$ and y' to $\dot{x}_2$

$$\Rightarrow \quad \dot{X}_1 = \frac{x_1}{\sqrt{x_1^2+x_2^2}} \cdot \left(\mu \cdot \sqrt{x_1^2+x_2^2} - \left(\sqrt{x_1^2+x_2^2}\right)^3\right) - X_2 \cdot \left(w + \nu \cdot \left(\sqrt{x_1^2+x_2^2}\right)^2\right)$$

$$= X_1 \cdot \mu - x_1^3 - x_1 x_2^2 - X_2 W - x_2 x_1^2 \nu - x_2^3 \nu$$

comparing this with $\dot{X} = F_1(\cancel{X}) =$

$$= \frac{1}{10} x_1 - x_2^3 - x_1 x_2^2 - x_1^2 x_2 - x_2 - x_1^3$$

We can see that: $\mu = \frac{1}{10}$, $w = 1$, $\nu = 1$

we also check so it is true for $\dot{X}_2$!

from 3 we get:

$$\dot{X}_2 = x_2 \cdot \mu - x_2 x_1^2 - x_2^3 + x_1 w + \nu x_1^3 + \nu x_1 x_2^2$$

comparing this with

$$\dot{X}_2 = F_2(\cancel{X}) = x_1 + \frac{1}{10} x_2 + x_1 x_2^2 + x_1^3 - x_2^3 - x_1^2 x_2$$

we see that $\mu = \frac{1}{10}$, $w = 1$ and $\nu = 1$

is true for this too!