## a) Make a plot of the (h,r) plane

In[∘]:=
```
Solve[x * (r - x) + h == 0, x]
```

Out[∘]= $\left\{\left\{x \to \frac{1}{2}\left(r - \sqrt{4h + r^2}\right)\right\}, \left\{x \to \frac{1}{2}\left(r + \sqrt{4h + r^2}\right)\right\}\right\}$

In[∘]:=
```
Solve[D[x * (r - x) + h, x] == 0, x]
```

Out[∘]= $\left\{\left\{x \to \frac{r}{2}\right\}\right\}$

In[∘]:=

```
x = r / 2
```

Out[∘]= $\frac{r}{2}$

In[∘]:=
```
Solve[x * (r - x) + h == 0, h]
```

Out[∘]= $\left\{\left\{h \to -\frac{r^2}{4}\right\}\right\}$

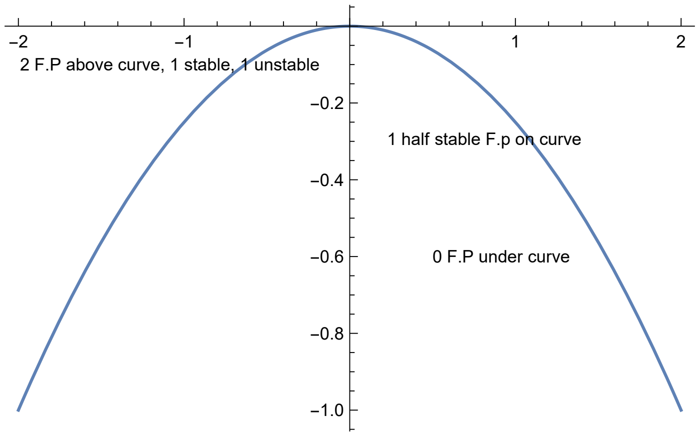
In[∘]:=
```
align[Right] = {1, 0};
align[Center] = {0, 0};
align[Left] = {-1, 0};

lText = Text["0 F.P under curve", {0.5, -0.6}, align[Left]];
cText =
  Text[" 2 F.P above curve, 1 stable, 1 unstable", {-1.1, -0.1}, align[Center]];
rText = Text["1 half stable F.p on curve", {1.4, -0.3}, align[Right]];

txt = Graphics[{lText, cText, rText}];
hc = Plot[-r^2 / 4, {r, -2, 2}];

Show[hc, txt]
```

Out[∘]=



```
Clear[x]
```

## c) Find analytically the bifurcation curve[h_c(r),r]

In[ ]:=
```
derivative = D[h + x * r - x^2, x];
sol = Solve[derivative == 0, x]
```

Out[ ]= $\left\{\left\{x \to \frac{r}{2}\right\}\right\}$

In[ ]:=
```
x = r / 2;
hc = Solve[x * (r - x) + h == 0, h];
hc
```

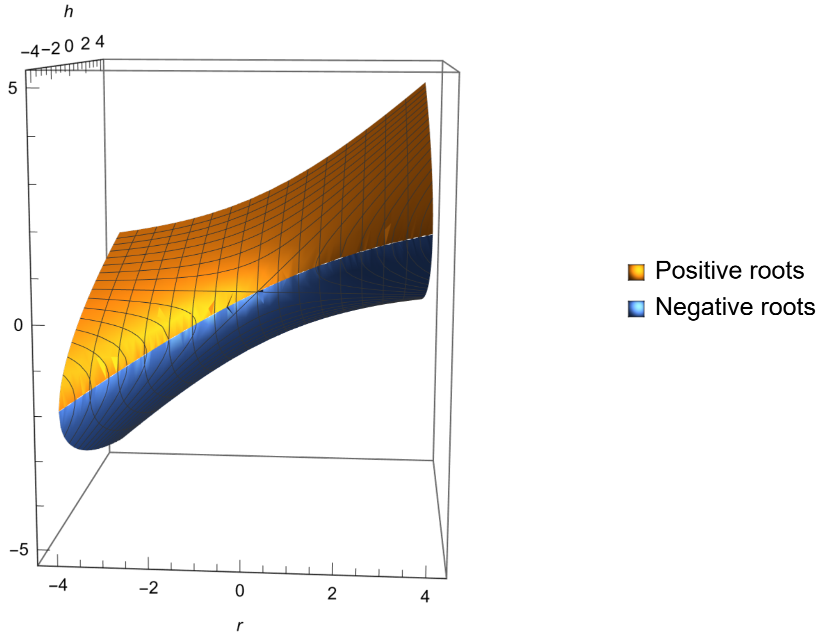
Out[ ]= $\left\{\left\{h \to -\frac{r^2}{4}\right\}\right\}$

```
(*From this we have hc, so the answer is just [-r^2/4,r]*)
```

### b) Make the 3d plot of (x*,h,r) where f(x*,h, r) = 0

In[11]:=
```
xstar1 = (r + Sqrt[4 h + r^2]) / 2;
(*xstar1 and xstar2 is given in the beggining of the pdf*)
xstar2 =  (r - Sqrt[4 h + r^2]) / 2;
ParametricPlot3D[{{r, h, xstar1}, {r, h, xstar2}}, {r, -4, 4}, {h, -4, 4},
 PlotLegends → {"Positive roots", "Negative roots"}, {AxesLabel → {r, h}}]
```

Out[12]=

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# d) find the transcritical bifurcation direction that depends on r

Start by taking the derivative of [h_c(r), r] and then normalize it

In[∘]:=
```
v1 = {-r^2 / 4, r}
v2 = D[v1, r]
```

Out[∘]= $\left\{-\frac{r^2}{4}, r\right\}$

Out[∘]= $\left\{-\frac{r}{2}, 1\right\}$

In[∘]:=
```
Normalize[v2]
```

Out[∘]= $\left\{-\frac{r}{2\sqrt{1+\frac{\text{Abs}[r]^2}{4}}}, \frac{1}{\sqrt{1+\frac{\text{Abs}[r]^2}{4}}}\right\}$