a) Plot the trajectories and classify the fixed points, for the dynamical system for different values of sigma

In[ ]:=
```
dotx[x_, y_, sigma_] := (sigma + 3) * x + 4 * y;
doty[x_, y_, sigma_] := -(9 / 4) * x + (sigma - 3) * y;
sol1 = Solve[dotx[x, y, -1] == 0 && doty[x, y, -1] == 0, {x, y}]
sol2 = Solve[dotx[x, y, 0] == 0 && doty[x, y, 0] == 0, {x, y}]
sol3 = Solve[dotx[x, y, 1] == 0 && doty[x, y, 1] == 0, {x, y}]
(*here is the solutions for the fixed points for the different sigma values*)
```

Out[ ]= {{x → 0, y → 0}}

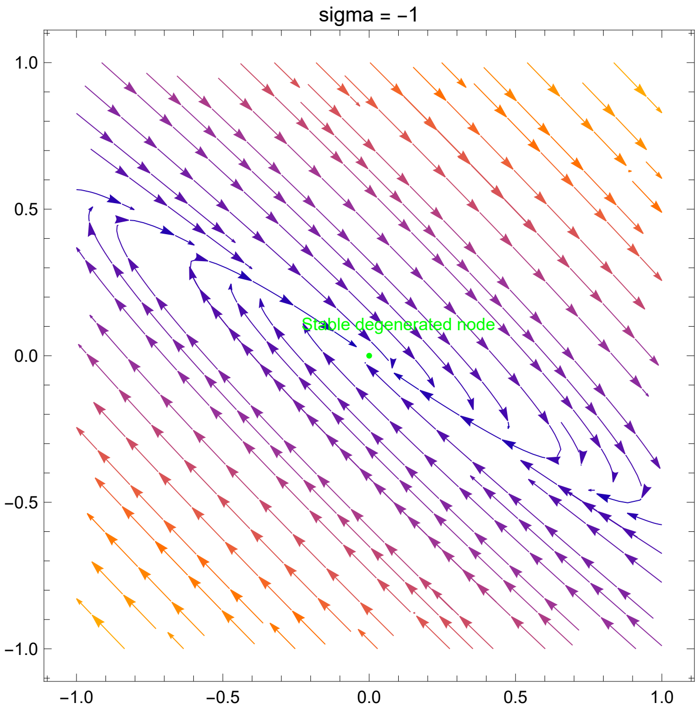
Solve: Equations may not give solutions for all "solve" variables.

Out[ ]= $\left\{\left\{y \to -\frac{3x}{4}\right\}\right\}$

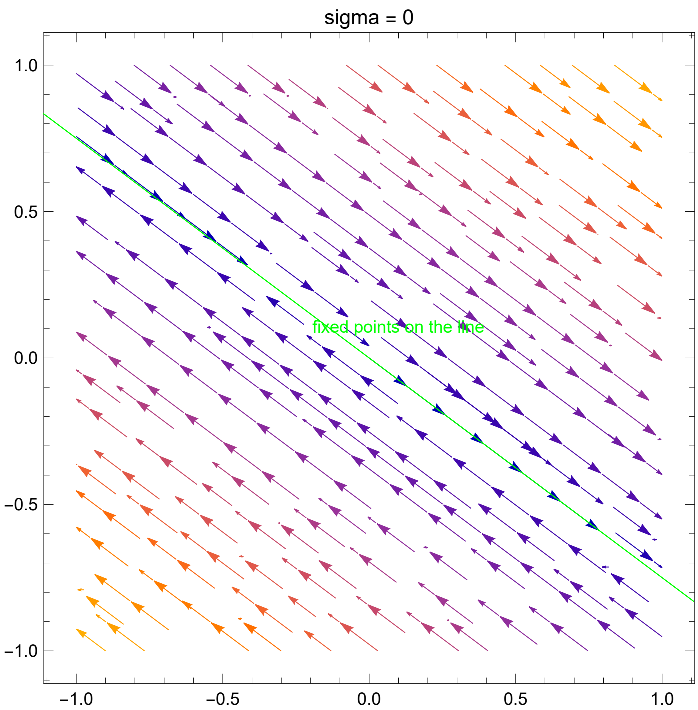
Out[ ]= {{x → 0, y → 0}}

In[ ]:=
```
plot1 = StreamPlot[{dotx[x, y, -1], doty[x, y, -1]}, {x, -1, 1}, {y, -1, 1},
   Epilog → {Green, Point[{0, 0}], Text["Stable degenerated node", {0.1, 0.1}]},
   PlotLabel → "sigma = -1"];

plot2 = StreamPlot[{dotx[x, y, 0], doty[x, y, 0]}, {x, -1, 1},
   {y, -1, 1}, Epilog → {Green, Line[{{16 / 3, -4}, {-16 / 3, 4}}],
     Text["fixed points on the line", {0.1, 0.1}]}, PlotLabel → "sigma = 0"];

plot3 = StreamPlot[{dotx[x, y, 1], doty[x, y, 1]}, {x, -1, 1}, {y, -1, 1},
   Epilog → {Green, Point[{0, 0}], Text["unstable degenerated node", {0.1, 0.1}]},
   PlotLabel → "sigma = 1"];

Show[plot1]
Show[plot2]
Show[plot3]
```

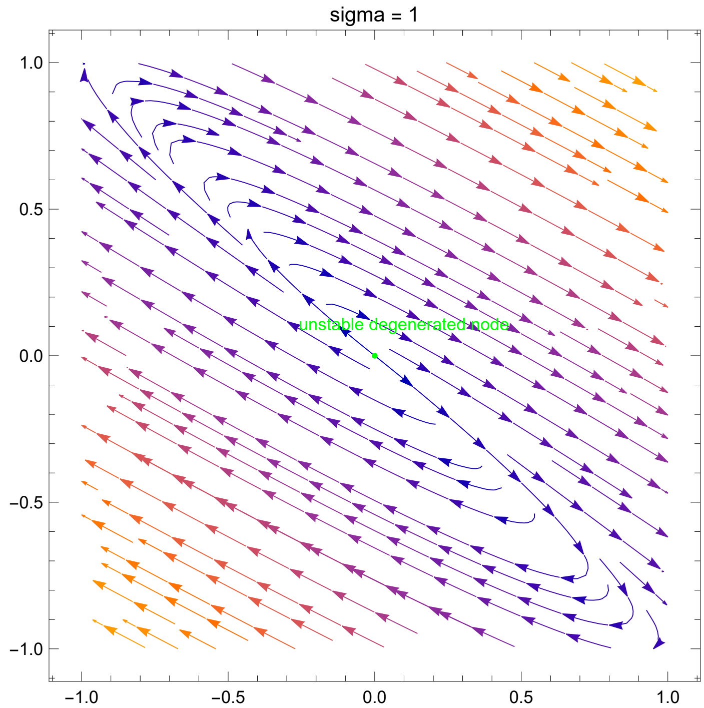
Out[ ]=


sigma = −1
Stable degenerated node
1.0
0.5
0.0
−0.5
−1.0

Out[ ]=


sigma = 0
fixed points on the line
1.0
0.5
0.0
−0.5
−1.0

Out[ ]=



b) Analytically compute eigenvalues of M_sigma in terms of sigma

In[ ]:=
```
Clear[sigma];
M = {{sigma + 3, 4}, {-9 / 4, sigma - 3}};
Eigenvalues[M]
```

Out[ ]= {sigma, sigma}

c) Compute all the eigenvectors of M_q, normalize to one and choose the x-components

In[ ]:=
```
eigenvector = Eigenvectors[M];
eigenvector = eigenvector[[1]];
v = Normalize[eigenvector];
v = v * -1
```

Out[ ]= $\left\{\frac{4}{5}, -\frac{3}{5}\right\}$

d) Compute the inverse matrix of M_q

In[ ]:=
```
inverseM = Inverse[M]
```

Out[ ]= $\left\{\left\{\frac{-3 + \text{sigma}}{\text{sigma}^2}, -\frac{4}{\text{sigma}^2}\right\}, \left\{\frac{9}{4\ \text{sigma}^2}, \frac{3 + \text{sigma}}{\text{sigma}^2}\right\}\right\}$

e) Give the value of sigma for which M_q is singular

M_q is singular when the inverse of M_q does not exist. Form d) we have the inverse of M_q and we can see that it not exist for sigma = 0, which is the answer.

f) For which values of c and d do you recover the dynamical system in the excercise above? choose c positive and write the result as the vector [c,d]

In[ ]:=
```
Solve[-c * d == 3 && d^2 == 4 && -c^2 == -9 / 4 && c * d == -3, {c, d}]
```

Out[ ]= $\left\{\left\{c \to -\frac{3}{2}, d \to 2\right\}, \left\{c \to \frac{3}{2}, d \to -2\right\}\right\}$

Choose the one with the positive c

g) Calculate the eigenvalues of the generalised dynamical system

In[ ]:=
```
M = {{sigma - c * d, d^2}, {-c^2, sigma + c * d}};
Eigenvalues[M]
```

Out[ ]= {sigma, sigma}

h) Analytically find the direction of the line of fixed points for sigma = 0

In[ ]:=
```
sigma = 0;
Amatrix = {{sigma - c * d, d^2}, {-c^2, sigma + c * d}};
eigenvector = Eigenvectors[Amatrix];
vector = eigenvector[[1]];
vector = Normalize[vector]
```

Out[ ]= $\left\{\frac{d}{c\sqrt{1+\text{Abs}\left[\frac{d}{c}\right]^2}}, \frac{1}{\sqrt{1+\text{Abs}\left[\frac{d}{c}\right]^2}}\right\}$