An empirical study of fixation for strategies in the Iterated Prisoner's Dilemma

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Abstract

The Iterated Prisoner's Dilemma is a well established framework for the study of emergent behaviour. In this paper an extensive numerical study of the evolutionary dynamics of this framework are presented.

Fixation probabilities for Moran processes are obtained for 164 different strategies.

To the authors knowledge this is the largest such study. It allows for insights about the behaviour and performance of strategies with regard to their survival in an evolutionary setting.

1 Introduction

Since the formulation of the Moran Process in [17], this model of evolutionary population dynamics has been used to gain insights about the evolutionary stability of strategies in a number of settings. Similarly since the first Iterated Prisoner's Dilemma (IPD) tournament described in [4] the Prisoner's dilemma has been used to understand the evolution of cooperative behaviour in complex systems.

The analytical models of a Moran process are based on the relative fitness between two strategies and take this to be a fixed value r [19]. This is a valid model for simple strategies of the Prisoner's Dilemma such as to *always cooperate* or *always defect*. This manuscript provides a detailed numerical analysis of **164** complex and adaptive strategies for the IPD. In this case the relative fitness of a strategy is dependent on the population distribution.

Further deviations from the analytical model occur when interactions between players are subject to uncertainty. This is referred to as noise and has been considered in the IPD setting in [7, 18, 27].

This work provides answers to the following questions:

1. What strategies are good invaders?
2. What strategies are good at resisting invasion?
3. How does the population size affect these findings?

Figure 1 shows a diagrammatic representation of the Moran process. This process is a stochastic birth death process on a finite population in which the population size stays constant over time. Individuals are **selected** according to a given fitness landscape. Once selected, a given individual is reproduced and similarly another individual is chosen to be removed from the population. In some settings mutation is also considered but without mutation (the case considered in this work) this process will arrive at an absorbing state where the population is entirely made up of a single individual. The probability with which a given strategy is the survivor is called the fixation probability. A more detailed analytic description of this is given in Section 3.

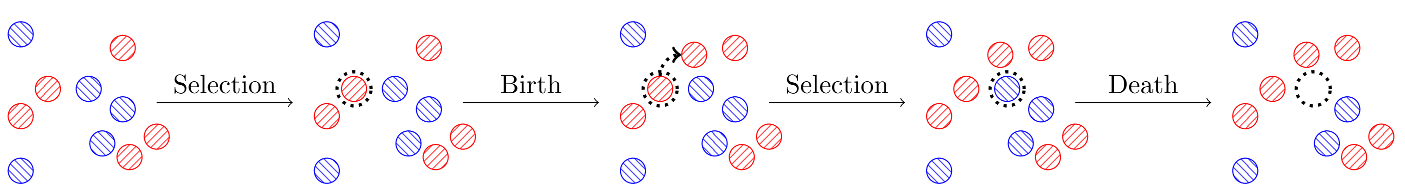


Figure 1: A diagrammatic representation of a Moran process

The Moran process was initially introduced in [17] in a genetic setting. It has since been used in a variety of settings including the understanding of the spread of cooperative behaviour. However, as stated before, these mainly consider non sophisticated strategies. Some work has looked at evolutionary stability of strategies within the Prisoner's Dilemma [12] but this is not done in the more widely used setting of the Moran process but in terms of infinite population stability.

In [6] Moran processes are looked at in a theoretic framework for a small subset of strategies. In [10] machine learning techniques are used to train a strategy capable of resisting invasion and also invade any memory one strategy. Recent work [8] has investigated the effect of memory length on strategy performance and the emergence of cooperation but this is not done in Moran process context and only considers specific cases of memory 2 strategies.

The contribution of this work is a detailed and extensive analysis of absorption probabilities for 164 strategies. These strategies and the numerical simulations are from [24] which is an open source research library written for the study of the IPD. The strategies and simulation frameworks are automatically tested in accordance to best research software practice. The large number of strategies are available thanks to the open source nature of the project with over 40 contributions made by different programmers. Thus by considering Moran processes with population size greater than 2 we are taking in to account the effect of complex population dynamics. By considering sophisticated strategies we are taking in to effect the reputation of a strategy during each interaction.

Section 2 will explain the methodological approach used, Section 3 will validate the methodology by comparing simulated results to analytical results. The main results of this manuscript are presented in Section 4 which will present a detailed analysis of all the data generated. Finally, Section 5 will conclude and offer future avenues for the work presented here.

2 Methodology

To carry out this large numerical experiment 164 strategies are used from [24]. These include 161 default strategies in the library at the time (excluding strategies classified as having a long run time and those that make use of the length of the game) as well as the following 3 finite state machine machine strategies [3]:

Appendix A shows all the players in question. More information about each player can be obtained in the documentation for [24]. There are 43 stochastic and 121 deterministic strategies. Their memory depth is shown in Table 1.

Memory Depth	0	1	2	3	4	5	6	9	10	11	12	16	20	40	200	∞
Count	3	31	12	8	2	6	1	1	5	1	1	2	2	2	1	86

Table 1: Memory depth

All strategies are paired and these pairs are used in 1000 repetitions of a Moran process assuming a starting population of $(N/2, N/2)$. This is repeated for even N between 2 and 14. The fixation probability is then estimated for each value of N.

Note that due to the high computational cost of these experiments, for any given interaction between two players within the Moran process the outcome is sampled from a pre computed cache of 1000 match outcomes. This is carried out using an software written for the purpose of this work. This has been implemented in [24] ensuring that it can be used to either reproduce the work or carry out further work.

Figure 2 shows the distribution of the number of outcomes between all strategy pairs. Tables 2 shows that 95% of the stochastic matches have less than 788 unique outcomes whilst the maximum number is 971. This ensures that using a set of cached results from 1000 precomputed matches is sufficient for the analysis taking place here.

	Outcome count
count	13530.00
mean	85.98
std	192.58
min	1.00
25%	1.00
50%	1.00
75%	36.00
95%	595.00
max	971.00

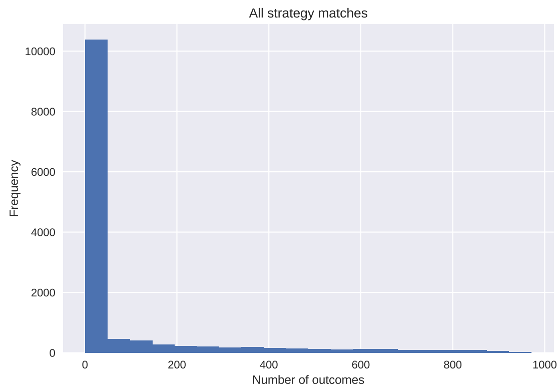
(a) All matches

	Outcome count
count	4753.00
mean	242.90
std	260.04
min	2.00
25%	28.00
50%	139.00
75%	394.00
95%	788.00
max	971.00

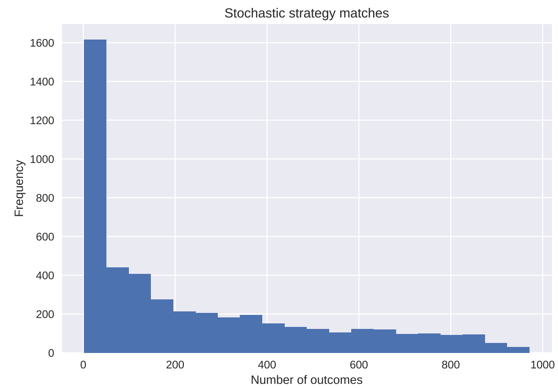
(b) Stochastic matches

Table 2: Summary statistics for the number of different match outcomes used as the cached results

Section 3 will validate the methodology used here against known theoretic results.



(a) All matches



(b) Stochastic matches

Figure 2: The distribution of the number of unique outcomes used as the cached results

3 Validation

As described in [19] Consider the payoff matrix:

$$M = \begin{pmatrix} a, b \\ c, d \end{pmatrix} \tag{1}$$

The expected payoffs of i players of the first type in a population with $N - i$ players of the second type are given by:

$$F_i = \frac{a(i-1) + b(N-i)}{N-1} \tag{2}$$

$$G_i = \frac{ci + d(N-i-1)}{N-1} \tag{3}$$

With an intensity of selection ω the fitness of both strategies is given by:

$$f_i = 1 - \omega + \omega F_i \tag{4}$$

$$g_i = 1 - \omega + \omega G_i \tag{5}$$

The transitions within the birth death process that underpins the Moran process are then given by:

$$p_{i,i+1} = \frac{if_i}{if_i + (N-i)g_i}\frac{N-i}{N} \tag{6}$$

$$p_{i,i-1} = \frac{(N-i)g_i}{if_i + (N-i)g_i}\frac{i}{N} \tag{7}$$

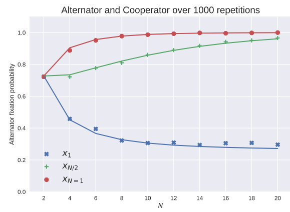
$$p_{ii} = 1 - p_{i,i+1} - p_{i,i-1} \tag{8}$$

Using this it is a known result that the fixation probability of the first strategy in a population of i individuals of the first type (and $N - i$ individuals of the second. We have:

$$x_i = \frac{1 + \sum_{j=1}^{i-1}\prod_{k=1}^{j}\gamma_j}{1 + \sum_{j=1}^{N-1}\prod_{k=1}^{j}\gamma_j} \tag{9}$$

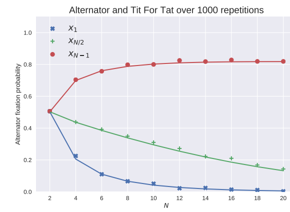
where:

$$\gamma_j = \frac{p_{j,j-1}}{p_{j,j+1}}$$

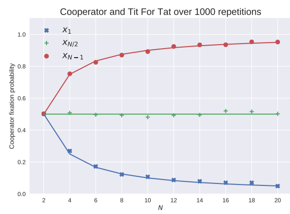


(a) Alternator and Cooperator

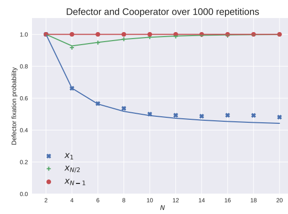
(b) Alternator and Defector



(c) Alternator and Tit For Tat

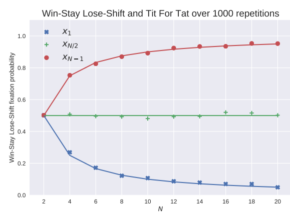


(d) Cooperator and Tit For Tat

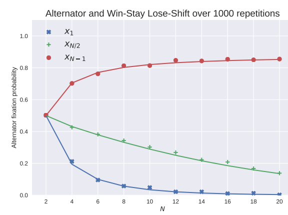


(e) Defector and Cooperator

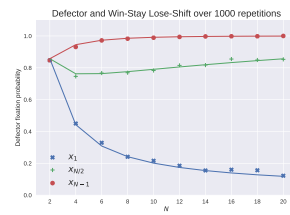
(f) Defector and Tit For Tat



(g) Win Stay Lose Shift and Tit For Tat



(h) Alternator and Win Stay Lose Shift



(i) Defector and Win Stay Lose Shift

Figure 3: Comparison of theoretic and actual Moran Process fixation probabilities for **deterministic** strategies

Using this comparisons of $x_1, x_{N/2}, x_{N-1}$ are shown in Figure 16. The points represent the simulated values and the line shows the theoretic value. Note that these are all deterministic strategies and show a perfect match up between the expected value of (9) and the actual Moran process for all strategies pairs.

Figure 17 shows the fixation probabilities for stochastic strategies. These are no longer a good match which highlights the weakness of the analytical formulae that relies on the average payoffs. A detailed analysis of the 164 strategies considered, using direct Moran processes will be shown in the next Section.

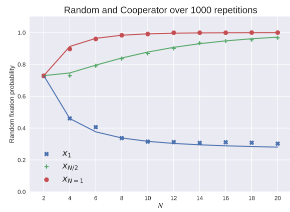
4 Empirical results

This section will outline the data analysis carried out:

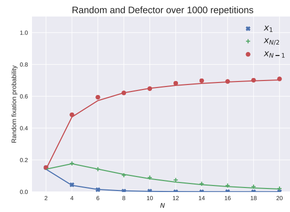
- Section 4.1 will consider the specific case of $N = 2$.
- Section 4.2 will investigate the effect of population size on the ability of a strategy to invade another population. This will highlight how complex strategies with long memories outperform simpler strategies.
- Section 4.3 will similarly investigate the ability to defend against an invasion.
- Section 4.4 will investigate the relationship between performance for differing population sizes. This highlights the importance of considering population dynamics over large populations.
- Section 4.5 will calculate the relative fitness of all strategies.

4.1 The special case of $N = 2$

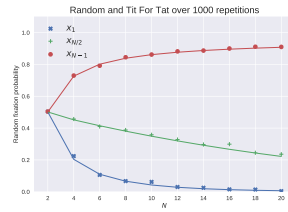
The main fixation probabilities of interest are x_1 and x_{N-1}, these reflect a strategy's ability to invade or resist invasion. For $N = 2$ these two cases coincide. Figure 5a shows all pairwise fixation probabilities for strategies on the vertical column when being matched against probabilities on the horizontal column. This is summarised in Figure 5b and Table 3.



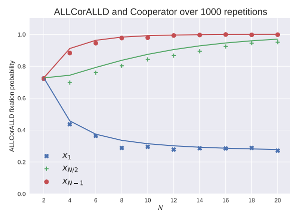
(a) Random and Cooperator



(b) Random and Defector

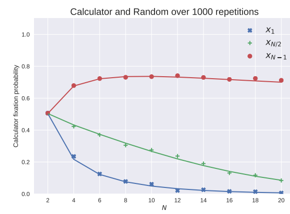


(c) Random and Tit For Tat

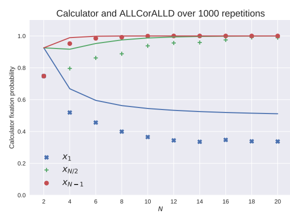


(d) All C or all D and Cooperator

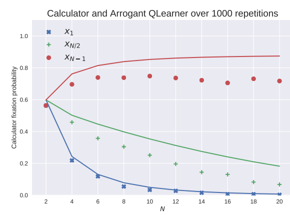
(e) All C or all D and Defector



(f) Calculator and Random



(g) Calculator and All C or all D



(h) Calculator and Arrogant Q learner

(i) All C or all D and Tit For Tat

Figure 4: Comparison of theoretic and actual Moran Process fixation probabilities for **stochastic** strategies

1. The top strategy is an extortionate Zero determinant strategy [20] with parameters $l = 1$ and $s = 1/4$.

2. The Collective strategy has a simple handshake mechanism (a cooperation followed by a defection on the first move). As long as the opponent plays the same handshake and does not defect in the future it cooperates. Otherwise it defects for all rounds [11]. This strategy was specifically designed for Evolutionary processes so it is perhaps also not surprising that it does well here.

3. The finite state machine stragegy

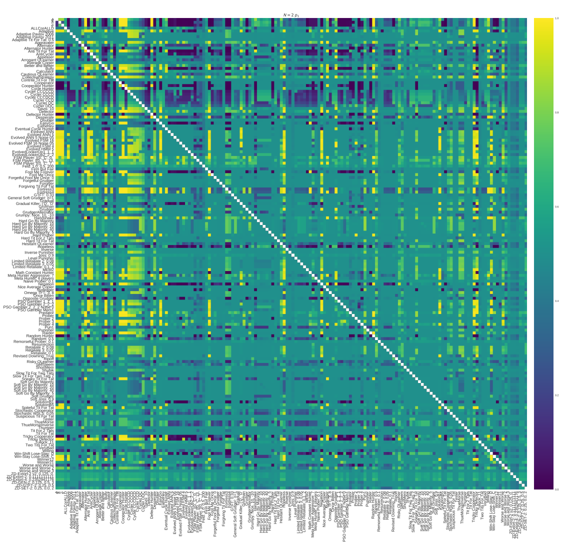
4. The Feld strategy is the corresponding strategy submitted to Axelrod's first tournament [4]: it punishes defections but otherwise defects with a random probability that decays over time.

5. The final strategy in the top five is another extortionate Zero determinant strategy [20] with parameters $l = p$.

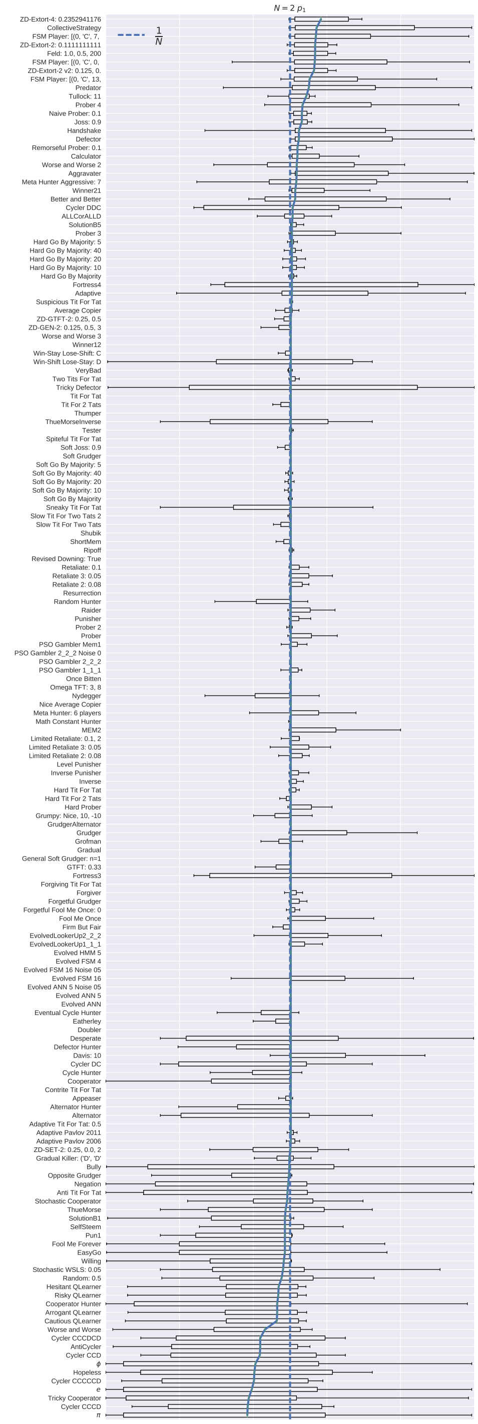
Player	Median p_1	Memory Depth	Stochastic
ZD-Extort-4: 0.23529411764705882, 0.25, 1	0.584	1	True
CollectiveStrategy	0.572	∞	False
FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'...	0.570	1	False
Feld: 1.0, 0.5, 200	0.568	200	True
ZD-Extort-2: 0.1111111111111111, 0.5	0.568	1	True

Table 3: Summary of top five strategies for $N = 2$

As will be demonstrated in Section 4.4 $N = 2$ is a particular case. In the next sections we will pay close attention to strategies who are strong invaders/resistors and shown diagrammatically in Figure 6.



(a) The pairwise fixation probabilities for $N = 2$



(b) The median fixation probabilities for $N = 2$

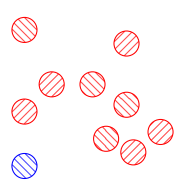
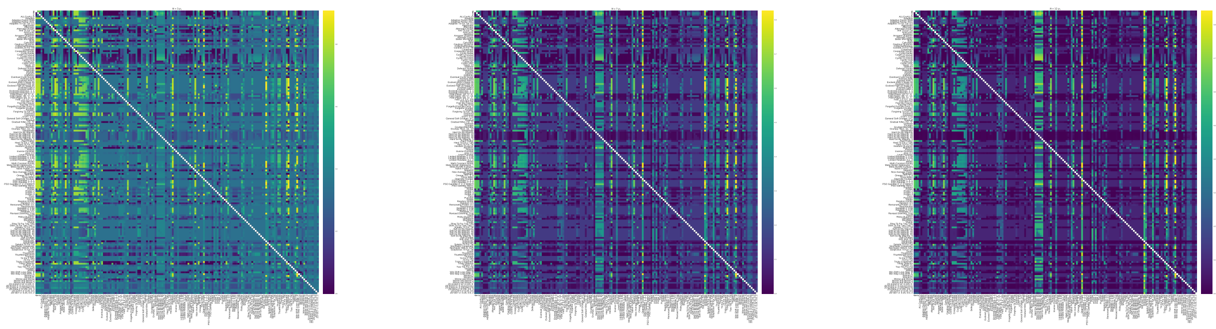


Figure 6: A single individual will successfully invade the population with probability x_1. The group of Individuals will successfully resist with probability x_{N-1}

4.2 Strong invaders

In this section x_i will be investigated: the probability of 1 individual of a given type successfully becoming fixated in a population of $N-1$ other individuals. Figures 7 shows these values for the players along the vertical axis when matched against the players on the horizontal axis. It can be seen that invasion is in general more challenging for $N=7$ and $N=12$ in comparison to $N=3$. This information is summarised in Figure 8 showing the median fixation as well as the neutral fixation for each given scenario.



(a) $N=3$ (b) $N=7$ (c) $N=12$

Figure 7: Pairwise fixation probability x_1 of all strategies

For $N \in \{3, 7, 12\}$ the top five strategies are given in Tables 4.

Player	Median p_1	Memory Depth	Stochastic
CollectiveStrategy	0.403	∞	False
Predator	0.396	9	False
Prober 4	0.368	∞	False
Remorseful Prober: 0.1	0.357	2	True
Worse and Worse 2	0.355	∞	True

(a) $N=3$

Player	Median p_1	Memory Depth	Stochastic
Prober 4	0.177	∞	False
CollectiveStrategy	0.170	∞	False
Worse and Worse 2	0.159	∞	True
Predator	0.158	9	False
Remorseful Prober: 0.1	0.146	2	True

(b) $N=7$

Player	Median p_1	Memory Depth	Stochastic
Prober 4	0.105	∞	False
Worse and Worse 2	0.093	∞	True
Remorseful Prober: 0.1	0.089	2	True
Predator	0.088	9	False
Tester	0.088	∞	False

(c) $N=12$

Table 4: Properties of top five invaders

It can be seen that apart from the Collective strategy, none of the strategies of Table 3 perform well for $N \in \{3, 7, 12\}$. The new high performing strategies are:

- Predator, a finite state machine described in [3].

(a) $N = 3$ (b) $N = 7$ (c) $N = 12$

Figure 8: Median probabilities x_1 of all strategies as well as the neutral fixation probability

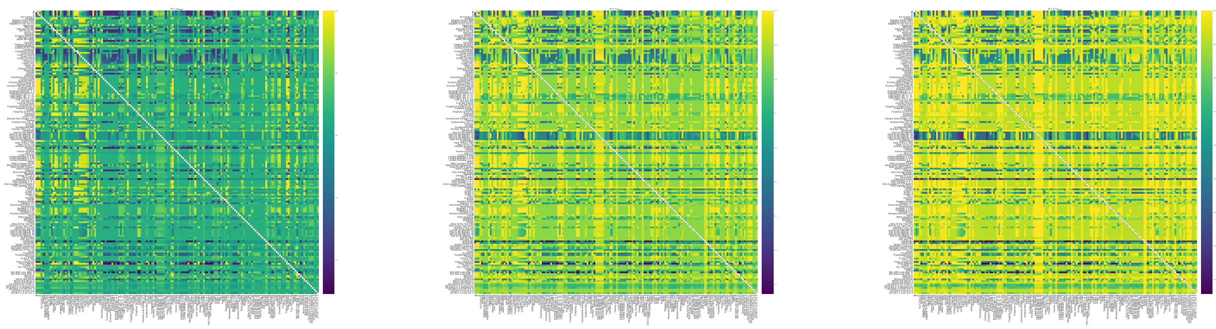
- Prober 4, complex strategy with an initial 20 move sequence of cooperations and defections [14]. This initial sequence serves as some kind of handshake.
- Remorseful Prober, a strategy that will not immediately retaliate when it recognises that the opponent is itself retaliating to a random defection [13].
- Worse and worse 2: plays tit for tat for 20 moves and then defects with with growing probability [14].
- Tester: a strategy submitted to the second of Axelrod's tournaments [5].

As well as noting that the memory length and complexity of these strategies are quite complex it is interesting to note that none of them are akin to memory one strategies. Most are not stochastic.

In the next section the performance in terms of x_{N-1} will be described: what strategies are particularly good at resisting an invasion.

4.3 Strong resistors

Figures 9 show x_{N-1} the players along the vertical axis when matched against the players on the horizontal axis. It can be seen that as the population size N increases the probability of resistance increases. This information is summarised in Figure 10 showing the median fixation as well as the neutral fixation for each given scenario.



(a) $N = 3$ (b) $N = 7$ (c) $N = 12$

Figure 9: Pairwise fixation probability x_{N-1} of all strategies

Table 5 shows the top five strategies when ranked according to x_{N-1} for $N \in \{3, 7, 12\}$. Once again none of the short memory strategies from Section 4.1 perform well for high N.

Three strategies have both high x_1 and high x_{N-1}:

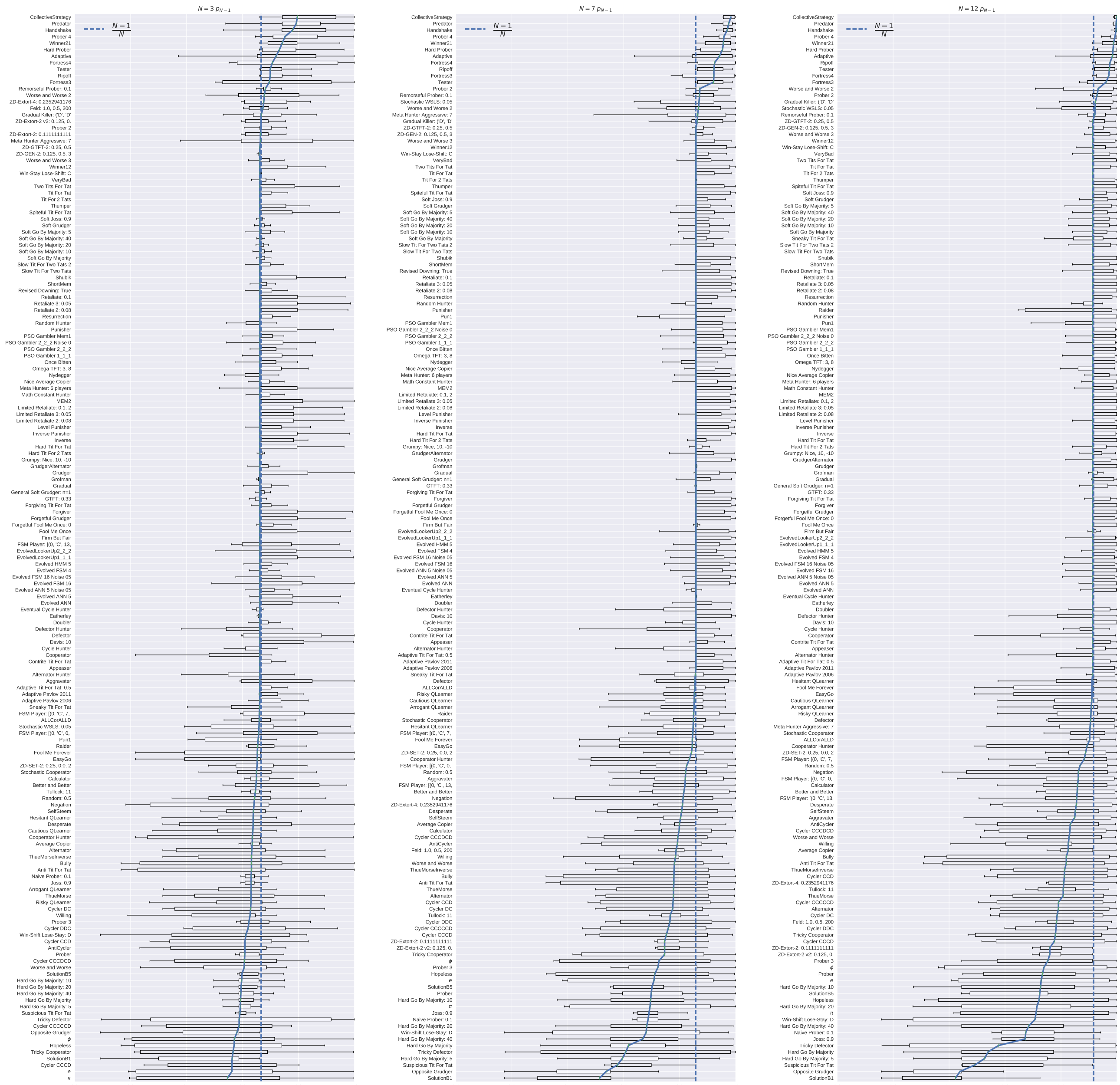
- Collective;
- Predator;
- Prober 4

However, Remorseful Prober, Worse and Worse 2 and Tester no longer do as well. There are two strategies that are only top performers in x_{N-1}:

- Handshake: a slightly less aggressive version of the Collective strategy [22]. As long as the initial sequence is played then it cooperates. Thus it will do well in a population consisting of many members of itself: just as the Collective strategy does. However it is not aggressive enough to invade other populations.
- Winner 21: a strategy that makes it's decision deterministically based on 1 round of it's own strategy and 2 of the opponents strategy [15].

Interestingly none of these strategies are deterministic: this is explained by the need of strategies to have a steady hand when interacting with their own kind. In essence: acting stochastically increase the chance of friendly fire.

It is evident through Sections 4.1, 4.2 and 4.3 that performance of strategies not only depends on the initial population distribution but also that there seems to be a difference depending on whether or not $N > 2$. This will be explored further in the next section.



(a) $N = 2$ (b) $N = 7$ (c) $N = 12$

Figure 10: Median probabilities x_{N-1} of all strategies as well as the neutral fixation probability

Player	Median p_{N-1}	Memory Depth	Stochastic
CollectiveStrategy	0.796	∞	False
Predator	0.792	9	False
Handshake	0.779	∞	False
Prober 4	0.752	∞	False
Winner21	0.742	2	False

(a) $N = 3$

Player	Median p_{N-1}	Memory Depth	Stochastic
CollectiveStrategy	0.983	∞	False
Predator	0.975	9	False
Handshake	0.970	∞	False
Prober 4	0.958	∞	False
Winner21	0.956	2	False

(b) $N = 7$

Player	Median p_{N-1}	Memory Depth	Stochastic
CollectiveStrategy	0.999	∞	False
Handshake	0.997	∞	False
Predator	0.997	9	False
Prober 4	0.993	∞	False
Winner21	0.988	2	False

(c) $N = 12$

Table 5: Properties of top five resistors

4.4 The effect of population size

Figures 11, 12 and 13 show the median rank of each strategy against population size. For all starting populations $i \in \{1, N/2, N-1\}$ the ranks of strategies are relatively stable across the different values of $N > 2$ however for $N = 2$ there is a distinct difference. This confirms what has been discussed in previous sections.

Tables 6, 7 and 8 show the same information for the strategies that rated high for $N = 2$ and $N = 12$.

Player	2	3	4	5	6	7	8	9	10	11	12
ZD-Extort-4: 0.23529411764705882, 0.25, 1	1.0	10.0	103.0	108.0	118.0	120.5	123.5	122.0	124.5	NaN	128.5
CollectiveStrategy	2.0	1.0	1.0	1.0	1.0	2.0	2.0	2.0	2.0	87.0	49.5
FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'...	3.0	105.5	103.0	106.5	112.5	114.0	115.0	110.0	112.0	NaN	114.0
Feld: 1.0, 0.5, 200	4.5	6.0	6.0	99.5	103.0	99.0	108.0	107.0	103.0	97.0	113.0
ZD-Extort-2: 0.1111111111111111, 0.5	4.5	13.5	112.0	113.5	119.5	123.0	125.0	126.5	134.0	NaN	142.5
Prober 4	11.0	3.0	3.0	2.0	2.0	1.0	1.0	1.0	1.0	82.0	1.0
Worse and Worse 2	18.5	5.0	4.0	4.0	4.0	3.0	5.5	3.5	4.0	NaN	2.0
Remorseful Prober: 0.1	16.5	4.0	5.0	5.0	5.0	5.0	93.0	5.0	6.5	84.0	3.0
Predator	9.0	2.0	2.0	3.0	3.0	4.0	3.0	3.5	3.0	88.5	4.5
Tester	84.0	13.5	8.5	51.0	8.5	6.5	5.5	49.5	6.5	NaN	4.5

Table 6: Ranks of some strategies according to x_1 for different population sizes

Tables 9a, 9b and 9c show the correlation coefficients of the ranks in of strategies in differing population size. This is shown graphically in Figure 14. It is immediate to note that how well a strategy performs in any Moran process for $N > 2$ has little to do with the performance for $N = 2$. This illustrates why the strong performance of zero determinant strategies predicted in [20] does not extend to larger populations. This was discussed theoretically in [1] however not observed empirically at the scale presented here.

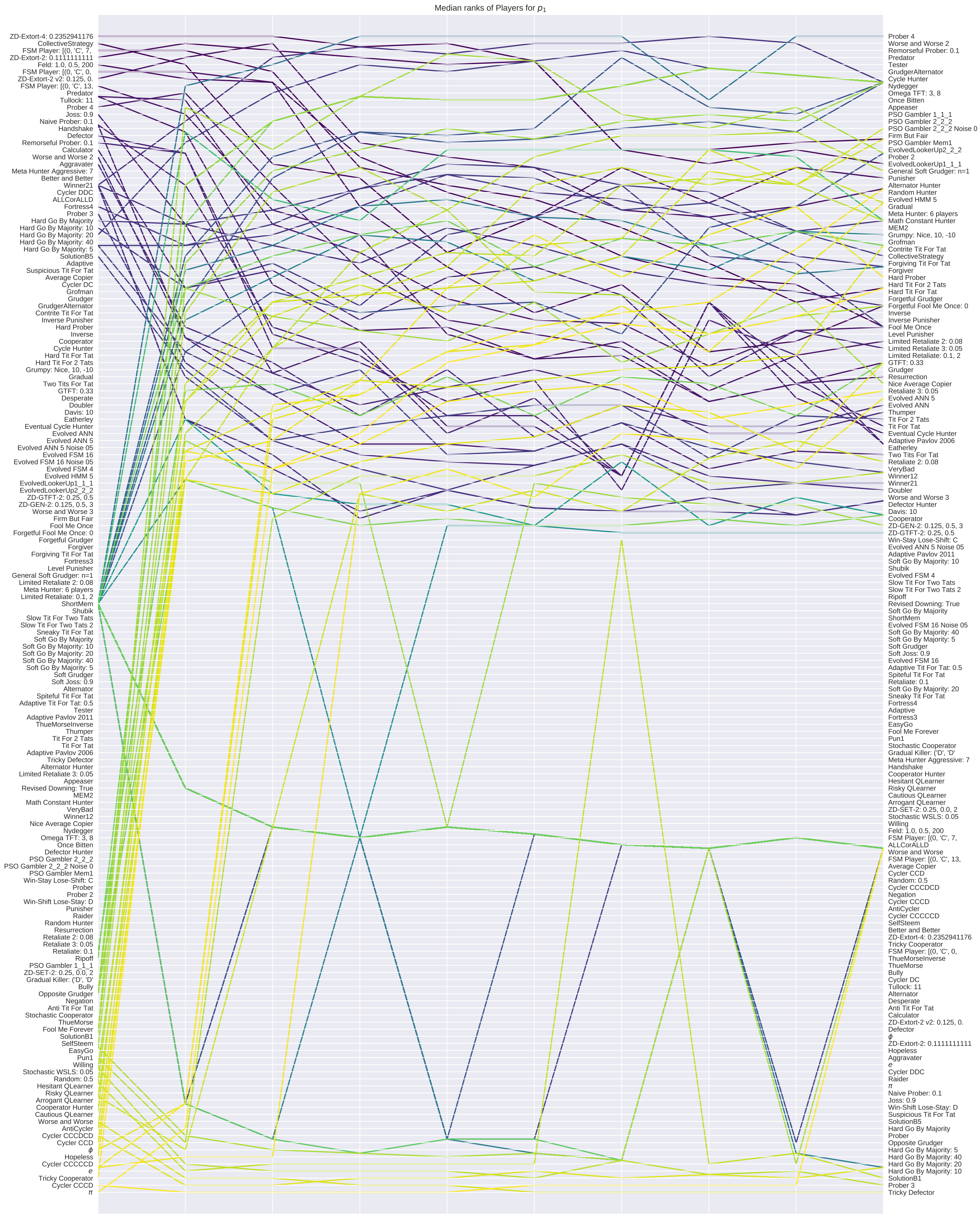


Figure 11: Ranks of all strategies according to x_1 for different population sizes

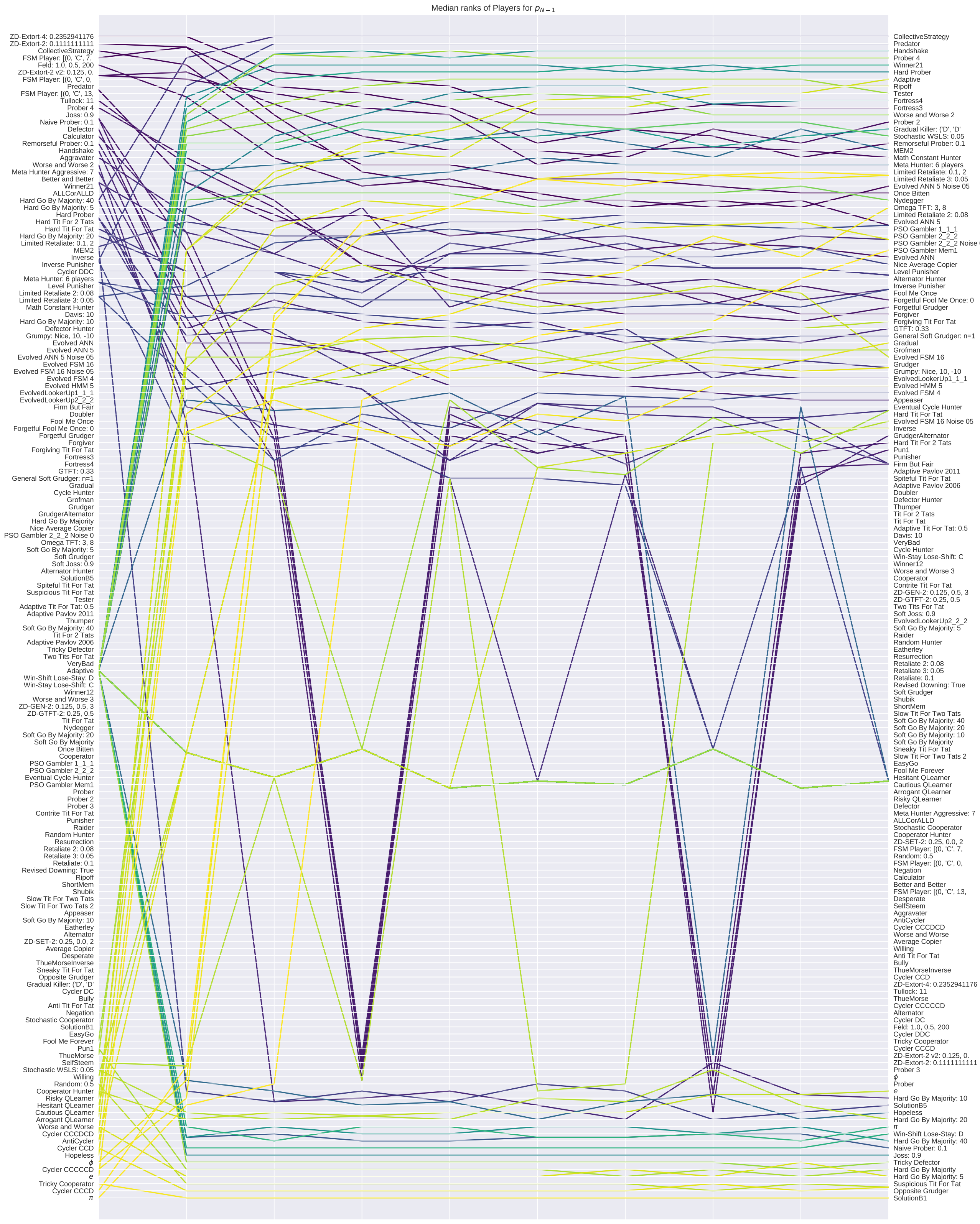


Figure 12: Ranks of all strategies according to x_{N-1} for different population sizes

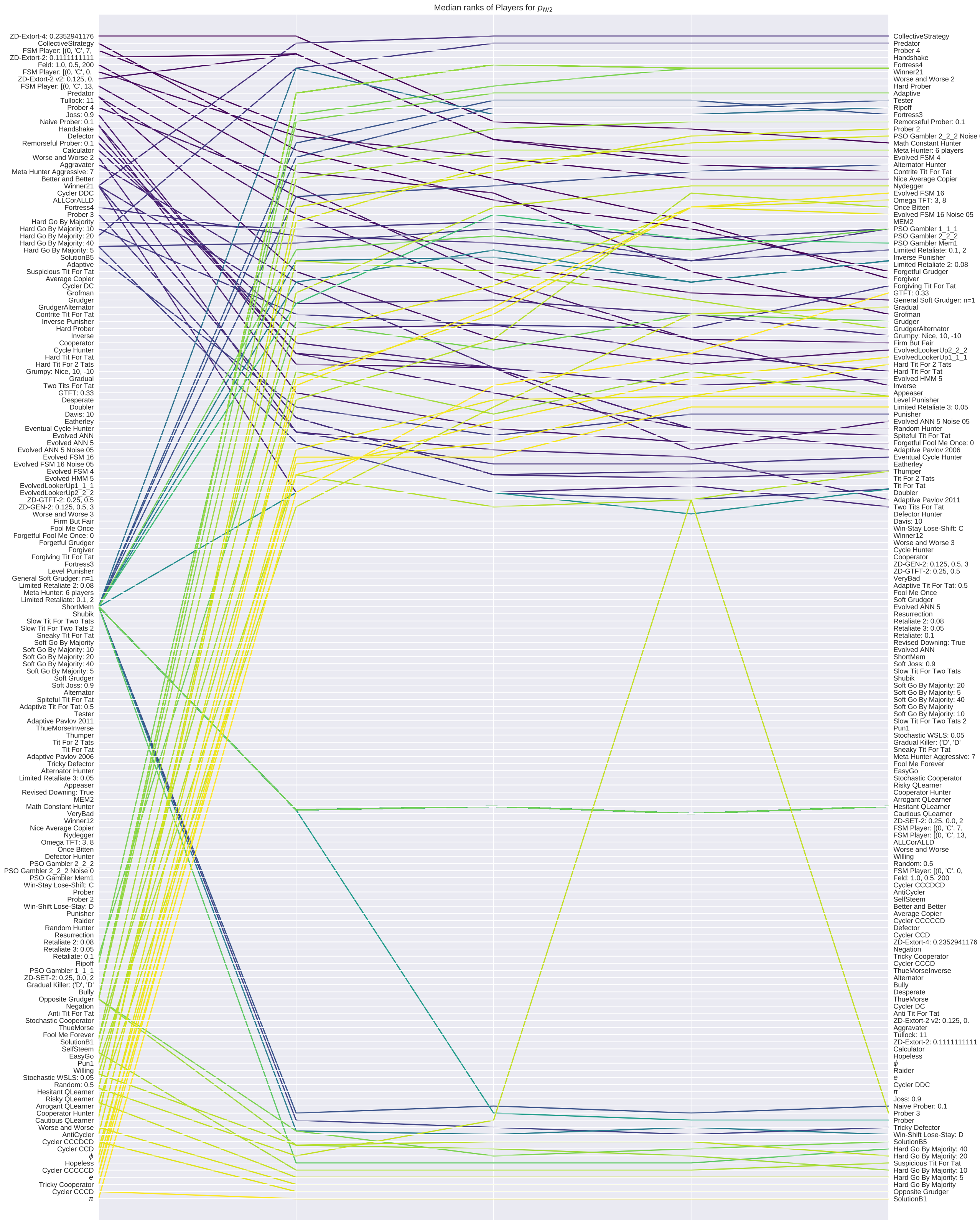


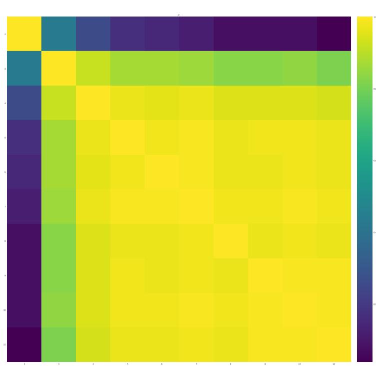
Figure 13: Ranks of all strategies according to $x_{N/2}$ for different population sizes

Player	2	3	4	5	6	7	8	9	10	11	12
ZD-Extort-4: 0.23529411764705882, 0.25, 1	1.0	10.0	103.0	108.0	118.0	120.5	123.5	122.0	124.5	NaN	128.5
CollectiveStrategy	2.0	1.0	1.0	1.0	1.0	2.0	2.0	2.0	2.0	87.0	49.5
FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'...	3.0	105.5	103.0	106.5	112.5	114.0	115.0	110.0	112.0	NaN	114.0
Feld: 1.0, 0.5, 200	4.5	6.0	6.0	99.5	103.0	99.0	108.0	107.0	103.0	97.0	113.0
ZD-Extort-2: 0.1111111111111111, 0.5	4.5	13.5	112.0	113.5	119.5	123.0	125.0	126.5	134.0	NaN	142.5
CollectiveStrategy	3.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Handshake	17.0	3.0	3.0	3.0	3.0	3.0	3.0	2.0	3.0	2.0	2.5
Predator	8.0	2.0	2.0	2.0	2.0	2.0	2.0	3.0	2.0	3.0	2.5
Prober 4	11.0	4.0	4.0	4.0	4.0	4.0	5.0	4.0	6.0	4.5	4.0
Winner21	22.0	5.0	5.0	5.0	5.0	5.0	4.0	5.0	4.0	NaN	5.0

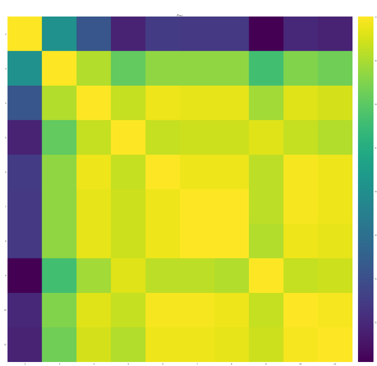
Table 7: Ranks of some strategies according to x_{N-1} for different population sizes

Player	2	4	6	8	10	12
ZD-Extort-4: 0.23529411764705882, 0.25, 1	1.0	103.0	118.0	123.5	124.5	128.5
CollectiveStrategy	2.0	1.0	1.0	2.0	2.0	49.5
FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'...	3.0	103.0	112.5	115.0	112.0	114.0
Feld: 1.0, 0.5, 200	4.5	6.0	103.0	108.0	103.0	113.0
ZD-Extort-2: 0.1111111111111111, 0.5	4.5	112.0	119.5	125.0	134.0	142.5
CollectiveStrategy	2.0	1.0	1.0	1.0	1.0	1.0
Predator	9.0	2.0	2.0	2.0	2.0	2.0
Prober 4	11.0	3.0	3.0	3.0	3.0	3.0
Handshake	14.5	4.0	4.0	4.0	4.0	4.0
Fortress4	29.0	8.5	8.0	7.0	5.0	5.0

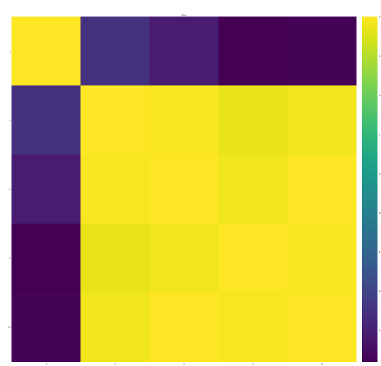
Table 8: Ranks of some strategies according to $x_{N/2}$ for different population sizes



(a) Rank based on x_1



(b) Rank based on x_{N-1}



(c) Rank based on $x_{N/2}$

Figure 14: Heatmap of correlation coefficients of rankings by population size

N	2	3	4	5	6	7	8	9	10	12
2	1.00	0.44	0.26	0.17	0.15	0.12	0.08	0.08	0.08	0.04
3	0.44	1.00	0.92	0.87	0.87	0.86	0.83	0.83	0.84	0.81
4	0.26	0.92	1.00	0.97	0.96	0.97	0.95	0.95	0.95	0.94
5	0.17	0.87	0.97	1.00	0.98	0.99	0.97	0.98	0.98	0.97
6	0.15	0.87	0.96	0.98	1.00	0.99	0.97	0.97	0.98	0.97
7	0.12	0.86	0.97	0.99	0.99	1.00	0.98	0.98	0.99	0.98
8	0.08	0.83	0.95	0.97	0.97	0.98	1.00	0.97	0.98	0.97
9	0.08	0.83	0.95	0.98	0.97	0.98	0.97	1.00	0.99	0.99
10	0.08	0.84	0.95	0.98	0.98	0.99	0.98	0.99	1.00	0.99
12	0.04	0.81	0.94	0.97	0.97	0.98	0.97	0.99	0.99	1.00

(a) Correlation coefficients for ranks for invasion

N	2	3	4	5	6	7	8	9	10	12
2	1.00	0.61	0.42	0.29	0.35	0.34	0.34	0.21	0.30	0.29
3	0.61	1.00	0.91	0.81	0.87	0.87	0.87	0.76	0.85	0.83
4	0.42	0.91	1.00	0.93	0.98	0.97	0.97	0.89	0.96	0.95
5	0.29	0.81	0.93	1.00	0.93	0.94	0.94	0.96	0.93	0.91
6	0.35	0.87	0.98	0.93	1.00	0.98	0.98	0.92	0.99	0.98
7	0.34	0.87	0.97	0.94	0.98	1.00	1.00	0.92	0.99	0.98
8	0.34	0.87	0.97	0.94	0.98	1.00	1.00	0.91	0.98	0.97
9	0.21	0.76	0.89	0.96	0.92	0.92	0.91	1.00	0.93	0.94
10	0.30	0.85	0.96	0.93	0.99	0.99	0.98	0.93	1.00	0.99
12	0.29	0.83	0.95	0.91	0.98	0.98	0.97	0.94	0.99	1.00

(b) Correlation coefficients for ranks for resistance

N	2	4	6	8	10
2	1.00	0.25	0.19	0.12	0.13
4	0.25	1.00	0.99	0.97	0.98
6	0.19	0.99	1.00	0.98	1.00
8	0.12	0.97	0.98	1.00	0.99
10	0.13	0.98	1.00	0.99	1.00

(c) Correlation coefficients for ranks for coexistance

Table 9: Correlation coefficients of rankings by population size

4.5 Relative fitness

Under the assumption of a constant relative fitness r between two strategies [19] the formula for x_i (for given N, r is:

$$x_i = x_i(r) = \frac{1 - \frac{1}{r^i}}{1 - \frac{1}{r^N}} \tag{10}$$

Figure 15 shows this function for $N = 10$ and $i \in \{1, 5, 10\}$.

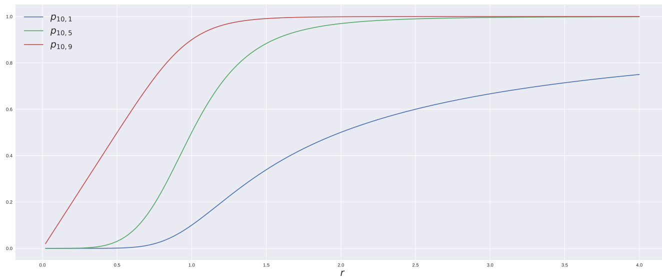


Figure 15: $x_i(r)$

The first and second derivative of (10) is given by equations (11) and (12).

$$\frac{dx_i}{dr} = \frac{r^{N-i-1}}{r^{2N} - 2r^N + 1} \left(-Nr^i + N + ir^N - i\right) \tag{11}$$

$$\frac{d^2x_i}{dr^2} = \frac{r^{N-i-2}}{\left(r^N - 1\right)^3} \left(2N^2 \left(r^i - 1\right) + N \left(r^N - 1\right) \left(N \left(r^i - 1\right) - 2i + r^i - 1\right) - i \left(i + 1\right) \left(r^N - 1\right)^2\right) \tag{12}$$

Using these, Halley's method [2] can be used to efficiently numerically invert $x_i(r)$ to obtain a theoretic relative fitness r that gives the calculated $x_i(r)$ between two strategies for a given N, i.

5 Conclusion

A detailed empirical analysis of 164 strategies of the IPD within a pairwise Moran process has been carried out. All $\binom{164}{2} = 13,366$ possible ordered pairs of strategies have been placed in a Moran process with different starting values allowing the each strategy to attempt to invade the other.

This is the largest such experiment carried out and has lead to many insights.

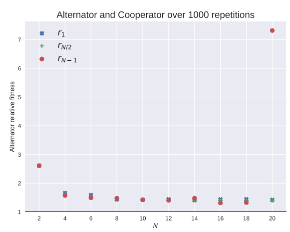
When studying evolutionary processes it is vital to consider $N > 2$ as the special case for $N = 2$ cannot be used to extrapolate performance in bigger populations. This was shown both observationally in Sections 4.2 and 4.3 but also by considering the correlation of the ranks in different population sizes in Section 4.4.

For $N = 2$, memory one strategies perform well, in particular as predicted by [20] zero determinant strategies rank highly. However, there are no memory one strategies in the top 5 performing strategies for $N > 3$. This is due to their lack of sophistication which allows them to recognise and adjust to their opponent.

It is felt that these findings are important for the ongoing understanding of population dynamics and offer evidence for some of the shortcomings of short memory which has started to be recognised by the community [8].

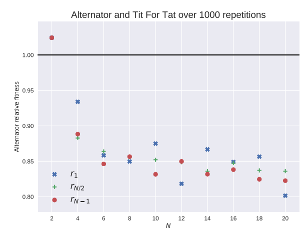
All source code for this work has been written in a sustainable manner: it is open source, under version control and tested which ensures that all results can be reproduced [21, 23, 26]. The raw data as well as the processed data has also been properly archived.

There are various areas for further work to build on this. Firstly, an analysis of the effect of noise would offer insights about the stability of the findings. It would also be possible to consider three or more types of strategy in the population and finally mutation would also offer an interesting dimension to explore.

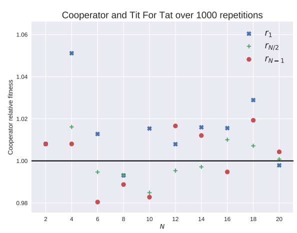


(a) Alternator and Cooperator

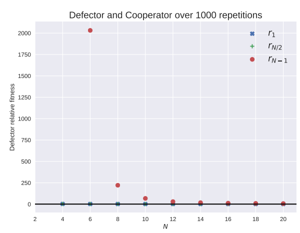
(b) Alternator and Defector



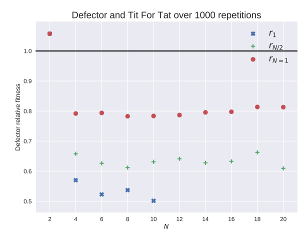
(c) Alternator and Tit For Tat



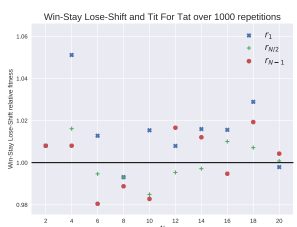
(d) Cooperator and Tit For Tat



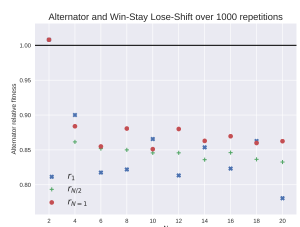
(e) Defector and Cooperator



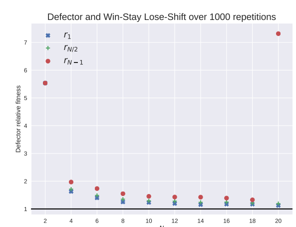
(f) Defector and Tit For Tat



(g) Win Stay Lose Shift and Tit For Tat

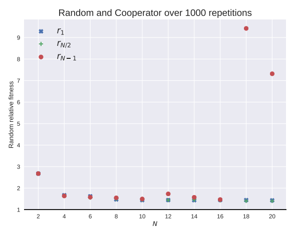


(h) Alternator and Win Stay Lose Shift

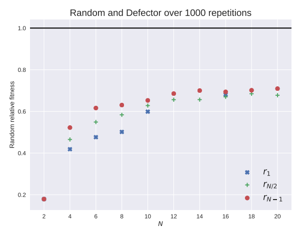


(i) Defector and Win Stay Lose Shift

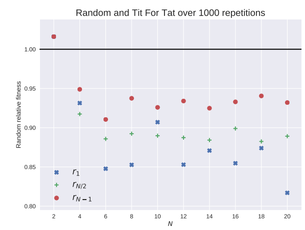
Figure 16: Estimated relative fitness for **deterministic** strategies



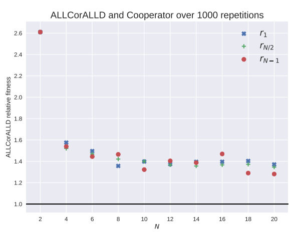
(a) Random and Cooperator



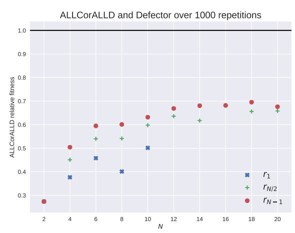
(b) Random and Defector



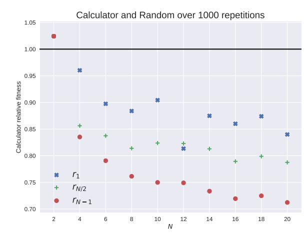
(c) Random and Tit For Tat



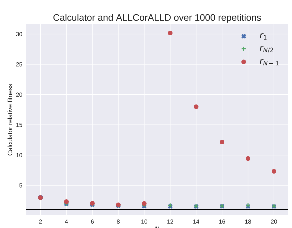
(d) All C or all D and Cooperator



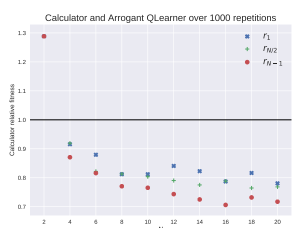
(e) All C or all D and Defector



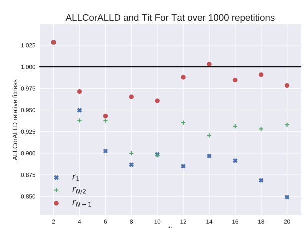
(f) Calculator and Random



(g) Calculator and All C or all D



(h) Calculator and Arrogant Q learner



(i) All C or all D and Tit For Tat

Figure 17: Estimated relative fitness for **stochastic** strategies

Acknowledgements

This work was performed using the computational facilities of the Advanced Research Computing @ Cardiff (ARCCA) Division, Cardiff University.

A variety of software libraries have been used in this work:

- The Axelrod library (IPD strategies and Moran processes) [24].
- The matplotlib library (visualisation) [9].
- The pandas and numpy libraries (data manipulation) [16, 25].

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A List of players

1. Random Hunter
2. Cooperator
3. Adaptive
4. Random: 0.5
5. Contrite Tit For Tat
6. Cycle Hunter
7. Adaptive Tit For Tat: 0.5
8. Cooperator Hunter
9. Stochastic Cooperator
10. Remorseful Prober: 0.1
11. Cycler CCCCCD
12. Tricky Cooperator
13. Win-Shift Lose-Stay: D
14. Aggravater
15. Tricky Defector
16. π
17. Resurrection
18. Stochastic WSLS: 0.05
19. Win-Stay Lose-Shift: C
20. ALLCorALLD
21. Tullock: 11
22. Alternator
23. Retaliate: 0.1
24. Cycler CCCD
25. Worse and Worse
26. Alternator Hunter
27. Retaliate 2: 0.08
28. Worse and Worse 2
29. AntiCycler
30. Winner21
31. Cycler CCD
32. Suspicious Tit For Tat
33. Retaliate 3: 0.05
34. Willing
35. Worse and Worse 3
36. Anti Tit For Tat
37. Adaptive Pavlov 2006
38. Revised Downing: True
39. Cycler DC
40. ZD-Extort-2: 0.1111111111111111, 0.5
41. Adaptive Pavlov 2011
42. ZD-Extort-2 v2: 0.125, 0.5, 1
43. Ripoff
44. VeryBad
45. Cycler DDC
46. Appeaser
47. Cycler CCCDCD
48. Winner12
49. SelfSteem
50. Risky QLearner

51. ZD-Extort-4: 0.23529411764705882, 0.25, 1
52. Arrogant QLearner
53. Thumper
54. ZD-GEN-2: 0.125, 0.5, 3
55. Defector
56. ZD-GTFT-2: 0.25, 0.5
57. Average Copier
58. Davis: 10
59. ShortMem
60. Meta Hunter: 6 players
61. ThueMorseInverse
62. Tester
63. Better and Better
64. Spiteful Tit For Tat
65. Shubik
66. Defector Hunter
67. ThueMorse
68. ZD-SET-2: 0.25, 0.0, 2
69. e
70. Slow Tit For Two Tats
71. Desperate
72. Bully
73. SolutionB1
74. Meta Hunter Aggressive: 7 players
75. Calculator
76. Slow Tit For Two Tats 2
77. Tit For Tat
78. Tit For 2 Tats
79. FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'), (1, 'C', 11, 'D'), (1, 'D', 11, 'D'), (2, 'C', 8, 'D'), (2, 'D', 8, 'C'), (3, 'C', 3, 'C'), (3, 'D', 12, 'D'), (4, 'C', 6, 'C'), (4, 'D', 3, 'C'), (5, 'C', 11, 'C'), (5, 'D', 8, 'D'), (6, 'C', 13, 'D'), (6, 'D', 14, 'C'), (7, 'C', 4, 'D'), (7, 'D', 2, 'D'), (8, 'C', 14, 'D'), (8, 'D', 8, 'D'), (9, 'C', 0, 'C'), (9, 'D', 10, 'D'), (10, 'C', 8, 'C'), (10, 'D', 15, 'C'), (11, 'C', 6, 'D'), (11, 'D', 5, 'D'), (12, 'C', 6, 'D'), (12, 'D', 9, 'D'), (13, 'C', 9, 'D'), (13, 'D', 8, 'D'), (14, 'C', 8, 'D'), (14, 'D', 13, 'D'), (15, 'C', 4, 'C'), (15, 'D', 5, 'C')], 1, C
80. Doubler
81. Sneaky Tit For Tat
82. SolutionB5
83. Soft Grudger
84. EasyGo
85. CollectiveStrategy
86. Soft Joss: 0.9
87. Eatherley
88. Cautious QLearner
89. Two Tits For Tat
90. Firm But Fair
91. Hard Go By Majority
92. Handshake
93. FSM Player: [(0, 'C', 13, 'D'), (0, 'D', 12, 'D'), (1, 'C', 3, 'D'), (1, 'D', 4, 'D'), (2, 'C', 14, 'D'), (2, 'D', 9, 'D'), (3, 'C', 0, 'C'), (3, 'D', 1, 'D'), (4, 'C', 1, 'D'), (4, 'D', 2, 'D'), (5, 'C', 12, 'C'), (5, 'D', 6, 'C'), (6, 'C', 1, 'C'), (6, 'D', 14, 'D'), (7, 'C', 12, 'D'), (7, 'D', 2, 'D'), (8, 'C', 7, 'D'), (8, 'D', 9, 'D'), (9, 'C', 8, 'D'), (9, 'D', 0, 'D'), (10, 'C', 2, 'C'), (10, 'D', 15, 'C'), (11, 'C', 7, 'D'), (11, 'D', 13, 'D'), (12, 'C', 3, 'C'), (12, 'D', 8, 'D'), (13, 'C', 7, 'C'), (13, 'D', 10, 'D'), (14, 'C', 10, 'D'), (14, 'D', 7, 'D'), (15, 'C', 15, 'C'), (15, 'D', 11, 'D')], 1, C
94. Fool Me Once
95. Prober 3
96. Eventual Cycle Hunter
97. Forgetful Grudger
98. Prober 4
99. Evolved ANN
100. Forgetful Fool Me Once: 0.05
101. Fool Me Forever
102. Hard Go By Majority: 10
103. Pun1
104. Evolved ANN 5
105. Fortress4
106. Hard Go By Majority: 20
107. Evolved ANN 5 Noise 05
108. PSO Gambler 2_2_2
109. Fortress3
110. Hard Go By Majority: 40
111. PSO Gambler 1_1_1
112. Evolved FSM 4
113. Forgiving Tit For Tat
114. Naive Prober: 0.1
115. GTFT: 0.33
116. Soft Go By Majority
117. Hard Go By Majority: 5
118. Nice Average Copier
119. Hard Prober
120. Forgiver
121. Opposite Grudger
122. General Soft Grudger: n=1,d=4,c=2
123. Evolved FSM 16
124. Hard Tit For 2 Tats
125. Soft Go By Majority: 10
126. Once Bitten
127. Hard Tit For Tat
128. EvolvedLookerUp2_2_2
129. Nydegger
130. Prober 2
131. Soft Go By Majority: 20
132. Hesitant QLearner
133. Evolved FSM 16 Noise 05
134. PSO Gambler 2_2_2 Noise 05

135. Omega TFT: 3, 8
136. FSM Player: [(0, 'C', 0, 'C'), (0, 'D', 3, 'C'), (1, 'C', 5, 'D'), (1, 'D', 0, 'C'), (2, 'C', 3, 'C'), (2, 'D', 2, 'D'), (3, 'C', 4, 'D'), (3, 'D', 6, 'D'), (4, 'C', 3, 'C'), (4, 'D', 1, 'D'), (5, 'C', 6, 'C'), (5, 'D', 3, 'D'), (6, 'C', 6, 'D'), (6, 'D', 6, 'D'), (7, 'C', 7, 'D'), (7, 'D', 5, 'C')], 1, C
137. EvolvedLookerUp1_1_1
138. Soft Go By Majority: 40
139. Inverse
140. Hopeless
141. MEM2
142. Soft Go By Majority: 5
143. Inverse Punisher
144. Predator
145. Joss: 0.9
146. ϕ
147. Gradual
148. Level Punisher
149. Gradual Killer: ('D', 'D', 'D', 'D', 'D', 'C', 'C')
150. Limited Retaliate: 0.1, 20
151. Grofman
152. Limited Retaliate 2: 0.08, 15
153. PSO Gambler Mem1
154. Negation
155. Punisher
156. Grudger
157. Prober
158. GrudgerAlternator
159. Limited Retaliate 3: 0.05, 20
160. Raider
161. Evolved HMM 5
162. Math Constant Hunter
163. Feld: 1.0, 0.5, 200
164. Grumpy: Nice, 10, -10