A numerical study of fixation probabilities for strategies in the Iterated Prisoner's Dilemma

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Abstract

The Iterated Prisoner's Dilemma is a well established framework for the study of emergent behaviour. In this paper an extensive numerical study of the evolutionary dynamics of this framework are presented.

Fixation probabilities for Moran processes are obtained for 172 different strategies. This is done in both a standard 200 turn interaction and a noisy setting.

To the authors knowledge this is the largest such study. It allows for insights about the behaviour and performance of strategies with regard to their survival in an evolutionary setting.

1 Introduction

Since the formulation of the Moran Process in [9], this model of evolutionary population dynamics has been used to gain insights about the evolutionary stability of strategies in a number of settings. Similarly since the first Iterated Prisoner's Dilemma (IPD) tournament described in [2] the Prisoner's dilemma has been used to understand the evolution of cooperative behaviour in complex systems.

The analytical models of a Moran process are based on the relative fitness between two strategies and take this to be a fixed value r [11]. This is a valid model for simple strategies of the Prisoner's Dilemma such as to always cooperate or always defect. This manuscript provides a detailed numerical analysis of 172 complex and adaptive strategies for the IPD. In this case the relative fitness of a strategy is dependent on the population distribution.

Further deviations from the analytical model occur when interactions between players are subject to uncertainty. This is referred to as noise and has been considered in the IPD setting in [4, 10, 14]. Noise is also considered here.

This work provides answers to the following questions:

- 1. What strategies are good invaders?
- 2. What strategies are good at resisting invasion?
- 3. How does the population size affect these findings?

Figure 1 shows a diagrammatic representation of the Moran process. The Moran process is a stochastic birth death process on a finite population in which the population size stays constant over time. Individuals are **selected** according to a given fitness landscape. Once selected, a given individual is reproduced and similarly another individual is chosen to be removed from the population. In some settings mutation is also considered but without mutation (the case considered in this work) this process will arrive at an absorbing state where the population is entirely made up of a single individual. The probability with which a given strategy is the survivor is called the absorption probability. A more detailed analytic description of this is given in Section 3.

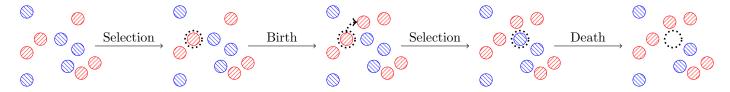


Figure 1: A diagrammatic representation of a Moran process

The Moran process was initially introduced in [9] in a genetic setting. It has sine been used in a variety of settings including the understanding of the spread of cooperative behaviour. However, as stated before, these mainly consider non sophisticated strategies. Some work has looked at evolutionary stability of strategies within the Prisoner's Dilemma [7]

but this is not done in the more widely used setting of the Moran process but in terms of infinite population stability. In [3] Moran processes are looked at in a theoretic framework for a small subset of strategies. In [6] machine learning techniques are used to train a strategy capable of resisting invasion and also invade any memory one strategy.

The contribution of this work is a detailed and extensive analysis of absorption probabilities for 172 strategies. These strategies and the numerical simulations are from [12] which is an open source research library written for the study of the IPD. The strategies and simulation frameworks are automatically tested in accordance to best practice. The large number of strategies are available thanks to the open source nature of the project with over 40 contributions made by different programmers.

Section 2 will explain the methodological approach used, Section 3 will validate the methodology by comparing simulated results to analytical results. The main results of this manuscript are presented in Section 4 which will present a detailed analysis of all the data generated. Finally, Section 5 will conclude and offer future avenues for the work presented here.

2 Methodology

To carry out this large numerical experiment 172 strategies are used from [12]. These include 169 default strategies in the library at the time (excluding strategies classified as having a long run time) as well as the following 3 finite state machine machine strategies [1]:

Appendix A shows all the players in question. More information about each player can be obtained in the documentation for [12]. The memory depth of the used strategies is shown in Table 1a.

Memory Depth	0	1	2	3	4	5	6	9	10	11	12	16	20	40	200	∞
Count	3	31	12	8	2	6	1	1	5	1	1	2	2	2	1	94

(a) Memory depth

Stochastic	Count
False	123
True	49

(b) Stochastic versus deterministic

Table 1: Summary of properties of used strategies

All strategies are paired and these pairs are used in 2000 repetitions of a Moran process assuming a starting population of (N/2, N/2). This is repeated for even N between 2 and 14. The fixation probability is then estimated for each value of N.

Note that due to the high computational cost of these experiments, for any given interaction between two players within the Moran process the outcome is sampled from a pre computed cache of 1000 match outcomes. This is carried out using the approximate Moran process implemented in [12].

As an example, Figure 2 shows the scores between two players that over the 1000 outcomes gives 971 different scores. A variety of software libraries have been used in this work:

- The Axelrod library (IPD strategies and Moran processes) [12].
- The matplotlib library (visualisation) [5].
- The pandas and numpy libraries (data manipulation) [8, 13].

Section 3 will validate this approach against theoretic results.

3 Validation

As described in [11] Consider the payoff matrix:

$$M = \begin{pmatrix} a, b \\ c, d \end{pmatrix} \tag{1}$$

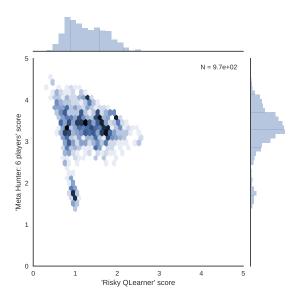


Figure 2: All possible scores for the pair of strategies that have the most different number of match outcomes

The expected payoffs of i players of the first type in a population with N-i players of the second type are given by:

$$F_i = \frac{a(i-1) + b(N-i)}{N-1} \tag{2}$$

$$G_i = \frac{ci + d(N - i - 1)}{N - 1} \tag{3}$$

With an intensity of selection ω the fitness of both strategies is given by:

$$f_i = 1 - \omega + \omega F_i \tag{4}$$

$$g_i = 1 - \omega + \omega G_i \tag{5}$$

The transitions within the birth death process that underpins the Moran process are then given by:

$$p_{i,i+1} = \frac{if_i}{if_i + (N-i)g_i} \frac{N-i}{N}$$
 (6)

$$p_{i,i-1} = \frac{(N-i)g_i}{if_i + (N-i)g_i} \frac{i}{N}$$
(7)

$$p_{ii} = 1 - p_{i,i+1} - p_{i,i-1} \tag{8}$$

Using this it is a known result that the fixation probability of the first strategy in a population of i individuals of the first type (and N-i individuals of the second. We have:

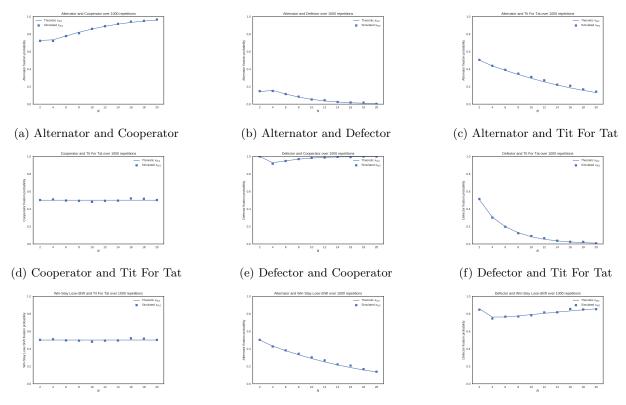
$$x_{i} = \frac{1 + \sum_{j=1}^{i-1} \prod_{k=1}^{j} \gamma_{j}}{1 + \sum_{j=1}^{N-1} \prod_{k=1}^{j} \gamma_{j}}$$
(9)

where:

$$\gamma_j = \frac{p_{j,j-1}}{p_{j,j+1}}$$

Using this comparisons of $x_{N/2}$ are shown in Figure 3. Note that these are all deterministic strategies and show a perfect match up between the expected value of (9) and the actual Moran process for all strategies pairs.

Figure 4 shows the fixation probabilities for stochastic strategies. These are no longer a good match which highlights the weakness of the analytical formulae that relies on the average payoffs. A detailed analysis of the 172 strategies considered will be shown in the next Section.



(g) Win Stay Lose Shift and Tit For Tat (h) Alternator and Win Stay Lose Shift (i) Defector and Win Stay Lose Shift

Figure 3: Comparison of theoretic and actual Moran Process fixation probabilities for deterministic strategies

4 Numerical results

Figures 6 and ?? shows the fixation rates of each player on the y axis against each player on the x axis.

Figure ?? and ?? show the distribution of the fixation rates for all strategies.

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Figures 11 and 12 show the median rank of each strategy against population size in the standard and noisy settings. Note that these ranks are not necessarily integers as group ties are given the average rank.

Tables 2, 3, 4 and 5 show the rankings across population size based for the top on bottom performers across the extreme population sizes.

Player				2	4	6	8	1	0 12	14
ZD-Extor	t-4: 0.2352941176470	5882, 0.2	25, 1 1	.0 10	7.0	122.0	127.0	127.	0 85.0	68.5
Meta Wir	nner Memory One: 31	players	2	2.0 113	3.5	120.5	126.0	134.	0 129.0	28.0
Feld: 1.0,	0.5, 200		3	3.0 100	6.0	116.0	122.0	126.	0 127.0	132.0
ZD-Extor	t-2: 0.1111111111111	111, 0.5	4	1.0 123	3.0	136.0	147.0	146.	0 138.0	144.0
ZD-Extor	t-2 v2: 0.125, 0.5, 1		5	5.0 123	1.0	133.0	146.0	147.	0 140.0	146.0
	Player	2	4	6		8	10	12	14	
	Cycler CCCCCD	168.0	153.0	149.0	144	.0 14	40.5	145.0	142.0	
	$e^{}$	169.0	171.0	158.0	156	.0 15	55.0	157.0	135.0	
	Cycler CCCD	170.0	157.0	150.0	149	.0 1	48.0	150.0	151.5	
	Tricky Cooperator	171.0	158.5	152.0	150	.0 1	36.0	159.0	101.0	
	π	172.0	172.0	159.0	159	0.018	56.0	147.5	131.0	

Table 2: Performance across population sizes of top and bottom performing strategies in population size N=2

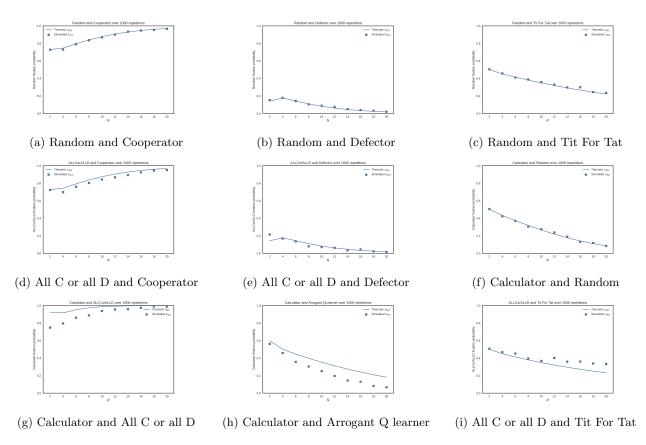


Figure 4: Comparison of theoretic and actual Moran Process fixation probabilities for **stochastic** strategies

Player	2	4	6	8	10	12	14
Fortress4	82.5	11.0	8.0	8.0	5.0	1.0	1.0
Fortress3	82.5	15.0	13.0	12.0	76.5	16.0	2.0
$EvolvedLooker Up 2_2_2$	82.5	59.5	59.5	58.5	76.5	62.5	3.0
Predator	11.0	2.0	2.0	2.0	2.0	2.0	4.0
Tricky Defector	82.5	169.0	170.0	162.0	161.0	3.0	5.0
Player	2	4	6	8	10	12	14
Player Hard Go By Majority: 40	32.0	4 163.5	6 165.5	8 167.5	10 167.0	12 167.5	14 167.0
-							
Hard Go By Majority: 40	32.0	163.5	165.5	167.5	167.0	167.5	167.0
Hard Go By Majority: 40 Hard Go By Majority: 5	32.0 32.0	163.5 163.5	165.5 165.5	167.5 167.5	167.0 167.0	167.5 167.5	167.0 169.5

Table 3: Performance across population sizes of top and bottom performing strategies in population size N=14

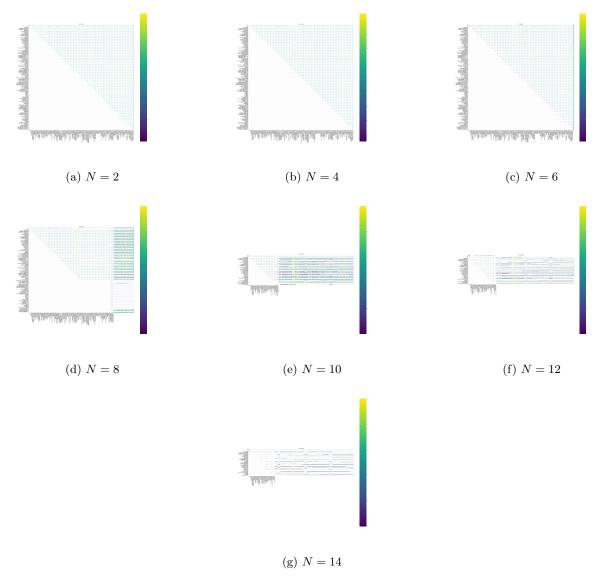


Figure 5: Fixation probabilities of all strategies $\,$

Player				2	4	6	8	10	12	14
MEM2				1.0	16.0	23.0	18.0	5.0	2.0	5.5
FSM Pl	ayer: $[(0, 'C', 0, 'C'),$	(0, D', 3)	3, 'C'	2.0	28.0	46.0	29.0	9.0	42.0	13.0
Retaliat	se 2: 0.08			3.5	19.5	43.0	33.5	48.0	63.0	63.0
Retaliat	se: 0.1			3.5	23.0	39.5	48.5	78.5	113.0	115.0
Predato	r			5.0	34.0	51.5	43.0	36.0	18.5	14.0
	Player	2	4	6	;	8	10	12	14	
	Arrogant QLearner	168.0	162.0	163.0	158	3.0 15	52.0	145.0	155.5	
	Cautious QLearner	170.0	158.0	165.0	156	6.0 - 15	63.0 1	138.0	138.0	
	Cooperator Hunter	170.0	157.0	146.0	143	3.0 13	85.0 1	141.0	127.0	
	Hesitant QLearner	170.0	162.0	164.0	159	0.0 13	34.0 1	129.0	121.0	
	Cycler CCCCCD	172.0	172.0	167.0	165	5.0 16	66.0	157.5	152.0	

Table 4: Performance across population sizes of top and bottom performing strategies in population size N=2 (with noise)

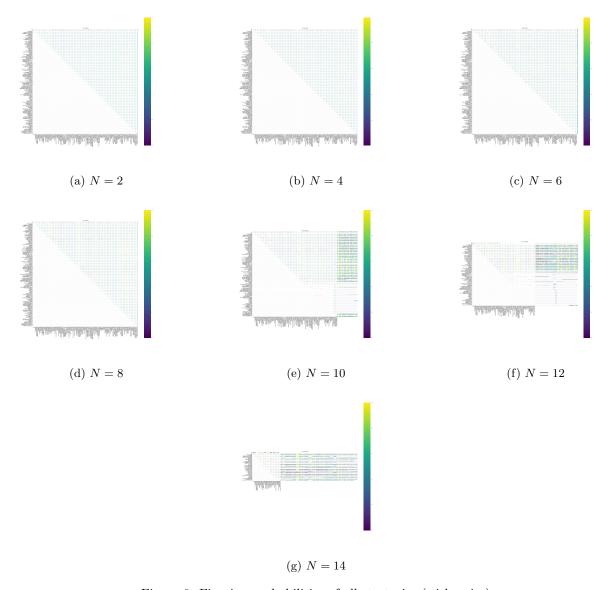


Figure 6: Fixation probabilities of all strategies (with noise)

Player	2	4	6	8	10	12	14
PSO Gambler 2_2_2	92.0	115.0	101.0	40.0	13.0	4.0	1.0
Prober 3	43.0	139.0	151.0	154.0	112.0	7.0	2.0
Tricky Defector	96.5	121.0	118.5	111.0	41.0	60.0	3.0
Prober	55.0	142.0	152.0	163.0	158.0	54.0	4.0
Fool Me Once	9.0	11.5	18.0	28.0	2.0	6.0	5.5
Player	2	4	6	8	10	12	2 14
Hard Go By Majority: 10	65.0	141.0	159.0	168.0	168.0	168.0	168.0
Hard Go By Majority: 20	72.5	145.5	162.0	167.0	170.0	169.5	5 169.0
Hard Go By Majority: 5	75.0	151.0	168.0	171.0	169.0	169.5	5 170.0
Hard Go By Majority: 40	71.0	149.0	169.5	170.0	171.0	171.0	171.0
Hard Go By Majority	76.0	159.0	172.0	172.0	172.0	172.0	172.0

Table 5: Performance across population sizes of top and bottom performing strategies in population size N=14 (with noise)

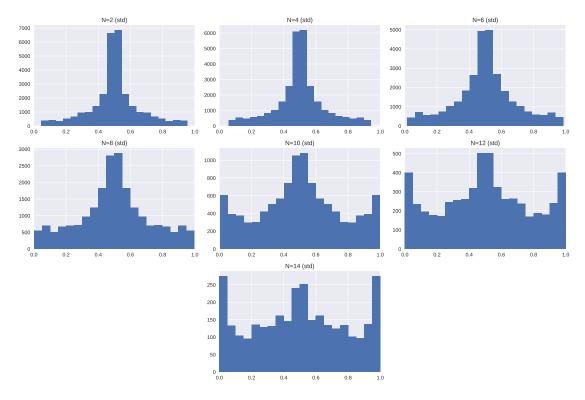


Figure 7: Distribution of fixation rates for all players

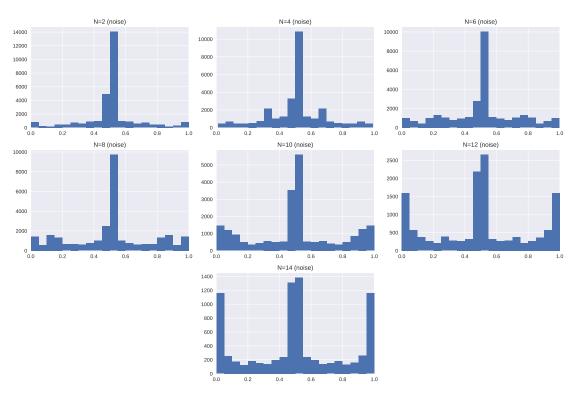


Figure 8: Distribution of fixation rates for all players (noise)

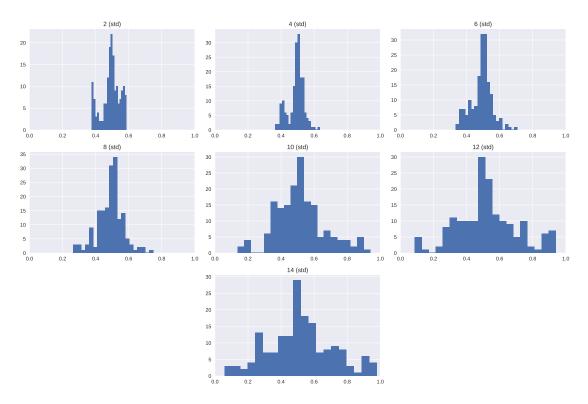


Figure 9: Distribution of median fixation rates for all players

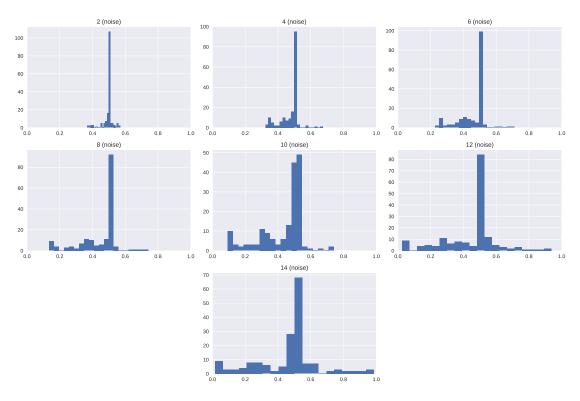


Figure 10: Distribution of median fixation rates for all players (noise)

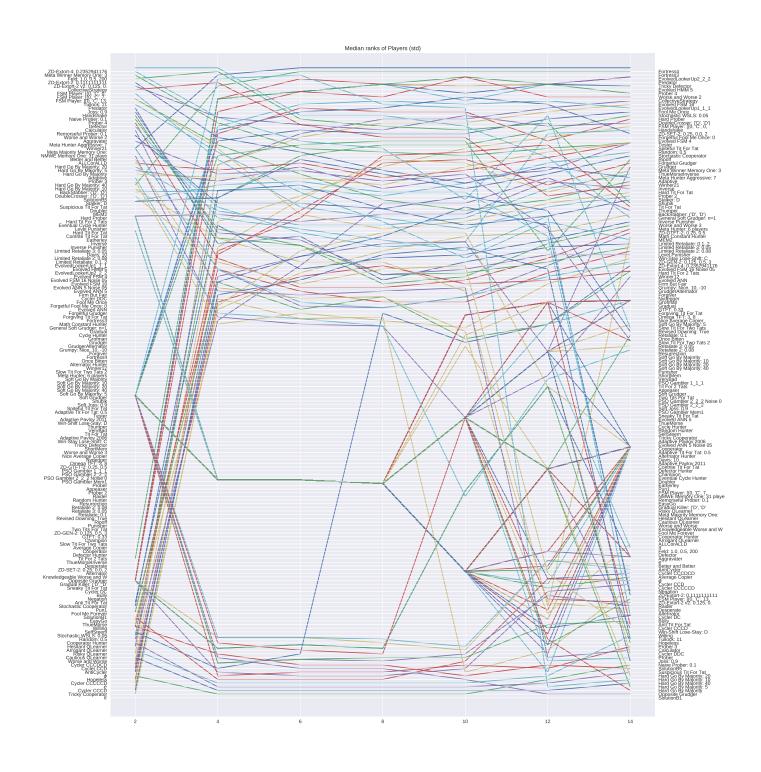


Figure 11: Median rank of players against population size

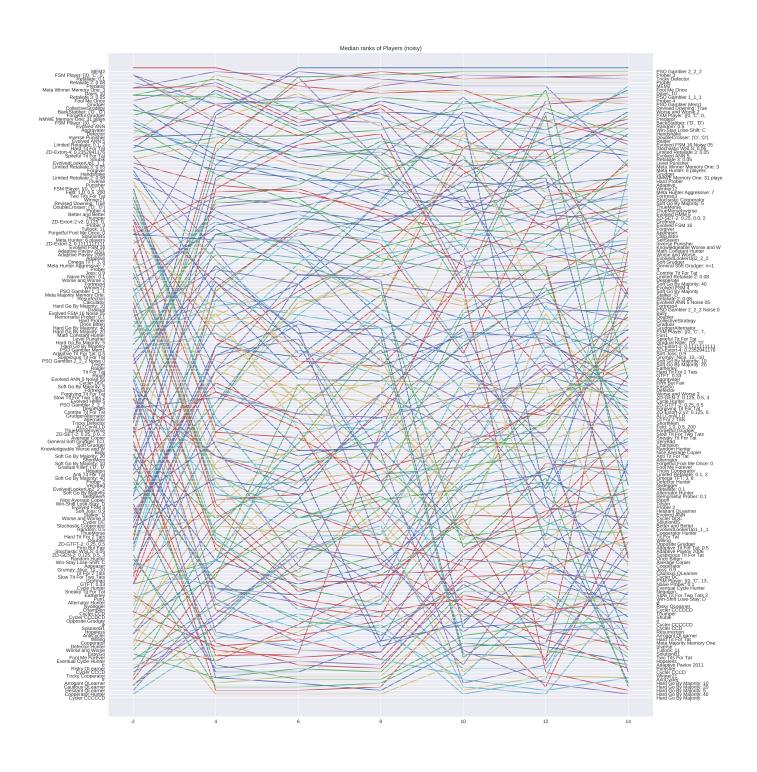


Figure 12: Median rank of players against population size with noise

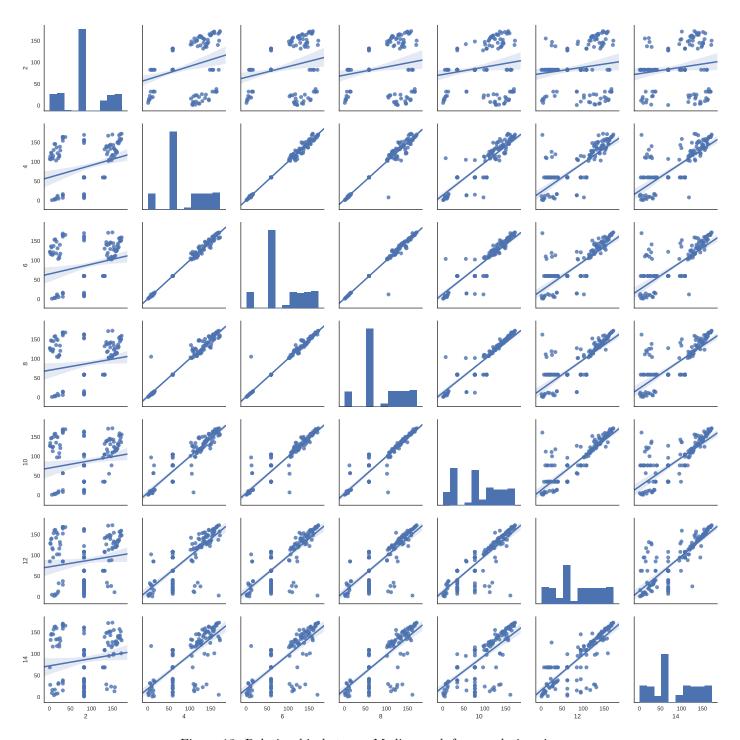


Figure 13: Relationship between Median rank for population size

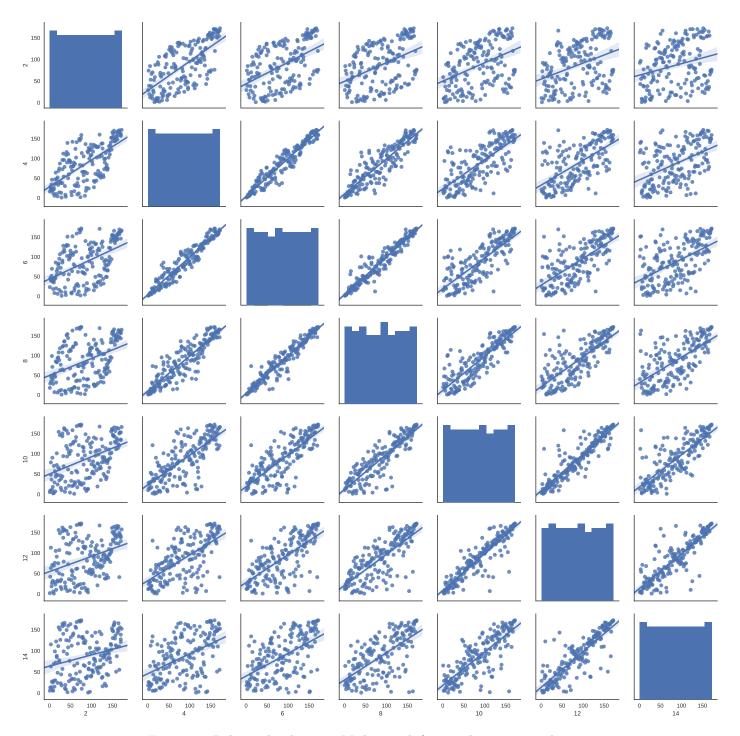


Figure 14: Relationship between Median rank for population size with noise

Table 6 shows the performance based on memory length.

Memory Depth 2 4 6 8 10 12 14 9 11.00 2.00 2.00 2.00 2.00 2.00 4.00 4 82.50 35.25 33.75 33.25 40.75 54.50 10.50 16 82.50 59.50 59.50 58.50 76.50 73.75 68.50 2 82.50 59.50 59.50 58.50 34.50 62.50 68.50 6 82.50 59.50 59.50 58.50 34.50 62.50 68.50 10 82.50 59.50 59.50 58.50 34.50 62.50 68.50 3 82.50 59.50 59.50 58.50 76.50 78.50 88.50 3 82.50 59.50 59.50 58.50 76.50 76.50 88.50 4 131.50 103.00 112.00 122.00 131.50 118.50 117.75 4 82.50 1								
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Memory Depth	2	4	6	8	10	12	14
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$\begin{array}{c ccccccccccccccccccccccccccccccccccc$								
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	16	82.50		59.50	58.50	76.50	78.50	39.25
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	∞	82.50	59.50	59.50	58.50	76.50	73.75	68.50
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	2	82.50	59.50	59.50	58.50	34.50	62.50	68.50
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	6	82.50	59.50	59.50	58.50	34.50	62.50	68.50
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	10	82.50	59.50	59.50	58.50	76.50	62.50	68.50
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	12	82.50	59.50	59.50	58.50	34.50	62.50	68.50
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	3	82.50	59.50	59.50	58.50	76.50	78.50	88.50
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	0	131.50	103.00	118.00	125.00	122.00	131.50	108.50
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	20	57.25	111.50	112.50	113.00	120.75	115.00	117.75
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	40	57.25	111.50	112.50	113.00	121.75	115.00	117.75
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	1	82.50	119.00	120.50	123.00	124.50	117.00	130.00
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	200	3.00	106.00	116.00	122.00	126.00	127.00	132.00
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	5	82.50	151.50	140.50	136.00	135.25	140.00	140.00
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	11	10.00	130.00	137.00	145.00	150.00	153.00	156.00
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$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Memory Depth	2	4	6	8	10	12	14
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		5.00	34.00	51.50	43.00	36.00	18 50	14.00
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12 70.00 11.50 12.00 73.00 121.00 130.00 134.00 5 89.00 134.50 127.75 135.00 131.25 135.50 136.00 11 44.50 111.00 129.00 144.00 150.00 146.00 159.00								
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11 44.50 111.00 129.00 144.00 150.00 146.00 159.00								

(b) Noisy

Table 6: Median rank by memory length

5 Conclusion

Further work:

- ullet Spatial structure;
- More than two types in the population;
- Modified Moran processes (Fermi selection);
- Mutation;

Acknowledgements

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A List of players

1. Adaptive	9. Adaptive Pavlov 2006	17. Calculator
2. Adaptive Tit For Tat: 0.5	10. Adaptive Pavlov 2011	18. Cautious QLearner
3. Aggravater	11. Appeaser	19. Champion
4. ALLCorALLD	12. Arrogant QLearner	20. CollectiveStrategy
5. Alternator	13. Average Copier	21. Contrite Tit For Tat
6. Alternator Hunter	14. Better and Better	22. Cooperator
7. AntiCycler	15. BackStabber: ('D', 'D')	23. Cooperator Hunter
8. Anti Tit For Tat	16. Bully	24. Cycle Hunter

25. Cycler CCCCCD	60.	General Soft	Grudger:	93.	Naive Prober: 0.1
26. Cycler CCCD	01	n=1,d=4,c=2		94.	MEM2
27. Cycler CCD		Soft Go By Majority		95.	Negation
28. Cycler DC	62.	Soft Go By Majority:	10	96.	Nice Average Copier
29. Cycler DDC	63.	Soft Go By Majority:	20	97.	Nydegger
30. Cycler CCCDCD	64.	Soft Go By Majority:	40	98.	Omega TFT: 3, 8
31. Davis: 10	65.	Soft Go By Majority:	5	99.	Once Bitten
32. Defector	66.	ϕ		100.	Opposite Grudger
33. Defector Hunter	67.	Gradual		101.	
34. Desperate	68.	Gradual Killer: ('D', 'I	D', 'D', 'D',		Predator
35. DoubleCrosser: ('D', 'D')		'D', 'C', 'C')			Prober
36. Doubler		Grofman			Prober 2
37. EasyGo	70.	Grudger			Prober 3
38. Eatherley	71.	${\bf Grudger Alternator}$			Prober 4
39. Eventual Cycle Hunter	72.	Grumpy: Nice, 10, -10	l		Pun1
40. Evolved ANN	73.	Handshake			PSO Gambler 1_1_1 PSO Gambler 2_2_2
41. Evolved ANN 5	74.	Hard Go By Majority			PSO Gambler 2.2.2 Noise 05
42. Evolved ANN 5 Noise 05	75.	Hard Go By Majority:	10		PSO Gambler Mem1
43. Evolved FSM 4	76.	Hard Go By Majority:	20		Punisher
44. Evolved FSM 16	77.	Hard Go By Majority:	40		Raider
45. Evolved FSM 16 Noise 05	78.	Hard Go By Majority:	5		Random: 0.5
46. EvolvedLookerUp1_1_1	79.	Hard Prober			Random Hunter
47. EvolvedLookerUp2_2_2	80.	Hard Tit For 2 Tats		116.	Remorseful Prober: 0.1
48. Evolved HMM 5	81.	Hard Tit For Tat		117.	Resurrection
	82.	Hesitant QLearner		118.	Retaliate: 0.1
49. Feld: 1.0, 0.5, 20050. Firm But Fair	83.	Hopeless		119.	Retaliate 2: 0.08
51. Fool Me Forever		Inverse		120.	Retaliate 3: 0.05
		Inverse Punisher		121.	Revised Downing: True
52. Fool Me Once		Joss: 0.9		122.	Ripoff
53. Forgetful Fool Me Once: 0.05		Knowledgeable Worse	and Worse	123.	Risky QLearner
54. Forgetful Grudger			and worse	124.	SelfSteem
55. Forgiver		Level Punisher	20	125.	ShortMem
56. Forgiving Tit For Tat		Limited Retaliate: 0.1		126.	Shubik
57. Fortress3		Limited Retaliate 2: 0			Slow Tit For Two Tats
58. Fortress4		Limited Retaliate 3: 0			Slow Tit For Two Tats 2
59. GTFT: 0.33	92.	Math Constant Hunter	r	129.	Sneaky Tit For Tat

- 130. Soft Grudger
- 131. Soft Joss: 0.9
- 132. SolutionB1
- 133. SolutionB5
- 134. Spiteful Tit For Tat
- 135. Stalker: D
- 136. Stochastic Cooperator
- 137. Stochastic WSLS: 0.05
- 138. Suspicious Tit For Tat
- 139. Tester
- 140. ThueMorse
- 141. ThueMorseInverse
- 142. Thumper
- 143. Tit For Tat
- 144. Tit For 2 Tats
- 145. Tricky Cooperator
- 146. Tricky Defector
- 147. Tullock: 11
- 148. Two Tits For Tat
- 149. VeryBad
- 150. Willing
- 151. Winner12
- 152. Winner21
- 153. Win-Shift Lose-Stay: D

- 154. Win-Stay Lose-Shift: C
- 155. Worse and Worse
- 156. Worse and Worse 2
- 157. Worse and Worse 3
- 158. ZD-Extort-2: 0.1111111111111111, 0.5
- 159. ZD-Extort-2 v2: 0.125, 0.5, 1
- 160. ZD-Extort-4: 0.23529411764705882,171. FSM Player: [(0, 'C', 13, 'D'), (0, 0.25, 1 'D', 12, 'D'), (1, 'C', 3, 'D'),
- 161. ZD-GTFT-2: 0.25, 0.5
- 162. ZD-GEN-2: 0.125, 0.5, 3
- 163. ZD-SET-2: 0.25, 0.0, 2
- 164. e
- 165. Meta Hunter: 6 players
- 166. Meta Hunter Aggressive: 7 players
- 167. Meta Majority Memory One: 31 players
- 168. Meta Winner Memory One: 31 players
- 169. NMWE Memory One: 31 players
- 170. FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'), (1, 'C', 11, 'D'), (1, 'D', 11, 'D'), (2, 'C', 8, 'D'), (2, 'D', 8, 'C'), (3, 'C', 3, 'C'), (3, 'D', 12, 'D'), (4, 'C', 6, 'C'), (4, 'D', 3, 'C'), (5, 'C', 11, 'C'), (5, 'D', 8, 'D'), (6, 'C', 13, 'D'), (6, 'D',

- 14, 'C'), (7, 'C', 4, 'D'), (7, 'D', 2, 'D'), (8, 'C', 14, 'D'), (8, 'D', 8, 'D'), (9, 'C', 0, 'C'), (9, 'D', 10, 'D'), (10, 'C', 8, 'C'), (10, 'D', 15, 'C'), (11, 'C', 6, 'D'), (11, 'D', 5, 'D'), (12, 'C', 6, 'D'), (12, 'D', 9, 'D'), (13, 'C', 9, 'D'), (13, 'D', 8, 'D'), (14, 'C', 8, 'D'), (14, 'D', 13, 'D'), (15, 'C', 4, 'C'), (15, 'D', 5, 'C')], 1, C
- 'D', 12, 'D'), (1, 'C', 3, 'D'), (1, 'D', 4, 'D'), (2, 'C', 14, 'D'), (2, 'D', 9, 'D'), (3, 'C', 0, 'C'), (3, 'D', 1, 'D'), (4, 'C', 1, 'D'), (4, 'D', 2, 'D'), (5, 'C', 12, 'C'), (5, 'D', 6, 'C'), (6, 'C', 1, 'C'), (6, 'D', 14, 'D'), (7, 'C', 12, 'D'), (7, 'D', 2, 'D'), (8, 'C', 7, 'D'), (8, 'D', 9, 'D'), (9, 'C', 8, 'D'), (9, 'D', 0, 'D'), (10, 'C', 2, 'C'), (10, 'D', 15, 'C'), (11, 'C', 7, 'D'), (11, 'D', 13, 'D'), (12, 'C', 3, 'C'), (12, 'D', 8, 'D'), (13, 'C', 7, 'C'), (13, 'D', 10, 'D'), (14, 'C', 10, 'D'), (14, 'D', 7, 'D'), (15, 'C', 15, 'C'), (15, 'D', 11, 'D')], 1, C
- 172. FSM Player: [(0, 'C', 0, 'C'), (0, 'D', 3, 'C'), (1, 'C', 5, 'D'), (1, 'D', 0, 'C'), (2, 'C', 3, 'C'), (2, 'D', 2, 'D'), (3, 'C', 4, 'D'), (3, 'D', 6, 'D'), (4, 'C', 3, 'C'), (4, 'D', 1, 'D'), (5, 'C', 6, 'C'), (5, 'D', 3, 'D'), (6, 'C', 6, 'D'), (6, 'D', 6, 'D'), (7, 'C', 7, 'D'), (7, 'D', 5, 'C')], 1, C