

An empirical study of fixation for strategies in the Iterated Prisoner's Dilemma

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Abstract

The Iterated Prisoner's Dilemma is a well established framework for the study of emergent behaviour. In this paper an extensive numerical study of the evolutionary dynamics of this framework are presented.

Fixation probabilities for Moran processes are obtained for 172 different strategies. This is done in both a standard 200 turn interaction and a noisy setting.

To the authors knowledge this is the largest such study. It allows for insights about the behaviour and performance of strategies with regard to their survival in an evolutionary setting.

1 Introduction

Since the formulation of the Moran Process in [15], this model of evolutionary population dynamics has been used to gain insights about the evolutionary stability of strategies in a number of settings. Similarly since the first Iterated Prisoner's Dilemma (IPD) tournament described in [3] the Prisoner's dilemma has been used to understand the evolution of cooperative behaviour in complex systems.

The analytical models of a Moran process are based on the relative fitness between two strategies and take this to be a fixed value r [17]. This is a valid model for simple strategies of the Prisoner's Dilemma such as to *always cooperate* or *always defect*. This manuscript provides a detailed numerical analysis of **164** complex and adaptive strategies for the IPD. In this case the relative fitness of a strategy is dependent on the population distribution.

Further deviations from the analytical model occur when interactions between players are subject to uncertainty. This is referred to as noise and has been considered in the IPD setting in [6, 16, 21].

This work provides answers to the following questions:

1. What strategies are good invaders?
2. What strategies are good at resisting invasion?
3. How does the population size affect these findings?

Figure 1 shows a diagrammatic representation of the Moran process. This process is a stochastic birth death process on a finite population in which the population size stays constant over time. Individuals are **selected** according to a given fitness landscape. Once selected, a given individual is reproduced and similarly another individual is chosen to be removed from the population. In some settings mutation is also considered but without mutation (the case considered in this work) this process will arrive at an absorbing state where the population is entirely made up of a single individual. The probability with which a given strategy is the survivor is called the fixation probability. A more detailed analytic description of this is given in Section 3.

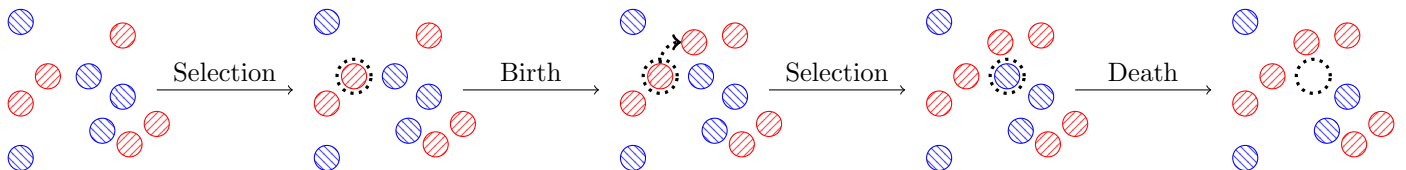


Figure 1: A diagrammatic representation of a Moran process

The Moran process was initially introduced in [15] in a genetic setting. It has since been used in a variety of settings including the understanding of the spread of cooperative behaviour. However, as stated before, these mainly consider non sophisticated strategies. Some work has looked at evolutionary stability of strategies within the Prisoner's Dilemma [11]

but this is not done in the more widely used setting of the Moran process but in terms of infinite population stability. In [5] Moran processes are looked at in a theoretic framework for a small subset of strategies. In [9] machine learning techniques are used to train a strategy capable of resisting invasion and also invade any memory one strategy. Recent work [7] has investigated the effect of memory length on strategy performance and the emergence of cooperation but this is not done in Moran process context and only considers specific cases of memory 2 strategies.

The contribution of this work is a detailed and extensive analysis of absorption probabilities for 164 strategies. These strategies and the numerical simulations are from [19] which is an open source research library written for the study of the IPD. The strategies and simulation frameworks are automatically tested in accordance to best research software practice. The large number of strategies are available thanks to the open source nature of the project with over 40 contributions made by different programmers. Thus by considering Moran processes with population size greater than 2 we are taking in to account the effect of complex population dynamics. By considering sophisticated strategies we are taking in to effect the reputation of a strategy during each interaction.

Section 2 will explain the methodological approach used, Section 3 will validate the methodology by comparing simulated results to analytical results. The main results of this manuscript are presented in Section 4 which will present a detailed analysis of all the data generated. Finally, Section 5 will conclude and offer future avenues for the work presented here.

2 Methodology

To carry out this large numerical experiment 164 strategies are used from [19]. These include 161 default strategies in the library at the time (excluding strategies classified as having a long run time and those that make use of the length of the game) as well as the following 3 finite state machine machine strategies [2]:

Appendix A shows all the players in question. More information about each player can be obtained in the documentation for [19]. There are 43 stochastic and 121 deterministic strategies. Their memory depth is shown in Table 1.

Memory Depth	0	1	2	3	4	5	6	9	10	11	12	16	20	40	200	∞
Count	3	31	12	8	2	6	1	1	5	1	1	2	2	2	1	86

Table 1: Memory depth

All strategies are paired and these pairs are used in 1000 repetitions of a Moran process assuming a starting population of $(N/2, N/2)$. This is repeated for even N between 2 and 14. The fixation probability is then estimated for each value of N .

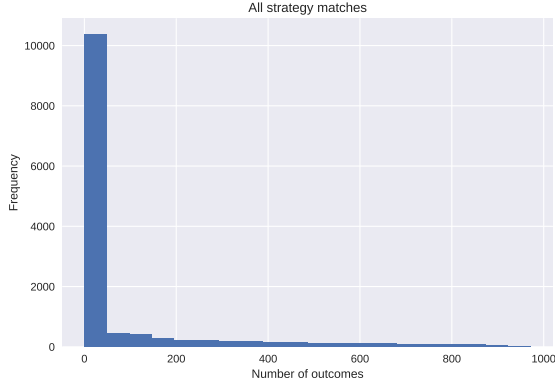
Note that due to the high computational cost of these experiments, for any given interaction between two players within the Moran process the outcome is sampled from a pre computed cache of 1000 match outcomes. This is carried out using an software written for the purpose of this work. This has been implemented in [19] ensuring that it can be used to either reproduce the work or carry out further work.

Figure 2 shows the distribution of the number of outcomes between all strategy pairs. Tables 2 shows that 95% of the stochastic matches have less than 788 unique outcomes whilst the maximum number is 971. This ensures that using a set of cached results from 1000 precomputed matches is sufficient for the analysis taking place here.

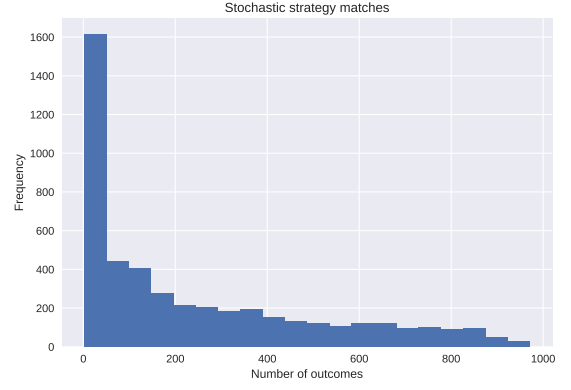
Outcome count		Outcome count	
count	13530.00	count	4753.00
mean	85.98	mean	242.90
std	192.58	std	260.04
min	1.00	min	2.00
25%	1.00	25%	28.00
50%	1.00	50%	139.00
75%	36.00	75%	394.00
95%	595.00	95%	788.00
max	971.00	max	971.00

(a) All matches
(b) Stochastic matches

Table 2: Summary statistics for the number of different match outcomes used as the cached results



(a) All matches



(b) Stochastic matches

Figure 2: The distribution of the number of unique outcomes used as the cached results

Section 3 will validate the methodology used here against known theoretic results.

3 Validation

As described in [17] Consider the payoff matrix:

$$M = \begin{pmatrix} a, b \\ c, d \end{pmatrix} \quad (1)$$

The expected payoffs of i players of the first type in a population with $N - i$ players of the second type are given by:

$$F_i = \frac{a(i-1) + b(N-i)}{N-1} \quad (2)$$

$$G_i = \frac{ci + d(N-i-1)}{N-1} \quad (3)$$

With an intensity of selection ω the fitness of both strategies is given by:

$$f_i = 1 - \omega + \omega F_i \quad (4)$$

$$g_i = 1 - \omega + \omega G_i \quad (5)$$

The transitions within the birth death process that underpins the Moran process are then given by:

$$p_{i,i+1} = \frac{if_i}{if_i + (N-i)g_i} \frac{N-i}{N} \quad (6)$$

$$p_{i,i-1} = \frac{(N-i)g_i}{if_i + (N-i)g_i} \frac{i}{N} \quad (7)$$

$$p_{ii} = 1 - p_{i,i+1} - p_{i,i-1} \quad (8)$$

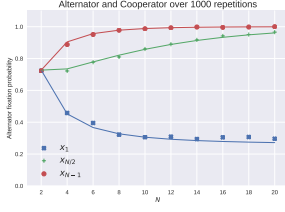
Using this it is a known result that the fixation probability of the first strategy in a population of i individuals of the first type (and $N - i$ individuals of the second. We have:

$$x_i = \frac{1 + \sum_{j=1}^{i-1} \prod_{k=1}^j \gamma_k}{1 + \sum_{j=1}^{N-1} \prod_{k=1}^j \gamma_k} \quad (9)$$

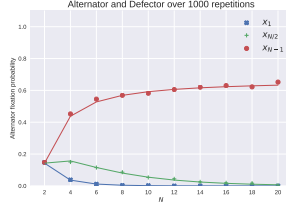
where:

$$\gamma_j = \frac{p_{j,j-1}}{p_{j,j+1}}$$

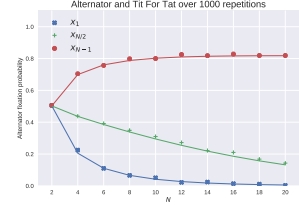
Using this comparisons of $x_1, x_{N/2}, x_{N-1}$ are shown in Figure 16. The points represent the simulated values and the line shows the theoretic value. Note that these are all deterministic strategies and show a perfect match up between the expected value of (9) and the actual Moran process for all strategies pairs.



(a) Alternator and Cooperator



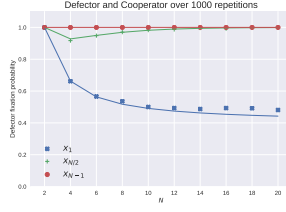
(b) Alternator and Defector



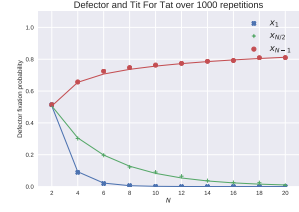
(c) Alternator and Tit For Tat



(d) Cooperator and Tit For Tat



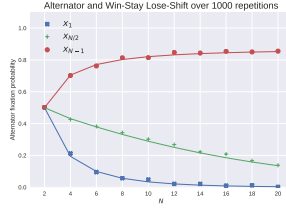
(e) Defector and Cooperator



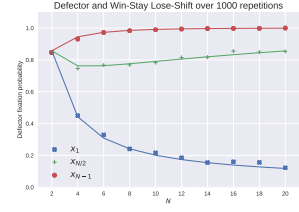
(f) Defector and Tit For Tat



(g) Win Stay Lose Shift and Tit For Tat



(h) Alternator and Win Stay Lose Shift



(i) Defector and Win Stay Lose Shift

Figure 3: Comparison of theoretic and actual Moran Process fixation probabilities for **deterministic** strategies

Figure 17 shows the fixation probabilities for stochastic strategies. These are no longer a good match which highlights the weakness of the analytical formulae that relies on the average payoffs. A detailed analysis of the 164 strategies considered, using direct Moran processes will be shown in the next Section.

4 Empirical results

This section will outline the data analysis carried out:

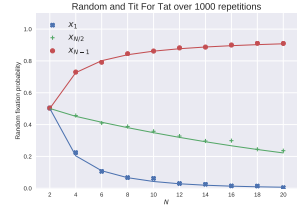
- Section 4.1 will consider the specific case of $N = 2$.
- Section 4.2 will investigate the effect of population size on the ability of a strategy to invade another population. This will highlight how complex strategies with long memories outperform simpler strategies.
- Section 4.3 will similarly investigate the ability to defend against an invasion.
- Section 4.4 will investigate the relationship between performance for differing population sizes. This highlights the importance of considering population dynamics over large populations.
- Section 4.5 will calculate the relative fitness of all strategies.



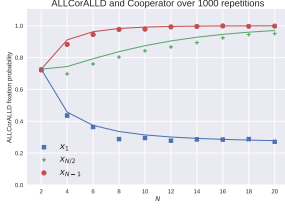
(a) Random and Cooperator



(b) Random and Defector



(c) Random and Tit For Tat



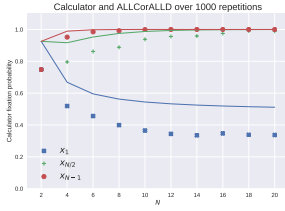
(d) All C or all D and Cooperator



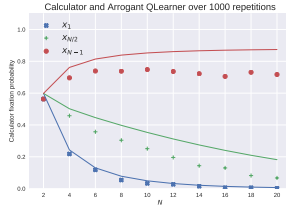
(e) All C or all D and Defector



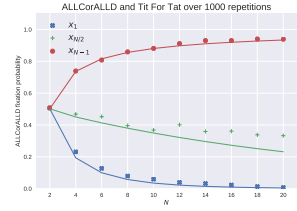
(f) Calculator and Random



(g) Calculator and All C or all D



(h) Calculator and Arrogant Q learner



(i) All C or all D and Tit For Tat

Figure 4: Comparison of theoretic and actual Moran Process fixation probabilities for **stochastic** strategies

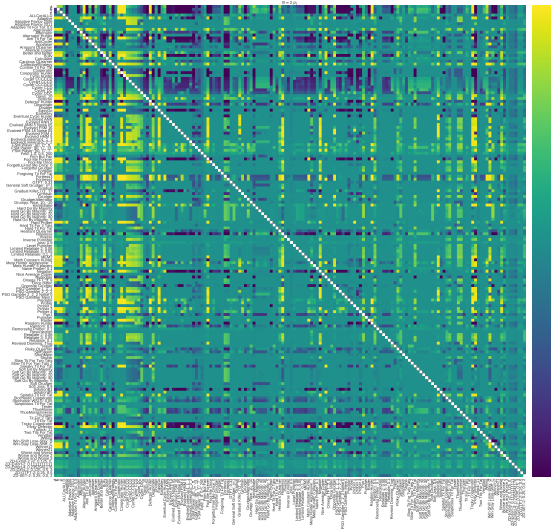
4.1 The special case of $N = 2$

The main fixation probabilities of interest are x_1 and x_{N-1} , these reflect a strategy's ability to invade or resist invasion. For $N = 2$ these two cases coincide. Figure 5a shows all pairwise fixation probabilities for strategies on the vertical column when being matched against probabilities on the horizontal column. This is summarised in Figure 5b and Table 3.

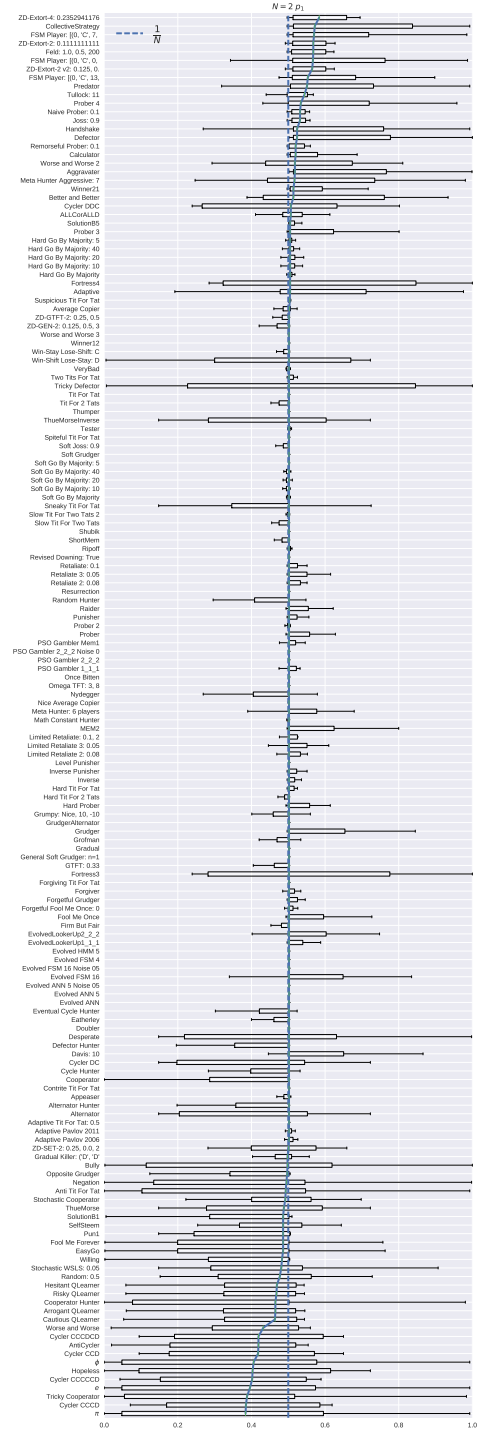
1. The top strategy is an extortionate Zero determinant strategy [18] with parameters $l = 1$ and $s = 1/4$.
2. The Collective strategy has a simple handshake mechanism (a cooperation followed by a defection on the first move). It then either cooperates or defects for all rounds [10]. This strategy was specifically designed for Evolutionary processes so it is perhaps also not surprising that it does well here.
3. The finite state machine strategy
4. The Feld strategy is the corresponding strategy submitted to Axelrod's first tournament [3]: it punishes defections but otherwise defects with a random probability that decays over time.
5. The final strategy in the top five is another extortionate Zero determinant strategy [18] with parameters $l = p$.

Player	Median p_1	Memory Depth	Stochastic
ZD-Extort-4: 0.23529411764705882, 0.25, 1	0.584	1	True
CollectiveStrategy	0.572	∞	False
FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C')...	0.570	1	False
Feld: 1.0, 0.5, 200	0.568	200	True
ZD-Extort-2: 0.1111111111111111, 0.5	0.568	1	True

Table 3: Summary of top five strategies for $N = 2$



(a) The pairwise fixation probabilities for $N = 2$



(b) The median fixation probabilities for $N = 2$

As will be demonstrated in Section 4.4 $N = 2$ is a particular case. In the next sections we will pay close attention to strategies who are strong invaders/resistors and shown diagrammatically in Figure 6.

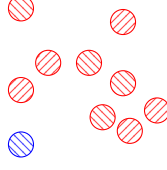


Figure 6: A single individual will successfully invade the population with probability x_1 . The group of Individuals will successfully resist with probability x_{N-1}

4.2 Strong invaders

In this section x_i will be investigated: the probability of 1 individual of a given type successfully becoming fixated in a population of $N - 1$ other individuals. Figures 7 shows these values for the players along the vertical axis when matched against the players on the horizontal axis. It can be seen that invasion is in general more challenging for $N = 7$ and $N = 12$ in comparison to $N = 3$. This information is summarised in Figure 8 showing the median fixation as well as the neutral fixation for each given scenario.

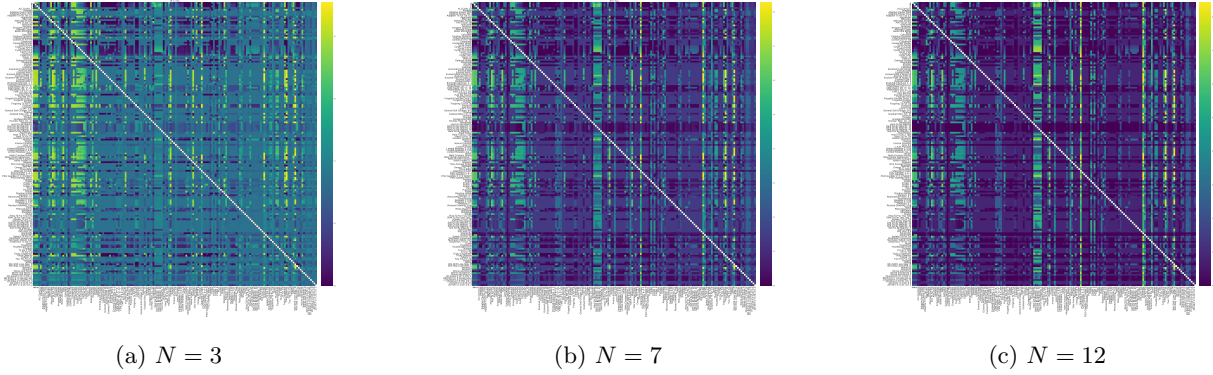


Figure 7: Pairwise fixation probability x_1 of all strategies

For $N \in \{3, 7, 12\}$ the top five strategies are given in Tables 5.

It can be seen that apart from the Collective strategy, none of the strategies of Table 3 perform well for $N \in \{3, 7, 12\}$. The new high performing strategies are:

- Predator, a finite state machine described in [2].
- Prober 4, complex strategy with an initial 20 move sequence of cooperations and defections [13]. This initial sequence serves as some kind of handshake.
- Remorseful Prober, a strategy that will not immediately retaliate when it recognises that the opponent is itself retaliating to a random defection [12].
- Worse and worse 2: plays tit for tat for 20 moves and then defects with with growing probability [13].
- Tester: a strategy submitted to the second of Axelrod's tournaments [4].

As well as noting that the memory length and complexity of these strategies are quite complex it is interesting to note that none of them are akin to memory one strategies. Most are not stochastic.

In the next section the performance in terms of x_{N-1} will be described: what strategies are particularly good at resisting an invasion.



Figure 8: Median probabilities x_1 of all strategies as well as the neutral fixation probability

Player	Median p_1	Memory Depth	Stochastic
CollectiveStrategy	0.403	∞	False
Predator	0.396	9	False
Prober 4	0.368	∞	False
Remorseful Prober: 0.1	0.357	2	True
Worse and Worse 2	0.355	∞	True

(a) $N = 3$

Player	Median p_1	Memory Depth	Stochastic
Prober 4	0.177	∞	False
CollectiveStrategy	0.170	∞	False
Worse and Worse 2	0.159	∞	True
Predator	0.158	9	False
Remorseful Prober: 0.1	0.146	2	True

(b) $N = 7$

Player	Median p_1	Memory Depth	Stochastic
Prober 4	0.105	∞	False
Worse and Worse 2	0.093	∞	True
Remorseful Prober: 0.1	0.089	2	True
Predator	0.088	9	False
Tester	0.088	∞	False

(c) $N = 12$

Table 4: Properties of top five invaders

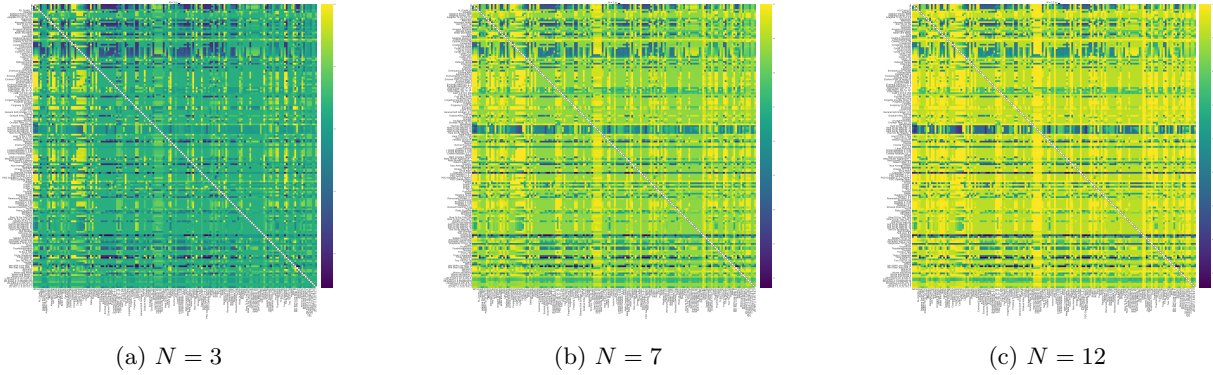
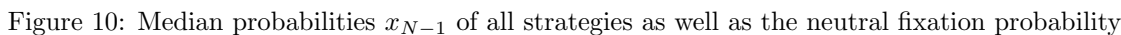


Figure 9: Pairwise fixation probability x_{N-1} of all strategies



Player	Median p_{N-1}	Memory Depth	Stochastic
CollectiveStrategy	0.796	∞	False
Predator	0.792	9	False
Handshake	0.779	∞	False
Prober 4	0.752	∞	False
Winner21	0.742	2	False

(a) $N = 3$

Player	Median p_{N-1}	Memory Depth	Stochastic
CollectiveStrategy	0.983	∞	False
Predator	0.975	9	False
Handshake	0.970	∞	False
Prober 4	0.958	∞	False
Winner21	0.956	2	False

(b) $N = 7$

Player	Median p_{N-1}	Memory Depth	Stochastic
CollectiveStrategy	0.999	∞	False
Handshake	0.997	∞	False
Predator	0.997	9	False
Prober 4	0.993	∞	False
Winner21	0.988	2	False

(c) $N = 12$

Table 5: Properties of top five strategies resistors

4.3 Strong resistors

4.4 The effect of population size

Figures 11, 12 and 13 show the median rank of each strategy against population size. Note that these ranks are not necessarily integers as group ties are given the average rank.

Tables 6a, 6b and 6c show the correlation coefficients of the ranks in of strategies in differing population size. This is shown graphically in Figure 14. It is immediate to note that how well a strategy performs in any Moran process for $N > 2$ has little to do with the performance for $N = 2$.

4.5 Relative fitness

Under the assumption of a constant relative fitness r between two strategies [17] the formula for x_i (for given N, r is:

$$x_i = x_i(r) = \frac{1 - \frac{1}{r^i}}{1 - \frac{1}{r^N}} \quad (10)$$

Figure 15 shows this function for $N = 10$ and $i \in \{1, 5, 10\}$.

The first and second derivative of (10) is given by equations (11) and (12).

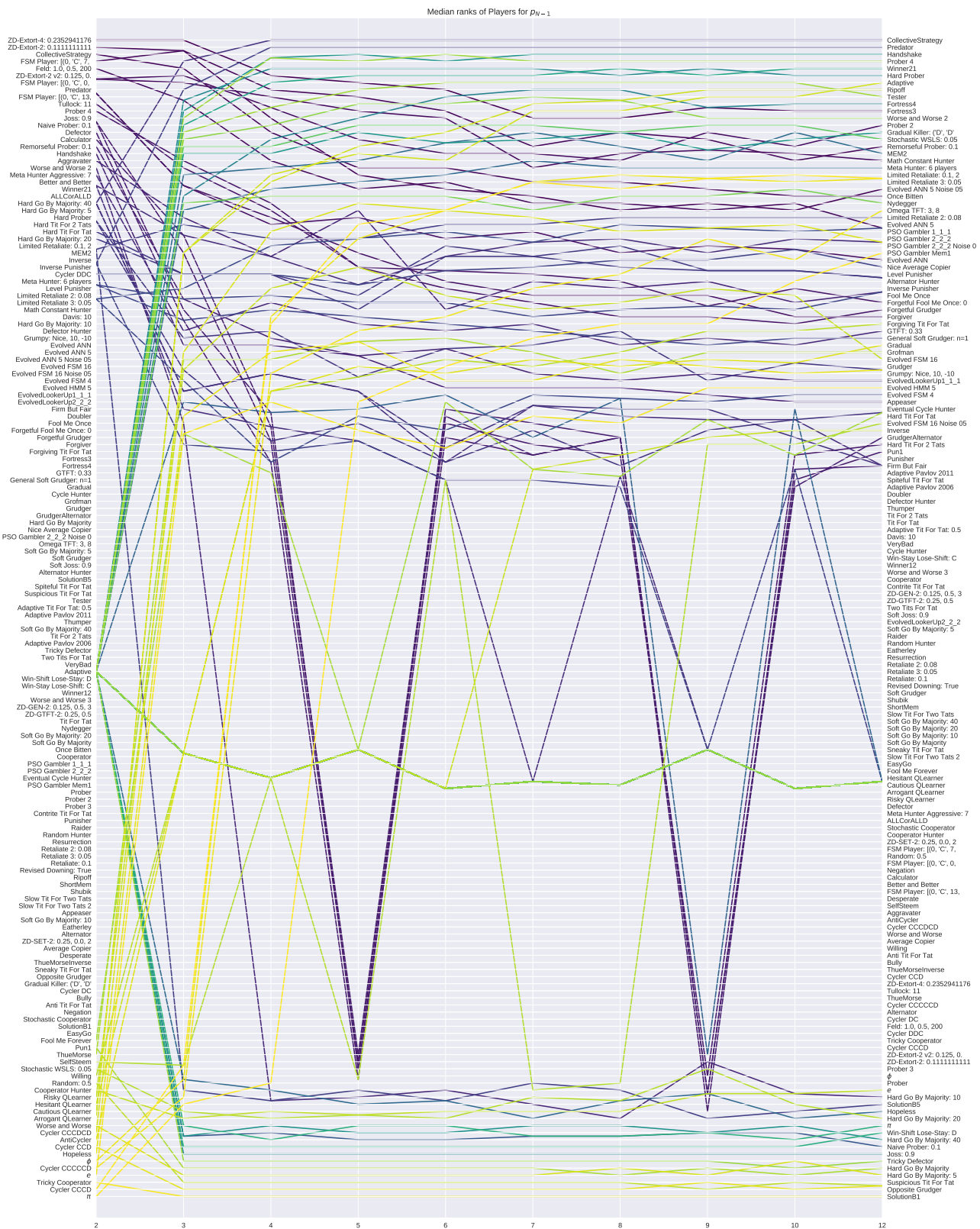
$$\frac{dx_i}{dr} = \frac{r^{N-i-1}}{r^{2N} - 2r^N + 1} (-Nr^i + N + ir^N - i) \quad (11)$$

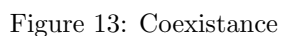
$$\frac{d^2x_i}{dr^2} = \frac{r^{N-i-2}}{(r^N - 1)^3} \left(2N^2 (r^i - 1) + N (r^N - 1) (N (r^i - 1) - 2i + r^i - 1) - i(i+1) (r^N - 1)^2 \right) \quad (12)$$

Using these, Halley's method [1] can be used to efficiently numerically invert $x_i(r)$ to obtain a theoretic relative fitness r that gives the calculated $x_i(r)$ between two strategies for a given N, i .



Figure 11: invade





N	2	3	4	5	6	7	8	9	10	12
2	1.00	0.44	0.26	0.17	0.15	0.12	0.08	0.08	0.08	0.04
3	0.44	1.00	0.92	0.87	0.87	0.86	0.83	0.83	0.84	0.81
4	0.26	0.92	1.00	0.97	0.96	0.97	0.95	0.95	0.95	0.94
5	0.17	0.87	0.97	1.00	0.98	0.99	0.97	0.98	0.98	0.97
6	0.15	0.87	0.96	0.98	1.00	0.99	0.97	0.97	0.98	0.97
7	0.12	0.86	0.97	0.99	0.99	1.00	0.98	0.98	0.99	0.98
8	0.08	0.83	0.95	0.97	0.97	0.98	1.00	0.97	0.98	0.97
9	0.08	0.83	0.95	0.98	0.97	0.98	0.97	1.00	0.99	0.99
10	0.08	0.84	0.95	0.98	0.98	0.99	0.98	0.99	1.00	0.99
12	0.04	0.81	0.94	0.97	0.97	0.98	0.97	0.99	0.99	1.00

(a) Correlation coefficients for ranks for invasion

N	2	3	4	5	6	7	8	9	10	12
2	1.00	0.61	0.42	0.29	0.35	0.34	0.34	0.21	0.30	0.29
3	0.61	1.00	0.91	0.81	0.87	0.87	0.87	0.76	0.85	0.83
4	0.42	0.91	1.00	0.93	0.98	0.97	0.97	0.89	0.96	0.95
5	0.29	0.81	0.93	1.00	0.93	0.94	0.94	0.96	0.93	0.91
6	0.35	0.87	0.98	0.93	1.00	0.98	0.98	0.92	0.99	0.98
7	0.34	0.87	0.97	0.94	0.98	1.00	1.00	0.92	0.99	0.98
8	0.34	0.87	0.97	0.94	0.98	1.00	1.00	0.91	0.98	0.97
9	0.21	0.76	0.89	0.96	0.92	0.92	0.91	1.00	0.93	0.94
10	0.30	0.85	0.96	0.93	0.99	0.99	0.98	0.93	1.00	0.99
12	0.29	0.83	0.95	0.91	0.98	0.98	0.97	0.94	0.99	1.00

(b) Correlation coefficients for ranks for resistance

N	2	4	6	8	10
2	1.00	0.25	0.19	0.12	0.13
4	0.25	1.00	0.99	0.97	0.98
6	0.19	0.99	1.00	0.98	1.00
8	0.12	0.97	0.98	1.00	0.99
10	0.13	0.98	1.00	0.99	1.00

(c) Correlation coefficients for ranks for coexistence

Table 6: Correlation coefficients of rankings by population size

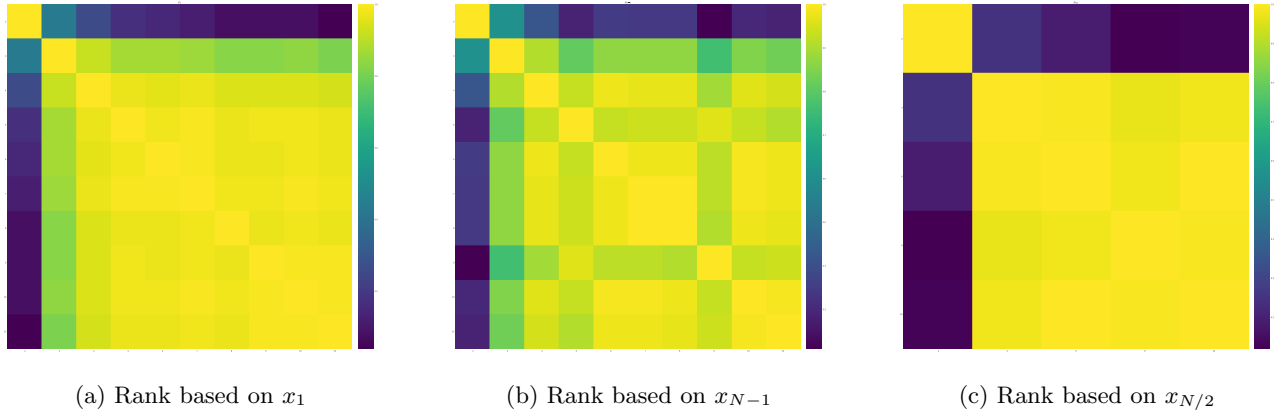


Figure 14: Heatmap of correlation coefficients of rankings by population size

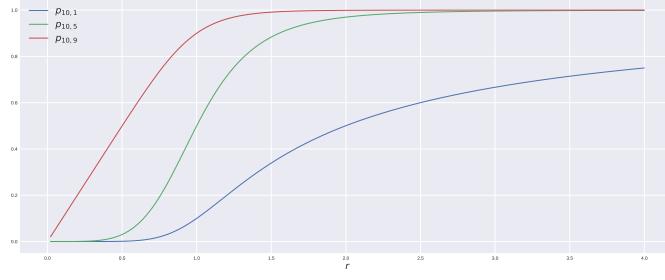
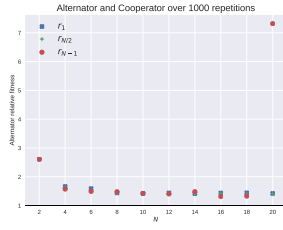
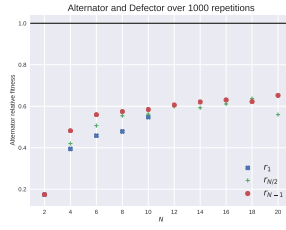


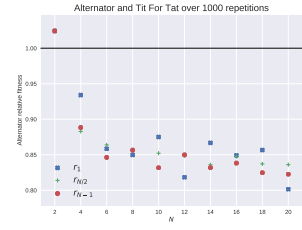
Figure 15: $x_i(r)$



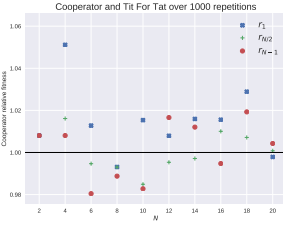
(a) Alternator and Cooperator



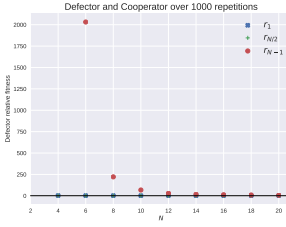
(b) Alternator and Defector



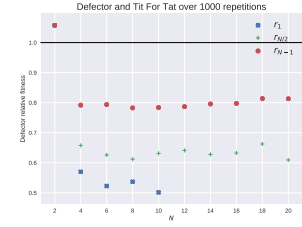
(c) Alternator and Tit For Tat



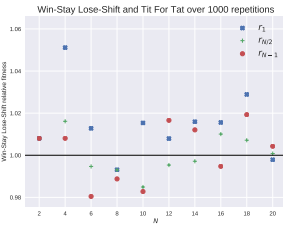
(d) Cooperator and Tit For Tat



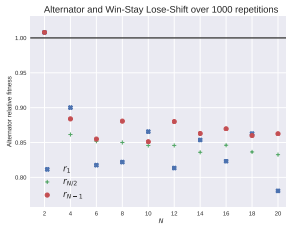
(e) Defector and Cooperator



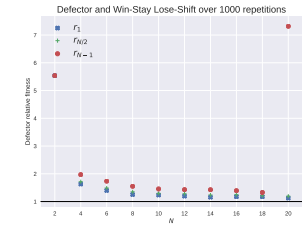
(f) Defector and Tit For Tat



(g) Win Stay Lose Shift and Tit For Tat

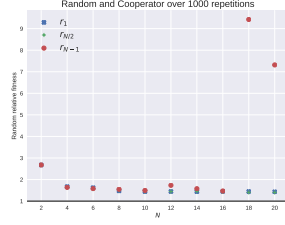


(h) Alternator and Win Stay Lose Shift

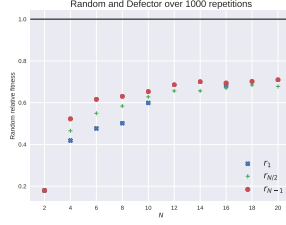


(i) Defector and Win Stay Lose Shift

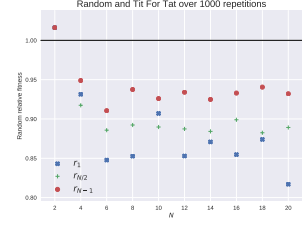
Figure 16: Estimated relative fitness for **deterministic** strategies



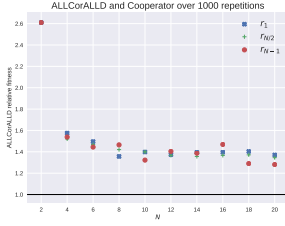
(a) Random and Cooperator



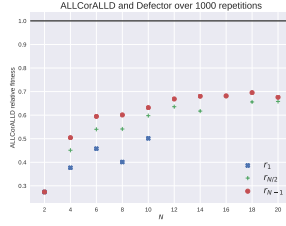
(b) Random and Defector



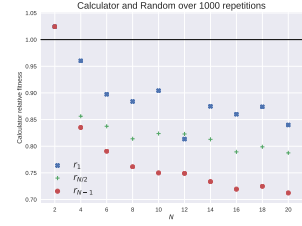
(c) Random and Tit For Tat



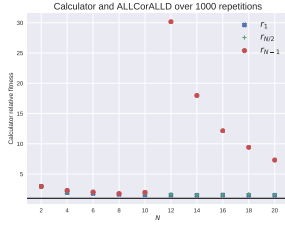
(d) All C or all D and Cooperator



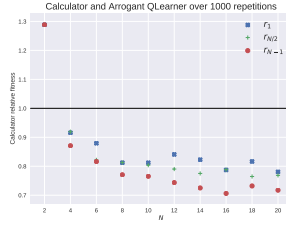
(e) All C or all D and Defector



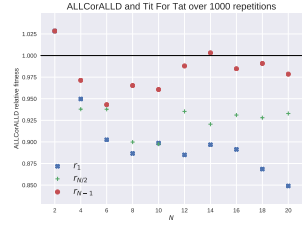
(f) Calculator and Random



(g) Calculator and All C or all D



(h) Calculator and Arrogant Q learner



(i) All C or all D and Tit For Tat

Figure 17: Estimated relative fitness for **stochastic** strategies

5 Conclusion

Further work:

- Spatial structure;
- More than two types in the population;
- Modified Moran processes (Fermi selection);
- Mutation;

Acknowledgements

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A variety of software libraries have been used in this work:

- The Axelrod library (IPD strategies and Moran processes) [19].
- The matplotlib library (visualisation) [8].
- The pandas and numpy libraries (data manipulation) [14, 20].

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A List of players

- | | | |
|--|--|--|
| 1. Hard Tit For Tat | 13. Predator | 30. Worse and Worse |
| 2. Negation | 14. FSM Player: [(0, 'C', 0, 'C'), (0, 'D', 3, 'C'), (1, 'C', 5, 'D'), (1, 'D', 0, 'C'), (2, 'C', 3, 'C'), (2, 'D', 2, 'D'), (3, 'C', 4, 'D'), (3, 'D', 6, 'D'), (4, 'C', 3, 'C'), (4, 'D', 1, 'D'), (5, 'C', 6, 'C'), (5, 'D', 3, 'D'), (6, 'C', 6, 'D'), (6, 'D', 6, 'D'), (7, 'C', 7, 'D'), (7, 'D', 5, 'C')], 1, C | 31. ZD-Extort-2: 0.1111111111111111, 0.5 |
| 3. Nice Average Copier | | 32. Worse and Worse 3 |
| 4. Nydegger | | 33. Win-Stay Lose-Shift: C |
| 5. Winner21 | | 34. Hard Tit For 2 Tats |
| 6. Winner12 | | 35. Tit For 2 Tats |
| 7. π | | 36. Hard Go By Majority |
| 8. Win-Shift Lose-Stay: D | 15. Prober 2 | 37. Cycle Hunter |
| 9. Omega TFT: 3, 8 | 16. Prober | 38. Fool Me Once |
| 10. Opposite Grudger | 17. Prober 3 | 39. ZD-GTFT-2: 0.25, 0.5 |
| 11. FSM Player: [(0, 'C', 13, 'D'), (0, 'D', 12, 'D'), (1, 'C', 3, 'D'), (1, 'D', 4, 'D'), (2, 'C', 14, 'D'), (2, 'D', 9, 'D'), (3, 'C', 0, 'C'), (3, 'D', 1, 'D'), (4, 'C', 1, 'D'), (4, 'D', 2, 'D'), (5, 'C', 12, 'C'), (5, 'D', 6, 'C'), (6, 'C', 1, 'C'), (6, 'D', 14, 'D'), (7, 'C', 12, 'D'), (7, 'D', 2, 'D'), (8, 'C', 7, 'D'), (8, 'D', 9, 'D'), (9, 'C', 8, 'D'), (9, 'D', 0, 'D'), (10, 'C', 2, 'C'), (10, 'D', 15, 'C'), (11, 'C', 7, 'D'), (11, 'D', 13, 'D'), (12, 'C', 3, 'C'), (12, 'D', 8, 'D'), (13, 'C', 7, 'C'), (13, 'D', 10, 'D'), (14, 'C', 10, 'D'), (14, 'D', 7, 'D'), (15, 'C', 15, 'C'), (15, 'D', 11, 'D')], 1, C | 40. Adaptive | |
| 12. Once Bitten | 18. Tricky Defector | 41. Cooperator Hunter |
| | 19. Tullock: 11 | 42. Forgiving Tit For Tat |
| | 20. VeryBad | 43. Handshake |
| | 21. Prober 4 | 44. Eventual Cycle Hunter |
| | 22. Willing | 45. Retaliate 3: 0.05 |
| | 23. Worse and Worse 2 | 46. Forgetful Fool Me Once: 0.05 |
| | 24. Pun1 | 47. Adaptive Tit For Tat: 0.5 |
| | 25. Two Tits For Tat | 48. Forgiver |
| | 26. Tricky Cooperator | 49. Revised Downing: True |
| | 27. PSO Gambler 2.2.2 | 50. Evolved ANN |
| | 28. PSO Gambler 1.1.1 | 51. Forgetful Grudger |
| | 29. Tit For Tat | |

52. Hard Go By Majority: 10
53. PSO Gambler 2.2.2 Noise 05
54. Cycler CCCCCD
55. Ripoff
56. Aggravater
57. Evolved ANN 5
58. Fortress3
59. Hard Go By Majority: 20
60. ZD-GEN-2: 0.125, 0.5, 3
61. ALLCorALLD
62. Hard Prober
63. Evolved ANN 5 Noise 05
64. SelfSteem
65. Alternator
66. Fortress4
67. Hard Go By Majority: 40
68. Cycler CCCD
69. Risky QLearner
70. FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'), (1, 'C', 11, 'D'), (1, 'D', 11, 'D'), (2, 'C', 8, 'D'), (2, 'D', 8, 'C'), (3, 'C', 3, 'C'), (3, 'D', 12, 'D'), (4, 'C', 6, 'C'), (4, 'D', 3, 'C'), (5, 'C', 11, 'C'), (5, 'D', 8, 'D'), (6, 'C', 13, 'D'), (6, 'D', 14, 'C'), (7, 'C', 4, 'D'), (7, 'D', 2, 'D'), (8, 'C', 14, 'D'), (8, 'D', 8, 'D'), (9, 'C', 0, 'C'), (9, 'D', 10, 'D'), (10, 'C', 8, 'C'), (10, 'D', 15, 'C'), (11, 'C', 6, 'D'), (11, 'D', 5, 'D'), (12, 'C', 6, 'D'), (12, 'D', 9, 'D'), (13, 'C', 9, 'D'), (13, 'D', 8, 'D'), (14, 'C', 8, 'D'), (14, 'D', 13, 'D'), (15, 'C', 4, 'C'), (15, 'D', 5, 'C')], 1, C
71. Meta Hunter Aggressive: 7 players
72. Alternator Hunter
73. Evolved FSM 4
74. Cycler CCD
75. Slow Tit For Two Tats 2
76. *e*
77. Soft Go By Majority
78. Hard Go By Majority: 5
79. ShortMem
80. AntiCycler
81. Hesitant QLearner
82. Cycler DC
83. Evolved FSM 16
84. GTFT: 0.33
85. Slow Tit For Two Tats
86. Anti Tit For Tat
87. Shubik
88. General Soft Grudger: n=1,d=4,c=2
89. Soft Grudger
90. ZD-Extort-2 v2: 0.125, 0.5, 1
91. Soft Go By Majority: 10
92. Hopeless
93. Adaptive Pavlov 2006
94. Sneaky Tit For Tat
95. Inverse
96. Cycler DDC
97. Evolved FSM 16 Noise 05
98. Adaptive Pavlov 2011
99. Soft Go By Majority: 20
100. Inverse Punisher
101. PSO Gambler Mem1
102. SolutionB5
103. Appeaser
104. EvolvedLookerUp2.2.2
105. Joss: 0.9
106. Cycler CCCDCD
107. EvolvedLookerUp1.1.1
108. Punisher
109. Soft Go By Majority: 40
110. SolutionB1
111. Arrogant QLearner
112. Soft Go By Majority: 5
113. Defector
114. Level Punisher
115. Spiteful Tit For Tat
116. Raider
117. Soft Joss: 0.9
118. Meta Hunter: 6 players
119. Average Copier
120. Davis: 10
121. Limited Retaliate: 0.1, 20
122. Stochastic Cooperator
123. ZD-Extort-4: 0.23529411764705882, 0.25, 1
124. ϕ
125. ZD-SET-2: 0.25, 0.0, 2
126. Better and Better
127. Limited Retaliate 2: 0.08, 15
128. Defector Hunter
129. Random Hunter
130. Stochastic WSLs: 0.05
131. Gradual
132. Limited Retaliate 3: 0.05, 20
133. Random: 0.5
134. Retaliate 2: 0.08
135. Remorseful Prober: 0.1
136. Suspicious Tit For Tat
137. Bully
138. Desperate
139. Gradual Killer: ('D', 'D', 'D', 'D', 'D', 'C', 'C')
140. Tester
141. CollectiveStrategy
142. Math Constant Hunter
143. Firm But Fair
144. Grofman
145. Feld: 1.0, 0.5, 200

- | | | |
|------------------------|---------------------------|----------------------------|
| 146. ThueMorse | 153. ThueMorseInverse | 160. Calculator |
| 147. Cautious QLearner | 154. Evolved HMM 5 | |
| 148. Naive Prober: 0.1 | 155. GrudgerAlternator | 161. Grumpy: Nice, 10, -10 |
| 149. Resurrection | 156. MEM2 | 162. Thumper |
| 150. Doubler | 157. Retaliate: 0.1 | |
| 151. Fool Me Forever | 158. Contrite Tit For Tat | 163. Cooperator |
| 152. Grudger | 159. EasyGo | 164. Eatherley |