

An empirical study of fixation for strategies in the Iterated Prisoner's Dilemma

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Abstract

The Iterated Prisoner's Dilemma is a well established framework for the study of emergent behaviour. In this paper an extensive numerical study of the evolutionary dynamics of this framework are presented.

Fixation probabilities for Moran processes are obtained for 172 different strategies. This is done in both a standard 200 turn interaction and a noisy setting.

To the authors knowledge this is the largest such study. It allows for insights about the behaviour and performance of strategies with regard to their survival in an evolutionary setting.

1 Introduction

Since the formulation of the Moran Process in [11], this model of evolutionary population dynamics has been used to gain insights about the evolutionary stability of strategies in a number of settings. Similarly since the first Iterated Prisoner's Dilemma (IPD) tournament described in [3] the Prisoner's dilemma has been used to understand the evolution of cooperative behaviour in complex systems.

The analytical models of a Moran process are based on the relative fitness between two strategies and take this to be a fixed value r [13]. This is a valid model for simple strategies of the Prisoner's Dilemma such as to *always cooperate* or *always defect*. This manuscript provides a detailed numerical analysis of **164** complex and adaptive strategies for the IPD. In this case the relative fitness of a strategy is dependent on the population distribution.

Further deviations from the analytical model occur when interactions between players are subject to uncertainty. This is referred to as noise and has been considered in the IPD setting in [5, 12, 17].

This work provides answers to the following questions:

1. What strategies are good invaders?
2. What strategies are good at resisting invasion?
3. How does the population size affect these findings?

Figure 1 shows a diagrammatic representation of the Moran process. This process is a stochastic birth death process on a finite population in which the population size stays constant over time. Individuals are **selected** according to a given fitness landscape. Once selected, a given individual is reproduced and similarly another individual is chosen to be removed from the population. In some settings mutation is also considered but without mutation (the case considered in this work) this process will arrive at an absorbing state where the population is entirely made up of a single individual. The probability with which a given strategy is the survivor is called the fixation probability. A more detailed analytic description of this is given in Section 3.

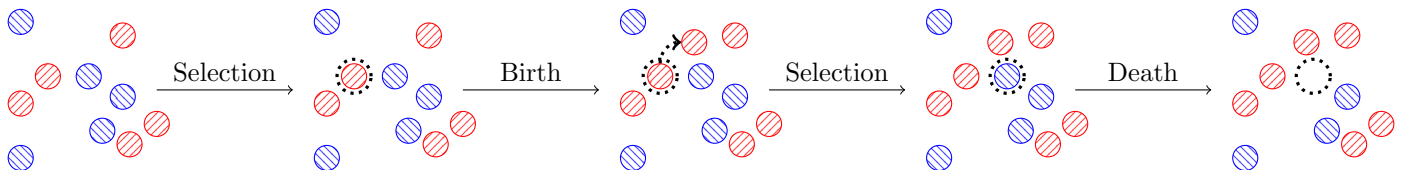


Figure 1: A diagrammatic representation of a Moran process

The Moran process was initially introduced in [11] in a genetic setting. It has since been used in a variety of settings including the understanding of the spread of cooperative behaviour. However, as stated before, these mainly consider non sophisticated strategies. Some work has looked at evolutionary stability of strategies within the Prisoner's Dilemma [9]

but this is not done in the more widely used setting of the Moran process but in terms of infinite population stability. In [4] Moran processes are looked at in a theoretic framework for a small subset of strategies. In [8] machine learning techniques are used to train a strategy capable of resisting invasion and also invade any memory one strategy. Recent work [6] has investigated the effect of memory length on strategy performance and the emergence of cooperation but this is not done in Moran process context and only considers specific cases of memory 2 strategies.

The contribution of this work is a detailed and extensive analysis of absorption probabilities for 164 strategies. These strategies and the numerical simulations are from [15] which is an open source research library written for the study of the IPD. The strategies and simulation frameworks are automatically tested in accordance to best research software practice. The large number of strategies are available thanks to the open source nature of the project with over 40 contributions made by different programmers. Thus by considering Moran processes with population size greater than 2 we are taking in to account the effect of complex population dynamics. By considering sophisticated strategies we are taking in to effect the reputation of a strategy during each interaction.

Section 2 will explain the methodological approach used, Section 3 will validate the methodology by comparing simulated results to analytical results. The main results of this manuscript are presented in Section 4 which will present a detailed analysis of all the data generated. Finally, Section 5 will conclude and offer future avenues for the work presented here.

2 Methodology

To carry out this large numerical experiment 164 strategies are used from [15]. These include 161 default strategies in the library at the time (excluding strategies classified as having a long run time and those that make use of the length of the game) as well as the following 3 finite state machine machine strategies [2]:

Appendix A shows all the players in question. More information about each player can be obtained in the documentation for [15]. There are 43 stochastic and 121 deterministic strategies. Their memory depth is shown in Table 1.

Memory Depth	0	1	2	3	4	5	6	9	10	11	12	16	20	40	200	∞
Count	3	31	12	8	2	6	1	1	5	1	1	2	2	2	1	86

Table 1: Memory depth

All strategies are paired and these pairs are used in 1000 repetitions of a Moran process assuming a starting population of $(N/2, N/2)$. This is repeated for even N between 2 and 14. The fixation probability is then estimated for each value of N .

Note that due to the high computational cost of these experiments, for any given interaction between two players within the Moran process the outcome is sampled from a pre computed cache of 1000 match outcomes. This is carried out using an software written for the purpose of this work. This has been implemented in [15] ensuring that it can be used to either reproduce the work or carry out further work.

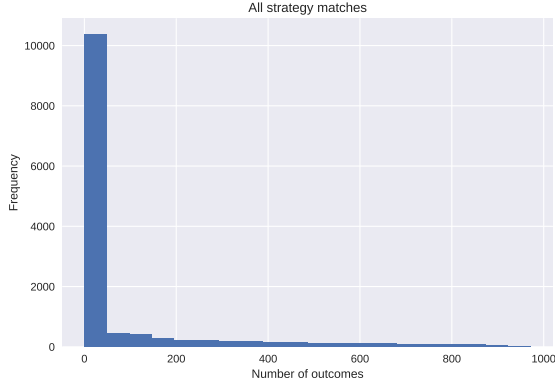
Figure 2 shows the distribution of the number of outcomes between all strategy pairs. Tables 2 shows that 95% of the stochastic matches have less than 788 unique outcomes whilst the maximum number is 971. This ensures that using a set of cached results from 1000 precomputed matches is sufficient for the analysis taking place here.

Outcome count		Outcome count	
count	13530.00	count	4753.00
mean	85.98	mean	242.90
std	192.58	std	260.04
min	1.00	min	2.00
25%	1.00	25%	28.00
50%	1.00	50%	139.00
75%	36.00	75%	394.00
95%	595.00	95%	788.00
max	971.00	max	971.00

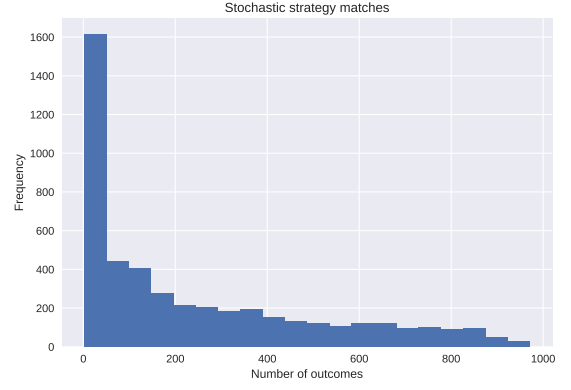
(a) All matches

(b) Stochastic matches

Table 2: Summary statistics for the number of different match outcomes used as the cached results



(a) All matches



(b) Stochastic matches

Figure 2: The distribution of the number of unique outcomes used as the cached results

Section 3 will validate the methodology used here against known theoretic results.

3 Validation

As described in [13] Consider the payoff matrix:

$$M = \begin{pmatrix} a, b \\ c, d \end{pmatrix} \quad (1)$$

The expected payoffs of i players of the first type in a population with $N - i$ players of the second type are given by:

$$F_i = \frac{a(i-1) + b(N-i)}{N-1} \quad (2)$$

$$G_i = \frac{ci + d(N-i-1)}{N-1} \quad (3)$$

With an intensity of selection ω the fitness of both strategies is given by:

$$f_i = 1 - \omega + \omega F_i \quad (4)$$

$$g_i = 1 - \omega + \omega G_i \quad (5)$$

The transitions within the birth death process that underpins the Moran process are then given by:

$$p_{i,i+1} = \frac{if_i}{if_i + (N-i)g_i} \frac{N-i}{N} \quad (6)$$

$$p_{i,i-1} = \frac{(N-i)g_i}{if_i + (N-i)g_i} \frac{i}{N} \quad (7)$$

$$p_{ii} = 1 - p_{i,i+1} - p_{i,i-1} \quad (8)$$

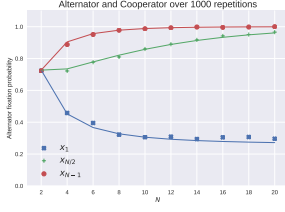
Using this it is a known result that the fixation probability of the first strategy in a population of i individuals of the first type (and $N - i$ individuals of the second. We have:

$$x_i = \frac{1 + \sum_{j=1}^{i-1} \prod_{k=1}^j \gamma_k}{1 + \sum_{j=1}^{N-1} \prod_{k=1}^j \gamma_k} \quad (9)$$

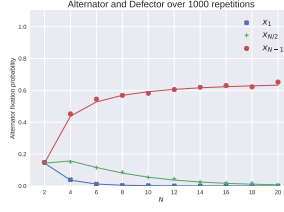
where:

$$\gamma_j = \frac{p_{j,j-1}}{p_{j,j+1}}$$

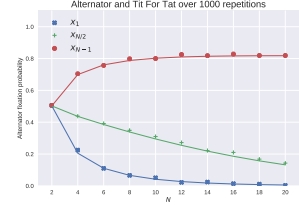
Using this comparisons of $x_1, x_{N/2}, x_{N-1}$ are shown in Figure 17. The points represent the simulated values and the line shows the theoretic value. Note that these are all deterministic strategies and show a perfect match up between the expected value of (9) and the actual Moran process for all strategies pairs.



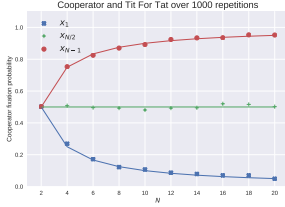
(a) Alternator and Cooperator



(b) Alternator and Defector



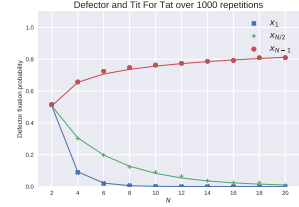
(c) Alternator and Tit For Tat



(d) Cooperator and Tit For Tat



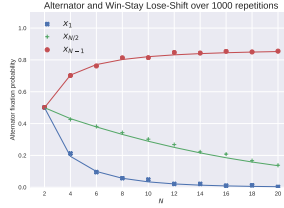
(e) Defector and Cooperator



(f) Defector and Tit For Tat



(g) Win Stay Lose Shift and Tit For Tat



(h) Alternator and Win Stay Lose Shift



(i) Defector and Win Stay Lose Shift

Figure 3: Comparison of theoretic and actual Moran Process fixation probabilities for **deterministic** strategies

Figure 18 shows the fixation probabilities for stochastic strategies. These are no longer a good match which highlights the weakness of the analytical formulae that relies on the average payoffs. A detailed analysis of the 164 strategies considered, using direct Moran processes will be shown in the next Section.

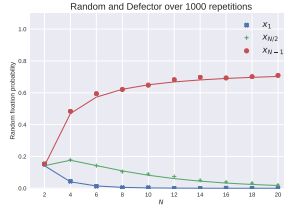
4 Empirical results

This section will outline the data analysis carried out:

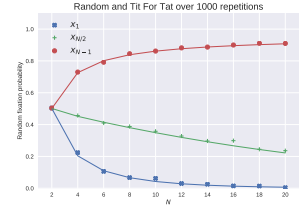
- Section 4.1 will consider the specific case of $N = 2$.
- Section 4.2 will investigate the effect of population size on the ability of a strategy to invade another population. This will highlight how complex strategies with long memories outperform simpler strategies.
- Section 4.3 will similarly investigate the ability to defend against an invasion.
- Section 4.4 will investigate the relationship between performance for differing population sizes. This highlights the importance of considering population dynamics over large populations.
- Section 4.5 will calculate the relative fitness of all strategies.



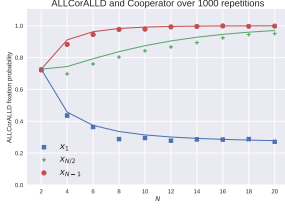
(a) Random and Cooperator



(b) Random and Defector



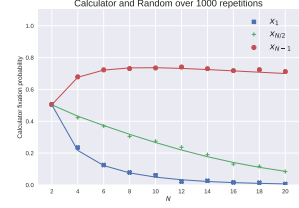
(c) Random and Tit For Tat



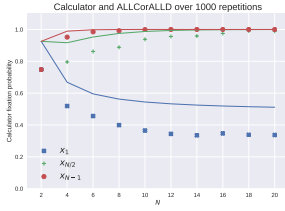
(d) All C or all D and Cooperator



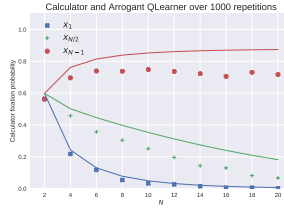
(e) All C or all D and Defector



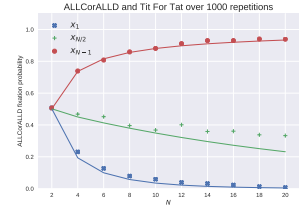
(f) Calculator and Random



(g) Calculator and All C or all D



(h) Calculator and Arrogant Q learner



(i) All C or all D and Tit For Tat

Figure 4: Comparison of theoretic and actual Moran Process fixation probabilities for **stochastic** strategies

4.1 The special case of $N = 2$

The main fixation probabilities of interest are x_1 and x_{N-1} , these reflect a strategy's ability to invade or resist invasion. For $N = 2$ these two cases coincide. Figure 5 shows all pairwise fixation probabilities for strategies on the vertical column when being matched against probabilities on the horizontal column. This is summarised in Figure 6 and Table 3. It can be seen that, as described in [14], zero determinant strategies perform well. The strategy with infinite memory **Collective** is a strategy with a handshake which implies that it will perform well against itself. There is also one of the trained finite state machines in the top five. The remaining strategy "Feld" is the corresponding strategy submitted to the first tournament by Axelrod [3]. All these strategies are stochastic.

Player	Median p_1	Memory Depth	Stochastic
ZD-Extort-4: 0.23529411764705882, 0.25, 1	0.584	1	True
CollectiveStrategy	0.572	∞	False
FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C')...	0.570	1	False
Feld: 1.0, 0.5, 200	0.568	200	True
ZD-Extort-2: 0.1111111111111111, 0.5	0.568	1	True

Table 3: Summary of top five strategies for $N = 2$

As will be demonstrated in Section 4.4 $N = 2$ is a particular case. In the next sections we will pay close attention to strategies who are strong invaders/resistors and shown diagrammatically in Figure 7.

4.2 Strong invaders

In this section x_i will be investigated: the probability of 1 individual of a given type successfully becoming fixated in a population of $N - 1$ other individuals. Figure 8 shows these values for the players along the vertical axis when matched against the players on the horizontal axis. It can be seen that invasion is in general more challenging for $N = 7$ and

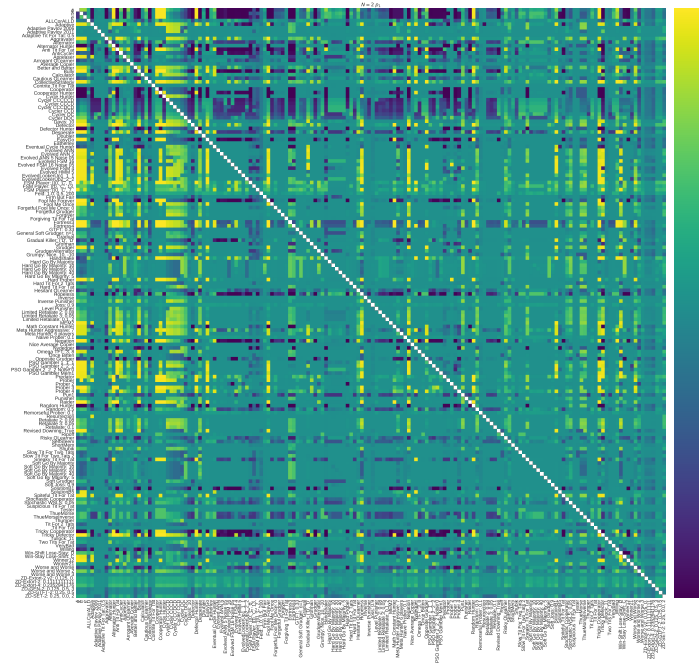


Figure 5: The pairwise fixation probabilities for $N = 2$

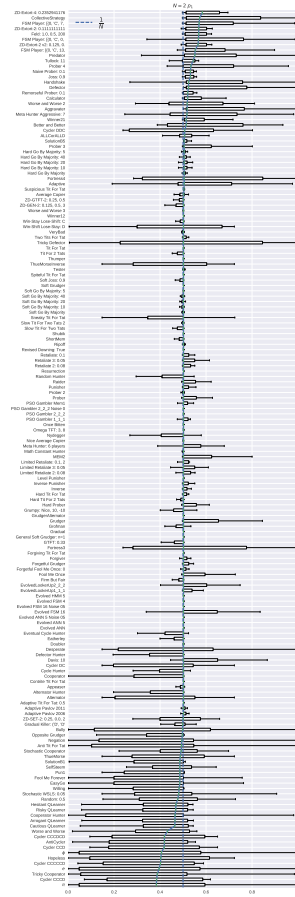


Figure 6: The median fixation probabilities for $N = 2$

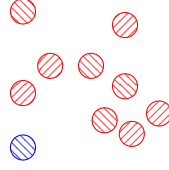


Figure 7: A single individual will successfully invade the population with probability x_1 . The group of Individuals will successfully resist with probability x_{N-1}

$N = 12$ in comparison to $N = 3$. This information is summarised in Figure 9 showing the median fixation as well as the neutral fixation for each given scenario.

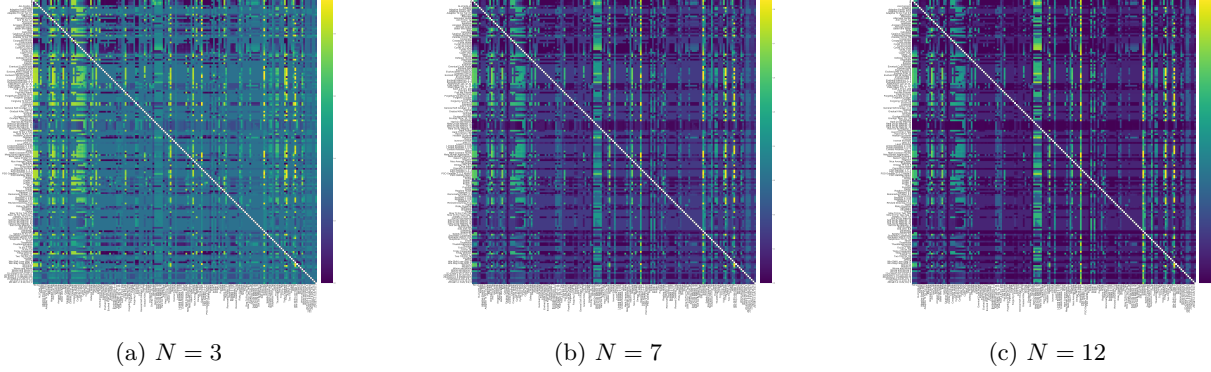


Figure 8: Pairwise fixation probability x_1 of all strategies

For $N \in \{3, 7, 12\}$ the top five strategies are given in Tables 5.

Player	Median p_1	Memory Depth	Stochastic
CollectiveStrategy	0.403	∞	False
Predator	0.396	9	False
Prober 4	0.368	∞	False
Remorseful Prober: 0.1	0.357	2	True
Worse and Worse 2	0.355	∞	True
(a) $N = 3$			
Player	Median p_1	Memory Depth	Stochastic
Prober 4	0.177	∞	False
CollectiveStrategy	0.170	∞	False
Worse and Worse 2	0.159	∞	True
Predator	0.158	9	False
Remorseful Prober: 0.1	0.146	2	True
(b) $N = 7$			
Player	Median p_1	Memory Depth	Stochastic
Prober 4	0.105	∞	False
Worse and Worse 2	0.093	∞	True
Remorseful Prober: 0.1	0.089	2	True
Predator	0.088	9	False
Tester	0.088	∞	False
(c) $N = 12$			

Table 4: Properties of top five invaders



Figure 9: Median probabilities x_1 of all strategies as well as the neutral fixation probability

4.3 Strong resistors

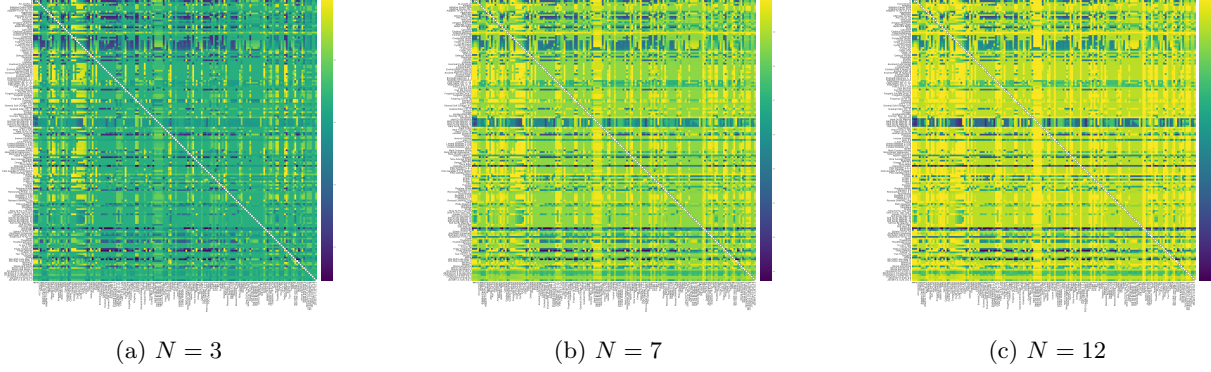


Figure 10: Pairwise fixation probability x_{N-1} of all strategies

Player	Median p_{N-1}	Memory Depth	Stochastic
CollectiveStrategy	0.403	∞	False
Predator	0.396	9	False
Prober 4	0.368	∞	False
Handshake	0.346	∞	False
Winner21	0.341	2	False

(a) $N = 3$

Player	Median p_{N-1}	Memory Depth	Stochastic
Prober 4	0.177	∞	False
CollectiveStrategy	0.170	∞	False
Predator	0.158	9	False
Winner21	0.145	2	False
Handshake	0.133	∞	False

(b) $N = 7$

Player	Median p_{N-1}	Memory Depth	Stochastic
Prober 4	0.105	∞	False
Predator	0.088	9	False
CollectiveStrategy	0.087	∞	False
Winner21	0.087	2	False
Handshake	0.069	∞	False

(c) $N = 12$

Table 5: Properties of top five strategies resistors

4.4 The effect of population size

Figures 12, 13 and 14 show the median rank of each strategy against population size. Note that these ranks are not necessarily integers as group ties are given the average rank.

Tables 6a, 6b and 6c show the correlation coefficients of the ranks in of strategies in differing population size. This is shown graphically in Figure 15. It is immediate to note that how well a strategy performs in any Moran process for $N > 2$ has little to do with the performance for $N = 2$.

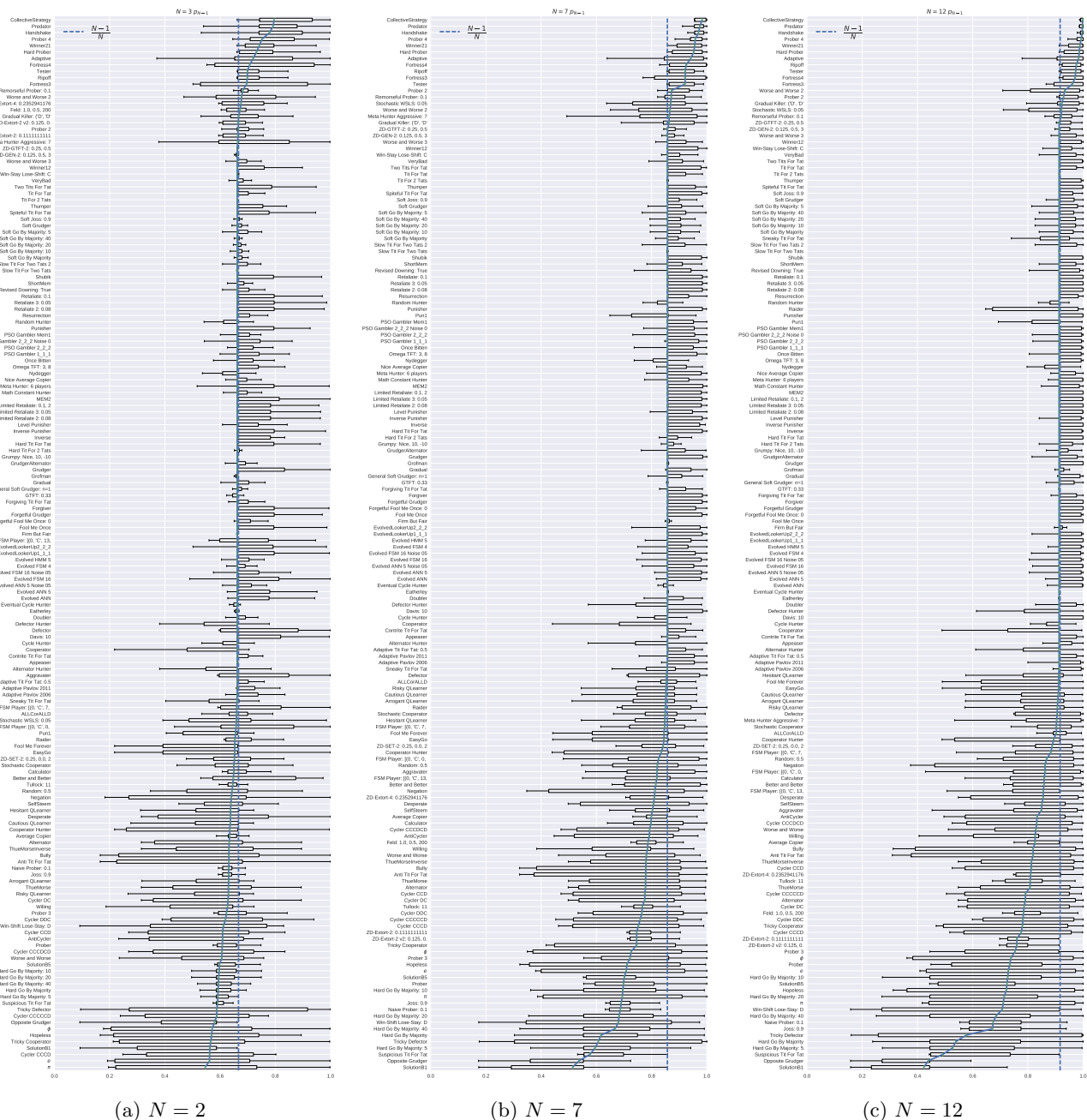
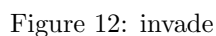


Figure 11: Median probabilities x_{N-1} of all strategies as well as the neutral fixation probability



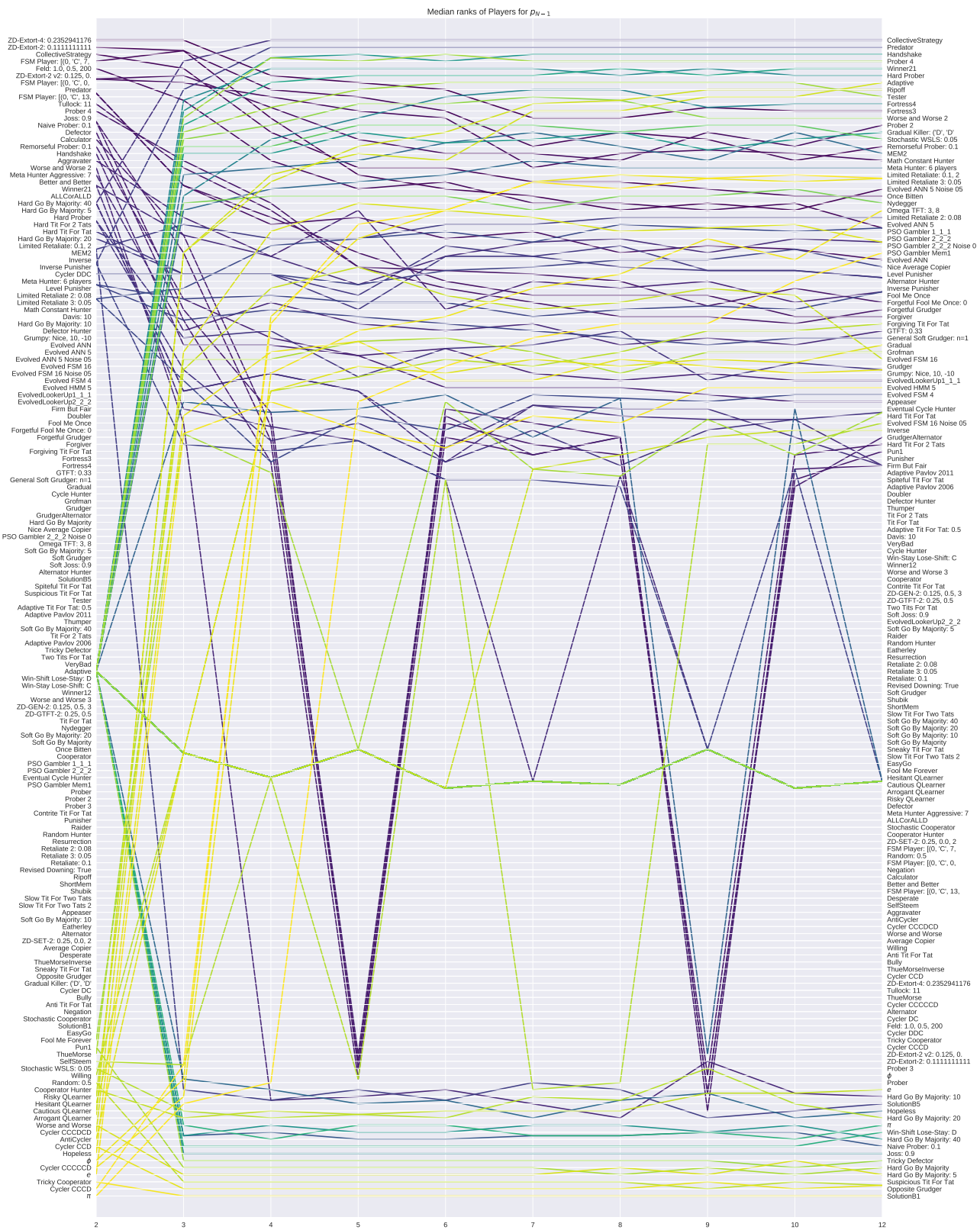


Figure 13: resist

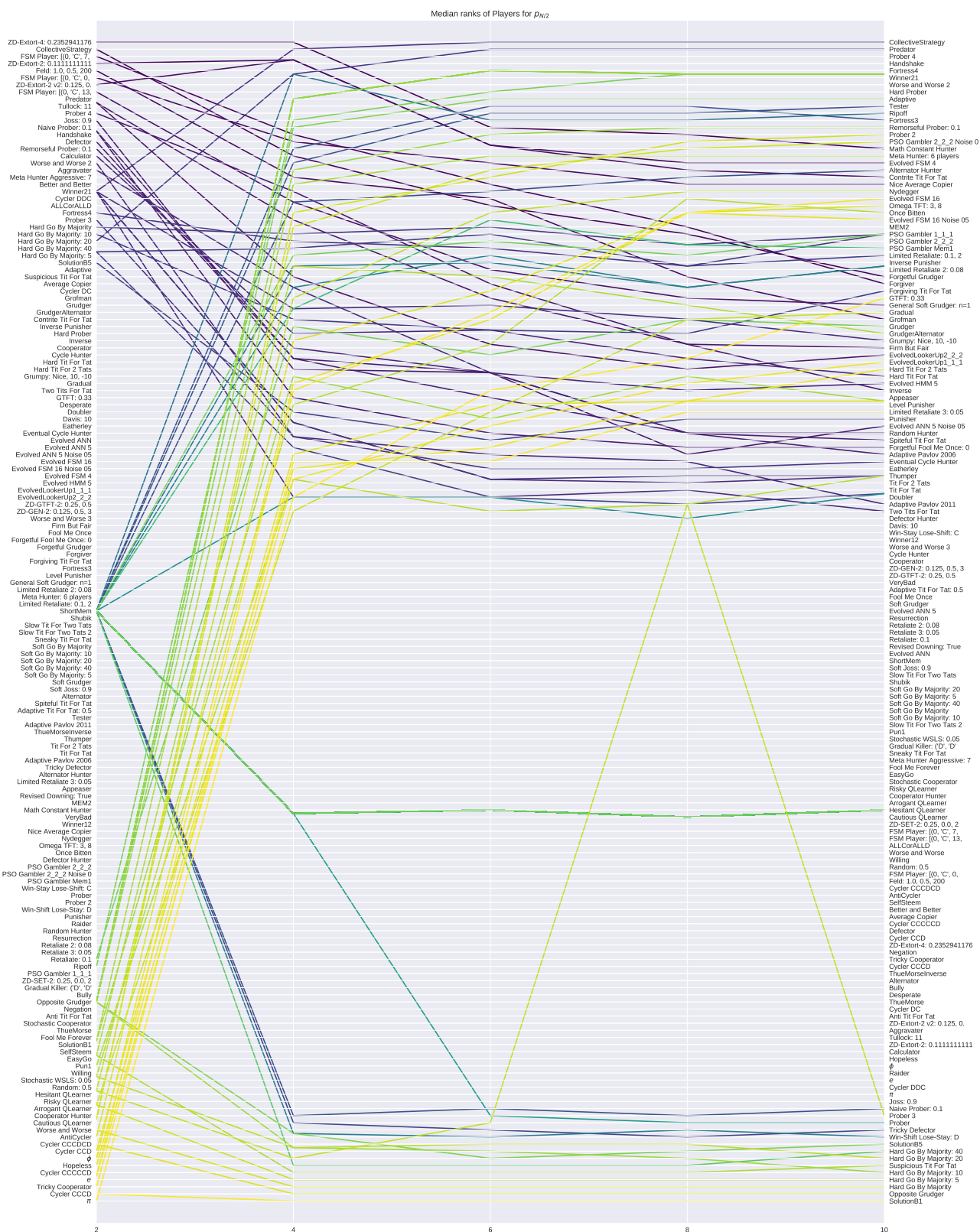


Figure 14: Coexistence

N	2	3	4	5	6	7	8	9	10	12
2	1.00	0.44	0.26	0.17	0.15	0.12	0.08	0.08	0.08	0.04
3	0.44	1.00	0.92	0.87	0.87	0.86	0.83	0.83	0.84	0.81
4	0.26	0.92	1.00	0.97	0.96	0.97	0.95	0.95	0.95	0.94
5	0.17	0.87	0.97	1.00	0.98	0.99	0.97	0.98	0.98	0.97
6	0.15	0.87	0.96	0.98	1.00	0.99	0.97	0.97	0.98	0.97
7	0.12	0.86	0.97	0.99	0.99	1.00	0.98	0.98	0.99	0.98
8	0.08	0.83	0.95	0.97	0.97	0.98	1.00	0.97	0.98	0.97
9	0.08	0.83	0.95	0.98	0.97	0.98	0.97	1.00	0.99	0.99
10	0.08	0.84	0.95	0.98	0.98	0.99	0.98	0.99	1.00	0.99
12	0.04	0.81	0.94	0.97	0.97	0.98	0.97	0.99	0.99	1.00

(a) Correlation coefficients for ranks for invasion

N	2	3	4	5	6	7	8	9	10	12
2	1.00	0.61	0.42	0.29	0.35	0.34	0.34	0.21	0.30	0.29
3	0.61	1.00	0.91	0.81	0.87	0.87	0.87	0.76	0.85	0.83
4	0.42	0.91	1.00	0.93	0.98	0.97	0.97	0.89	0.96	0.95
5	0.29	0.81	0.93	1.00	0.93	0.94	0.94	0.96	0.93	0.91
6	0.35	0.87	0.98	0.93	1.00	0.98	0.98	0.92	0.99	0.98
7	0.34	0.87	0.97	0.94	0.98	1.00	1.00	0.92	0.99	0.98
8	0.34	0.87	0.97	0.94	0.98	1.00	1.00	0.91	0.98	0.97
9	0.21	0.76	0.89	0.96	0.92	0.92	0.91	1.00	0.93	0.94
10	0.30	0.85	0.96	0.93	0.99	0.99	0.98	0.93	1.00	0.99
12	0.29	0.83	0.95	0.91	0.98	0.98	0.97	0.94	0.99	1.00

(b) Correlation coefficients for ranks for resistance

N	2	4	6	8	10
2	1.00	0.25	0.19	0.12	0.13
4	0.25	1.00	0.99	0.97	0.98
6	0.19	0.99	1.00	0.98	1.00
8	0.12	0.97	0.98	1.00	0.99
10	0.13	0.98	1.00	0.99	1.00

(c) Correlation coefficients for ranks for coexistence

Table 6: Correlation coefficients of rankings by population size

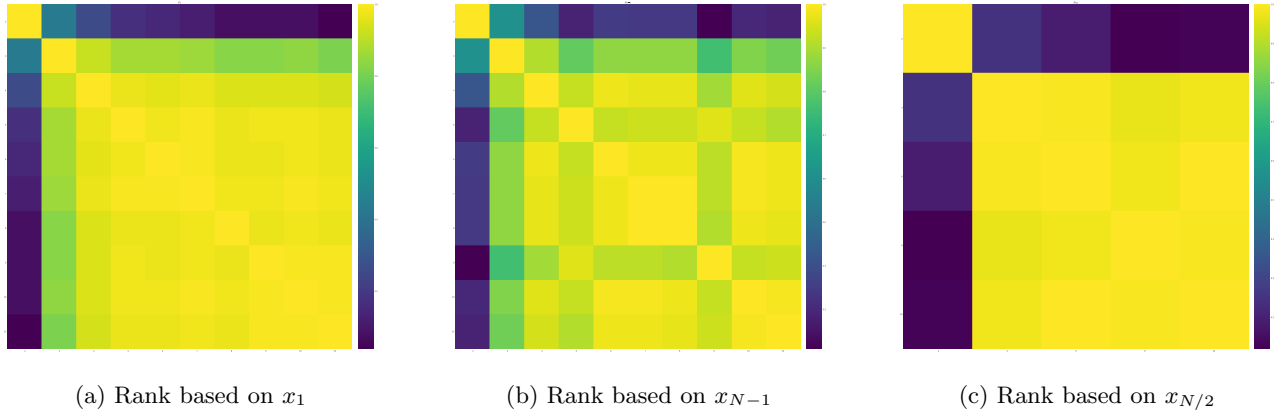


Figure 15: Heatmap of correlation coefficients of rankings by population size

4.5 Relative fitness

Under the assumption of a constant relative fitness r between two strategies [13] the formula for x_i (for given N, r is:

$$x_i = x_i(r) = \frac{1 - \frac{1}{r^i}}{1 - \frac{1}{r^N}} \quad (10)$$

Figure 16 shows this function for $N = 10$ and $i \in \{1, 5, 10\}$.

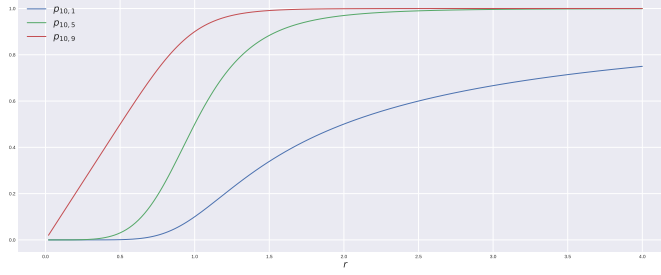


Figure 16: $x_i(r)$

The first and second derivative of (10) is given by equations (11) and (12).

$$\frac{dx_i}{dr} = \frac{r^{N-i-1}}{r^{2N} - 2r^N + 1} (-Nr^i + N + ir^N - i) \quad (11)$$

$$\frac{d^2x_i}{dr^2} = \frac{r^{N-i-2}}{(r^N - 1)^3} \left(2N^2 (r^i - 1) + N (r^N - 1) (N (r^i - 1) - 2i + r^i - 1) - i(i+1) (r^N - 1)^2 \right) \quad (12)$$

Using these, Halley's method [1] can be used to efficiently numerically invert $x_i(r)$ to obtain a theoretic relative fitness r that gives the calculated $x_i(r)$ between two strategies for a given N, i .

5 Conclusion

Further work:

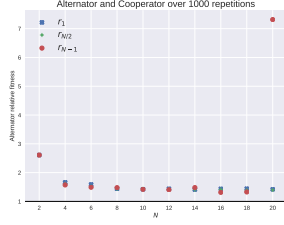
- Spatial structure;
- More than two types in the population;
- Modified Moran processes (Fermi selection);
- Mutation;

Acknowledgements

This work was performed using the computational facilities of the Advanced Research Computing @ Cardiff (ARCCA) Division, Cardiff University.

A variety of software libraries have been used in this work:

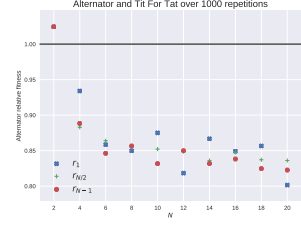
- The Axelrod library (IPD strategies and Moran processes) [15].
- The matplotlib library (visualisation) [7].
- The pandas and numpy libraries (data manipulation) [10, 16].



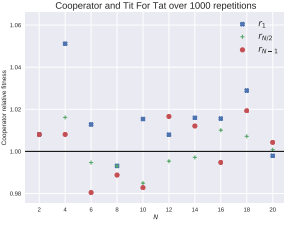
(a) Alternator and Cooperator



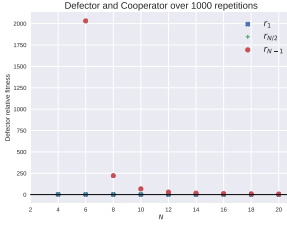
(b) Alternator and Defector



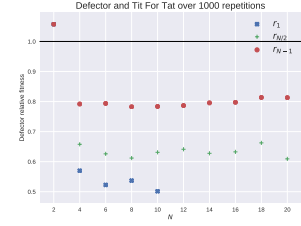
(c) Alternator and Tit For Tat



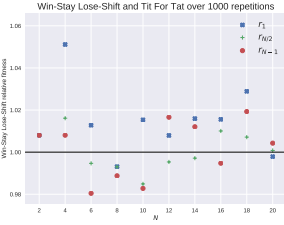
(d) Cooperator and Tit For Tat



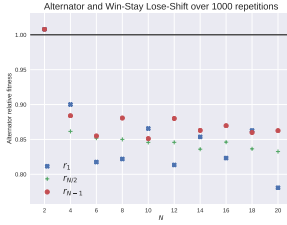
(e) Defector and Cooperator



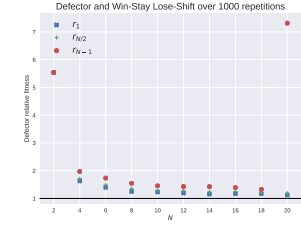
(f) Defector and Tit For Tat



(g) Win Stay Lose Shift and Tit For Tat

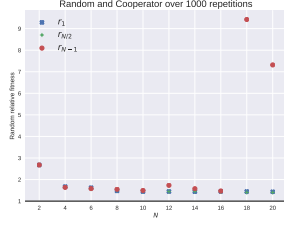


(h) Alternator and Win Stay Lose Shift

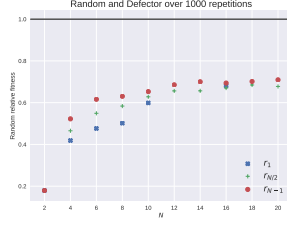


(i) Defector and Win Stay Lose Shift

Figure 17: Estimated relative fitness for **deterministic** strategies



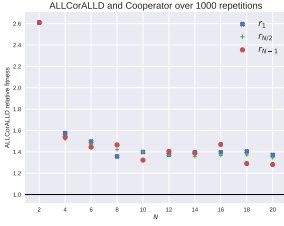
(a) Random and Cooperator



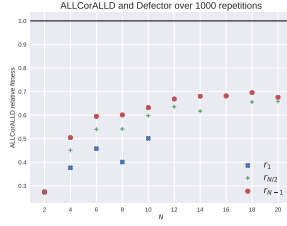
(b) Random and Defector



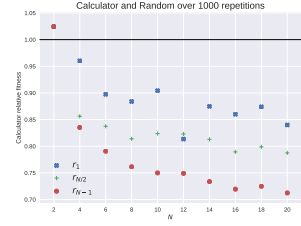
(c) Random and Tit For Tat



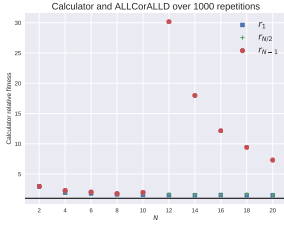
(d) All C or all D and Cooperator



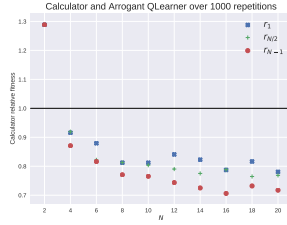
(e) All C or all D and Defector



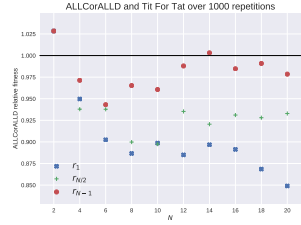
(f) Calculator and Random



(g) Calculator and All C or all D



(h) Calculator and Arrogant Q learner



(i) All C or all D and Tit For Tat

Figure 18: Estimated relative fitness for **stochastic** strategies

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A List of players

- | | | |
|------------------------|---|--|
| 1. Hard Tit For Tat | 7. π | 'D', 9, 'D'), (3, 'C', 0, 'C'), (3, 'D', 1, 'D'), (4, 'C', 1, 'D'), (4, 'D', 2, 'D'), (5, 'C', 12, 'C'), (5, 'D', 6, 'C'), (6, 'C', 1, 'C'), (6, 'D', 14, 'D'), (7, 'C', 12, 'D'), (7, 'D', 2, 'D'), (8, 'C', 7, 'D'), (8, 'D', 9, 'D'), (9, 'C', 8, 'D'), (9, 'D', 0, 'D'), (10, 'C', 2, 'C'), (10, 'D', 15, 'C'), (11, 'C', 7, 'D'), (11, 'D', 13, 'D'), (12, 'C', 3, 'C'), (12, 'D', 8, |
| 2. Negation | 8. Win-Shift Lose-Stay: D | |
| 3. Nice Average Copier | 9. Omega TFT: 3, 8 | |
| 4. Nydegger | 10. Opposite Grudger | |
| 5. Winner21 | 11. FSM Player: [(0, 'C', 13, 'D'), (0, 'D', 12, 'D'), (1, 'C', 3, 'D'), (1, 'D', 4, 'D'), (2, 'C', 14, 'D'), (2, | |
| 6. Winner12 | | |

- 'D'), (13, 'C', 7, 'C'), (13, 'D', 10, 'D'), (14, 'C', 10, 'D'), (14, 'D', 7, 'D'), (15, 'C', 15, 'C'), (15, 'D', 11, 'D')], 1, C
12. Once Bitten
 13. Predator
 14. FSM Player: [(0, 'C', 0, 'C'), (0, 'D', 3, 'C'), (1, 'C', 5, 'D'), (1, 'D', 0, 'C'), (2, 'C', 3, 'C'), (2, 'D', 2, 'D'), (3, 'C', 4, 'D'), (3, 'D', 6, 'D'), (4, 'C', 3, 'C'), (4, 'D', 1, 'D'), (5, 'C', 6, 'C'), (5, 'D', 3, 'D'), (6, 'C', 6, 'D'), (6, 'D', 6, 'D'), (7, 'C', 7, 'D'), (7, 'D', 5, 'C')], 1, C
 15. Prober 2
 16. Prober
 17. Prober 3
 18. Tricky Defector
 19. Tullock: 11
 20. VeryBad
 21. Prober 4
 22. Willing
 23. Worse and Worse 2
 24. Pun1
 25. Two Tits For Tat
 26. Tricky Cooperator
 27. PSO Gambler 2.2_2
 28. PSO Gambler 1.1_1
 29. Tit For Tat
 30. Worse and Worse
 31. ZD-Extort-2: 0.1111111111111111, 0.5
 32. Worse and Worse 3
 33. Win-Stay Lose-Shift: C
 34. Hard Tit For 2 Tats
 35. Tit For 2 Tats
 36. Hard Go By Majority
 37. Cycle Hunter
 38. Fool Me Once
 39. ZD-GTFT-2: 0.25, 0.5
 40. Adaptive
 41. Cooperator Hunter
 42. Forgiving Tit For Tat
 43. Handshake
 44. Eventual Cycle Hunter
 45. Retaliate 3: 0.05
 46. Forgetful Fool Me Once: 0.05
 47. Adaptive Tit For Tat: 0.5
 48. Forgiver
 49. Revised Downing: True
 50. Evolved ANN
 51. Forgetful Grudger
 52. Hard Go By Majority: 10
 53. PSO Gambler 2.2_2 Noise 05
 54. Cyclor CCCCCD
 55. Ripoff
 56. Aggravater
 57. Evolved ANN 5
 58. Fortress3
 59. Hard Go By Majority: 20
 60. ZD-GEN-2: 0.125, 0.5, 3
 61. ALLCorALLD
 62. Hard Prober
 63. Evolved ANN 5 Noise 05
 64. SelfSteem
 65. Alternator
 66. Fortress4
 67. Hard Go By Majority: 40
 68. Cyclor CCCD
 69. Risky QLearner
 70. FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'), (1, 'C', 11, 'D'), (1, 'D', 11, 'D'), (2, 'C', 8, 'D'), (2, 'D', 8, 'C'), (3, 'C', 3, 'C'), (3, 'D', 12, 'D'), (4, 'C', 6, 'C'), (4, 'D', 3, 'C'), (5, 'C', 11, 'C'), (5, 'D', 8, 'D'), (6, 'C', 13, 'D'), (6, 'D', 14, 'C'), (7, 'C', 4, 'D'), (7, 'D', 2, 'D'), (8, 'C', 14, 'D'), (8, 'D', 8, 'D'), (9, 'C', 0, 'C'), (9, 'D', 10, 'D'), (10, 'C', 8, 'C'), (10, 'D', 15, 'C'), (11, 'C', 6, 'D'), (11, 'D', 5, 'D'), (12, 'C', 6, 'D'), (12, 'D', 9, 'D'), (13, 'C', 9, 'D'), (13, 'D', 8, 'D'), (14, 'C', 8, 'D'), (14, 'D', 13, 'D'), (15, 'C', 4, 'C'), (15, 'D', 5, 'C')], 1, C
 71. Meta Hunter Aggressive: 7 players
 72. Alternator Hunter
 73. Evolved FSM 4
 74. Cyclor CCD
 75. Slow Tit For Two Tats 2
 76. e
 77. Soft Go By Majority
 78. Hard Go By Majority: 5
 79. ShortMem
 80. AntiCyclor
 81. Hesitant QLearner
 82. Cyclor DC
 83. Evolved FSM 16
 84. GTFT: 0.33
 85. Slow Tit For Two Tats
 86. Anti Tit For Tat
 87. Shubik
 88. General Soft Grudger: $n=1, d=4, c=2$
 89. Soft Grudger
 90. ZD-Extort-2 v2: 0.125, 0.5, 1
 91. Soft Go By Majority: 10
 92. Hopeless
 93. Adaptive Pavlov 2006

94. Sneaky Tit For Tat	118. Meta Hunter: 6 players	141. CollectiveStrategy
95. Inverse	119. Average Copier	142. Math Constant Hunter
96. Cycler DDC	120. Davis: 10	143. Firm But Fair
97. Evolved FSM 16 Noise 05	121. Limited Retaliate: 0.1, 20	144. Grofman
98. Adaptive Pavlov 2011	122. Stochastic Cooperator	145. Feld: 1.0, 0.5, 200
99. Soft Go By Majority: 20	123. ZD-Extort-4: 0.23529411764705882, 0.25, 1	146. ThueMorse
100. Inverse Punisher	124. ϕ	147. Cautious QLearner
101. PSO Gambler Mem1	125. ZD-SET-2: 0.25, 0.0, 2	148. Naive Prober: 0.1
102. SolutionB5	126. Better and Better	149. Resurrection
103. Appeaser	127. Limited Retaliate 2: 0.08, 15	150. Doubler
104. EvolvedLookerUp2.2_2	128. Defector Hunter	151. Fool Me Forever
105. Joss: 0.9	129. Random Hunter	152. Grudger
106. Cycler CCCDCD	130. Stochastic WSLs: 0.05	153. ThueMorseInverse
107. EvolvedLookerUp1.1_1	131. Gradual	154. Evolved HMM 5
108. Punisher	132. Limited Retaliate 3: 0.05, 20	155. GrudgerAlternator
109. Soft Go By Majority: 40	133. Random: 0.5	156. MEM2
110. SolutionB1	134. Retaliate 2: 0.08	157. Retaliate: 0.1
111. Arrogant QLearner	135. Remorseful Prober: 0.1	158. Contrite Tit For Tat
112. Soft Go By Majority: 5	136. Suspicious Tit For Tat	159. EasyGo
113. Defector	137. Bully	160. Calculator
114. Level Punisher	138. Desperate	161. Grumpy: Nice, 10, -10
115. Spiteful Tit For Tat	139. Gradual Killer: ('D', 'D', 'D', 'D', 'D', 'C', 'C')	162. Thumper
116. Raider	140. Tester	163. Cooperator
117. Soft Joss: 0.9		164. Eatherley