Evolving Invaders and Resistors for the Iterated Prisoner's Dilemma

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Abstract

We present insights and empirical results from an extensive numerical study of the evolutionary dynamics of the iterated prisoner's dilemma. Fixation probabilities for Moran processes are obtained for all pairs of 164 different strategies including classics such as TitForTat, zero determinant strategies, and many more sophisticated strategies. Players with long memories and sophisticated behaviours outperform many strategies that perform well in a two player setting. Moreover we introduce several strategies trained with evolutionary algorithms to excel at the Moran process. These strategies are excellent invaders and resistors of invasion and in some cases naturally evolve handshaking mechanisms to resist invasion. The best invaders were those trained to maximize total payoff while the best resistors invoke handshake machanisms. This suggests that while maximizing individual payoff can lead to the evolution of cooperation through invasion, the relatively weak invasion resistance of payoff maximizing strategies are not as evolutionarily stable as strategies employing handshake mechanisms.

1 Introduction

The Prisoner's Dilemma (PD) [8] is a fundamental two player game used to model a large variety of strategic interactions. Each player can choose between cooperation (C) or defection (D). The decisions are made simultaneously and independently. The payoffs of the game are defined by the matrix $\begin{pmatrix} R & S \\ T & P \end{pmatrix}$, where $T > R > P > S$ and $2R > T + S$. The PD is a one round game, but is commonly studied in a manner where the prior outcomes matter. This extended form is called the Iterated Prisoner's Dilemma (IPD). As described in [6, 12, 20] a number of strategies have been developed to take advantage of the history of play. Recently, some strategies referred to as zero Determinant strategies [20] can manipulate some players through extortionate mechanisms.

The Moran Process [18] is a model of evolutionary population dynamics that has been used to gain insights about the evolutionary stability in a number of settings (more details given in Section 1.1). Several earlier works have studied iterated games in the context of the prisoner's dilemma [19, 24], however these often make simplifying assumptions and/or do not consider sophisticated behaviour: only considering strategies that either cooperate or defect, or are limited to classes of strategies such as memory-one strategies that only use the previous round of play.

This manuscript provides a detailed numerical analysis of agent-based simulations of **164** complex and adaptive strategies for the IPD. This is made possible by the Axelrod library [25], an effort to provide software for reproducible research for the IPD. The library now contains over 186 parameterized strategies including classics like TitForTat and WinStayLoseShift, as well as recent variants such as OmegaTFT, zero determinant and other memory one strategies, strategies based on finite state machines, lookup tables, neural networks, and other machine learning based strategies, and a collection of novel strategies. Not all strategies have been considered for this study: excluded are those that make use of knowledge of the number of turns in a match and others that have a high computational run time. The large number of strategies are available thanks to the open source nature of the project with over 40 contributors from around the world, made by programmers and researchers [12]. Three of the considered strategies are finite state machines trained specifically for Moran processes (described further in Section 1.2.

In addition to providing a large collection of strategies, the Axelrod library can conduct matches, tournaments and population dynamics with variations including noise and spatial structure. The strategies and simulation frameworks are automatically tested to an extraordinarily high degree of coverage in accordance with best research software practices.

Using the Axelrod library and the many strategies it contains, we obtain fixation probabilities for all pairs of strategies, identifying those that are effective invaders and those resistant to invasion, for population sizes $N = 2$ to $N = 14$. Moreover we present a number of strategies that were created via reinforcement algorithms (evolutionary and particle swarm algorithms) that are among the best invaders and resistors of invasion known to date, and show that handshaking mechanims naturally arise from these processes as an invasion-resistance mechanism.

Recent work has argued that agent-based simulations can provide insights in evolutionary game theory not available via direct mathematical analysis [2]. The results and insights contained in this paper would be difficult to derive analytically.

In particular the following questions are addressed:

1. What strategies are good invaders?
2. What strategies are good at resisting invasion?
3. How does the population size affect these findings?

While the results agree with some of the published literature, it is found that:

1. Zero determinant strategies are not particularly effective for $N > 2$
2. Complex strategies can be effective, and in fact can naturally evolve through evolutionary processes to outperform designed strategies.
3. Strong resistors specifically evolve or have a handshake mechanism.
4. Strong invaders are generally cooperative strategies that do not defect first but retaliate to varying degrees of intensity against strategies that defect.
5. Strategies evolved to maximize their total payout can be strong invaders and achieve mutual cooperation with many other strategies.

1.1 The Moran Process

Figure 1 shows a diagrammatic representation of the Moran process, a stochastic birth death process on a finite population in which the population size stays constant over time. Individuals are **selected** according to a given fitness landscape. Once selected, the individual is reproduced and similarly another individual is chosen to be removed from the population. In some settings mutation is also considered but without mutation (the case considered in this work) this process will arrive at an absorbing state where the population is entirely made up of players of one strategy. The probability with which a given strategy takes over a the population is called the *fixation probability*. A more detailed analytic description of this is given in Section 2. In our simulations offspring do not inherit any knowledge or history from parent replicants.

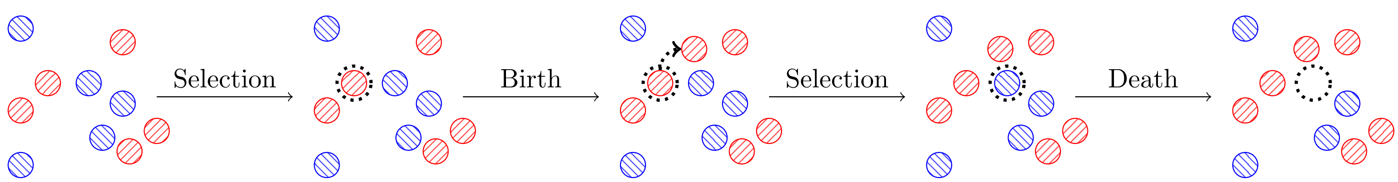


Figure 1: A diagrammatic representation of a Moran process

The Moran process was initially introduced in [18]. It has since been used in a variety of settings including the understanding of the spread of cooperative and non-cooperative behaviour such as cancer [27] and the emergence of cooperative behaviour in spatial topologies [4]. However these works mainly consider non-sophisticated strategies. Some work has looked at evolutionary stability of strategies within the Prisoner's Dilemma [15] but this is not done in the more widely used setting of the Moran process, rather in terms of infinite population stability. In [7] Moran processes are studied in a theoretical framework for a small subset of strategies. The subset included memory one strategies, strategies that recall the events of the previous round only.

Of particular interest are the Zero determinant strategies introduced in [20]. It was argued in [24] that generous ZD strategies are robust against invading strategies. However, in [13] a strategy using machine learning techniques was capable of resisting invasion and also able to invade any memory one strategy. Recent work [9] has investigated the effect of memory length on strategy performance and the emergence of cooperation but this is not done in a Moran process context and only considers specific cases of memory 2 strategies. In [1] it was recognised that many zero determinant strategies do not fare well against themselves. This is a disadvantage for the Moran process where the best strategies cooperate well with other players using the same strategy.

1.2 Strategies considered

To carry out this large numerical experiment, 164 strategies, listed (with their properties) in Appendix A are used from the Axelrod library. There are 43 stochastic and 121 deterministic strategies. Their memory depth, defined by the number of rounds of history used by the strategy each round, is shown in Table 1. The memory depth is infinite if the strategy uses the entire history of play (whatever its length). For example, a strategy that utilizes a handshaking mechanism where the opponents actions on the first few rounds of play determines the strategies subsequent behavior would have infinite memory depth.

A number of these strategies have been trained with reinforcement learning algorithms prior to this study and not specifically for the Moran process.

- Evolved ANN: a neural network based strategy;
- Evolved LookerUp: a lookup table based strategy;
- PSO Gambler: a stochastic version of the lookup table based strategy;
- Evolved HMM: a hidden Markov model based strategy.

A part from the PSO Gambler strategy, which was trained using a particle swarm optimisation algorithm, these strategies are trained with an evolutionary algorithm that perturbs strategy parameters and optimizes the mean total score against all other opponents [3]. They were trained to win IPD tournaments by maximizing their mean total payoffs against a variety of opponents. Variation is introduced via mutation and crossover of parameters, and the best performing strategies are carried to the next generation along with new variants. Similar methods appear in the literature [5].

More information about each player can be obtained in the documentation for [25] and a detailed description of the performance of these strategies in IPD tournaments will be described in upcoming manuscript(s).

All of the training code is available in the Axelrod repository with documentation to train new strategies easily. Training typically takes less than 100 generations and can be completed within several hours on commodity hardware.

There are three further strategies trained specifically for this study; Trained FSM 1, 2, and 3 (TF1 - TF3). These are based on finite state machines of 16, 16, and 8 states respectively (see Figures 2, 3 and 4).

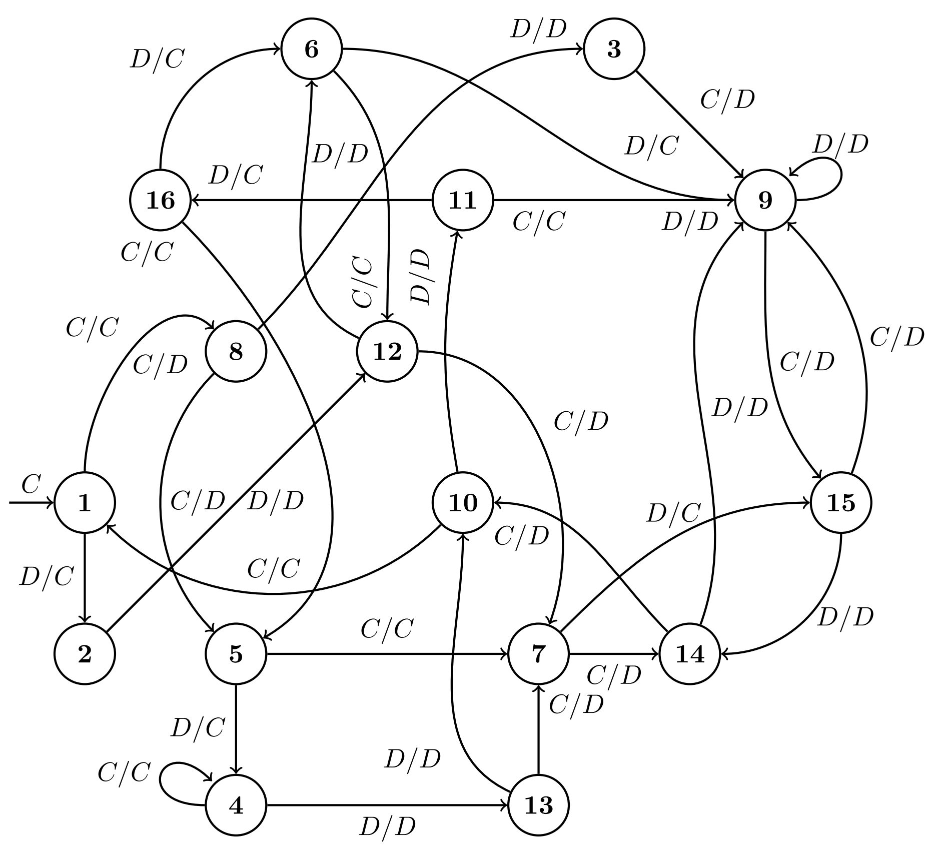


Figure 2: TF1: a 16 state finite state machine with a handshake leading to mutual cooperation at state 4.

As opposed to the previously described strategies, these strategies were trained with the objective function of **mean fixation probabilities for Moran processes** starting at initial population states consisting of $N/2$ individuals of the training candidates and $N/2$ individuals of an opponent strategy, taken from a selection of 150 opponents from the axelrod library:

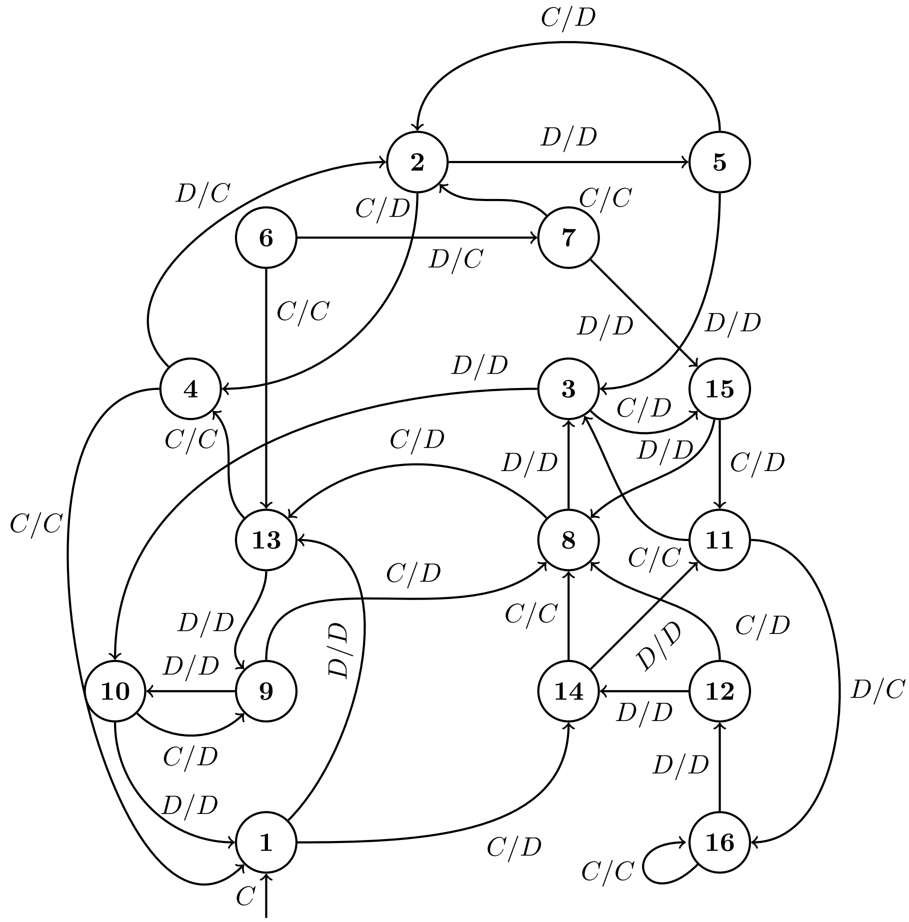


Figure 3: TF2: a 16 state finite state machine with a handshake leading to mutual cooperation at state 16.

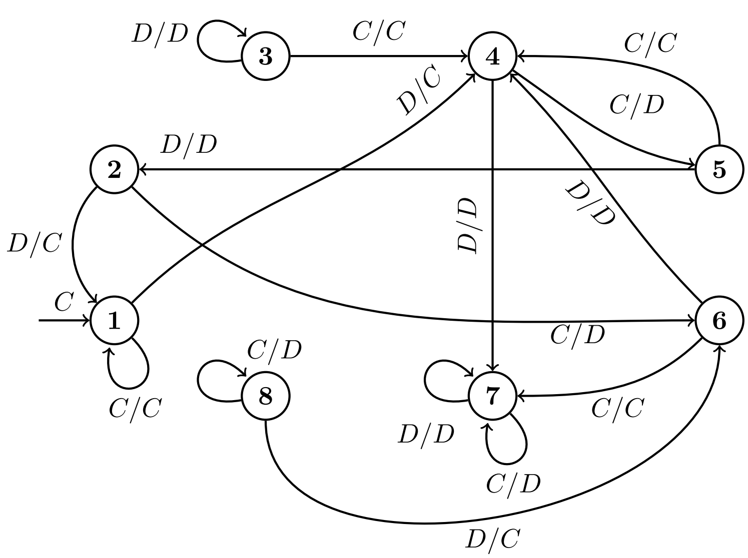


Figure 4: TF3: an 8 state finite state machine.

- TF1 $N = 12$, 0% noise, 10000 repetitions per matchup
- TF2 $N = 10$, 0% noise, 10000 repetitions per matchup
- TF3 $N = 8$, 1% noise, 100 repetitions per matchup

Each matchup of players was run to fixation for the specified number of repetitions to estimate the absorption probabilities. The trained algorithms were run for less than 50 generations. Training data for this is available at [11].

TF3 cooperates and defects with various cycles depending on the opponent's actions. TF3 will mutually cooperate with any strategy and only tolerates a few defections before defecting for the rest of match. It is similar to but not exactly the same as Fool Me Once, a strategy that cooperates until the opponent has defected twice (not necessarily consecutively), and defects indefinitely thereafter. Though a product of training with a Moran objective. It differs from TF1 and TF2 by lacking a handshake mechanism.

TF2 always starts with CD and will defect against opponents that start with DD. It plays CDD against itself and then cooperates thereafter. Cooperation can be rescued after a failed handshake by a complex sequence of plays which eventually results in mutual cooperation with Firm but Fair, Fortress3, Fortress4, and Grofman (always) and Evolved HMM 5 and GTFT (depending on the random seed). TF2 defects against all other players in the study, barring unusual cases arising from particular randomizations.

TF1 has an initial handshake of CCD and cooperates if the opponent matches. However if the opponent later defects, TF1 will respond in kind, so the handshake is not permanent. Only one player (Prober 4 [16]) manages to achieve cooperation with TF1 after about 20 rounds of play. TF1 is functionally very similar to a strategy known as "Collective Strategy", which has a handshake of CD and cooperates with opponents that matched the handshake until they defect, defecting thereafter if the opponent ever defects [14]. This strategy was specifically designed for evolutionary processes.

For both TF1 and TF2 a handshake mechanism naturally emerges from the structure of the underlying finite state machine. This behavior is an outcome of the evolutionary process and is in no way hard-coded or included via an additional mechanism.

Memory Depth	0	1	2	3	4	5	6	9	10	11	12	16	20	40	200	∞
Count	3	28	12	8	2	6	1	1	5	1	1	2	2	2	1	89

Table 1: Memory depth

1.3 Data collection

Each strategy pair is run for 1000 repetitions of the Moran process to fixation with starting population distributions of $(1, N-1)$, $(N/2, N/2)$ and $(N-1, 1)$, for N from 2 through 14. The fixation probability is then empirically computed for each combination of starting distribution and value of N. The axelrod library can carry out exact simulations of the Moran process. Since some of the strategies have a high computational cost or are stochastic, samples are taken from a large number of match outcomes for the pairs of players for use in computing fitnesses in the Moran process. This approach was verified to agree with unsampled calculations to a high degree of accuracy in specific cases. This is described in Algorithms 1 and 2.

Algorithm 1 Data Collection

for player one **in** players list **do**
  for player two **in** (players list - player one) **do**
    pair $\leftarrow$ (player one, player two)
    for starting population distributions **in** $[(1, N-1), (\frac{N}{2}, \frac{N}{2}), (N-1, 1)]$ **do**
      while repetitions ≤ 1000 **do**
        simulate moran process*(pair, starting distribution)
      end while
      return fixation probabilities
    end for
  end for
end for

Algorithm 2 Moran process

```
initial population ← (pair, starting distribution)
population ← initial population
while repetitions ≤ max repetitions do
    for player in population do
        for opponent in (population - player) do
            match ← (player, opponent)
            results ← cache (match)
        end for
    end for
    population ← sorted(results)
    parent ← selected randomly in proportion to total match payoffs
    child ← parent
    kill off ← random player from population
    population ← child replaces kill off
end while
```

Section 2 will further validate the methodology by comparing simulated results to analytical results in some cases. The main results of this manuscript are presented in Section 3 which will present a detailed analysis of all the data generated. Finally, Section 5 will conclude and offer future avenues for the work presented here.

2 Validation

As described in [19] consider the payoff matrix:

$$M = \begin{pmatrix} a, b \\ c, d \end{pmatrix} \tag{1}$$

The expected payoffs of i players of the first type in a population with $N-i$ players of the second type are given by:

$$f_i = \frac{a(i-1) + b(N-i)}{N-1} \tag{2}$$

$$g_i = \frac{ci + d(N-i-1)}{N-1} \tag{3}$$

The transitions within the birth death process that underpins the Moran process are then given by:

$$p_{i,i+1} = \frac{if_i}{if_i + (N-i)g_i}\frac{N-i}{N} \tag{4}$$

$$p_{i,i-1} = \frac{(N-i)g_i}{if_i + (N-i)g_i}\frac{i}{N} \tag{5}$$

$$p_{ii} = 1 - p_{i,i+1} - p_{i,i-1} \tag{6}$$

Using this it is a known result [4] that the fixation probability of the first strategy in a population of i individuals of the first type (and $N-i$ individuals of the second::

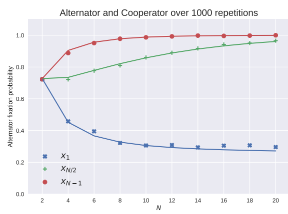
$$x_i = \frac{1 + \sum_{j=1}^{i-1}\prod_{k=1}^{j}\gamma_j}{1 + \sum_{j=1}^{N-1}\prod_{k=1}^{j}\gamma_j} \tag{7}$$

where:

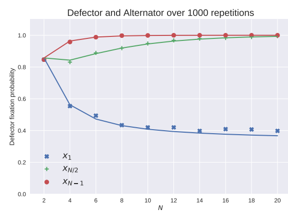
$$\gamma_j = \frac{p_{j,j-1}}{p_{j,j+1}}$$

A neutral strategy will have fixation probability $x_i = i/N$.

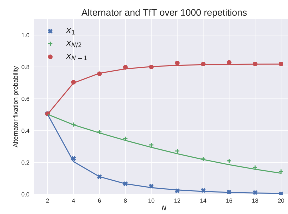
Comparisons of $x_1, x_{N/2}, x_{N-1}$ are shown in Figure 5. The points represent the simulated values and the line shows the theoretical value. Note that these are all deterministic strategies and show a perfect match up between the expected value of (7) and the actual Moran process for all strategies pairs. Figure 6 shows the fixation probabilities for stochastic strategies. These are no longer a good match which highlights the weakness of assuming a given interaction between two IPD strategies can be summarised with a set of utilities as shown in (1). For any given pair of strategies it is possible to obtain $p_{i,i-1}, p_{i,i+1}, p_{ii}$ exactly (as opposed to the approximations offered by (4), (5) and (6). Obtaining these requires particular analysis for a given pair and can be quite a complex endeavour for stochastic strategies with long memory: this is not necessary for the purposes of this work. All data generated for this validation exercise can be found at [11].



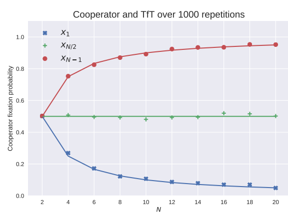
(a) Alternator and Cooperator



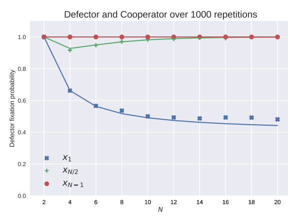
(b) Defector and Alternator



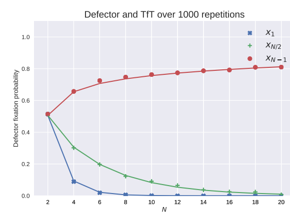
(c) Alternator and Tit For Tat



(d) Cooperator and Tit For Tat



(e) Defector and Cooperator



(f) Defector and Tit For Tat

(g) Win Stay Lose Shift and Tit For Tat

(h) Alternator and Win Stay Lose Shift

(i) Defector and Win Stay Lose Shift

Figure 5: Comparison of theoretic and actual Moran Process fixation probabilities for **deterministic** strategies

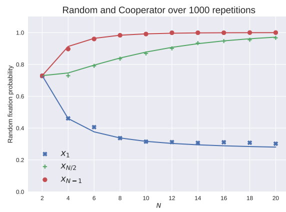
3 Empirical results

This section outlines the data analysis carried out:

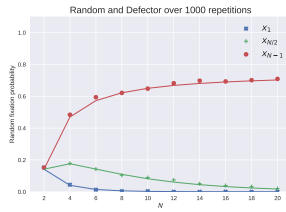
- Section 3.1 considers the specific case of $N = 2$.
- Section 3.2 investigates the effect of population size on the ability of a strategy to invade another population. This will highlight how complex strategies with long memories outperform simpler strategies.
- Section 3.3 similarly investigates the ability to defend against an invasion.
- Section 3.4 investigates the relationship between performance for differing population sizes as well as taking a close look at Zero determinant strategies [20].

3.1 The special case of $N = 2$

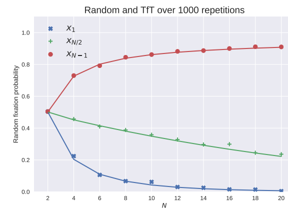
When $N = 2$ the Moran process is effectively a measure of the distribution of relative mean payoffs over all possible matches between two players. The strategy that scores higher than the other more often will fixate more often. For $N = 2$



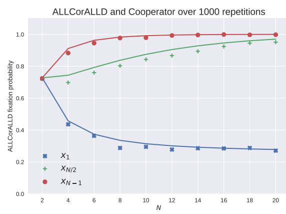
(a) Random and Cooperator



(b) Random and Defector

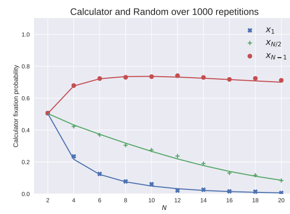


(c) Random and Tit For Tat

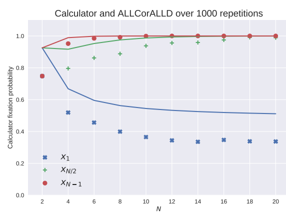


(d) All C or all D and Cooperator

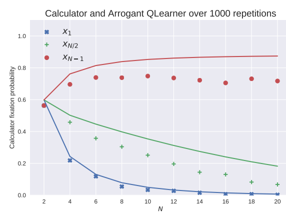
(e) All C or all D and Defector



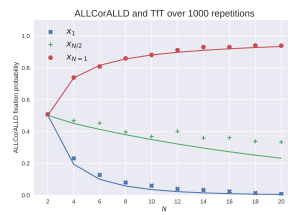
(f) Calculator and Random



(g) Calculator and All C or all D



(h) Calculator and Arrogant Q learner



(i) All C or all D and Tit For Tat

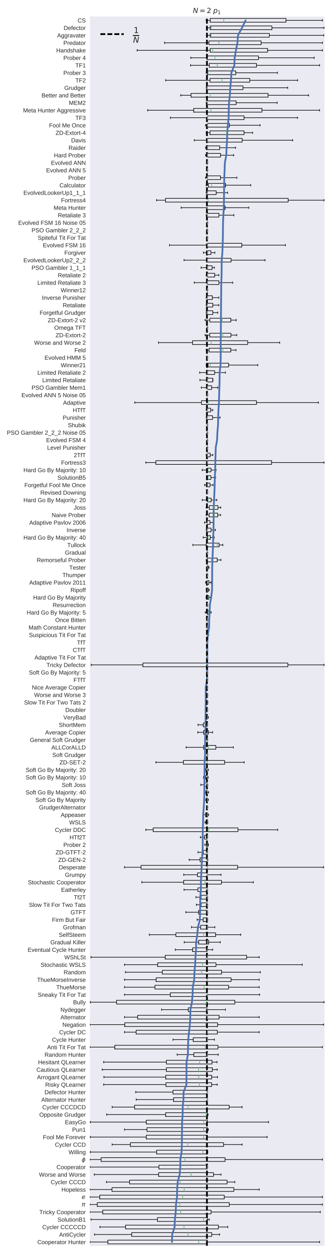
Figure 6: Comparison of theoretic and actual Moran Process fixation probabilities for **stochastic** strategies

the two cases of x_1 and x_{N-1} coincide, but will be considered separately for larger N in sections 3.2 and 3.3. Figure 7a shows all fixation probabilities for the strategies considered. The top 16 (10%) strategies are shown in Table 7b. The top five ranking strategies are:

1. The top strategy is the Collective Strategy (CS) which has a simple handshake mechanism described above.
2. Defector: it always defects. As it has little potential interaction with itself (recall that $N = 2$), its aggressiveness is rewarded.
3. Aggravater, which plays like Grudger (responding to any defections with unconditional defections throughout) however starts by playing 3 defections.
4. Predator, a finite state machine described in [5].
5. Handshake, a slightly less aggressive version of the Collective strategy [22]. As long as the initial sequence is played then it cooperates. Thus it will do well in a population consisting of many members of itself just as the Collective Strategy does. The difference is that CS will defect after the handshake if the opponent defects while handshake will not.

It is also noted that TF1, TF2 and TF3 all perform well. This is also the N for which a zero determinant strategy does appear in the top 10% ranking strategies: ZD-extort-4. The performance of zero determinant strategies will be examined more closely in Section 3.4.

As will be demonstrated in Section 3.4 the results for $N = 2$ differ from those of larger N. Hence these results do not concur with the literature which suggests that zero Determinant strategies should be effective for larger population sizes, but these analysis consider stationary behaviour, while this work runs for a fixed number of rounds. [24] The stationarity assumption allows for a deterministic payoff matrix leading to the conclusions about zero determinant strategies in the space of memory-one strategies that do not generalize to this context.



(a) The fixation probabilities for $N = 2$

	Player	Mean p_1
1	CS	0.6651
2	Defector	0.6496
3	Aggravater	0.6328
4	Predator	0.6301
5	Handshake	0.6240
6	Prober 4	0.6183
7	**TF1**	0.6171
8	Prober 3	0.6044
9	**TF2**	0.6026
10	Grudger	0.5996
11	Better and Better	0.5980
12	MEM2	0.5942
13	Meta Hunter Aggressive	0.5933
14	**TF3**	0.5927
15	Fool Me Once	0.5892
16	ZD-Extort-4	0.5867

(b) Top strategies for $N = 2$ (neutral fixation is $p = 0.5$).

Figure 7: Performance of strategies for $N = 2$

3.2 Strong Invaders

In this section the focus is on the ability of a mutant strategy to invade: the probability of one individual of a given type successfully fixating in a population of $N-1$ other individuals, denoted by x_1. The ranks of each strategy for all considered values of N according to mean x_1 are shown in Figure 8.

The fixation probabilities are shown in Figures 9a, 9b and 9c for $N \in \{3, 7, 14\}$ showing the mean fixation as well as the neutral fixation for each given scenario.

The top 16 strategies are given in Tables 2.

	Player	Mean p_1
1	CS	0.4478
2	Grudger	0.4313
3	MEM2	0.4278
4	**TF3**	0.4267
5	Prober 4	0.4242
6	Fool Me Once	0.4242
7	Davis	0.4218
8	Predator	0.4210
9	Evolved ANN 5	0.4163
10	Evolved ANN	0.4163
11	Evolved FSM 16	0.4154
12	Meta Hunter	0.4140
13	**TF1**	0.4139
14	PSO Gambler 2_2_2	0.4134
15	EvolvedLookerUp1_1_1	0.4113
16	Evolved FSM 16 Noise 05	0.4107

(a) $N = 3$

	Player	Mean p_1
1	Evolved FSM 16	0.2523
2	PSO Gambler 2_2_2	0.2467
3	Fool Me Once	0.2459
4	Evolved ANN 5	0.2450
5	Evolved ANN	0.2449
6	EvolvedLookerUp2_2_2	0.2443
7	Grudger	0.2442
8	MEM2	0.2436
9	**TF3**	0.2430
10	PSO Gambler 1_1_1	0.2404
11	CS	0.2395
12	Evolved FSM 16 Noise 05	0.2394
13	Evolved HMM 5	0.2390
14	Meta Hunter	0.2385
15	Davis	0.2379
16	PSO Gambler Mem1	0.2348

(b) $N = 7$

	Player	Mean p_1
1	Evolved FSM 16	0.2096
2	PSO Gambler 2_2_2	0.2042
3	EvolvedLookerUp2_2_2	0.2014
4	Evolved ANN	0.2014
5	Evolved ANN 5	0.2004
6	Evolved HMM 5	0.1972
7	PSO Gambler 1_1_1	0.1955
8	Fool Me Once	0.1955
9	Evolved FSM 16 Noise 05	0.1943
10	PSO Gambler Mem1	0.1920
11	Evolved FSM 4	0.1918
12	Meta Hunter	0.1869
13	Evolved ANN 5 Noise 05	0.1858
14	Omega TFT	0.1849
15	Fortress4	0.1848
16	**TF3**	0.1846

(c) $N = 14$

Table 2: Top invaders for $N \in \{3, 7, 14\}$

It can be seen that apart from CS, none of the strategies of Table 7b perform well for $N \in \{3, 7, 14\}$. The new top performing strategies are:

- Grudger (which only performs well for $N = 3$), starts by cooperating but will defect if at any point the opponent has defected.
- MEM2, an infinite memory strategy that switches between TFT, TF2T, and Defector [15].
- TF3, the finite state machine trained specifically for Moran processes described in Section 1.
- Prober 4, a strategy which starts with a specific 20 move sequence of cooperations and defections [16]. This initial sequence serves as approximate handshake.
- PSO Gambler and Evolved Lookerup 2 2 2: are strategies that make use of a lookup table mapping the first 2 moves of the opponent as well as the last 2 moves of both players to an action. The PSO gambler is a stochastic version which maps those states to probabilities of cooperating. The lookerup was described in [12].
- The Evolved ANN strategies are neural networks that map a number of attributes (first move, number of cooperations, last move, etc.) to an action. Both of these have been trained using an evolutionary algorithm.

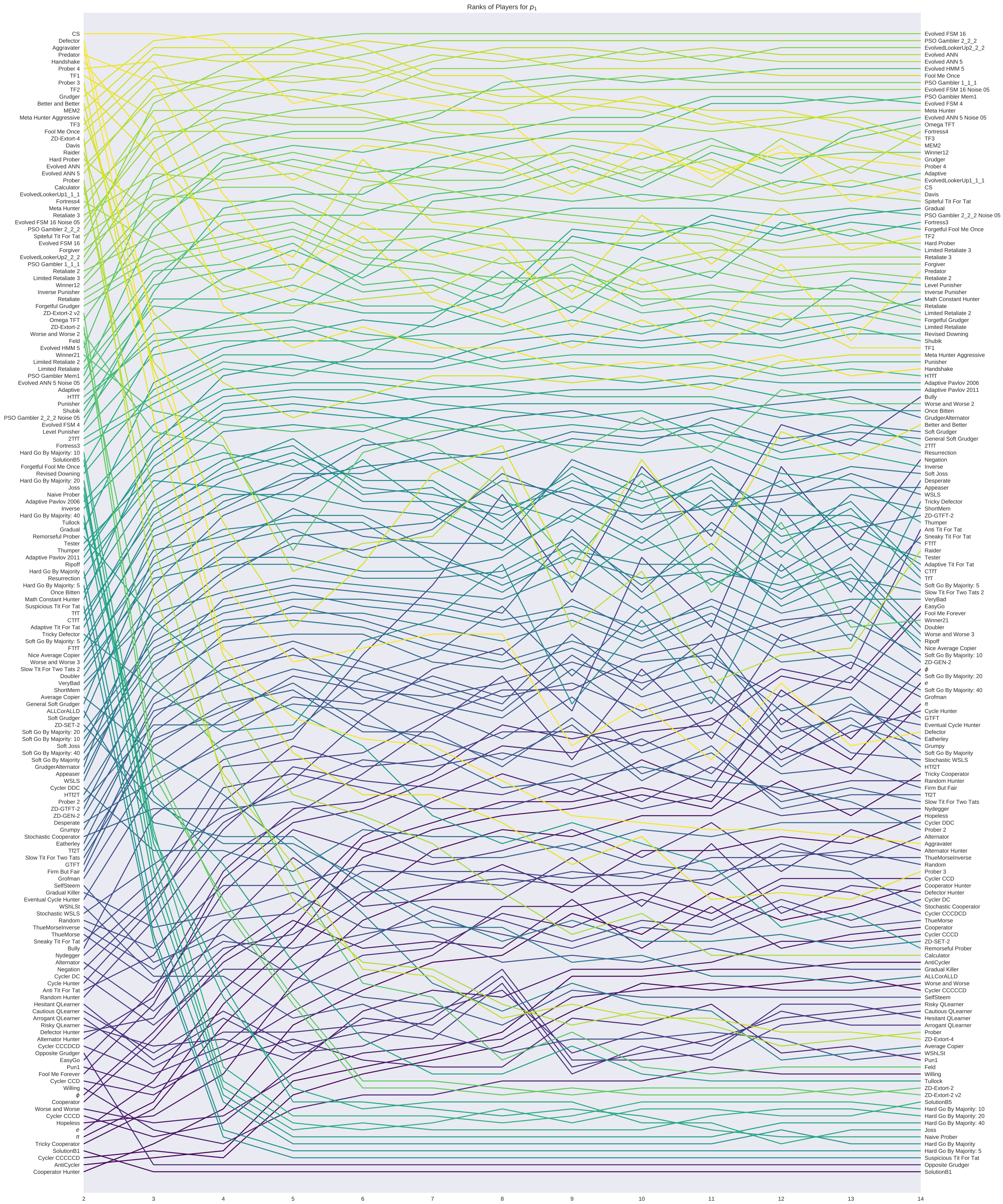


Figure 8: Ranks of all strategies according to x_1 for different population sizes

(a) $N = 3$ (b) $N = 7$ (c) $N = 14$

Figure 9: The fixation probabilities x_1

- The Evolved FSM 16 is a 16 state finite state machine trained to perform well in tournaments.

Only one of the above strategies is stochastic although close inspection of the source code of PSO Gambler shows that it makes stochastic decisions rarely, and is functionally very similar to its deterministic cousin Evolved Looker Up. The PSO Gambler Mem1 strategy is a memory one strategy that has been trained to maximise its utility and does perform well. Apart from TF3, the finite state machines trained specifically for Moran processes do not appear in the top 5, while strategies trained for tournaments do. This is due to the nature of invasion: most of the opponents will initially be different strategies. The next section will consider the converse situation.

3.3 Strong resistors

In addition to identifying good invaders, strategies resistant to invasion by other strategies are identified by examining the distribution of x_{N-1} for each strategy. The ranks of each strategy for all considered values of N according to mean x_{N-1} are shown in Figures 10.

The fixation probabilities are shown in Figures 11a, 9b and 11c for $N \in \{3, 7, 14\}$ showing the mean fixation as well as the neutral fixation for each given scenario.

Table 3 shows the top strategies when ranked according to x_{N-1} for $N \in \{3, 7, 14\}$. Once again none of the short memory strategies from Section 3.1 perform well for high N.

	Player	Mean p_{N-1}
1	CS	0.8359
2	Predator	0.8121
3	**TF1**	0.8087
4	Handshake	0.8014
5	**TF2**	0.7957
6	Prober 4	0.7905
7	Grudger	0.7612
8	Hard Prober	0.7582
9	**TF3**	0.7570
10	MEM2	0.7554
11	Davis	0.7536
12	Winner21	0.7529
13	Fool Me Once	0.7489
14	Fortress4	0.7467
15	Retaliate 3	0.7448
16	EvolvedLookerUp1_1_1	0.7422

(a) $N = 3$

	Player	Mean p_{N-1}
1	CS	0.9765
2	**TF1**	0.9714
3	**TF2**	0.9677
4	Predator	0.9677
5	Handshake	0.9547
6	Prober 4	0.9540
7	Winner21	0.9392
8	Hard Prober	0.9331
9	Fortress4	0.9255
10	Grudger	0.9198
11	**TF3**	0.9189
12	Davis	0.9186
13	Ripoff	0.9183
14	Tester	0.9176
15	MEM2	0.9165
16	Retaliate 3	0.9161

(b) $N = 7$

	Player	Mean p_{N-1}
1	CS	0.9984
2	**TF1**	0.9973
3	**TF2**	0.9949
4	Predator	0.9941
5	Prober 4	0.9863
6	Handshake	0.9812
7	Winner21	0.9778
8	Hard Prober	0.9731
9	Fortress4	0.9726
10	Ripoff	0.9669
11	Tester	0.9662
12	Grudger	0.9592
13	**TF3**	0.9589
14	Davis	0.9588
15	Retaliate 3	0.9580
16	Retaliate	0.9576

(c) $N = 14$

Table 3: Top resistors for $N \in \{3, 7, 14\}$

Interestingly none of these strategies are stochastic: this is explained by the need of strategies to have a steady hand when interacting with their own kind. Acting stochastically increases the chance of friendly fire. However it is possible to design a strategy with a stochastic or error-correcting handshake that is an excellent resistor even in noisy environments [13].

There are are only two new strategies that appear in the top ranks for x_{N-1}: TF1 and TF2. These two strategies are with CS the strongest resistors. They all have handshakes, and whilst the handshakes of CS and Handshake (which ranks highly for the smaller values of N) were programmed, the handshakes of TF1 and TF2 evolved through an evolutionary process without any priming.

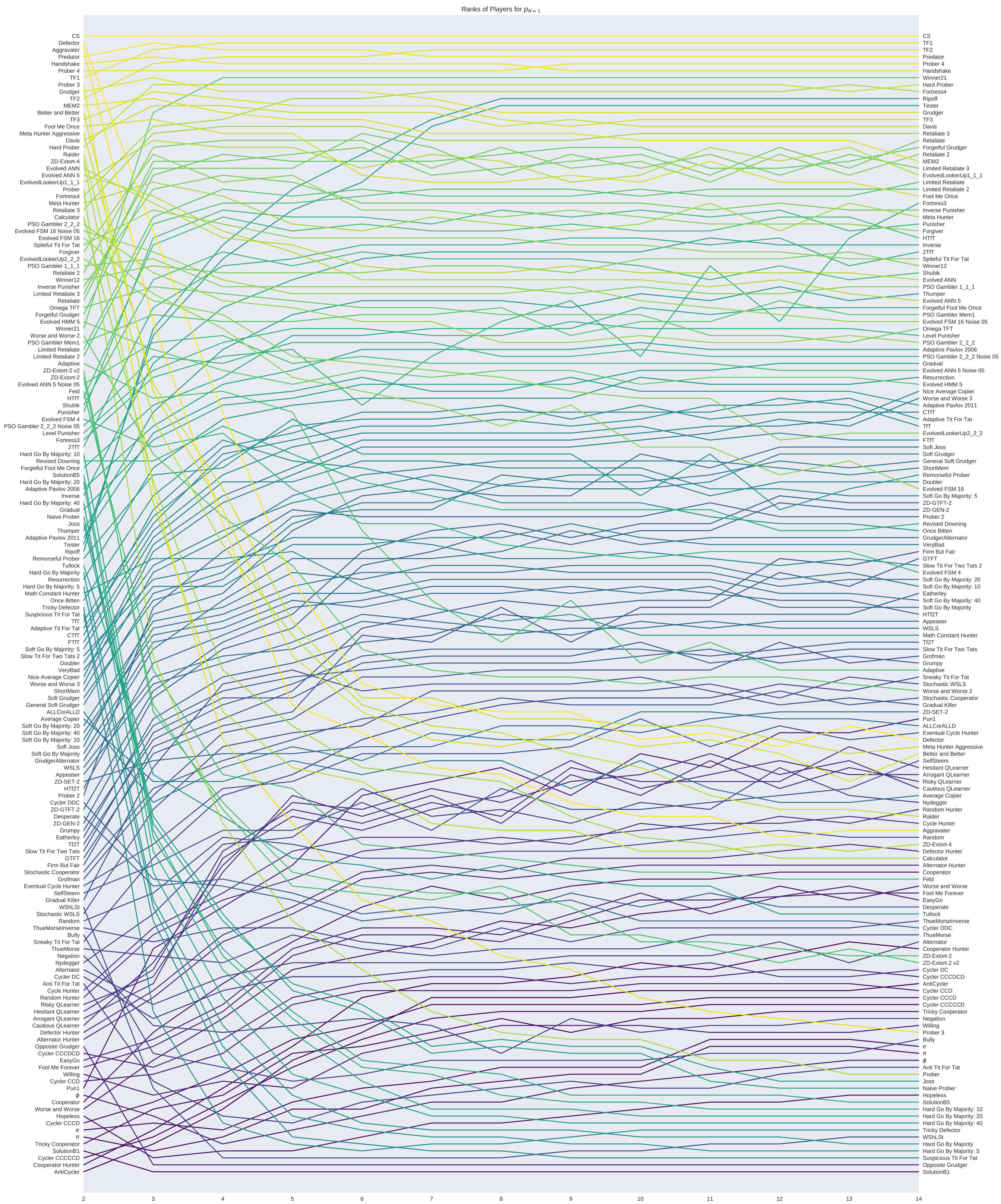
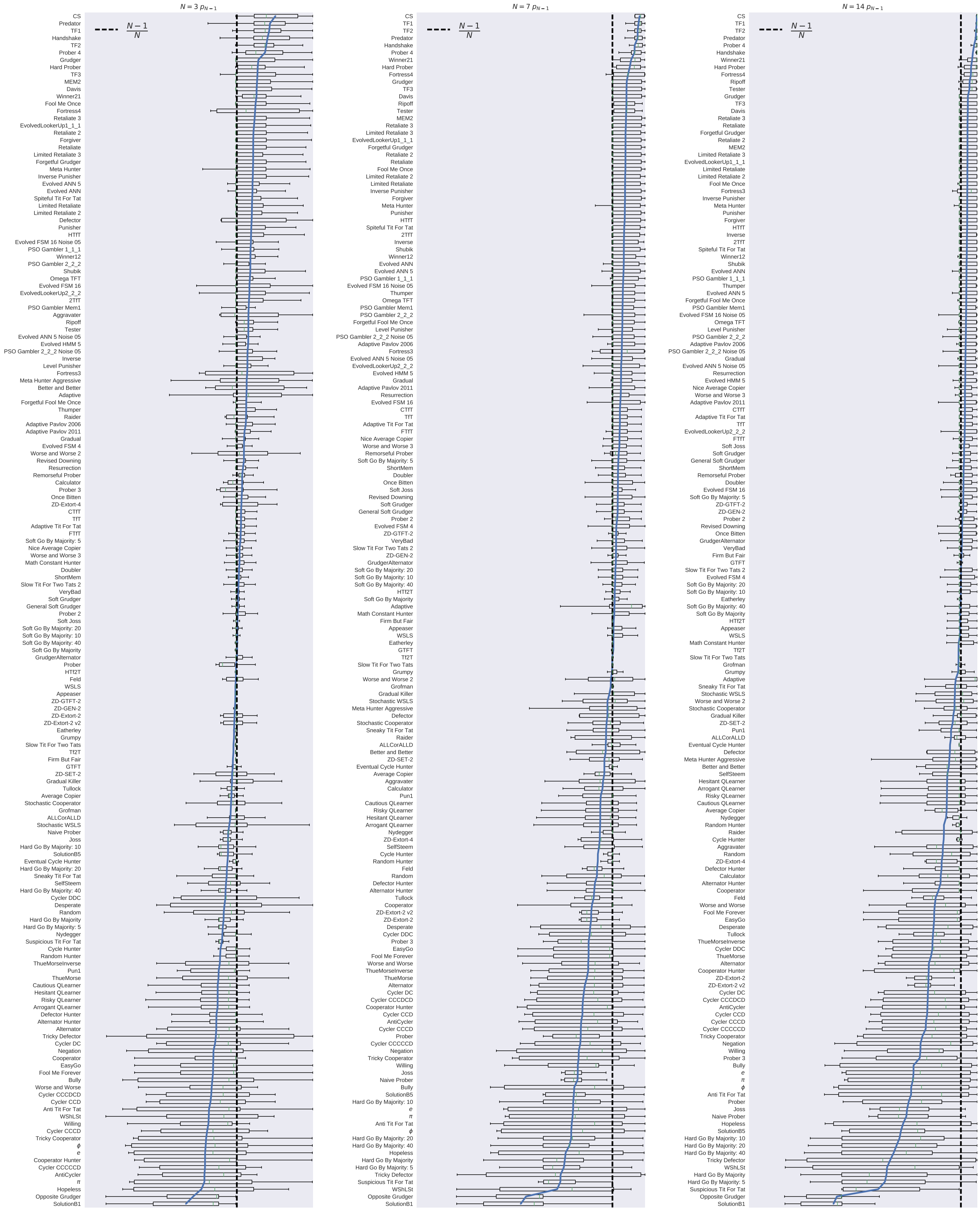


Figure 10: Ranks of all strategies according to x_{N-1} for different population sizes



(a) $N = 3$ (b) $N = 7$ (c) $N = 14$

Figure 11: The fixation probabilities x_{N-1}

As described in Section 3.2 the strategies trained with the payoff maximizing objective are among the best invaders in the library however they are not as resistant to invasion as the strategies trained using a Moran objective function. These strategies include trained finite state machine strategies, but they do not appear to have handshaking mechanisms. Therefore it is reasonable to conclude that the objective function is the cause of the emergence of handshaking mechanisms.

The payoff maximizing strategies typically will not defect before the opponent's first defection, possibly because the training strategy collection contains some strategies such as Grudger and Fool Me Once that retaliate harshly by defecting for the remainder of the match if the opponent has more than a small number of cumulative defections. Paradoxically it is advantageous to defect (as a signal) in order to achieve mutual cooperation with opponents using the same strategy but not with other opponents. Nevertheless an evolutionary process is able to tunnel through the costs and risks associated to early defections to find more optimal solutions, so it is not surprising in hindsight that handshaking strategies emerge from the evolutionary training process.

A handshake requires at least one defection and there is selective pressure to defect as few times as possible to achieve the self-recognition mechanism. It is also unwise to defect on the first move as some strategies additionally retaliate first round defections. So the handshakes used by TF1, TF2, and CS are in some sense optimal.

It is evident through Sections 3.1, 3.2 and 3.3 that performance of strategies not only depends on the initial population distribution but also that there seems to be a difference depending on whether or not $N > 2$. This will be explored further in the next section, looking not only at x_1 and x_{N-1} but also consider $x_{N/2}$.

3.4 The effect of population size

To complement Figures 8 and 10 Figure 12 shows the rank of each strategy based on $x_{N/2}$. Tables 4, 5 and 6 show the same information for a selection of strategies:

- The strategies that ranked highly for $N = 2$;
- The strategies that ranked highly for $N = 14$;
- The zero determinant strategies.

For all starting populations $i \in \{1, N/2, N-1\}$ the ranks of strategies are relatively stable across the different values of $N > 2$ however for $N = 2$ there is a distinct difference. This highlights that there is little that can be inferred about the evolutionary performance of a strategy in a large population from its performance in a small population.

This is confirmed by the performance of the zero determinant strategies. While some do rank relatively highly for $N = 2$ (ZD-extort-4 has rank 16) this rank does not translate to larger populations.

Player	2	3	4	5	6	7	8	9	10	11	12	13	14
CS	1	1	2	11	9	11	13	21	16	22	17	25	23
Defector	2	43	80	91	89	87	87	103	97	105	94	103	101
Aggravater	3	50	89	99	102	103	108	113	114	115	115	116	117
Predator	4	8	24	35	28	33	31	43	36	43	34	45	35
Handshake	5	17	40	46	43	46	46	49	48	49	47	50	49
Evolved FSM 16	31	11	6	2	1	1	1	1	1	1	1	1	1
PSO Gambler 2_2_2	29	14	10	6	4	2	2	2	2	2	2	2	2
EvolvedLookerUp2_2_2	33	18	11	9	10	6	6	5	3	5	3	3	3
Evolved ANN	20	10	8	7	8	5	3	3	4	3	4	4	4
Evolved ANN 5	21	9	7	8	7	4	5	4	5	4	5	5	5
ZD-Extort-4	16	81	107	120	135	136	142	140	142	142	144	144	145
ZD-Extort-2 v2	41	105	126	140	152	152	153	152	153	153	153	152	153
ZD-Extort-2	43	107	125	139	151	151	152	153	152	152	152	153	152
ZD-SET-2	100	111	117	117	122	127	131	128	131	131	130	132	131
ZD-GTFT-2	112	92	82	80	81	82	84	72	81	71	78	72	70
ZD-GEN-2	113	96	87	83	85	88	90	82	87	82	86	83	91

Table 4: Invasion: Fixation ranks of some strategies according to x_1 for different population sizes

Figure 13 show the correlation coefficients of the ranks of strategies in differing population size. How well a strategy performs in any Moran process for $N > 2$ has little to do with the performance for $N = 2$. This illustrates why the strong performance of zero determinant strategies predicted in [20] does not extend to larger populations. This was discussed theoretically in [1] and observed empirically in these simulations.

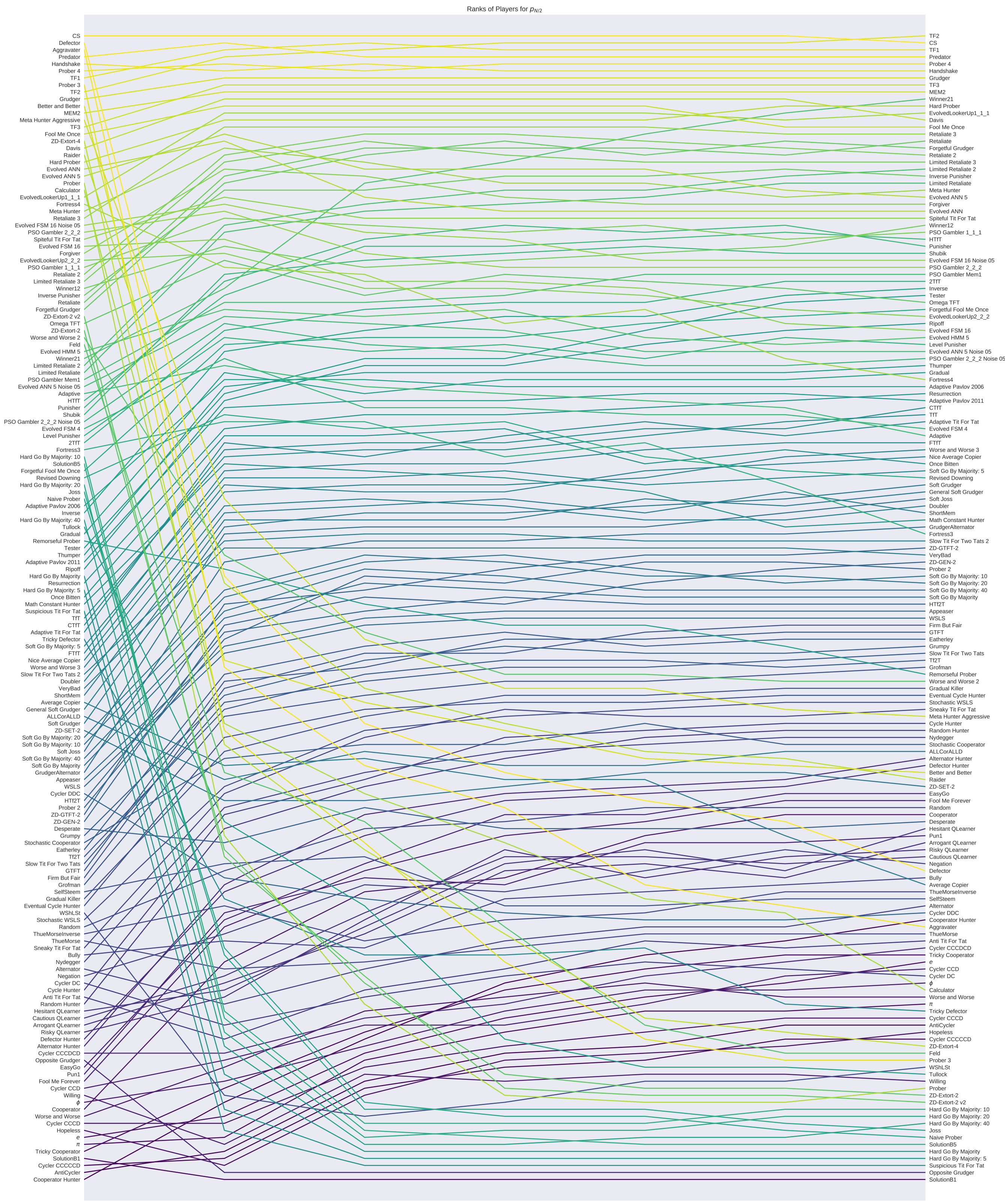


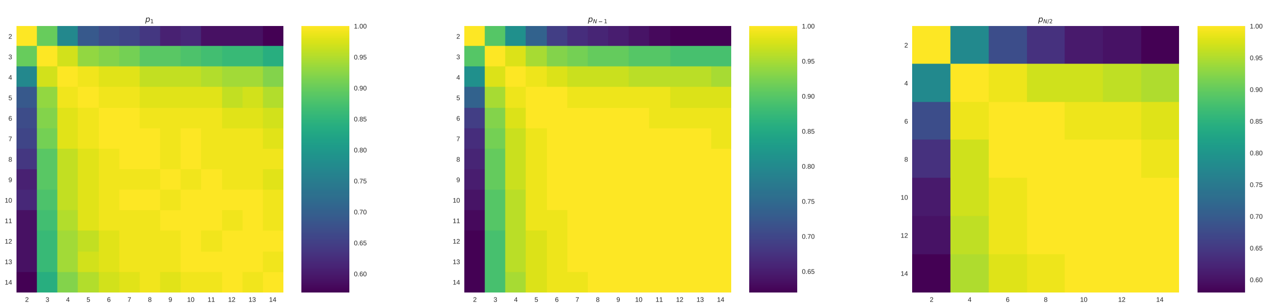
Figure 12: Fixation ranks of all strategies according to $x_{N/2}$ for different population sizes

Player	2	3	4	5	6	7	8	9	10	11	12	13	14
CS	1	1	1	1	1	1	1	1	1	1	1	1	1
Defector	2	29	55	79	94	97	98	98	102	101	103	100	102
Aggravater	3	42	71	97	101	106	107	111	113	113	116	115	115
Predator	4	2	3	3	3	4	4	4	4	4	4	4	4
Handshake	5	4	5	5	5	5	5	6	6	6	6	6	6
TF1	7	3	2	2	2	2	2	2	2	2	2	2	2
TF2	10	5	4	4	4	3	3	3	3	3	3	3	3
Prober 4	6	6	6	6	6	6	6	5	5	5	5	5	5
ZD-Extort-4	19	68	98	106	108	114	115	115	118	118	117	118	117
ZD-Extort-2 v2	49	98	111	121	123	124	124	130	130	132	134	132	134
ZD-Extort-2	50	97	112	123	124	125	123	126	131	131	132	133	133
ZD-SET-2	108	105	104	104	103	103	100	100	101	99	98	98	98
ZD-GTFT-2	112	95	88	84	75	72	71	73	71	71	67	68	68
ZD-GEN-2	114	96	89	86	77	75	72	74	72	72	68	69	69

Table 5: Resistance: Fixation ranks of some strategies according to x_{N-1} for different population sizes

Player	2	4	6	8	10	12	14
CS	1	1	1	1	1	1	2
Defector	2	78	99	106	110	113	120
Aggravater	3	91	105	111	122	125	128
Predator	4	2	4	4	4	4	4
Handshake	5	6	5	6	6	6	6
TF2	9	4	3	2	2	2	1
TF1	7	3	2	3	3	3	3
Prober 4	6	5	6	5	5	5	5
ZD-Extort-4	16	102	117	129	141	143	145
ZD-Extort-2 v2	41	118	135	151	152	152	153
ZD-Extort-2	43	117	136	149	151	151	152
ZD-SET-2	100	110	110	108	106	106	108
ZD-GTFT-2	112	82	80	77	75	75	74
ZD-GEN-2	113	85	81	82	79	77	76

Table 6: Ranks of some strategies according to $x_{N/2}$ for different population sizes



(a) Rank based on x_1 (b) Rank based on x_{N-1} (c) Rank based on $x_{N/2}$

Figure 13: Heatmap of correlation coefficients of rankings by population size

4 Discussion

Training strategies to excel at the Moran process leads to the evolution of cooperation, but only with like individuals in the case of TF1 and TF2. This may have significant implications for human social interactions such as the evolution of ingroup/outgroup mechanisms and other sometimes costly rituals that reinforce group behavior.

While TF1 and TF2 are competent invaders, the best invaders in the study do not appear to employ strict handshakes, and are generally cooperative strategies. TF3, which does not use a handshake, is a better invader than TF1 and TF2 but not as good of a resistor. Nevertheless it was the result of the same kind of training procceses and is a better combined invader-resistor than the invaders that were trained previously to maximize payout.

The strategies trained to maximize payoff in head-to-head matches are generally cooperative and are effective invaders. Combined with the fact that handshaking strategies are stronger resisters, this suggests that while maximizing individual payoff can lead to the evolution of cooperation, these strategies are not the most evolutionarily stable in the long run. A strategy with a handshaking mechanism is still capable of invading and is more resistant to subsequent invasions. Moreover, the best resistor of the payoff maximally trained strategies (Evolved Looker Up 1_1_1), which always defects if the opponent defects in the first round, is effectively employing a one-shot handshake of C. These insights also suggest that a strategy aware of the population distribution could choose to become a handshaker at a critical threshold and use a different strategy for invasion when in the minority. Information about the population distribution was not available to our strategies.

We did not attempt other objective functions that may serve to select for both invasion and resistance better than training at a starting population of $(N/2, N/2)$. Nevertheless our results suggest that there is not much room for improvement. Any handshake more sophisticated than C necessarily involves a defection. (A handshake consisting of a longer sequence of cooperations is effectively a grudger.) For TF3 or EvolvedLookerUp1_1_1 to become better resistors they need a longer or more strict handshake. But if this handshake involves a defection then likely the invasion ability is diminished for $N > 2$: the top invaders for larger N are nice strategies that do not defect before their opponents. This is because good invaders still need to cooperate with themselves, and so in the absense of a handshake mechanism or knowledge of the population distribution, a strategy must be generally cooperative. Aggressive strategies are only effective invaders for the smallest N, dropping down dramatically as the population size increases.

We did, however, attempt to evolve CS using FSM and lookup table based players, which resulted in some very similar strategies. In particular we evolved a lookup strategy that had a handshake of DC and played TFT with other players after a correct handshake while defecting otherwise, which is quite close in function to CS (full grudging is not possible with a lookup table).

Finally we note that it may be possible to achieve similar results with smaller capacity finite state machine players.

5 Conclusion

A detailed empirical analysis of 164 strategies of the IPD within a pairwise Moran process has been carried out. All $\binom{164}{2} = 13,366$ possible ordered pairs of strategies have been placed in a Moran process with different starting values allowing the each strategy to attempt to invade the other. This is the largest such experiment carried out and has lead to many insights.

When studying evolutionary processes it is vital to consider $N > 2$ since results for $N = 2$ cannot be used to extrapolate performance in larger populations. This was shown both observationally in Sections 3.2 and 3.3 but also by considering the correlation of the ranks in different population sizes in Section 3.4.

Memory one strategies do not perform well in general in this study. There are no memory one strategies in the top 5 performing strategies for $N > 3$. This is due at least partly to their lack of ability to recognise their opponents. More sophisticated strategies prove to be high performers for invasion: these are infinite memory strategies which have been trained using a number of reinforcement learning algorithms. Interestingly they have been trained to perform well in tournaments and not Moran processes which highlights the potentially for improvement.

One of the major findings discussed in Section 3.3, is the ability of strategies with a handshake mechanism to resist invasion. This was not only revealed for CS (a human designed strategy) but also for two FSM strategies (TF1 and TF2) specifically trained through an evolutionary process. In these two cases, the handshake mechanism was a product of the evolutionary process.

These findings are important for the ongoing understanding of population dynamics and offer evidence for some of the shortcomings of low memory which has started to be recognised by the community [9].

All source code for this work has been written in a sustainable manner: it is open source, under version control and tested which ensures that all results can be reproduced [21, 23, 28]. The raw data as well as the processed data has also been properly archived and can be found at [11].

There are many opportunities build on this work. In particular, an analysis of the effect of noise should offer insights regarding the stability of the findings, particularly for the handshaking strategies. They may be less dominant for larger amounts of noise since the handshaking mechanisms may become brittle. There are many other variations to explore including populations with more than one type, spatial structure, and mutation.

Acknowledgements

This work was performed using the computational facilities of the Advanced Research Computing @ Cardiff (ARCCA) Division, Cardiff University.

A variety of software libraries have been used in this work:

- The axelrod library (IPD strategies and Moran processes) [25].
- The matplotlib library (visualisation) [10].
- The pandas and numpy libraries (data manipulation) [17, 26].

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A List of players

1. ϕ - *Deterministic* - *Memory depth*: ∞
2. π - *Deterministic* - *Memory depth*: ∞
3. e - *Deterministic* - *Memory depth*: ∞
4. ALLCorALLD - *Stochastic* - *Memory depth*: 1
5. Adaptive - *Deterministic* - *Memory depth*: ∞
6. Adaptive Pavlov 2006 - *Deterministic* - *Memory depth*: ∞
7. Adaptive Pavlov 2011 - *Deterministic* - *Memory depth*: ∞
8. Adaptive Tit For Tat: 0.5 - *Deterministic* - *Memory depth*: ∞
9. Aggravater - *Deterministic* - *Memory depth*: ∞
10. Alternator - *Deterministic* - *Memory depth*: 1
11. Alternator Hunter - *Deterministic* - *Memory depth*: ∞
12. Anti Tit For Tat - *Deterministic* - *Memory depth*: 1
13. AntiCycler - *Deterministic* - *Memory depth*: ∞
14. Appeaser - *Deterministic* - *Memory depth*: ∞
15. Arrogant QLearner - *Stochastic* - *Memory depth*: ∞
16. Average Copier - *Stochastic* - *Memory depth*: ∞
17. Better and Better - *Stochastic* - *Memory depth*: ∞
18. Bully - *Deterministic* - *Memory depth*: 1
19. Calculator - *Stochastic* - *Memory depth*: ∞
20. Cautious QLearner - *Stochastic* - *Memory depth*: ∞
21. CollectiveStrategy (**CS**) - *Deterministic* - *Memory depth*: ∞
22. Contrite Tit For Tat (**CTfT**) - *Deterministic* - *Memory depth*: 3
23. Cooperator - *Deterministic* - *Memory depth*: 0

24. Cooperator Hunter - *Deterministic* - *Memory depth*: ∞

25. Cycle Hunter - *Deterministic* - *Memory depth*: ∞

26. Cycler CCCCCD - *Deterministic* - *Memory depth*: 5

27. Cycler CCCD - *Deterministic* - *Memory depth*: 3

28. Cycler CCCDCD - *Deterministic* - *Memory depth*: 5

29. Cycler CCD - *Deterministic* - *Memory depth*: 2

30. Cycler DC - *Deterministic* - *Memory depth*: 1

31. Cycler DDC - *Deterministic* - *Memory depth*: 2

32. Davis: 10 - *Deterministic* - *Memory depth*: ∞

33. Defector - *Deterministic* - *Memory depth*: 0

34. Defector Hunter - *Deterministic* - *Memory depth*: ∞

35. Desperate - *Stochastic* - *Memory depth*: 1

36. Doubler - *Deterministic* - *Memory depth*: ∞

37. EasyGo - *Deterministic* - *Memory depth*: ∞

38. Eatherley - *Stochastic* - *Memory depth*: ∞

39. Eventual Cycle Hunter - *Deterministic* - *Memory depth*: ∞

40. Evolved ANN - *Deterministic* - *Memory depth*: ∞

41. Evolved ANN 5 - *Deterministic* - *Memory depth*: ∞

42. Evolved ANN 5 Noise 05 - *Deterministic* - *Memory depth*: ∞

43. Evolved FSM 16 - *Deterministic* - *Memory depth*: 16

44. Evolved FSM 16 Noise 05 - *Deterministic* - *Memory depth*: 16

45. Evolved FSM 4 - *Deterministic* - *Memory depth*: 4

46. Evolved HMM 5 - *Stochastic* - *Memory depth*: 5

47. EvolvedLookerUp1_1_1 - *Deterministic* - *Memory depth*: ∞

48. EvolvedLookerUp2_2_2 - *Deterministic* - *Memory depth*: ∞

49. FSM Player: [(0, 'C', 0, 'C'), (0, 'D', 3, 'C'), (1, 'C', 5, 'D'), (1, 'D', 0, 'C'), (2, 'C', 3, 'C'), (2, 'D', 2, 'D'), (3, 'C', 4, 'D'), (3, 'D', 6, 'D'), (4, 'C', 3, 'C'), (4, 'D', 1, 'D'), (5, 'C', 6, 'C'), (5, 'D', 3, 'D'), (6, 'C', 6, 'D'), (6, 'D', 6, 'D'), (7, 'C', 7, 'D'), (7, 'D', 5, 'C')], 0, C (**TF3**) - *Deterministic* - *Memory depth*: ∞

50. FSM Player: [(0, 'C', 13, 'D'), (0, 'D', 12, 'D'), (1, 'C', 3, 'D'), (1, 'D', 4, 'D'), (2, 'C', 14, 'D'), (2, 'D', 9, 'D'), (3, 'C', 0, 'C'), (3, 'D', 1, 'D'), (4, 'C', 1, 'D'), (4, 'D', 2, 'D'), (5, 'C', 12, 'C'), (5, 'D', 6, 'C'), (6, 'C', 1, 'C'), (6, 'D', 14, 'D'), (7, 'C', 12, 'D'), (7, 'D', 2, 'D'), (8, 'C', 7, 'D'), (8, 'D', 9, 'D'), (9, 'C', 8, 'D'), (9, 'D', 0, 'D'), (10, 'C', 2, 'C'), (10, 'D', 15, 'C'), (11, 'C', 7, 'D'), (11, 'D', 13, 'D'), (12, 'C', 3, 'C'), (12, 'D', 8, 'D'), (13, 'C', 7, 'C'), (13, 'D', 10, 'D'), (14, 'C', 10, 'D'), (14, 'D', 7, 'D'), (15, 'C', 15, 'C'), (15, 'D', 11, 'D')], 0, C (**TF2**) - *Deterministic* - *Memory depth*: ∞

51. FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'), (1, 'C', 11, 'D'), (1, 'D', 11, 'D'), (2, 'C', 8, 'D'), (2, 'D', 8, 'C'), (3, 'C', 3, 'C'), (3, 'D', 12, 'D'), (4, 'C', 6, 'C'), (4, 'D', 3, 'C'), (5, 'C', 11, 'C'), (5, 'D', 8, 'D'), (6, 'C', 13, 'D'), (6, 'D', 14, 'C'), (7, 'C', 4, 'D'), (7, 'D', 2, 'D'), (8, 'C', 14, 'D'), (8, 'D', 8, 'D'), (9, 'C', 0, 'C'), (9, 'D', 10, 'D'), (10, 'C', 8, 'C'), (10, 'D', 15, 'C'), (11, 'C', 6, 'D'), (11, 'D', 5, 'D'), (12, 'C', 6, 'D'), (12, 'D', 9, 'D'), (13, 'C', 9, 'D'), (13, 'D', 8, 'D'), (14, 'C', 8, 'D'), (14, 'D', 13, 'D'), (15, 'C', 4, 'C'), (15, 'D', 5, 'C')], 0, C (**TF1**) - *Deterministic* - *Memory depth*: ∞

52. Feld: 1.0, 0.5, 200 - *Stochastic* - *Memory depth*: 200

53. Firm But Fair - *Stochastic* - *Memory depth*: 1

54. Fool Me Forever - *Deterministic* - *Memory depth*: ∞

55. Fool Me Once - *Deterministic* - *Memory depth*: ∞

56. Forgetful Fool Me Once: 0.05 - *Stochastic* - *Memory depth*: ∞

57. Forgetful Grudger - *Deterministic* - *Memory depth*: 10

58. Forgiver - *Deterministic* - *Memory depth*: ∞

59. Forgiving Tit For Tat (**FTfT**) - *Deterministic* - *Memory depth*: ∞

60. Fortress3 - *Deterministic* - *Memory depth*: 3

61. Fortress4 - *Deterministic* - *Memory depth*: 4

62. GTFT: 0.33 - *Stochastic* - *Memory depth*: 1

63. General Soft Grudger: n=1,d=4,c=2 - *Deterministic* - *Memory depth*: ∞

64. Gradual - *Deterministic* - *Memory depth*: ∞

65. Gradual Killer: ('D', 'D', 'D', 'D', 'D', 'C', 'C') - *Deterministic* - *Memory depth*: ∞

66. Grofman - *Stochastic* - *Memory depth*: ∞

67. Grudger - *Deterministic* - *Memory depth*: ∞

68. GrudgerAlternator - *Deterministic* - *Memory depth*: ∞

69. Grumpy: Nice, 10, -10 - *Deterministic* - *Memory depth*: ∞
70. Handshake - *Deterministic* - *Memory depth*: ∞
71. Hard Go By Majority - *Deterministic* - *Memory depth*: ∞
72. Hard Go By Majority: 10 - *Deterministic* - *Memory depth*: 10
73. Hard Go By Majority: 20 - *Deterministic* - *Memory depth*: 20
74. Hard Go By Majority: 40 - *Deterministic* - *Memory depth*: 40
75. Hard Go By Majority: 5 - *Deterministic* - *Memory depth*: 5
76. Hard Prober - *Deterministic* - *Memory depth*: ∞
77. Hard Tit For 2 Tats (**HTf2T**) - *Deterministic* - *Memory depth*: 3
78. Hard Tit For Tat (**HTfT**) - *Deterministic* - *Memory depth*: 3
79. Hesitant QLearner - *Stochastic* - *Memory depth*: ∞
80. Hopeless - *Stochastic* - *Memory depth*: 1
81. Inverse - *Stochastic* - *Memory depth*: ∞
82. Inverse Punisher - *Deterministic* - *Memory depth*: ∞
83. Joss: 0.9 - *Stochastic* - *Memory depth*: 1
84. Level Punisher - *Deterministic* - *Memory depth*: ∞
85. Limited Retaliate 2: 0.08, 15 - *Deterministic* - *Memory depth*: ∞
86. Limited Retaliate 3: 0.05, 20 - *Deterministic* - *Memory depth*: ∞
87. Limited Retaliate: 0.1, 20 - *Deterministic* - *Memory depth*: ∞
88. MEM2 - *Deterministic* - *Memory depth*: ∞
89. Math Constant Hunter - *Deterministic* - *Memory depth*: ∞
90. Meta Hunter Aggressive: 7 players - *Deterministic* - *Memory depth*: ∞
91. Meta Hunter: 6 players - *Deterministic* - *Memory depth*: ∞
92. Naive Prober: 0.1 - *Stochastic* - *Memory depth*: 1
93. Negation - *Stochastic* - *Memory depth*: 1
94. Nice Average Copier - *Stochastic* - *Memory depth*: ∞
95. Nydegger - *Deterministic* - *Memory depth*: 3
96. Omega TFT: 3, 8 - *Deterministic* - *Memory depth*: ∞
97. Once Bitten - *Deterministic* - *Memory depth*: 12
98. Opposite Grudger - *Deterministic* - *Memory depth*: ∞
99. PSO Gambler 1_1_1 - *Stochastic* - *Memory depth*: ∞
100. PSO Gambler 2_2_2 - *Stochastic* - *Memory depth*: ∞
101. PSO Gambler 2_2_2 Noise 05 - *Stochastic* - *Memory depth*: ∞
102. PSO Gambler Mem1 - *Stochastic* - *Memory depth*: 1
103. Predator - *Deterministic* - *Memory depth*: 9
104. Prober - *Deterministic* - *Memory depth*: ∞
105. Prober 2 - *Deterministic* - *Memory depth*: ∞
106. Prober 3 - *Deterministic* - *Memory depth*: ∞
107. Prober 4 - *Deterministic* - *Memory depth*: ∞
108. Pun1 - *Deterministic* - *Memory depth*: 2
109. Punisher - *Deterministic* - *Memory depth*: ∞
110. Raider - *Deterministic* - *Memory depth*: 3
111. Random Hunter - *Deterministic* - *Memory depth*: ∞
112. Random: 0.5 - *Stochastic* - *Memory depth*: 0
113. Remorseful Prober: 0.1 - *Stochastic* - *Memory depth*: 2
114. Resurrection - *Deterministic* - *Memory depth*: 1
115. Retaliate 2: 0.08 - *Deterministic* - *Memory depth*: ∞
116. Retaliate 3: 0.05 - *Deterministic* - *Memory depth*: ∞
117. Retaliate: 0.1 - *Deterministic* - *Memory depth*: ∞
118. Revised Downing: True - *Deterministic* - *Memory depth*: ∞
119. Ripoff - *Deterministic* - *Memory depth*: 2
120. Risky QLearner - *Stochastic* - *Memory depth*: ∞
121. SelfSteem - *Stochastic* - *Memory depth*: ∞
122. ShortMem - *Deterministic* - *Memory depth*: 10
123. Shubik - *Deterministic* - *Memory depth*: ∞
124. Slow Tit For Two Tats - *Deterministic* - *Memory depth*: 2
125. Slow Tit For Two Tats 2 - *Deterministic* - *Memory depth*: 2
126. Sneaky Tit For Tat - *Deterministic* - *Memory depth*: ∞

127. Soft Go By Majority - *Deterministic* - *Memory depth*: ∞

128. Soft Go By Majority: 10 - *Deterministic* - *Memory depth*: 10

129. Soft Go By Majority: 20 - *Deterministic* - *Memory depth*: 20

130. Soft Go By Majority: 40 - *Deterministic* - *Memory depth*: 40

131. Soft Go By Majority: 5 - *Deterministic* - *Memory depth*: 5

132. Soft Grudger - *Deterministic* - *Memory depth*: 6

133. Soft Joss: 0.9 - *Stochastic* - *Memory depth*: 1

134. SolutionB1 - *Deterministic* - *Memory depth*: 3

135. SolutionB5 - *Deterministic* - *Memory depth*: 5

136. Spiteful Tit For Tat - *Deterministic* - *Memory depth*: ∞

137. Stochastic Cooperator - *Stochastic* - *Memory depth*: 1

138. Stochastic WSLS: 0.05 - *Stochastic* - *Memory depth*: 1

139. Suspicious Tit For Tat - *Deterministic* - *Memory depth*: 1

140. Tester - *Deterministic* - *Memory depth*: ∞

141. ThueMorse - *Deterministic* - *Memory depth*: ∞

142. ThueMorseInverse - *Deterministic* - *Memory depth*: ∞

143. Thumper - *Deterministic* - *Memory depth*: 2

144. Tit For 2 Tats (**Tf2T**) - *Deterministic* - *Memory depth*: 2

145. Tit For Tat (**TfT**) - *Deterministic* - *Memory depth*: 1

146. Tricky Cooperator - *Deterministic* - *Memory depth*: 10

147. Tricky Defector - *Deterministic* - *Memory depth*: ∞

148. Tullock: 11 - *Stochastic* - *Memory depth*: 11

149. Two Tits For Tat (**2TfT**) - *Deterministic* - *Memory depth*: 2

150. VeryBad - *Deterministic* - *Memory depth*: ∞

151. Willing - *Stochastic* - *Memory depth*: 1

152. Win-Shift Lose-Stay: D (**WShLSt**) - *Deterministic* - *Memory depth*: 1

153. Win-Stay Lose-Shift: C (**WSLS**) - *Deterministic* - *Memory depth*: 1

154. Winner12 - *Deterministic* - *Memory depth*: 2

155. Winner21 - *Deterministic* - *Memory depth*: 2

156. Worse and Worse - *Stochastic* - *Memory depth*: ∞

157. Worse and Worse 2 - *Stochastic* - *Memory depth*: ∞

158. Worse and Worse 3 - *Stochastic* - *Memory depth*: ∞

159. ZD-Extort-2 v2: 0.125, 0.5, 1 - *Stochastic* - *Memory depth*: 1

160. ZD-Extort-2: 0.1111111111111111, 0.5 - *Stochastic* - *Memory depth*: 1

161. ZD-Extort-4: 0.23529411764705882, 0.25, 1 - *Stochastic* - *Memory depth*: 1

162. ZD-GEN-2: 0.125, 0.5, 3 - *Stochastic* - *Memory depth*: 1

163. ZD-GTFT-2: 0.25, 0.5 - *Stochastic* - *Memory depth*: 1

164. ZD-SET-2: 0.25, 0.0, 2 - *Stochastic* - *Memory depth*: 1