An empirical study of fixation for strategies in the Iterated Prisoner's Dilemma

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Abstract

The Iterated Prisoner's Dilemma is a well established framework for the study of emergent behaviour. In this paper an extensive numerical study of the evolutionary dynamics of this framework are presented.

Fixation probabilities for Moran processes are obtained for 172 different strategies. This is done in both a standard 200 turn interaction and a noisy setting.

To the authors knowledge this is the largest such study. It allows for insights about the behaviour and performance of strategies with regard to their survival in an evolutionary setting.

1 Introduction

Since the formulation of the Moran Process in [11], this model of evolutionary population dynamics has been used to gain insights about the evolutionary stability of strategies in a number of settings. Similarly since the first Iterated Prisoner's Dilemma (IPD) tournament described in [3] the Prisoner's dilemma has been used to understand the evolution of cooperative behaviour in complex systems.

The analytical models of a Moran process are based on the relative fitness between two strategies and take this to be a fixed value r [13]. This is a valid model for simple strategies of the Prisoner's Dilemma such as to *always cooperate* or *always defect*. This manuscript provides a detailed numerical analysis of **164** complex and adaptive strategies for the IPD. In this case the relative fitness of a strategy is dependent on the population distribution.

Further deviations from the analytical model occur when interactions between players are subject to uncertainty. This is referred to as noise and has been considered in the IPD setting in [5, 12, 17].

This work provides answers to the following questions:

1. What strategies are good invaders?
2. What strategies are good at resisting invasion?
3. How does the population size affect these findings?

Figure 1 shows a diagrammatic representation of the Moran process. This process is a stochastic birth death process on a finite population in which the population size stays constant over time. Individuals are **selected** according to a given fitness landscape. Once selected, a given individual is reproduced and similarly another individual is chosen to be removed from the population. In some settings mutation is also considered but without mutation (the case considered in this work) this process will arrive at an absorbing state where the population is entirely made up of a single individual. The probability with which a given strategy is the survivor is called the fixation probability. A more detailed analytic description of this is given in Section 3.

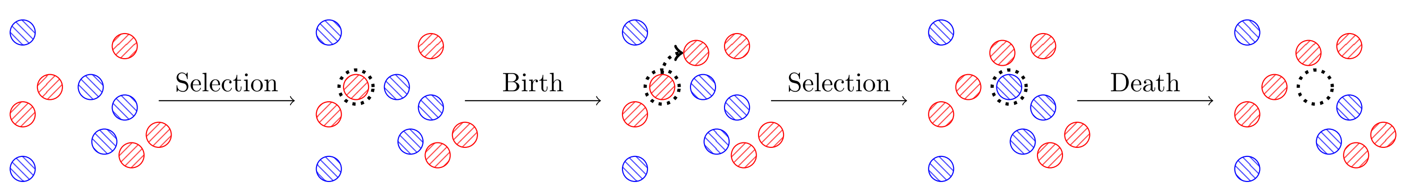


Figure 1: A diagrammatic representation of a Moran process

The Moran process was initially introduced in [11] in a genetic setting. It has since been used in a variety of settings including the understanding of the spread of cooperative behaviour. However, as stated before, these mainly consider non sophisticated strategies. Some work has looked at evolutionary stability of strategies within the Prisoner's Dilemma [9] but this is not done in the more widely used setting of the Moran process but in terms of infinite population stability. In [4] Moran processes are looked at in a theoretic framework for a small subset of strategies. In [8] machine learning techniques are used to train a strategy capable of resisting invasion and also invade any memory one strategy. Recent work [6] has investigated the effect of memory length on strategy performance and the emergence of cooperation but this is not done in Moran process context and only considers specific cases of memory 2 strategies.

The contribution of this work is a detailed and extensive analysis of absorption probabilities for 164 strategies. These strategies and the numerical simulations are from [15] which is an open source research library written for the study of the IPD. The strategies and simulation frameworks are automatically tested in accordance to best research software practice. The large number of strategies are available thanks to the open source nature of the project with over 40 contributions made by different programmers. Thus by considering Moran processes with population size greater than 2 we are taking in to account the effect of complex population dynamics. By considering sophisticated strategies we are taking in to effect the reputation of a strategy during each interaction.

Section 2 will explain the methodological approach used, Section 3 will validate the methodology by comparing simulated results to analytical results. The main results of this manuscript are presented in Section 4 which will present a detailed analysis of all the data generated. Finally, Section 5 will conclude and offer future avenues for the work presented here.

2 Methodology

To carry out this large numerical experiment 164 strategies are used from [15]. These include 161 default strategies in the library at the time (excluding strategies classified as having a long run time and those that make use of the length of the game) as well as the following 3 finite state machine machine strategies [2]:

Appendix A shows all the players in question. More information about each player can be obtained in the documentation for [15]. There are 43 stochastic and 121 deterministic strategies. Their memory depth is shown in Table 1.

Memory Depth	0	1	2	3	4	5	6	9	10	11	12	16	20	40	200	∞
Count	3	31	12	8	2	6	1	1	5	1	1	2	2	2	1	86

Table 1: Memory depth

All strategies are paired and these pairs are used in 1000 repetitions of a Moran process assuming a starting population of $(N/2, N/2)$. This is repeated for even N between 2 and 14. The fixation probability is then estimated for each value of N.

Note that due to the high computational cost of these experiments, for any given interaction between two players within the Moran process the outcome is sampled from a pre computed cache of 1000 match outcomes. This is carried out using an software written for the purpose of this work. This has been implemented in [15] ensuring that it can be used to either reproduce the work or carry out further work.

Figure 2 shows the distribution of the number of outcomes between all strategy pairs. Tables 2 shows that 95% of the stochastic matches have less than 788 unique outcomes whilst the maximum number is 971. This ensures that using a set of cached results from 1000 precomputed matches is sufficient for the analysis taking place here.

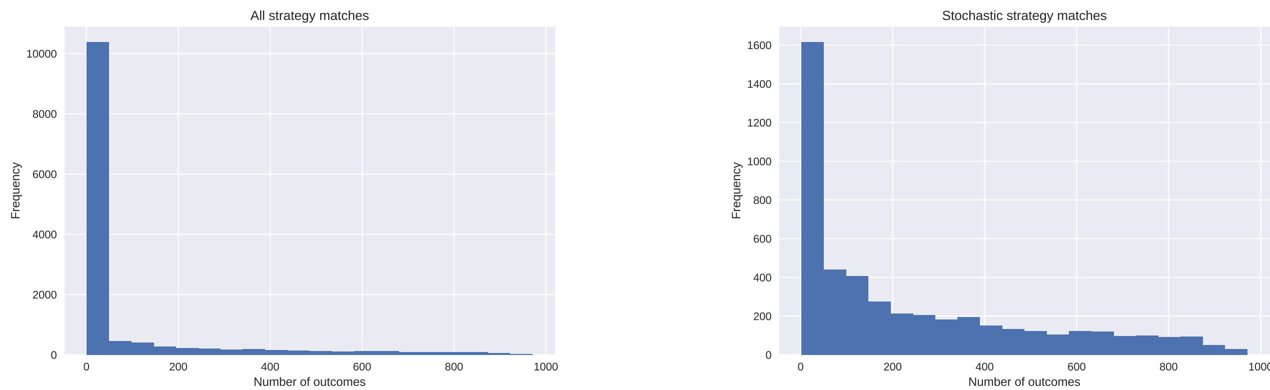
	Outcome count
count	13530.00
mean	85.98
std	192.58
min	1.00
25%	1.00
50%	1.00
75%	36.00
95%	595.00
max	971.00

(a) All matches

	Outcome count
count	4753.00
mean	242.90
std	260.04
min	2.00
25%	28.00
50%	139.00
75%	394.00
95%	788.00
max	971.00

(b) Stochastic matches

Table 2: Summary statistics for the number of different match outcomes used as the cached results



(a) All matches

(b) Stochastic matches

Figure 2: The distribution of the number of unique outcomes used as the cached results

Section 3 will validate the methodology used here against known theoretic results.

3 Validation

As described in [13] Consider the payoff matrix:

$$M = \begin{pmatrix} a, b \\ c, d \end{pmatrix} \tag{1}$$

The expected payoffs of i players of the first type in a population with $N - i$ players of the second type are given by:

$$F_i = \frac{a(i-1) + b(N-i)}{N-1} \tag{2}$$

$$G_i = \frac{ci + d(N-i-1)}{N-1} \tag{3}$$

With an intensity of selection ω the fitness of both strategies is given by:

$$f_i = 1 - \omega + \omega F_i \tag{4}$$

$$g_i = 1 - \omega + \omega G_i \tag{5}$$

The transitions within the birth death process that underpins the Moran process are then given by:

$$p_{i,i+1} = \frac{if_i}{if_i + (N-i)g_i}\frac{N-i}{N} \tag{6}$$

$$p_{i,i-1} = \frac{(N-i)g_i}{if_i + (N-i)g_i}\frac{i}{N} \tag{7}$$

$$p_{ii} = 1 - p_{i,i+1} - p_{i,i-1} \tag{8}$$

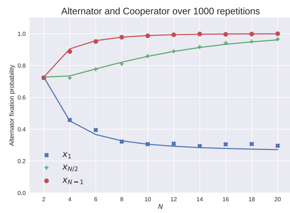
Using this it is a known result that the fixation probability of the first strategy in a population of i individuals of the first type (and $N - i$ individuals of the second. We have:

$$x_i = \frac{1 + \sum_{j=1}^{i-1}\prod_{k=1}^{j}\gamma_j}{1 + \sum_{j=1}^{N-1}\prod_{k=1}^{j}\gamma_j} \tag{9}$$

where:

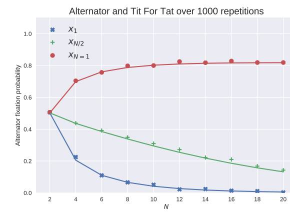
$$\gamma_j = \frac{p_{j,j-1}}{p_{j,j+1}}$$

Using this comparisons of $x_1, x_{N/2}, x_{N-1}$ are shown in Figure 17. The points represent the simulated values and the line shows the theoretic value. Note that these are all deterministic strategies and show a perfect match up between the expected value of (9) and the actual Moran process for all strategies pairs.

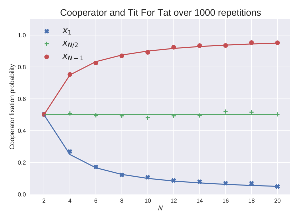


(a) Alternator and Cooperator

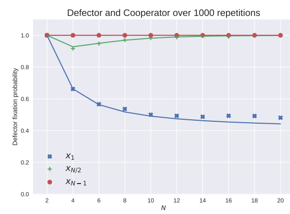
(b) Alternator and Defector



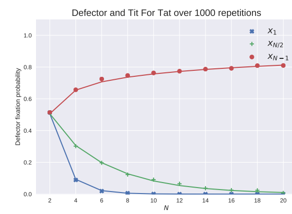
(c) Alternator and Tit For Tat



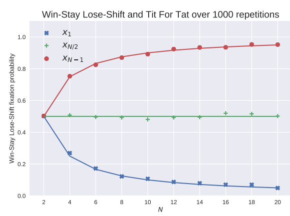
(d) Cooperator and Tit For Tat



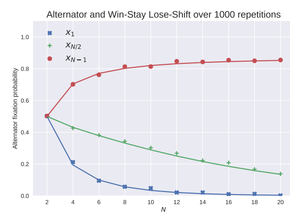
(e) Defector and Cooperator



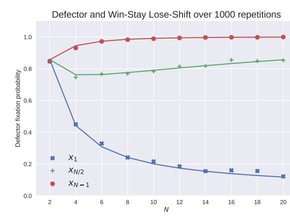
(f) Defector and Tit For Tat



(g) Win Stay Lose Shift and Tit For Tat



(h) Alternator and Win Stay Lose Shift



(i) Defector and Win Stay Lose Shift

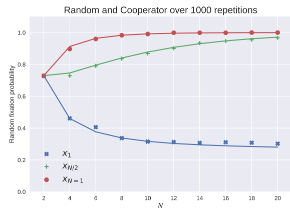
Figure 3: Comparison of theoretic and actual Moran Process fixation probabilities for **deterministic** strategies

Figure 18 shows the fixation probabilities for stochastic strategies. These are no longer a good match which highlights the weakness of the analytical formulae that relies on the average payoffs. A detailed analysis of the 164 strategies considered, using direct Moran processes will be shown in the next Section.

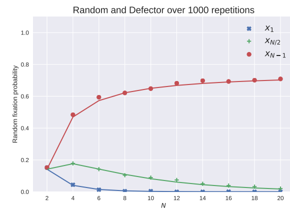
4 Empirical results

This section will outline the data analysis carried out:

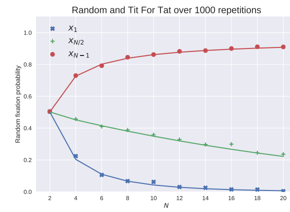
- Section 4.1 will consider the specific case of $N = 2$.
- Section 4.2 will investigate the effect of population size on the ability of a strategy to invade another population. This will highlight how complex strategies with long memories outperform simpler strategies.
- Section 4.3 will similarly investigate the ability to defend against an invasion.
- Section 4.4 will investigate the relationship between performance for differing population sizes. This highlights the importance of considering population dynamics over large populations.
- Section 4.5 will calculate the relative fitness of all strategies.



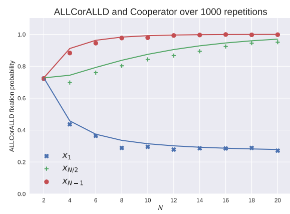
(a) Random and Cooperator



(b) Random and Defector

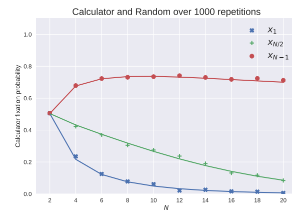


(c) Random and Tit For Tat

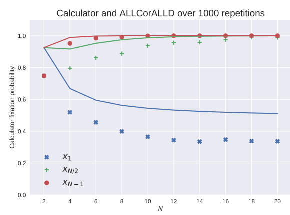


(d) All C or all D and Cooperator

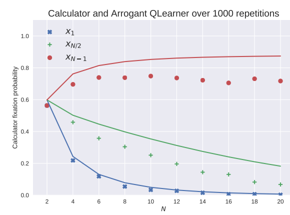
(e) All C or all D and Defector



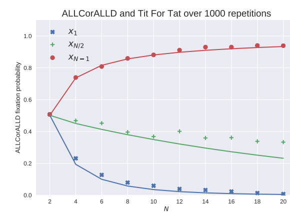
(f) Calculator and Random



(g) Calculator and All C or all D



(h) Calculator and Arrogant Q learner



(i) All C or all D and Tit For Tat

Figure 4: Comparison of theoretic and actual Moran Process fixation probabilities for **stochastic** strategies

4.1 The special case of $N = 2$

The main fixation probabilities of interest are x_1 and x_{N-1}, these reflect a strategy's ability to invade or resist invasion. For $N = 2$ these two cases coincide. Figure 5 shows all pairwise fixation probabilities for strategies on the vertical column when being matched against probabilities on the horizontal column. This is summarised in Figure 6 and Table 3 It can be seen that, as described in in [14], zero determinant strategies perform well. The strategy with infinite memory `Collective` is a strategy with a handshake which implies that it will perform well against itself.. There is also one of the trained finite state machines in the top five. The remaining strategy "Feld" is the corresponding strategy submitted to the first tournament by Axelrod [3]. All these strategies are stochastic.

Player	Median p_1	Memory Depth	Stochastic
ZD-Extort-4: 0.23529411764705882, 0.25, 1	0.584	1	True
CollectiveStrategy	0.572	∞	False
FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'...	0.570	1	False
Feld: 1.0, 0.5, 200	0.568	200	True
ZD-Extort-2: 0.1111111111111111, 0.5	0.568	1	True

Table 3: Summary of top five strategies for $N = 2$

As will be demonstrated in Section 4.4 $N = 2$ is a particular case. In the next sections we will pay close attention to strategies who are strong invaders/resistors and shown diagrammatically in Figure 7.

4.2 Strong invaders

In this section x_i will be investigated: the probability of 1 individual of a given type successfully becoming fixated in a population of $N - 1$ other individuals. Figures 8 shows these values for the players along the vertical axis when matched against the players on the horizontal axis. It can be seen that invasion is in general more challenging for $N = 7$ and

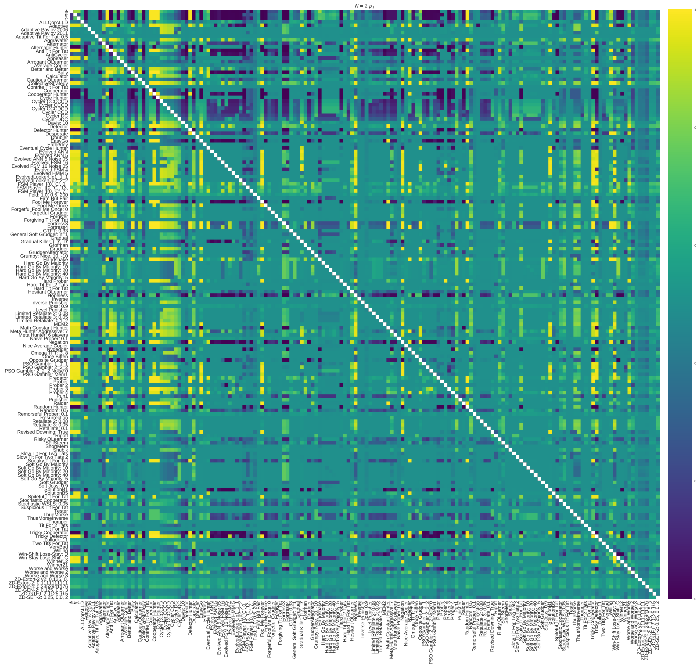


Figure 5: The pairwise fixation probabilities for $N = 2$

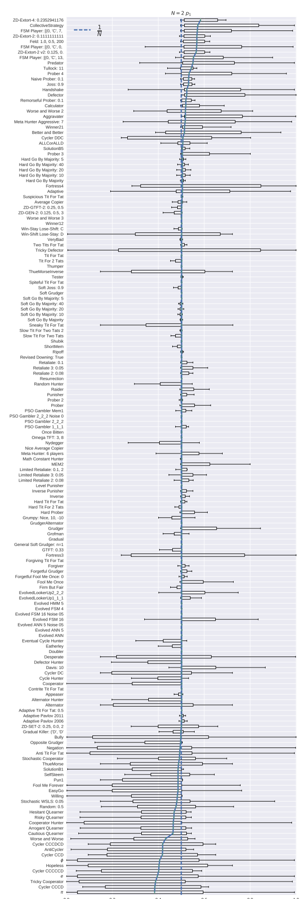


Figure 6: The median fixation probabilities for $N = 2$

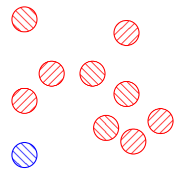
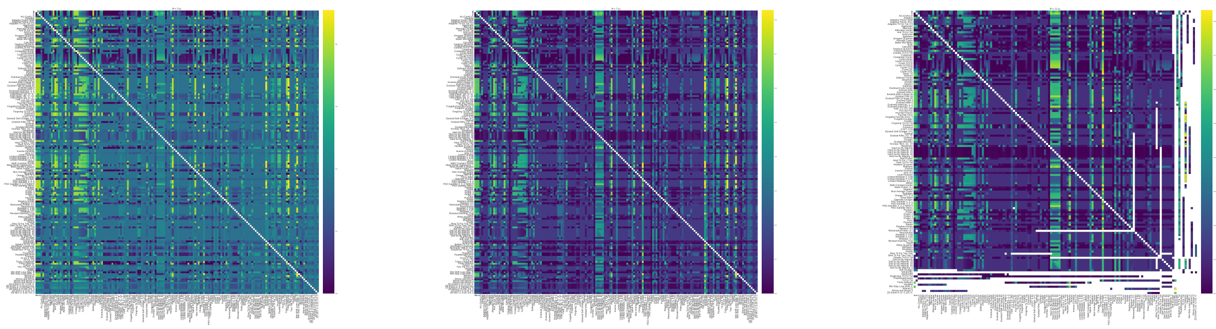


Figure 7: A single individual will successfully invade the population with probability x_1. The group of Individuals will successfully resist with probability x_{N-1}

$N = 11$ in comparison to $N = 3$. This information is summarised in Figure 9 showing the median fixation as well as the neutral fixation for each given scenario.



(a) $N = 3$ (b) $N = 7$ (c) $N = 11$

Figure 8: Pairwise fixation probability x_1 of all strategies

For $N \in \{3, 7, 11\}$ the top five strategies are given in Tables 5.

Player	Median p_1	Memory Depth	Stochastic
CollectiveStrategy	0.403	∞	False
Predator	0.396	9	False
Prober 4	0.368	∞	False
Remorseful Prober: 0.1	0.357	2	True
Worse and Worse 2	0.355	∞	True

(a) $N = 3$

Player	Median p_1	Memory Depth	Stochastic
Prober 4	0.177	∞	False
CollectiveStrategy	0.170	∞	False
Worse and Worse 2	0.159	∞	True
Predator	0.158	9	False
Remorseful Prober: 0.1	0.146	2	True

(b) $N = 7$

Player	Median p_1	Memory Depth	Stochastic
Worse and Worse 3	0.125	∞	True
ThueMorseInverse	0.122	∞	False
GrudgerAlternator	0.115	∞	False
Hard Tit For Tat	0.115	3	False
Inverse	0.115	∞	True

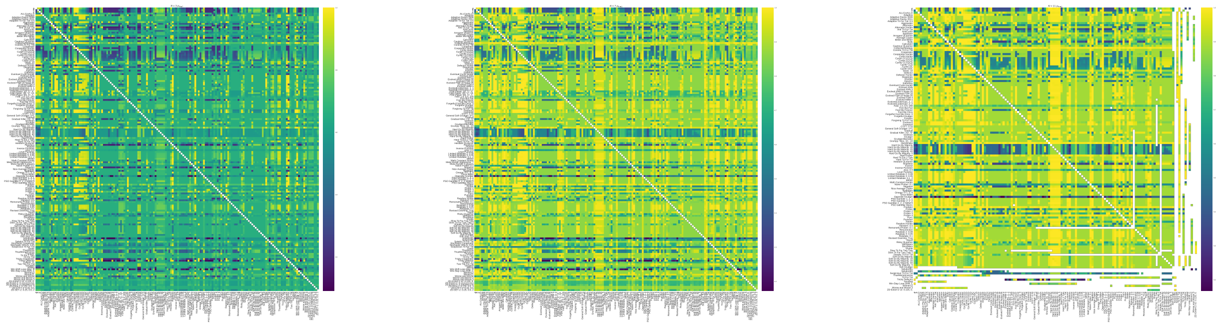
(c) $N = 11$

Table 4: Properties of top five invaders

(a) $N = 3$ (b) $N = 7$ (c) $N = 11$

Figure 9: Median probabilities x_1 of all strategies as well as the neutral fixation probability

4.3 Strong resistors



(a) $N = 3$ (b) $N = 7$ (c) $N = 11$

Figure 10: Pairwise fixation probability x_{N-1} of all strategies

Player	Median p_{N-1}	Memory Depth	Stochastic
CollectiveStrategy	0.403	∞	False
Predator	0.396	9	False
Prober 4	0.368	∞	False
Handshake	0.346	∞	False
Winner21	0.341	2	False

(a) $N = 3$

Player	Median p_{N-1}	Memory Depth	Stochastic
Prober 4	0.177	∞	False
CollectiveStrategy	0.170	∞	False
Predator	0.158	9	False
Winner21	0.145	2	False
Handshake	0.133	∞	False

(b) $N = 7$

Player	Median p_{N-1}	Memory Depth	Stochastic
Adaptive	0.111	∞	False
Prober 4	0.108	∞	False
CollectiveStrategy	0.089	∞	False
Predator	0.084	9	False
Handshake	0.081	∞	False

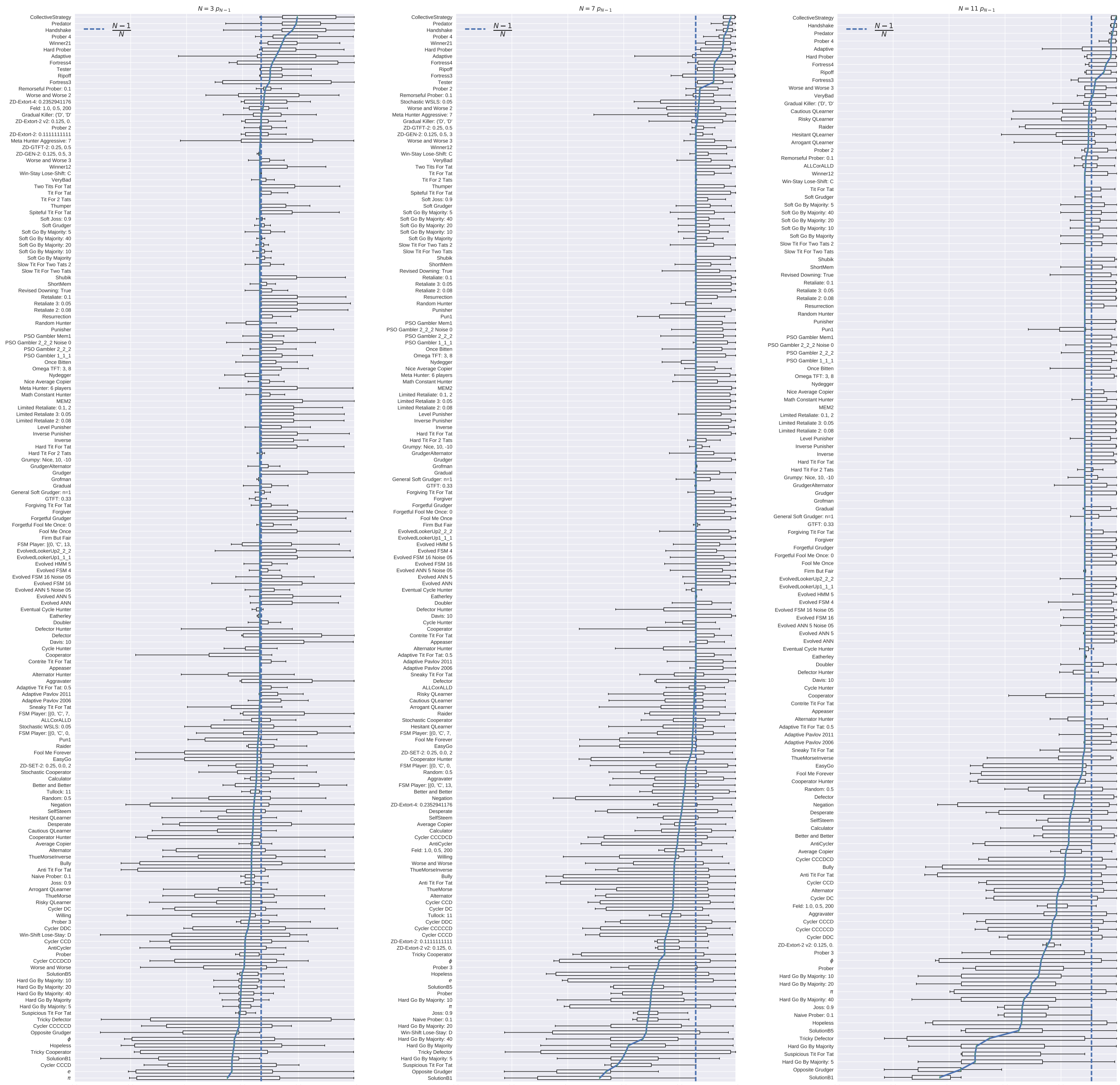
(c) $N = 11$

Table 5: Properties of top five strategies resistors

4.4 The effect of population size

Figures 12, 13 and 14 show the median rank of each strategy against population size. Note that these ranks are not necessarily integers as group ties are given the average rank.

Tables 6a, 6b and 6c show the correlation coefficients of the ranks in of strategies in differing population size. This is shown graphically in Figure 15. It is immediate to note that how well a strategy performs in any Moran process for $N > 2$ has little to do with the performance for $N = 2$.



(a) $N = 2$ (b) $N = 7$ (c) $N = 11$

Figure 11: Median probabilities x_{N-1} of all strategies as well as the neutral fixation probability

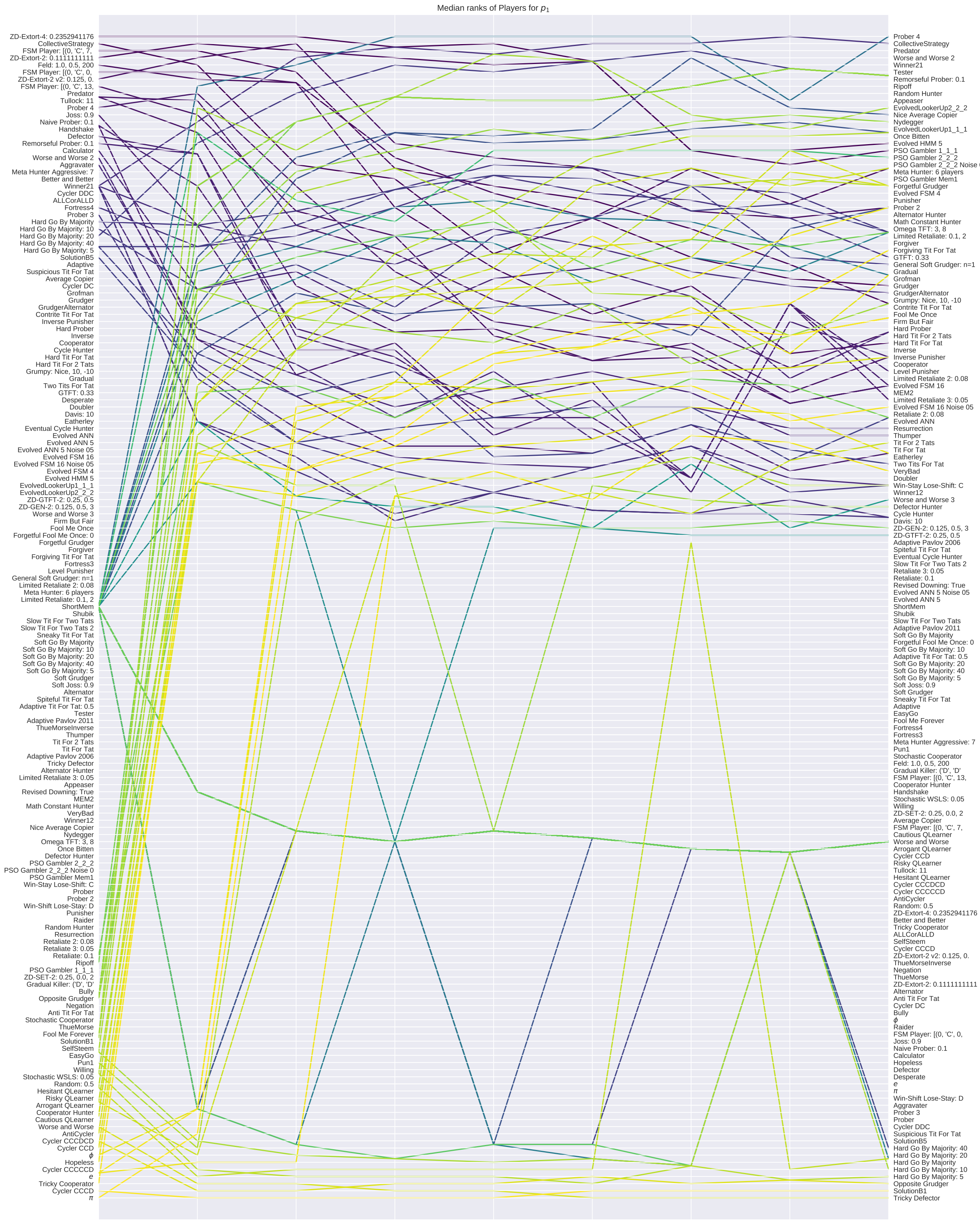


Figure 12: invade

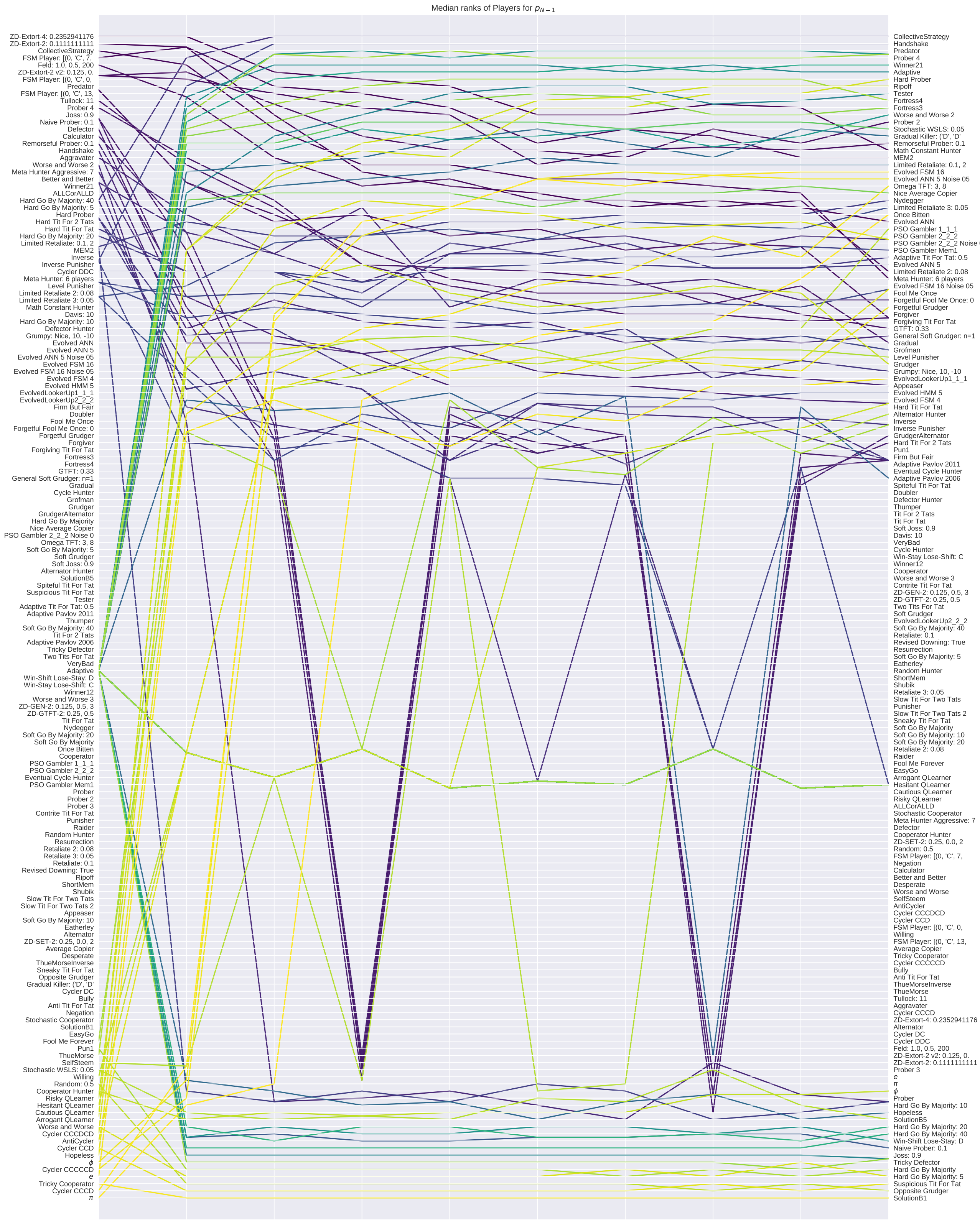


Figure 13: resist

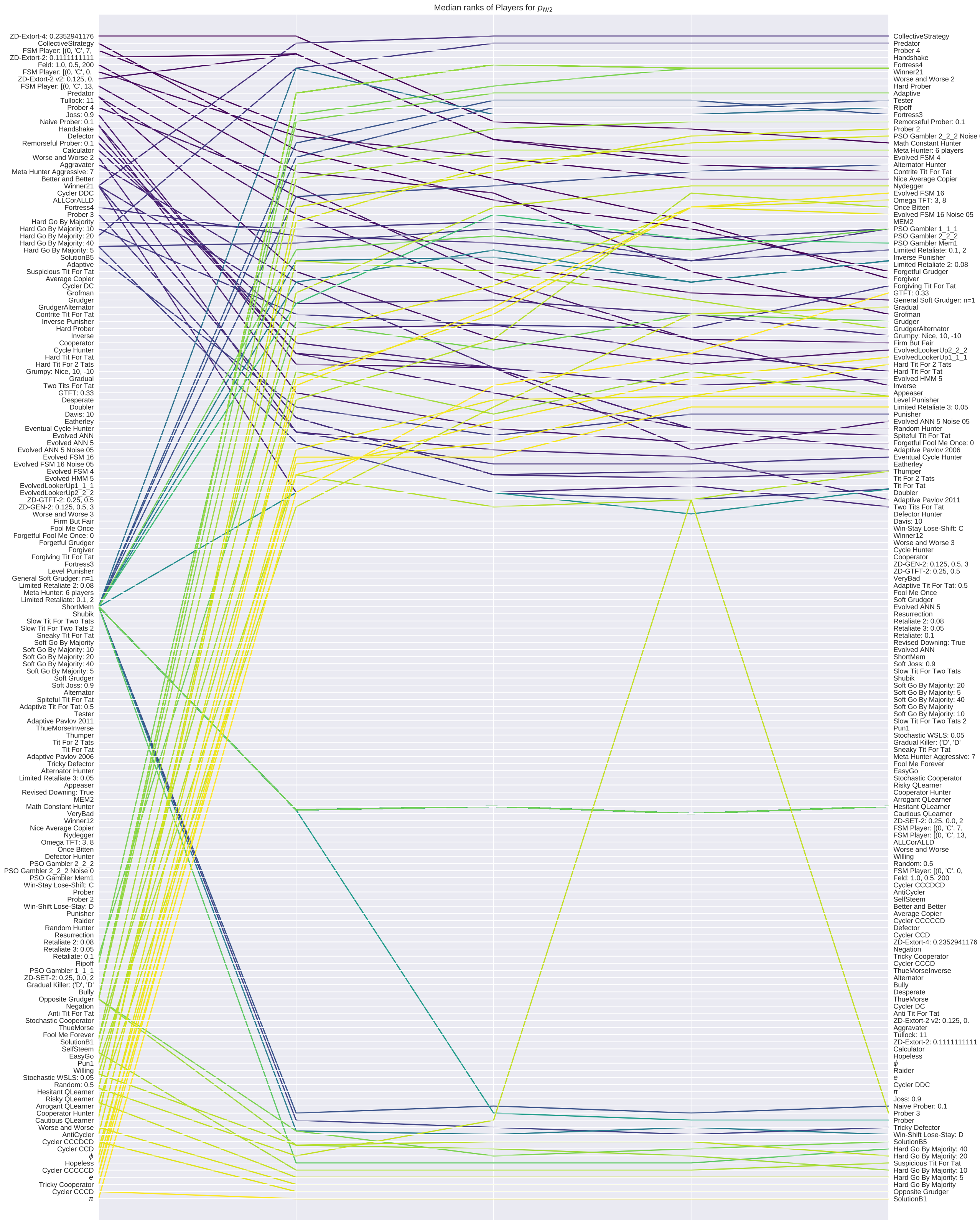


Figure 14: Coexistance

N	2	3	4	5	6	7	8	9	10
2	1.00	0.44	0.26	0.17	0.15	0.12	0.08	0.08	0.08
3	0.44	1.00	0.92	0.87	0.87	0.86	0.83	0.83	0.84
4	0.26	0.92	1.00	0.97	0.96	0.97	0.95	0.95	0.95
5	0.17	0.87	0.97	1.00	0.98	0.99	0.97	0.98	0.98
6	0.15	0.87	0.96	0.98	1.00	0.99	0.97	0.97	0.98
7	0.12	0.86	0.97	0.99	0.99	1.00	0.98	0.98	0.99
8	0.08	0.83	0.95	0.97	0.97	0.98	1.00	0.97	0.98
9	0.08	0.83	0.95	0.98	0.97	0.98	0.97	1.00	0.99
10	0.08	0.84	0.95	0.98	0.98	0.99	0.98	0.99	1.00

(a) Correlation coefficients for ranks for invasion

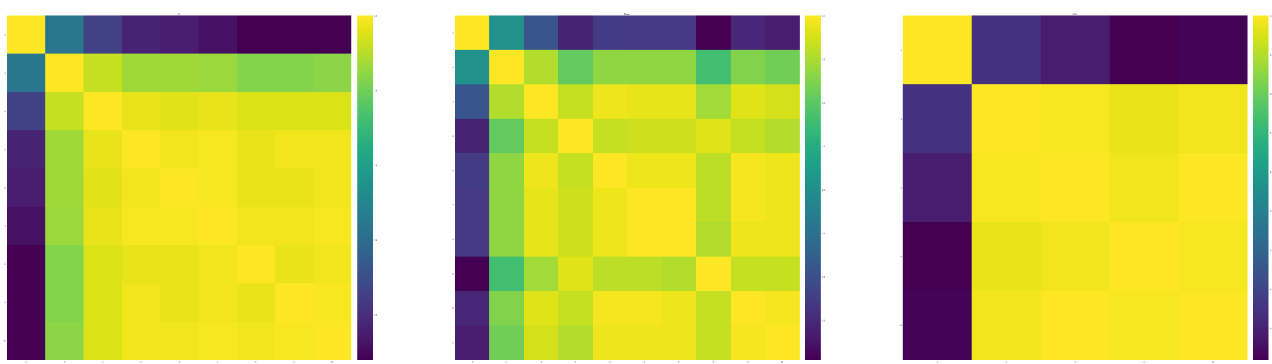
N	2	3	4	5	6	7	8	9	10	12
2	1.00	0.61	0.42	0.29	0.35	0.34	0.34	0.21	0.30	0.27
3	0.61	1.00	0.91	0.81	0.87	0.87	0.87	0.76	0.85	0.83
4	0.42	0.91	1.00	0.93	0.98	0.97	0.97	0.89	0.96	0.95
5	0.29	0.81	0.93	1.00	0.93	0.94	0.94	0.96	0.93	0.91
6	0.35	0.87	0.98	0.93	1.00	0.98	0.98	0.92	0.99	0.98
7	0.34	0.87	0.97	0.94	0.98	1.00	1.00	0.92	0.99	0.98
8	0.34	0.87	0.97	0.94	0.98	1.00	1.00	0.91	0.98	0.98
9	0.21	0.76	0.89	0.96	0.92	0.92	0.91	1.00	0.93	0.93
10	0.30	0.85	0.96	0.93	0.99	0.99	0.98	0.93	1.00	0.99
12	0.27	0.83	0.95	0.91	0.98	0.98	0.98	0.93	0.99	1.00

(b) Correlation coefficients for ranks for resistance

N	2	4	6	8	10
2	1.00	0.25	0.19	0.12	0.13
4	0.25	1.00	0.99	0.97	0.98
6	0.19	0.99	1.00	0.98	1.00
8	0.12	0.97	0.98	1.00	0.99
10	0.13	0.98	1.00	0.99	1.00

(c) Correlation coefficients for ranks for coexistance

Table 6: Correlation coefficients



(a) Rank based on x_1 (b) Rank based on x_{N-1} (c) Rank based on $x_{N/2}$

Figure 15: Correlation coefficients for ranks

4.5 Relative fitness

Under the assumption of a constant relative fitness r between two strategies [13] the formula for x_i (for given N, r is:

$$x_i = x_i(r) = \frac{1 - \frac{1}{r^i}}{1 - \frac{1}{r^N}} \tag{10}$$

Figure 16 shows this function for $N = 10$ and $i \in \{1, 5, 10\}$.

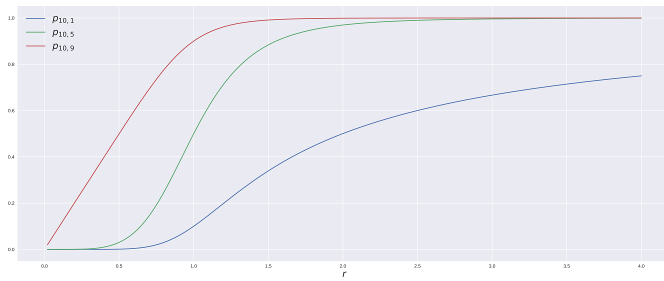


Figure 16: $x_i(r)$

The first and second derivative of (10) is given by equations (11) and (12).

$$\frac{dx_i}{dr} = \frac{r^{N-i-1}}{r^{2N} - 2r^N + 1}\left(-Nr^i + N + ir^N - i\right) \tag{11}$$

$$\frac{d^2x_i}{dr^2} = \frac{r^{N-i-2}}{\left(r^N - 1\right)^3}\left(2N^2\left(r^i - 1\right) + N\left(r^N - 1\right)\left(N\left(r^i - 1\right) - 2i + r^i - 1\right) - i\left(i + 1\right)\left(r^N - 1\right)^2\right) \tag{12}$$

Using these, Halley's method [1] can be used to efficiently numerically invert $x_i(r)$ to obtain a theoretic relative fitness r that gives the calculated $x_i(r)$ between two strategies for a given N, i.

5 Conclusion

Further work:

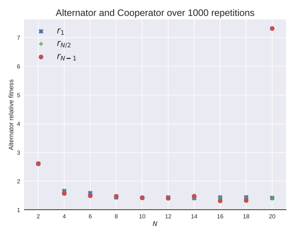
- Spatial structure;
- More than two types in the population;
- Modified Moran processes (Fermi selection);
- Mutation;

Acknowledgements

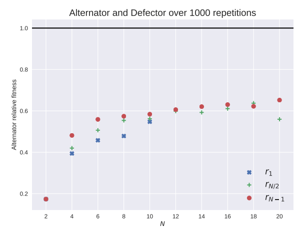
This work was performed using the computational facilities of the Advanced Research Computing @ Cardiff (ARCCA) Division, Cardiff University.

A variety of software libraries have been used in this work:

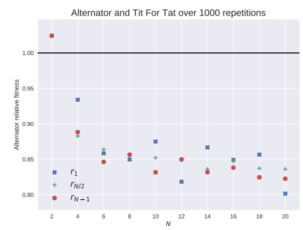
- The Axelrod library (IPD strategies and Moran processes) [15].
- The matplotlib library (visualisation) [7].
- The pandas and numpy libraries (data manipulation) [10, 16].



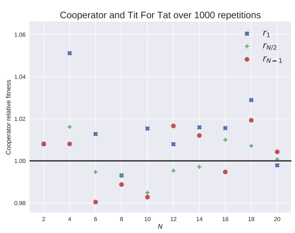
(a) Alternator and Cooperator



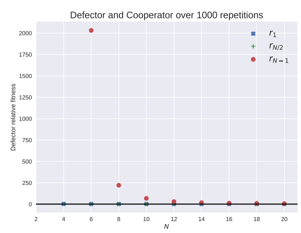
(b) Alternator and Defector



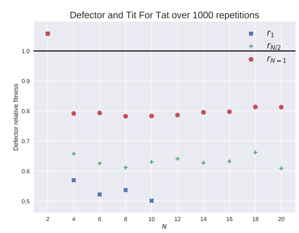
(c) Alternator and Tit For Tat



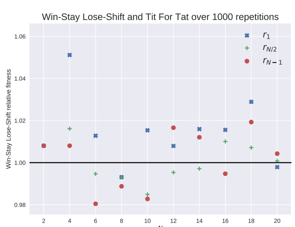
(d) Cooperator and Tit For Tat



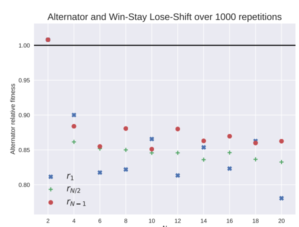
(e) Defector and Cooperator



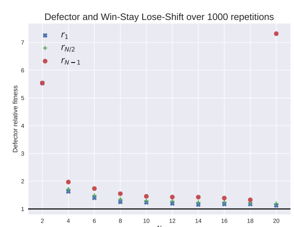
(f) Defector and Tit For Tat



(g) Win Stay Lose Shift and Tit For Tat

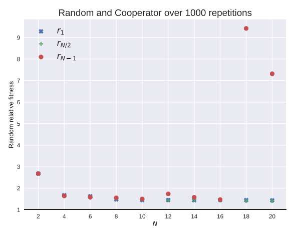


(h) Alternator and Win Stay Lose Shift

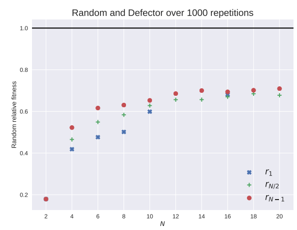


(i) Defector and Win Stay Lose Shift

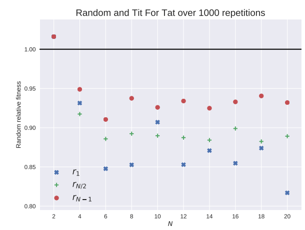
Figure 17: Estimated relative fitness for **deterministic** strategies



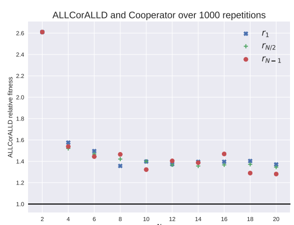
(a) Random and Cooperator



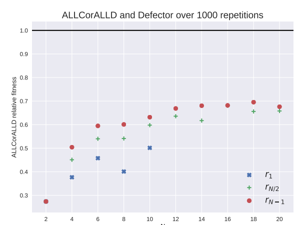
(b) Random and Defector



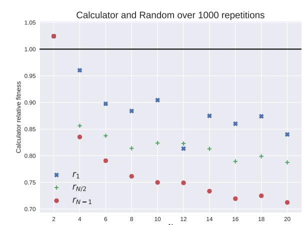
(c) Random and Tit For Tat



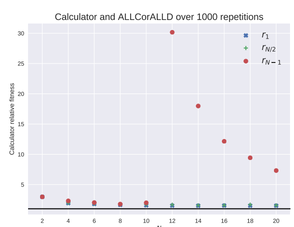
(d) All C or all D and Cooperator



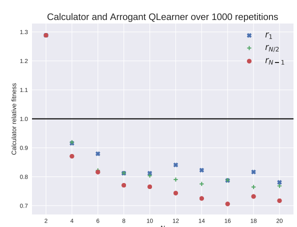
(e) All C or all D and Defector



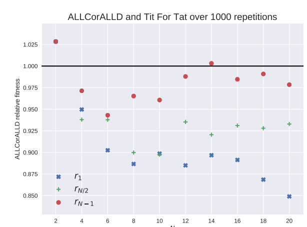
(f) Calculator and Random



(g) Calculator and All C or all D



(h) Calculator and Arrogant Q learner



(i) All C or all D and Tit For Tat

Figure 18: Estimated relative fitness for **stochastic** strategies

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A List of players

1. Retaliate 3: 0.05
2. Adaptive
3. PSO Gambler 2_2_2
4. Revised Downing: True
5. Adaptive Tit For Tat: 0.5
6. Negation
7. Ripoff
8. Aggravater
9. Prober 3
10. SelfSteem
11. ALLCorALLD
12. Nydegger
13. Risky QLearner
14. Alternator
15. Alternator Hunter
16. PSO Gambler 2_2_2 Noise 05
17. ShortMem
18. AntiCycler

19. Shubik

20. Anti Tit For Tat

21. FSM Player: [(0, 'C', 0, 'C'), (0, 'D', 3, 'C'), (1, 'C', 5, 'D'), (1, 'D', 0, 'C'), (2, 'C', 3, 'C'), (2, 'D', 2, 'D'), (3, 'C', 4, 'D'), (3, 'D', 6, 'D'), (4, 'C', 3, 'C'), (4, 'D', 1, 'D'), (5, 'C', 6, 'C'), (5, 'D', 3, 'D'), (6, 'C', 6, 'D'), (6, 'D', 6, 'D'), (7, 'C', 7, 'D'), (7, 'D', 5, 'C')], 1, C

22. Once Bitten

23. Adaptive Pavlov 2006

24. Slow Tit For Two Tats

25. Adaptive Pavlov 2011

26. Slow Tit For Two Tats 2

27. Nice Average Copier

28. Appeaser

29. PSO Gambler Mem1

30. Punisher

31. Sneaky Tit For Tat

32. Raider

33. Arrogant QLearner

34. Soft Grudger

35. Average Copier

36. Omega TFT: 3, 8

37. Soft Joss: 0.9

38. Better and Better

39. π

40. Random Hunter

41. FSM Player: [(0, 'C', 13, 'D'), (0, 'D', 12, 'D'), (1, 'C', 3, 'D'), (1, 'D', 4, 'D'), (2, 'C', 14, 'D'), (2, 'D', 9, 'D'), (3, 'C', 0, 'C'), (3, 'D', 1, 'D'), (4, 'C', 1, 'D'), (4, 'D', 2, 'D'), (5, 'C', 12, 'C'), (5, 'D', 6, 'C'), (6, 'C', 1, 'C'), (6, 'D', 14, 'D'), (7, 'C', 12, 'D'), (7, 'D', 2, 'D'), (8, 'C', 7, 'D'), (8, 'D', 9, 'D'), (9, 'C', 8, 'D'), (9, 'D', 0, 'D'), (10, 'C', 2, 'C'), (10, 'D', 15, 'C'), (11, 'C', 7, 'D'), (11, 'D', 13, 'D'), (12, 'C', 3, 'C'), (12, 'D', 8, 'D'), (13, 'C', 7, 'C'), (13, 'D', 10, 'D'), (14, 'C', 10, 'D'), (14, 'D', 7, 'D'), (15, 'C', 15, 'C'), (15, 'D', 11, 'D')], 1, C

42. Opposite Grudger

43. Random: 0.5

44. SolutionB1

45. Pun1

46. Prober

47. Bully

48. Predator

49. Remorseful Prober: 0.1

50. SolutionB5

51. Resurrection

52. Calculator

53. Prober 2

54. Hard Tit For 2 Tats

55. Spiteful Tit For Tat

56. Prober 4

57. Retaliate: 0.1

58. Retaliate 2: 0.08

59. PSO Gambler 1_1_1

60. Grumpy: Nice, 10, -10

61. CollectiveStrategy

62. Stochastic Cooperator

63. Cautious QLearner

64. Cooperator

65. Fool Me Once

66. Hard Go By Majority: 10

67. Hard Go By Majority

68. Stochastic WSLS: 0.05

69. Worse and Worse 3

70. Hard Go By Majority: 40

71. Cooperator Hunter

72. Eventual Cycle Hunter

73. Firm But Fair

74. Contrite Tit For Tat

75. Forgetful Fool Me Once: 0.05

76. Math Constant Hunter

77. Cycle Hunter

78. Suspicious Tit For Tat

79. ZD-Extort-2: 0.1111111111111111, 0.5

80. Evolved ANN

81. Fool Me Forever

82. Hard Go By Majority: 5

83. MEM2

84. ZD-Extort-2 v2: 0.125, 0.5, 1

85. Forgetful Grudger

86. Cycler CCCCCD

87. Tester

88. Evolved ANN 5

89. Hard Tit For Tat

90. Forgiver

91. Fortress3

92. ThueMorse

93. Evolved ANN 5 Noise 05

94. Forgiving Tit For Tat

95. Cycler CCCD

96. ThueMorseInverse

97. Soft Go By Majority

98. ZD-Extort-4: 0.23529411764705882, 0.25, 1

99. Evolved FSM 4

100. Fortress4

101. ZD-GEN-2: 0.125, 0.5, 3

102. Thumper

103. FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'), (1, 'C', 11, 'D'), (1, 'D', 11, 'D'), (2, 'C', 8, 'D'), (2, 'D', 8, 'C'), (3, 'C', 3, 'C'), (3, 'D', 12, 'D'), (4, 'C', 6, 'C'), (4, 'D', 3, 'C'), (5, 'C', 11, 'C'), (5, 'D', 8, 'D'), (6, 'C', 13, 'D'), (6, 'D', 14, 'C'), (7, 'C', 4, 'D'), (7, 'D', 2, 'D'), (8, 'C', 14, 'D'), (8, 'D', 8, 'D'), (9, 'C', 0, 'C'), (9, 'D', 10, 'D'), (10, 'C', 8, 'C'), (10, 'D', 15, 'C'), (11, 'C', 6, 'D'), (11, 'D', 5, 'D'), (12, 'C', 6, 'D'), (12, 'D', 9, 'D'), (13, 'C', 9, 'D'), (13, 'D', 8, 'D'), (14, 'C', 8, 'D'), (14, 'D', 13, 'D'), (15, 'C', 4, 'C'), (15, 'D', 5, 'C')], 1, C
104. ZD-GTFT-2: 0.25, 0.5
105. Cycler CCD
106. GTFT: 0.33
107. Tit For 2 Tats
108. Meta Hunter: 6 players
109. Hesitant QLearner
110. Evolved FSM 16
111. General Soft Grudger: n=1,d=4,c=2
112. Joss: 0.9
113. Tullock: 11
114. Cycler DC
115. Soft Go By Majority: 10
116. Inverse
117. ZD-SET-2: 0.25, 0.0, 2
118. Limited Retaliate 2: 0.08, 15
119. Tricky Defector
120. Naive Prober: 0.1
121. Meta Hunter Aggressive: 7 players
122. Evolved FSM 16 Noise 05
123. Tricky Cooperator
124. e
125. Cycler DDC
126. Limited Retaliate 3: 0.05, 20
127. Hopeless
128. VeryBad
129. Soft Go By Majority: 20
130. EvolvedLookerUp2_2_2
131. Two Tits For Tat
132. Tit For Tat
133. Cycler CCCDCD
134. EvolvedLookerUp1_1_1
135. Soft Go By Majority: 40
136. Level Punisher
137. Hard Prober
138. Limited Retaliate: 0.1, 20
139. Winner21
140. Defector
141. Soft Go By Majority: 5
142. Davis: 10
143. Handshake
144. ϕ
145. Inverse Punisher
146. Winner12
147. Gradual
148. Defector Hunter
149. Willing
150. Desperate
151. Gradual Killer: ('D', 'D', 'D', 'D', 'D', 'C', 'C')
152. Hard Go By Majority: 20
153. Win-Shift Lose-Stay: D
154. Grofman
155. Win-Stay Lose-Shift: C
156. Doubler
157. Grudger
158. Evolved HMM 5
159. EasyGo
160. Worse and Worse
161. Feld: 1.0, 0.5, 200
162. GrudgerAlternator
163. Worse and Worse 2
164. Eatherley