An Empirical Study of Invasion and Resistance for Iterated Prisoner's Dilemma Strategies

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Abstract

The Iterated Prisoner's Dilemma is a well established framework for the study of emergent behaviour. In this paper an extensive numerical study of the evolutionary dynamics of this framework are presented. Fixation probabilities for Moran processes are obtained for 164 different strategies. We find that players with long memories and sophisticated behaviours outperform many strategies that perform well in a two player setting.

1 Introduction

The Prisoner's Dilemma (PD) [8] is a fundamental two player game used to model a large variety of strategic interactions. Each player can choose between cooperation (C) or defection (D). The decisions are made simultaneously and independently. The payoffs of the game are defined by the matrix $\begin{pmatrix} R & S \\ T & P \end{pmatrix}$, where $T > R > P > S$ and $2R > T + S$. The PD is a one round game, but is commonly studied in a manner where the prior outcomes matter. This extended form is called the Iterated Prisoner's Dilemma (IPD).

The Moran Process [18] is a model of evolutionary population dynamics that has been used to gain insights about the evolutionary stability and fixation of strategies in a number of settings. Similarly since the first Iterated Prisoner's Dilemma (IPD) tournament described in [5], the Prisoner's dilemma has been used to understand the evolution of cooperative behaviour in complex systems. Several earlier works have studied iterated games in the context of the prisoner's dilemma [19, 24].

This manuscript provides a detailed numerical analysis of **164** complex and adaptive strategies for the IPD. This is made possible by the Axelrod library [25], an effort to provide software for reproducible research for the IPD. The library now contains over 150 parameterized strategies including classics like TitForTat and WinStayLoseShift, as well as recent variants such as OmegaTFT, zero determinant and other memory one strategies, strategies based on finite state machines, lookup tables, neural networks, and other machine learning based strategies, and a collection of novel strategies. The library can conduct matches, tournaments and population dynamics with variations including noise and spatial structure.

We present fixation probabilities for all pairs of strategies in the library, identifying those are effective invaders and those resistant to invasion, for population sizes $N = 2$ to $N = 14$.

In particular we address the following questions:

1. What strategies are good invaders?
2. What strategies are good at resisting invasion?
3. How does the population size affect these findings?

While our results agree with some of the published literature, we find that:

1. Zero determinant strategies are not particularly effective for $N > 2$
2. Long memory strategies are generally more effective than short memory strategies
3. Complex strategies can be effective

1.1 The Moran Process

Figure 1 shows a diagrammatic representation of the Moran process, a stochastic birth death process on a finite population in which the population size stays constant over time. Individuals are **selected** according to a given fitness landscape. Once selected, the individual is reproduced and similarly another individual is chosen to be removed from the population.

In some settings mutation is also considered but without mutation (the case considered in this work) this process will arrive at an absorbing state where the population is entirely made up of players of one strategy. The probability with which a given strategy is the survivor is called the fixation probability. A more detailed analytic description of this is given in Section 3.

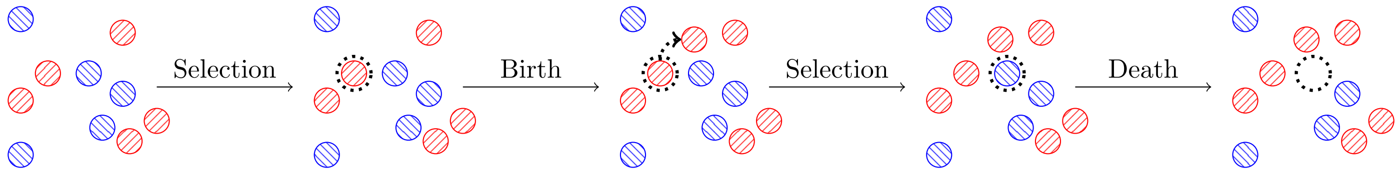


Figure 1: A diagrammatic representation of a Moran process

The Moran process was initially introduced in [18]. It has since been used in a variety of settings including the understanding of the spread of cooperative and non-cooperative behaviour such as cancer [27] and the emergence of cooperative behaviour in spatial topologies [3]. However these works mainly consider non-sophisticated strategies. Some work has looked at evolutionary stability of strategies within the Prisoner's Dilemma [13] but this is not done in the more widely-used setting of the Moran process, rather in terms of infinite population stability. In [7] Moran processes are studied in a theoretical framework for a small subset of strategies. The subset included memory one strategies, strategies that recall the events of the previous round only.

Of particular interest are the zero determinant strategies introduced in [20] and highly-praised [24] it was argued that generous ZD strategies are robust against invading strategies. However, in [11] a strategy using machine learning techniques was capable of resisting invasion and also able to invade any memory one strategy. Recent work [9] has investigated the effect of memory length on strategy performance and the emergence of cooperation but this is not done in Moran process context and only considers specific cases of memory 2 strategies. In [1] it was recognized that many zero determinant strategies do not fare well against themselves. This is a disadvantage for the Moran process where the best strategies cooperate well with other players using the same strategy.

1.2 The Axelrod Library

The contribution of this work is a detailed and extensive analysis of absorption probabilities for 164 strategies. These strategies and the numerical simulations are from [25] which is an open source research library written for the study of the IPD. The strategies and simulation frameworks are automatically tested to an extraordinarily high degree of coverage in accordance with best research software practices. The large number of strategies are available thanks to the open source nature of the project with over 40 contributions made by different programmers and researchers.

Section 2 will explain the methodological approach used, Section 3 will validate the methodology by comparing simulated results to analytical results in some cases. The main results of this manuscript are presented in Section 4 which will present a detailed analysis of all the data generated. Finally, Section 5 will conclude and offer future avenues for the work presented here.

2 Methodology

To carry out this large numerical experiment, 164 strategies are used from [25]. These include 161 default strategies in the library at the time (excluding strategies classified as having a long run time and those that make use of the length of the game) as well as the following 3 finite state machine machine strategies [4]:

Appendix A shows all the players in question. More information about each player can be obtained in the documentation for [25]. There are 43 stochastic and 121 deterministic strategies. Their memory depth, defined by the number of rounds of history used by the strategy each round, is shown in Table 1. The memory depth is infinite if the strategy uses the entire history of play (whatever its length). For example, a strategy that utilizes a handshaking mechanism where the opponents actions on the first few rounds of play determines the strategies subsequent behavior would have infinite memory depth.

Each strategy pair is run for 1000 runs of the Moran process to fixation with starting population distributions of $(1, N-1)$, $(N/2, N/2)$ and $(N-1, 1)$, for N from 2 through 14. The fixation probability is then empirically computed for each combination of starting distribution and value of N.

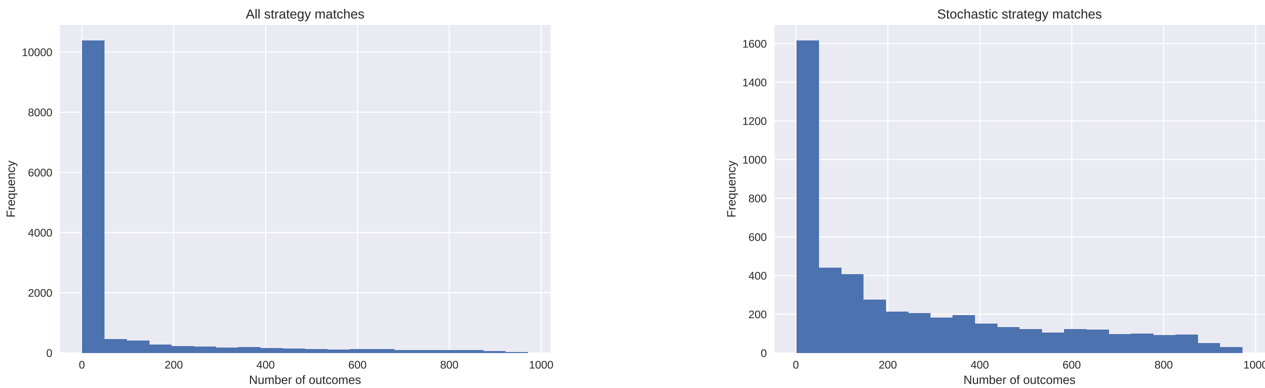
Our software can carry out exact simulations of the Moran process. Since some of the strategies have a high computational cost or are stochastic, we sample from a large number of match outcomes for the pairs of players for use in

Memory Depth	0	1	2	3	4	5	6	9	10	11	12	16	20	40	200	∞
Count	3	31	12	8	2	6	1	1	5	1	1	2	2	2	1	86

Table 1: Memory depth

computing fitnesses in the Moran process. This approach was verified to agree with unsampled calculations to a high degree of accuracy in specific cases.

Figure 2 shows the distribution of the number of outcomes between all strategy pairs. Tables 2 shows that 95% of the stochastic matches have less than 788 unique outcomes whilst the maximum number is 971. This ensures that using a set of cached results from 1000 precomputed matches is sufficient for the analysis taking place here.



(a) All matches

(b) Stochastic matches

Figure 2: The distribution of the number of unique outcomes used as the cached results

	Outcome count
count	13530.00
mean	85.98
std	192.58
min	1.00
25%	1.00
50%	1.00
75%	36.00
95%	595.00
max	971.00

(a) All matches

	Outcome count
count	4753.00
mean	242.90
std	260.04
min	2.00
25%	28.00
50%	139.00
75%	394.00
95%	788.00
max	971.00

(b) Stochastic matches

Table 2: Summary statistics for the number of different match outcomes used as the cached results

Section 3 will validate the methodology used here against known theoretic results.

3 Validation

As described in [19] consider the payoff matrix:

$$M = \begin{pmatrix} a, b \\ c, d \end{pmatrix} \tag{1}$$

The expected payoffs of i players of the first type in a population with $N-i$ players of the second type are given by:

$$F_i = \frac{a(i-1)+b(N-i)}{N-1} \tag{2}$$

$$G_i = \frac{ci+d(N-i-1)}{N-1} \tag{3}$$

With an intensity of selection ω the fitness of both strategies is given by:

$$f_i = 1-\omega+\omega F_i \tag{4}$$

$$g_i = 1-\omega+\omega G_i \tag{5}$$

The transitions within the birth death process that underpins the Moran process are then given by:

$$p_{i,i+1} = \frac{if_i}{if_i+(N-i)g_i}\frac{N-i}{N} \tag{6}$$

$$p_{i,i-1} = \frac{(N-i)g_i}{if_i+(N-i)g_i}\frac{i}{N} \tag{7}$$

$$p_{ii} = 1-p_{i,i+1}-p_{i,i-1} \tag{8}$$

Using this it is a known result that the fixation probability of the first strategy in a population of i individuals of the first type (and $N-i$ individuals of the second. We have:

$$x_i = \frac{1+\sum_{j=1}^{i-1}\prod_{k=1}^{j}\gamma_j}{1+\sum_{j=1}^{N-1}\prod_{k=1}^{j}\gamma_j} \tag{9}$$

where:

$$\gamma_j = \frac{p_{j,j-1}}{p_{j,j+1}}$$

A neutral strategy will have fixation probability $x_i = i/N$. Alternatively, we can frame the outcomes in terms of relative fitness. For a strategy of relative fitness r the fixation probability is well-known to be

$$x_i = \frac{1-r^{-i}}{1-r^{-N}}$$

We can use this formula to compute a value r_i that produces the observed value x_i, noting that for arbitrary strategies the value of this effective relative fitness is dependent on i and N.

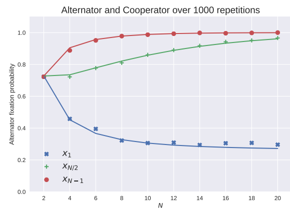
Comparisons of $x_1, x_{N/2}, x_{N-1}$ are shown in Figure 16. The points represent the simulated values and the line shows the theoretical value. Note that these are all deterministic strategies and show a perfect match up between the expected value of (9) and the actual Moran process for all strategies pairs.

Figure 17 shows the fixation probabilities for stochastic strategies. These are no longer a good match which highlights the weakness of the analytical formulae that relies on the average payoffs. A detailed analysis of the 164 strategies considered, using direct Moran processes will be shown in the next Section.

4 Empirical results

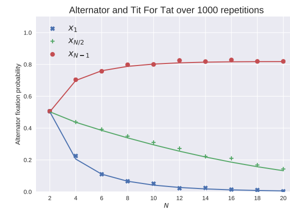
This section outlines the data analysis carried out:

- Section 4.1 considers the specific case of $N = 2$.
- Section 4.2 investigates the effect of population size on the ability of a strategy to invade another population. This will highlight how complex strategies with long memories outperform simpler strategies.
- Section 4.3 similarly investigates the ability to defend against an invasion.
- Section 4.4 investigates the relationship between performance for differing population sizes.
- Section 4.5 calculates the relative fitness of all strategies.

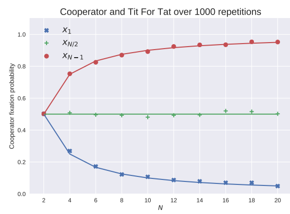


(a) Alternator and Cooperator

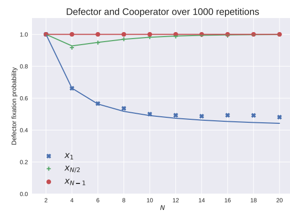
(b) Alternator and Defector



(c) Alternator and Tit For Tat

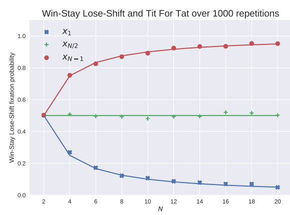


(d) Cooperator and Tit For Tat

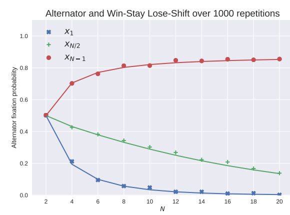


(e) Defector and Cooperator

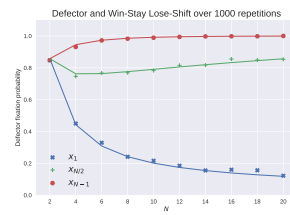
(f) Defector and Tit For Tat



(g) Win Stay Lose Shift and Tit For Tat



(h) Alternator and Win Stay Lose Shift



(i) Defector and Win Stay Lose Shift

Figure 3: Comparison of theoretic and actual Moran Process fixation probabilities for **deterministic** strategies

4.1 The special case of $N = 2$

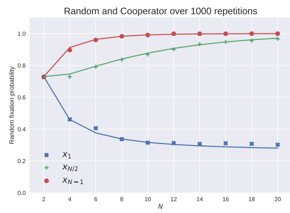
When $N = 2$ the Moran process is effectively a measure of the relative mean payoffs over all possible matches between two players. The strategy that scores higher than the other more often will fixate more often.

Overall the main fixation probabilities of interest are x_1 and x_{N-1}, these reflect a strategy's ability to invade or resist invasion; for $N = 2$ these two cases coincide. Figure 5a shows all pairwise fixation probabilities for strategies on the vertical column when being matched against probabilities on the horizontal column. This is summarised in Figure 5b and Table 3.

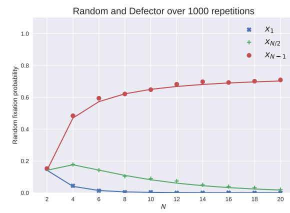
1. The top strategy is an extortionate Zero determinant strategy [20] with parameters $l = 1$ and $s = 1/4$.
2. The Collective strategy has a simple handshake mechanism (a cooperation followed by a defection on the first move). As long as the opponent plays the same handshake and does not defect in the future it cooperates. Otherwise it defects for all rounds [12]. This strategy was specifically designed for evolutionary processes.
3. The finite state machine strategy
4. The Feld strategy is the corresponding strategy submitted to Axelrod's first tournament [5]: it punishes defections but otherwise defects with a random probability that decays over time.
5. The final strategy in the top five is another extortionate Zero determinant strategy [20] with parameters $l = p$.

As will be demonstrated in Section 4.4 the results for $N = 2$ differ from those of larger N. Hence our results do not concur with the literature which suggests that Zero Determinant strategies should be effective for larger population sizes, but we note that those analysis run each match to stationarity, while our matches run for a fixed number of rounds.

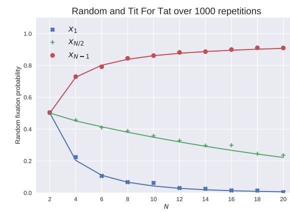
In the next sections we pay close attention to strategies who are strong invaders/resistors and shown diagrammatically in Figure 6.



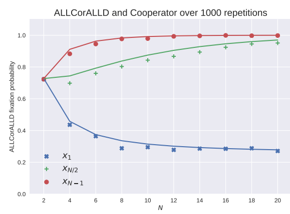
(a) Random and Cooperator



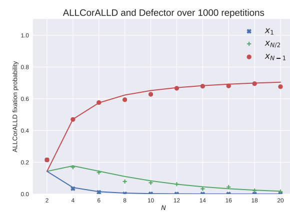
(b) Random and Defector



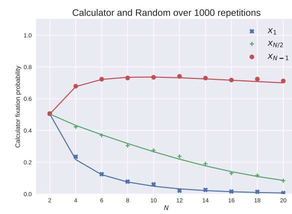
(c) Random and Tit For Tat



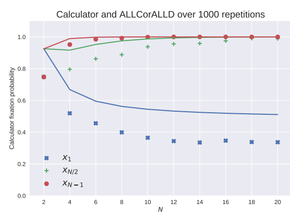
(d) All C or all D and Cooperator



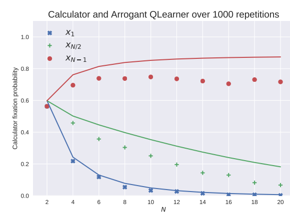
(e) All C or all D and Defector



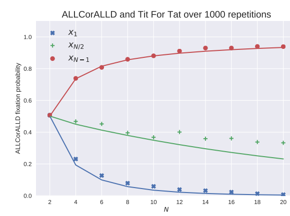
(f) Calculator and Random



(g) Calculator and All C or all D



(h) Calculator and Arrogant Q learner

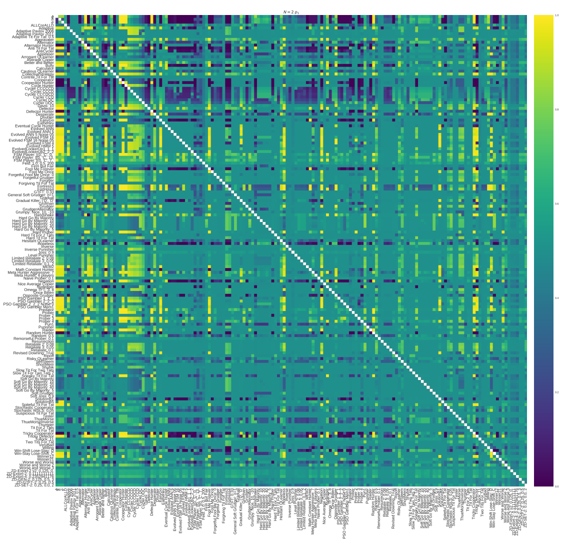


(i) All C or all D and Tit For Tat

Figure 4: Comparison of theoretic and actual Moran Process fixation probabilities for **stochastic** strategies

Player	Median p_1	Memory Depth	Stochastic
ZD-Extort-4: 0.23529411764705882, 0.25, 1	0.584	1	True
CollectiveStrategy	0.572	∞	False
FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'...	0.570	1	False
Feld: 1.0, 0.5, 200	0.568	200	True
ZD-Extort-2: 0.1111111111111111, 0.5	0.568	1	True

Table 3: Summary of top five strategies for $N = 2$



(a) The pairwise fixation probabilities for $N = 2$

(b) The median fixation probabilities for $N = 2$

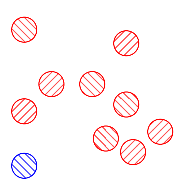
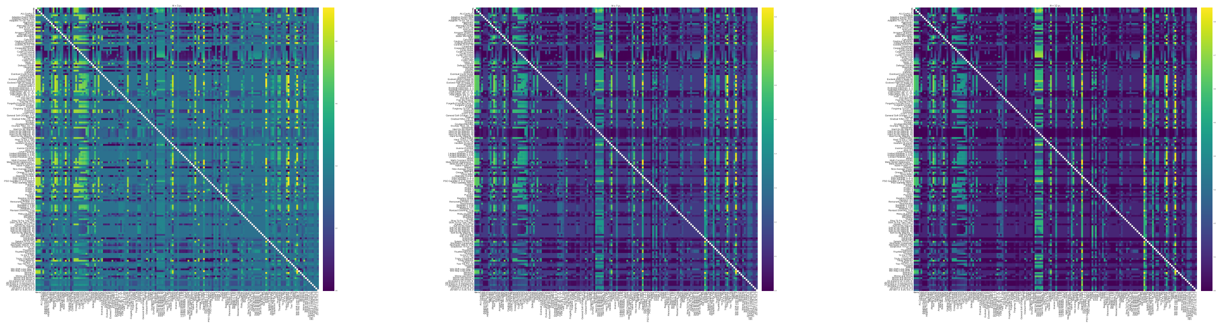


Figure 6: A single individual will successfully invade the population with probability x_1. The group of Individuals will successfully resist with probability x_{N-1}

4.2 Strong invaders

In this section we focus on the ability of a mutant strategy to invade: the probability of 1 individual of a given type successfully becoming fixated in a population of $N-1$ other individuals, denoted by x_i. Figures 7 shows these values for the players along the vertical axis when matched against the players on the horizontal axis. For population size N, a neutral mutant has fixation probability $x_1 = 1/N$. This information is summarised in Figure 8 showing the median fixation as well as the neutral fixation for each given scenario.



(a) $N = 3$ (b) $N = 7$ (c) $N = 12$

Figure 7: Pairwise fixation probability x_1 of all strategies

For $N \in \{3, 7, 12\}$ the top five strategies are given in Tables 4.

Player	Median p_1	Memory Depth	Stochastic
CollectiveStrategy	0.403	∞	False
Predator	0.396	9	False
Prober 4	0.368	∞	False
Remorseful Prober: 0.1	0.357	2	True
Worse and Worse 2	0.355	∞	True

(a) $N = 3$

Player	Median p_1	Memory Depth	Stochastic
Prober 4	0.177	∞	False
CollectiveStrategy	0.170	∞	False
Worse and Worse 2	0.159	∞	True
Predator	0.158	9	False
Remorseful Prober: 0.1	0.146	2	True

(b) $N = 7$

Player	Median p_1	Memory Depth	Stochastic
Prober 4	0.105	∞	False
Worse and Worse 2	0.093	∞	True
Remorseful Prober: 0.1	0.089	2	True
Predator	0.088	9	False
Tester	0.088	∞	False

(c) $N = 12$

Table 4: Properties of top five invaders

It can be seen that apart from the Collective Strategy, none of the strategies of Table 3 perform well for $N \in \{3, 7, 12\}$. The new high performing strategies are:

- Predator, a finite state machine described in [4].

(a) $N = 3$ (b) $N = 7$ (c) $N = 12$

Figure 8: Median probabilities x_1 of all strategies as well as the neutral fixation probability

- Prober 4, complex strategy with an initial 20 move sequence of cooperations and defections [15]. This initial sequence serves as approximate handshake.

- Remorseful Prober, a strategy that will not immediately retaliate when it recognises that the opponent is retaliating to a random defection [14].

- Worse and Worse 2: plays Tit For Tat for 20 moves and then defects with with growing probability [15].

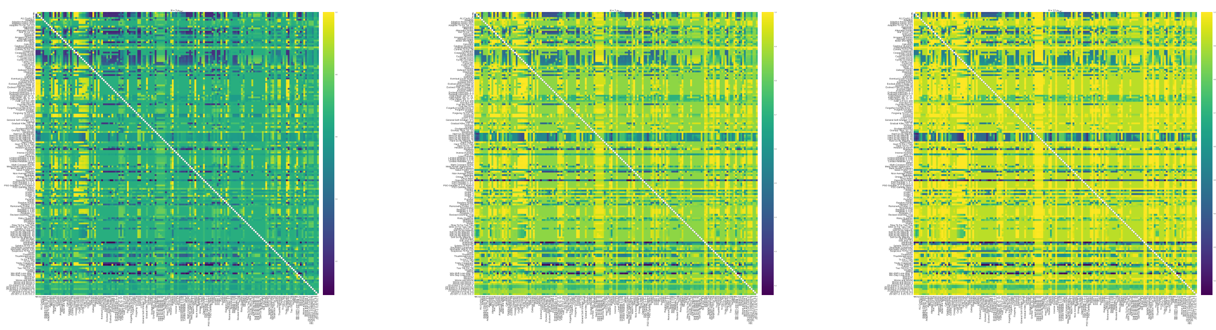
- Tester: a strategy submitted to the second of Axelrod's tournaments [6].

As well as noting that the memory length and complexity of these strategies are much greater than one, it is interesting to note that none of them are akin to memory one strategies. Most are not stochastic.

4.3 Strong resistors

In addition to identifying good invaders, we also identify strategies resistant to invasion by other strategies by examining the distribution of x_{N-1} for each strategy. Note that this is equivalent to looking at x_1 for all opponents.

Figures 9 show x_{N-1} the players along the vertical axis when matched against the players on the horizontal axis. This information is summarised in Figure 10 showing the median fixation as well as the neutral fixation for each given scenario.



(a) $N = 3$ (b) $N = 7$ (c) $N = 12$

Figure 9: Pairwise fixation probability x_{N-1} of all strategies

Table 5 shows the top five strategies when ranked according to x_{N-1} for $N \in \{3, 7, 12\}$. Once again none of the short memory strategies from Section 4.1 perform well for high N.

Three strategies have both high x_1 and high x_{N-1}:

- Collective
- Predator
- Prober 4

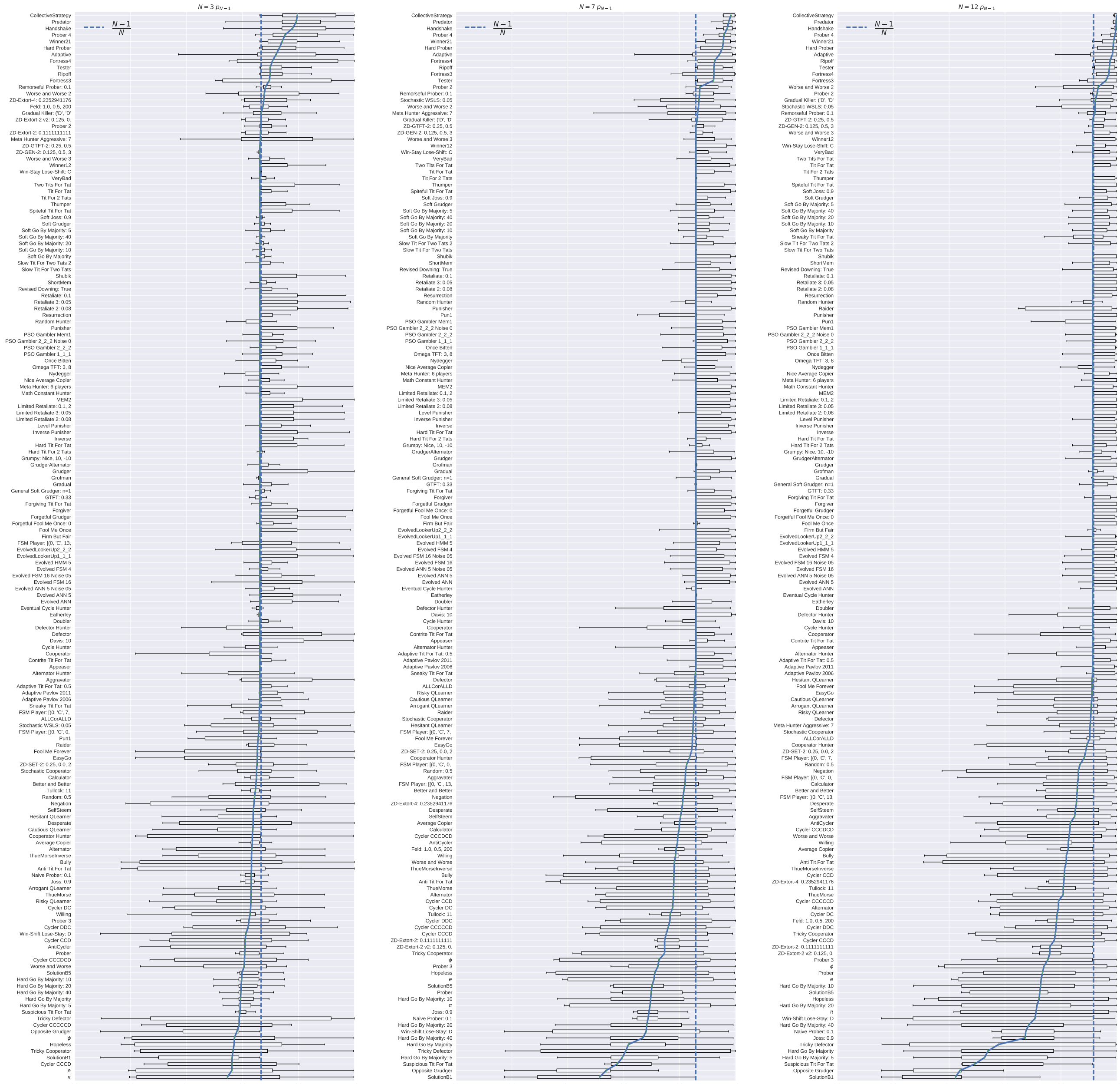
However, Remorseful Prober, Worse and Worse 2 and Tester no longer do as well. There are two strategies that are only top performers in x_{N-1}:

- Handshake: a slightly less aggressive version of the Collective strategy [22]. As long as the initial sequence is played then it cooperates. Thus it will do well in a population consisting of many members of itself: just as the Collective strategy does. However it is not aggressive enough to invade other populations.

- Winner 21: a strategy that makes its decision deterministically based on 1 round of its own strategy and 2 of the opponent's strategy [16].

Interestingly none of these strategies are stochastic: this is explained by the need of strategies to have a steady hand when interacting with their own kind. In essence: acting stochastically increase the chance of friendly fire. However we note that it is possible to design a strategy with a "stochastic handshake" [11].

It is evident through Sections 4.1, 4.2 and 4.3 that performance of strategies not only depends on the initial population distribution but also that there seems to be a difference depending on whether or not $N > 2$. This will be explored further in the next section.



(a) $N = 2$ (b) $N = 7$ (c) $N = 12$

Figure 10: Median probabilities x_{N-1} of all strategies as well as the neutral fixation probability

Player	Median p_{N-1}	Memory Depth	Stochastic
CollectiveStrategy	0.796	∞	False
Predator	0.792	9	False
Handshake	0.779	∞	False
Prober 4	0.752	∞	False
Winner21	0.742	2	False

(a) $N = 3$

Player	Median p_{N-1}	Memory Depth	Stochastic
CollectiveStrategy	0.983	∞	False
Predator	0.975	9	False
Handshake	0.970	∞	False
Prober 4	0.958	∞	False
Winner21	0.956	2	False

(b) $N = 7$

Player	Median p_{N-1}	Memory Depth	Stochastic
CollectiveStrategy	0.999	∞	False
Handshake	0.997	∞	False
Predator	0.997	9	False
Prober 4	0.993	∞	False
Winner21	0.988	2	False

(c) $N = 12$

Table 5: Properties of top five resistors

4.4 The effect of population size

Figures 11, 12 and 13 show the median rank of each strategy against population size. For all starting populations $i \in \{1, N/2, N-1\}$ the ranks of strategies are relatively stable across the different values of $N > 2$ however for $N = 2$ there is a distinct difference. This confirms what has been discussed in previous sections.

Tables 6, 7 and 8 show the same information for the strategies that rated high for $N = 2$ and $N = 12$.

Player	2	3	4	5	6	7	8	9	10	11	12
ZD-Extort-4: 0.23529411764705882, 0.25, 1	1.0	10.0	103.0	108.0	118.0	120.5	123.5	122.0	124.5	129.0	128.5
CollectiveStrategy	2.0	1.0	1.0	1.0	1.0	2.0	2.0	2.0	2.0	48.5	49.5
FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'...	3.0	105.5	103.0	106.5	112.5	114.0	115.0	110.0	112.0	110.0	114.0
Feld: 1.0, 0.5, 200	4.5	6.0	6.0	99.5	103.0	99.0	108.0	107.0	103.0	109.0	113.0
ZD-Extort-2: 0.1111111111111111, 0.5	4.5	13.5	112.0	113.5	119.5	123.0	125.0	126.5	134.0	130.0	142.5
Prober 4	11.0	3.0	3.0	2.0	2.0	1.0	1.0	1.0	1.0	1.0	1.0
Worse and Worse 2	18.5	5.0	4.0	4.0	4.0	3.0	5.5	3.5	4.0	48.5	2.0
Remorseful Prober: 0.1	16.5	4.0	5.0	5.0	5.0	5.0	93.0	5.0	6.5	97.0	3.0
Predator	9.0	2.0	2.0	3.0	3.0	4.0	3.0	3.5	3.0	3.0	4.5
Tester	84.0	13.5	8.5	51.0	8.5	6.5	5.5	49.5	6.5	48.5	4.5

Table 6: Ranks of some strategies according to x_1 for different population sizes

Tables 9a, 9b and 9c show the correlation coefficients of the ranks in of strategies in differing population size. This is shown graphically in Figure 14. It is immediate to note that how well a strategy performs in any Moran process for $N > 2$ has little to do with the performance for $N = 2$. This illustrates why the strong performance of zero determinant strategies predicted in [20] does not extend to larger populations. This was discussed theoretically in [1] however not observed empirically at the scale presented here.

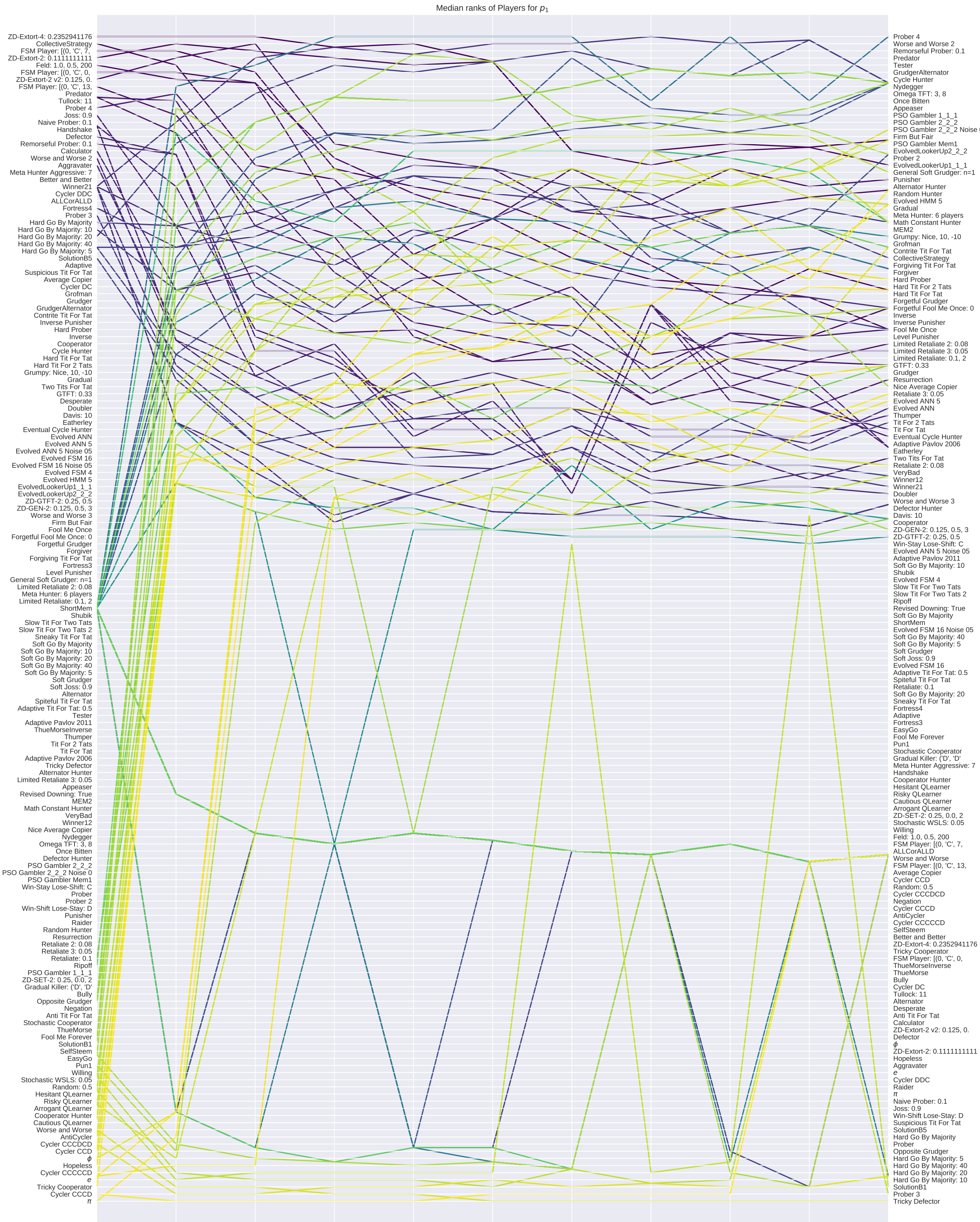


Figure 11: Ranks of all strategies according to x_1 for different population sizes

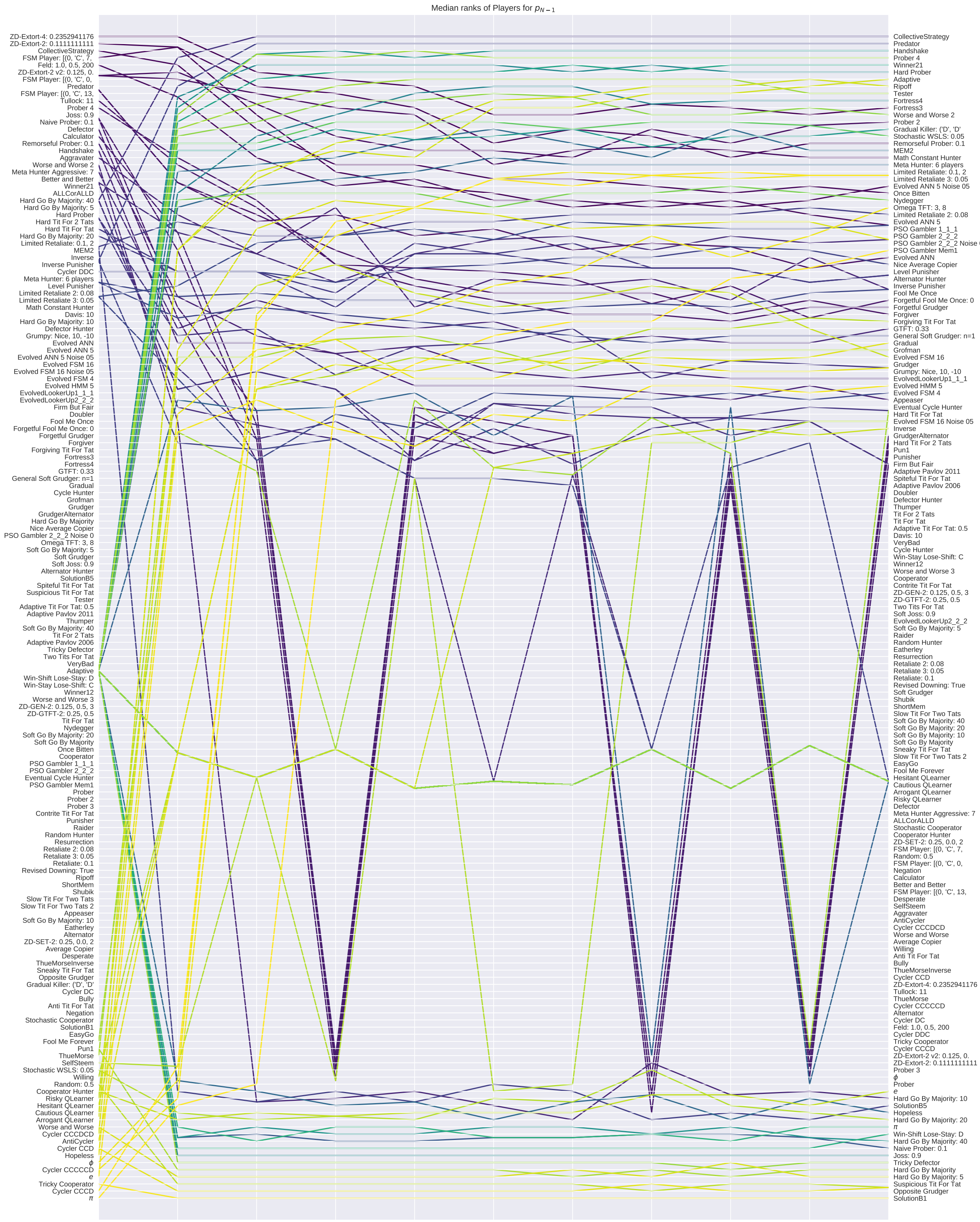


Figure 12: Ranks of all strategies according to x_{N-1} for different population sizes

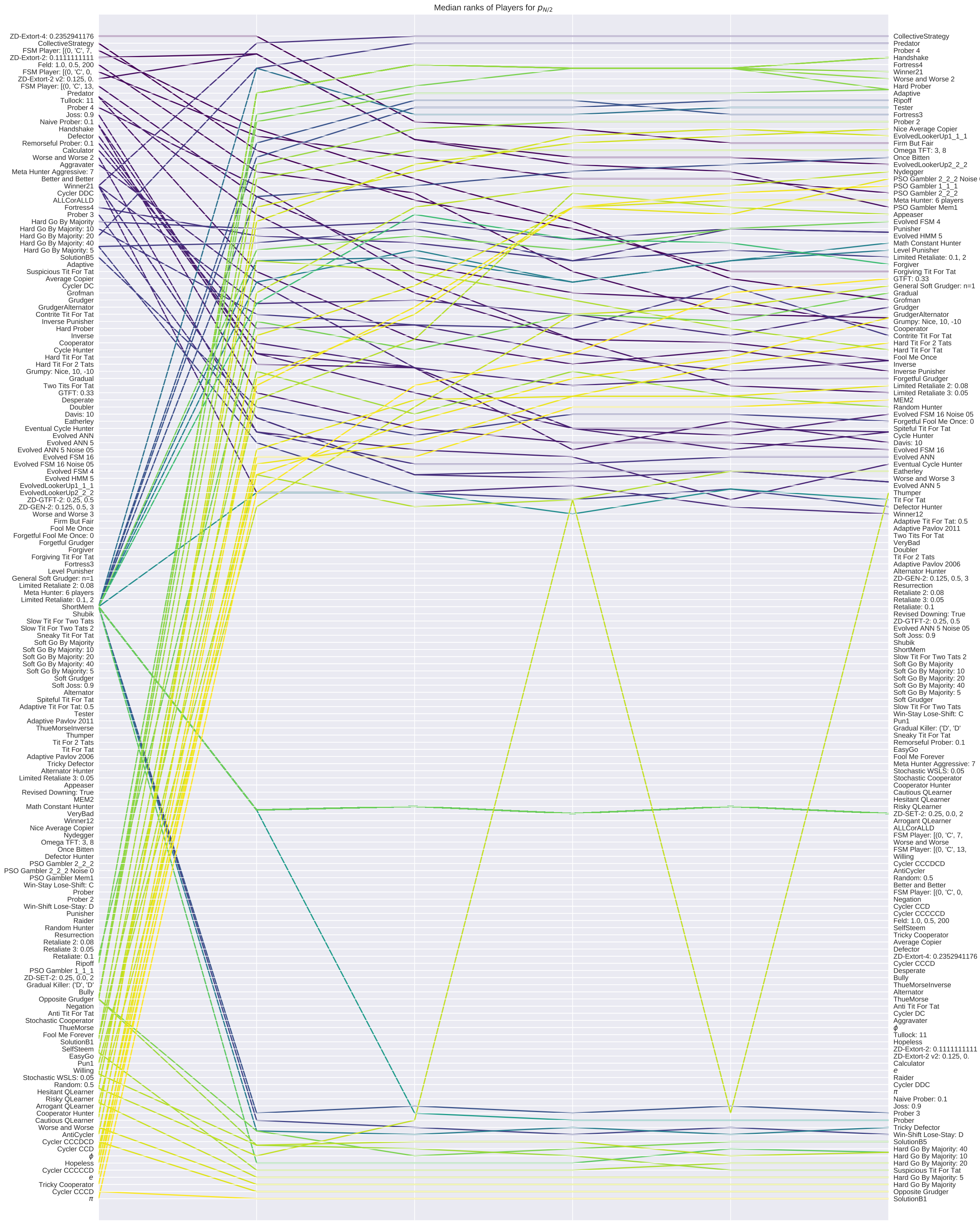


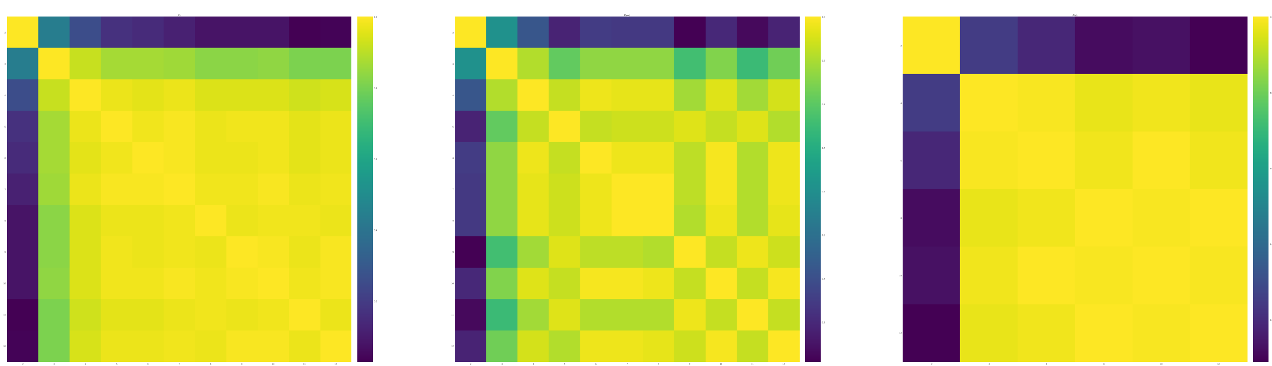
Figure 13: Ranks of all strategies according to $x_{N/2}$ for different population sizes

Player	2	3	4	5	6	7	8	9	10	11	12
ZD-Extort-4: 0.23529411764705882, 0.25, 1	1.0	10.0	103.0	108.0	118.0	120.5	123.5	122.0	124.5	129.0	128.5
CollectiveStrategy	2.0	1.0	1.0	1.0	1.0	2.0	2.0	2.0	2.0	48.5	49.5
FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'...	3.0	105.5	103.0	106.5	112.5	114.0	115.0	110.0	112.0	110.0	114.0
Feld: 1.0, 0.5, 200	4.5	6.0	6.0	99.5	103.0	99.0	108.0	107.0	103.0	109.0	113.0
ZD-Extort-2: 0.1111111111111111, 0.5	4.5	13.5	112.0	113.5	119.5	123.0	125.0	126.5	134.0	130.0	142.5
CollectiveStrategy	3.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Handshake	17.0	3.0	3.0	3.0	3.0	3.0	3.0	2.0	3.0	3.0	2.5
Predator	8.0	2.0	2.0	2.0	2.0	2.0	2.0	3.0	2.0	2.0	2.5
Prober 4	11.0	4.0	4.0	4.0	4.0	4.0	5.0	4.0	6.0	4.0	4.0
Winner21	22.0	5.0	5.0	5.0	5.0	5.0	4.0	5.0	4.0	5.0	5.0

Table 7: Ranks of some strategies according to x_{N-1} for different population sizes

Player	2	4	6	8	10	12
ZD-Extort-4: 0.23529411764705882, 0.25, 1	1.0	103.0	118.0	123.5	124.5	128.5
CollectiveStrategy	2.0	1.0	1.0	2.0	2.0	49.5
FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'...	3.0	103.0	112.5	115.0	112.0	114.0
Feld: 1.0, 0.5, 200	4.5	6.0	103.0	108.0	103.0	113.0
ZD-Extort-2: 0.1111111111111111, 0.5	4.5	112.0	119.5	125.0	134.0	142.5
CollectiveStrategy	2.0	1.0	1.0	1.0	1.0	1.0
Predator	9.0	2.0	2.0	2.0	2.0	2.0
Prober 4	11.0	3.0	3.0	3.0	3.0	3.0
Handshake	14.5	4.0	4.0	4.0	4.0	4.0
Fortress4	29.0	8.5	8.0	7.0	5.0	5.0

Table 8: Ranks of some strategies according to $x_{N/2}$ for different population sizes



(a) Rank based on x_1 (b) Rank based on x_{N-1} (c) Rank based on $x_{N/2}$

Figure 14: Heatmap of correlation coefficients of rankings by population size

N	2	3	4	5	6	7	8	9	10	11	12
2	1.00	0.44	0.26	0.17	0.15	0.12	0.08	0.08	0.08	0.03	0.04
3	0.44	1.00	0.92	0.87	0.87	0.86	0.83	0.83	0.84	0.81	0.81
4	0.26	0.92	1.00	0.97	0.96	0.97	0.95	0.95	0.95	0.93	0.94
5	0.17	0.87	0.97	1.00	0.98	0.99	0.97	0.98	0.98	0.96	0.97
6	0.15	0.87	0.96	0.98	1.00	0.99	0.97	0.97	0.98	0.96	0.97
7	0.12	0.86	0.97	0.99	0.99	1.00	0.98	0.98	0.99	0.97	0.98
8	0.08	0.83	0.95	0.97	0.97	0.98	1.00	0.97	0.98	0.98	0.97
9	0.08	0.83	0.95	0.98	0.97	0.98	0.97	1.00	0.99	0.97	0.99
10	0.08	0.84	0.95	0.98	0.98	0.99	0.98	0.99	1.00	0.98	0.99
11	0.03	0.81	0.93	0.96	0.96	0.97	0.98	0.97	0.98	1.00	0.97
12	0.04	0.81	0.94	0.97	0.97	0.98	0.97	0.99	0.99	0.97	1.00

(a) Correlation coefficients for ranks for invasion

N	2	3	4	5	6	7	8	9	10	11	12
2	1.00	0.61	0.42	0.29	0.35	0.34	0.34	0.21	0.30	0.23	0.29
3	0.61	1.00	0.91	0.81	0.87	0.87	0.87	0.76	0.85	0.75	0.83
4	0.42	0.91	1.00	0.93	0.98	0.97	0.97	0.89	0.96	0.89	0.95
5	0.29	0.81	0.93	1.00	0.93	0.94	0.94	0.96	0.93	0.96	0.91
6	0.35	0.87	0.98	0.93	1.00	0.98	0.98	0.92	0.99	0.91	0.98
7	0.34	0.87	0.97	0.94	0.98	1.00	1.00	0.92	0.99	0.91	0.98
8	0.34	0.87	0.97	0.94	0.98	1.00	1.00	0.91	0.98	0.91	0.97
9	0.21	0.76	0.89	0.96	0.92	0.92	0.91	1.00	0.93	0.98	0.94
10	0.30	0.85	0.96	0.93	0.99	0.99	0.98	0.93	1.00	0.93	0.99
11	0.23	0.75	0.89	0.96	0.91	0.91	0.91	0.98	0.93	1.00	0.93
12	0.29	0.83	0.95	0.91	0.98	0.98	0.97	0.94	0.99	0.93	1.00

(b) Correlation coefficients for ranks for resistance

N	2	4	6	8	10	12
2	1.00	0.25	0.19	0.12	0.13	0.09
4	0.25	1.00	0.99	0.97	0.98	0.97
6	0.19	0.99	1.00	0.98	1.00	0.98
8	0.12	0.97	0.98	1.00	0.99	1.00
10	0.13	0.98	1.00	0.99	1.00	0.99
12	0.09	0.97	0.98	1.00	0.99	1.00

(c) Correlation coefficients for ranks for coexistance

Table 9: Correlation coefficients of rankings by population size

4.5 Relative fitness

Under the assumption of a constant relative fitness r between two strategies [19] the formula for x_i (for given N, r is:

$$x_i = x_i(r) = \frac{1 - \frac{1}{r^i}}{1 - \frac{1}{r^N}} \tag{10}$$

Figure 15 shows this function for $N = 10$ and $i \in \{1, 5, 10\}$.

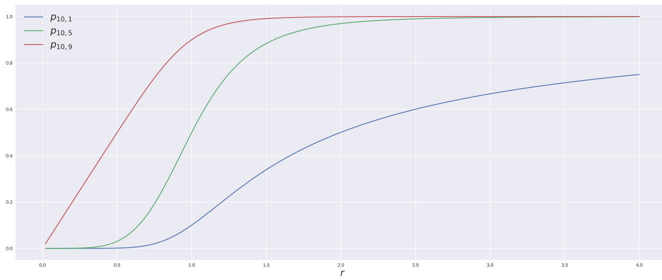


Figure 15: $x_i(r)$

The first and second derivative of (10) is given by equations (11) and (12).

$$\frac{dx_i}{dr} = \frac{r^{N-i-1}}{r^{2N} - 2r^N + 1} \left(-Nr^i + N + ir^N - i\right) \tag{11}$$

$$\frac{d^2x_i}{dr^2} = \frac{r^{N-i-2}}{\left(r^N - 1\right)^3} \left(2N^2 \left(r^i - 1\right) + N \left(r^N - 1\right) \left(N \left(r^i - 1\right) - 2i + r^i - 1\right) - i \left(i + 1\right) \left(r^N - 1\right)^2\right) \tag{12}$$

Using these, Halley's method [2] can be used to efficiently numerically invert $x_i(r)$ to obtain a theoretic relative fitness r that gives the calculated $x_i(r)$ between two strategies for a given N, i.

5 Conclusion

A detailed empirical analysis of 164 strategies of the IPD within a pairwise Moran process has been carried out. All $\binom{164}{2} = 13,366$ possible ordered pairs of strategies have been placed in a Moran process with different starting values allowing the each strategy to attempt to invade the other.

This is the largest such experiment carried out and has lead to many insights.

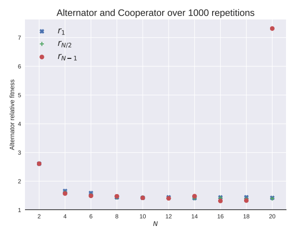
When studying evolutionary processes it is vital to consider $N > 2$ as the special case for $N = 2$ cannot be used to extrapolate performance in bigger populations. This was shown both observationally in Sections 4.2 and 4.3 but also by considering the correlation of the ranks in different population sizes in Section 4.4.

For $N = 2$, memory one strategies perform well, in particular as predicted by [20] zero determinant strategies rank highly. However, there are no memory one strategies in the top 5 performing strategies for $N > 3$. This is due to their lack of sophistication which allows them to recognise and adjust to their opponent.

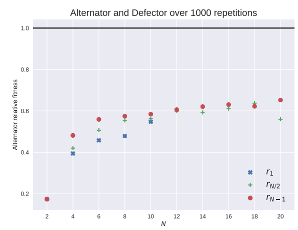
It is felt that these findings are important for the ongoing understanding of population dynamics and offer evidence for some of the shortcomings of short memory which has started to be recognised by the community [9].

All source code for this work has been written in a sustainable manner: it is open source, under version control and tested which ensures that all results can be reproduced [21, 23, 28]. The raw data as well as the processed data has also been properly archived.

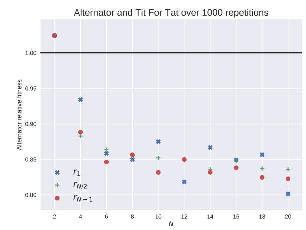
There are various areas for further work to build on this. Firstly, an analysis of the effect of noise would offer insights about the stability of the findings. It would also be possible to consider three or more types of strategy in the population and finally mutation would also offer an interesting dimension to explore.



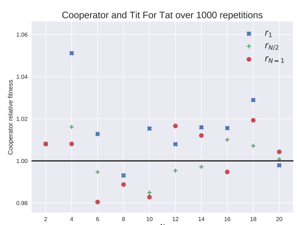
(a) Alternator and Cooperator



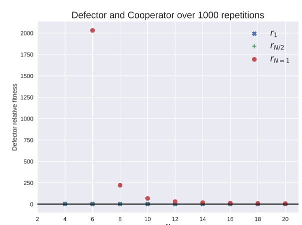
(b) Alternator and Defector



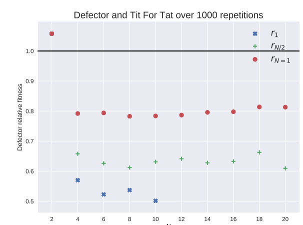
(c) Alternator and Tit For Tat



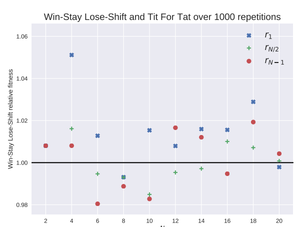
(d) Cooperator and Tit For Tat



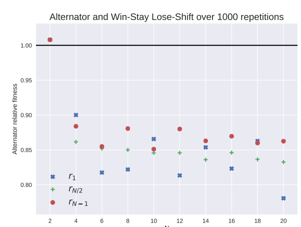
(e) Defector and Cooperator



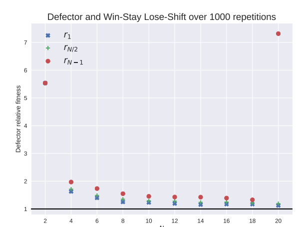
(f) Defector and Tit For Tat



(g) Win Stay Lose Shift and Tit For Tat

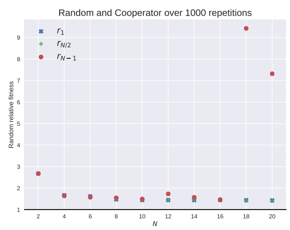


(h) Alternator and Win Stay Lose Shift

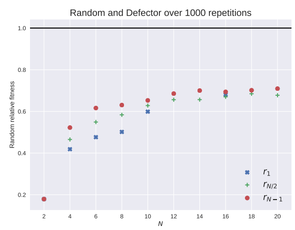


(i) Defector and Win Stay Lose Shift

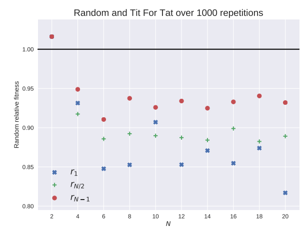
Figure 16: Estimated relative fitness for **deterministic** strategies



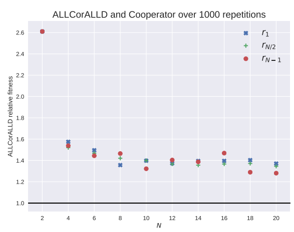
(a) Random and Cooperator



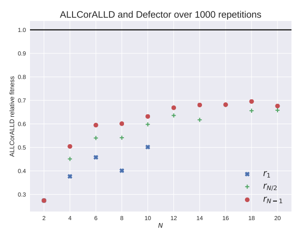
(b) Random and Defector



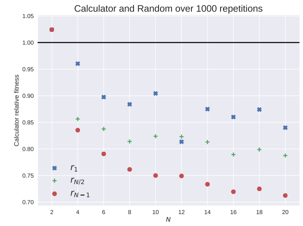
(c) Random and Tit For Tat



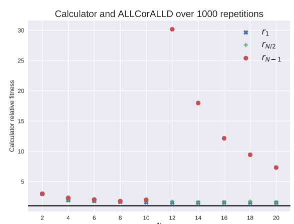
(d) All C or all D and Cooperator



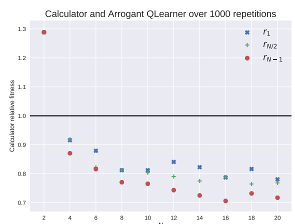
(e) All C or all D and Defector



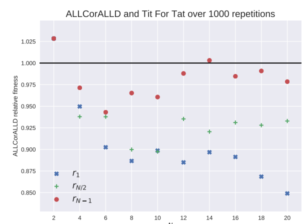
(f) Calculator and Random



(g) Calculator and All C or all D



(h) Calculator and Arrogant Q learner



(i) All C or all D and Tit For Tat

Figure 17: Estimated relative fitness for **stochastic** strategies

Acknowledgements

This work was performed using the computational facilities of the Advanced Research Computing @ Cardiff (ARCCA) Division, Cardiff University.

A variety of software libraries have been used in this work:

- The Axelrod library (IPD strategies and Moran processes) [25].
- The matplotlib library (visualisation) [10].
- The pandas and numpy libraries (data manipulation) [17, 26].

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A List of players

1. Cooperator Hunter
2. Retaliate 3: 0.05
3. Adaptive
4. Adaptive Tit For Tat: 0.5
5. Cycle Hunter
6. Revised Downing: True
7. Ripoff
8. Aggravater
9. Cycler CCCCCD
10. ALLCorALLD
11. Alternator
12. SelfSteem
13. *e*
14. Meta Hunter Aggressive: 7 players
15. Alternator Hunter
16. Cycler CCCD
17. Risky QLearner
18. AntiCycler
19. Sneaky Tit For Tat
20. Anti Tit For Tat
21. Cycler CCD
22. Slow Tit For Two Tats 2
23. Slow Tit For Two Tats
24. Cycler DC
25. Shubik
26. Adaptive Pavlov 2006
27. Soft Grudger
28. ShortMem
29. Adaptive Pavlov 2011
30. Cycler DDC
31. Soft Joss: 0.9
32. Spiteful Tit For Tat
33. Appeaser
34. Cycler CCCDCD
35. SolutionB5
36. Arrogant QLearner
37. SolutionB1
38. Defector
39. Stochastic Cooperator
40. Average Copier
41. Davis: 10
42. ZD-Extort-4: 0.23529411764705882, 0.25, 1

43. FSM Player: [(0, 'C', 7, 'C'), (0, 'D', 1, 'C'), (1, 'C', 11, 'D'), (1, 'D', 11, 'D'), (2, 'C', 8, 'D'), (2, 'D', 8, 'C'), (3, 'C', 3, 'C'), (3, 'D', 12, 'D'), (4, 'C', 6, 'C'), (4, 'D', 3, 'C'), (5, 'C', 11, 'C'), (5, 'D', 8, 'D'), (6, 'C', 13, 'D'), (6, 'D', 14, 'C'), (7, 'C', 4, 'D'), (7, 'D', 2, 'D'), (8, 'C', 14, 'D'), (8, 'D', 8, 'D'), (9, 'C', 0, 'C'), (9, 'D', 10, 'D'), (10, 'C', 8, 'C'), (10, 'D', 15, 'C'), (11, 'C', 6, 'D'), (11, 'D', 5, 'D'), (12, 'C', 6, 'D'), (12, 'D', 9, 'D'), (13, 'C', 9, 'D'), (13, 'D', 8, 'D'), (14, 'C', 8, 'D'), (14, 'D', 13, 'D'), (15, 'C', 4, 'C'), (15, 'D', 5, 'C')], 1, C
44. Better and Better
45. Stochastic WSLS: 0.05
46. Defector Hunter
47. ZD-GEN-2: 0.125, 0.5, 3
48. Bully
49. Desperate
50. ZD-GTFT-2: 0.25, 0.5
51. Suspicious Tit For Tat
52. ZD-Extort-2 v2: 0.125, 0.5, 1
53. Tester
54. Calculator
55. ZD-SET-2: 0.25, 0.0, 2
56. FSM Player: [(0, 'C', 13, 'D'), (0, 'D', 12, 'D'), (1, 'C', 3, 'D'), (1, 'D', 4, 'D'), (2, 'C', 14, 'D'), (2, 'D', 9, 'D'), (3, 'C', 0, 'C'), (3, 'D', 1, 'D'), (4, 'C', 1, 'D'), (4, 'D', 2, 'D'), (5, 'C', 12, 'C'), (5, 'D', 6, 'C'), (6, 'C', 1, 'C'), (6, 'D', 14, 'D'), (7, 'C', 12, 'D'), (7, 'D', 2, 'D'), (8, 'C', 7, 'D'), (8, 'D', 9, 'D'), (9, 'C', 8, 'D'), (9, 'D', 0, 'D'), (10, 'C', 2, 'C'), (10, 'D', 15, 'C'), (11, 'C', 7, 'D'), (11, 'D', 13, 'D'), (12, 'C', 3, 'C'), (12, 'D', 8, 'D'), (13, 'C', 7, 'C'), (13, 'D', 10, 'D'), (14, 'C', 10, 'D'), (14, 'D', 7, 'D'), (15, 'C', 15, 'C'), (15, 'D', 11, 'D')], 1, C
57. ThueMorse
58. CollectiveStrategy
59. Doubler
60. Cautious QLearner
61. ThueMorseInverse
62. EasyGo
63. Cooperator
64. Thumper
65. Contrite Tit For Tat
66. Eatherley
67. Meta Hunter: 6 players
68. Fool Me Once
69. Hard Go By Majority
70. Tit For Tat
71. Eventual Cycle Hunter
72. Tit For 2 Tats
73. Handshake
74. Fortress3
75. Evolved ANN
76. Forgiving Tit For Tat
77. PSO Gambler 2_2_2 Noise 05
78. Forgiver
79. Hard Go By Majority: 10
80. Negation
81. Tricky Cooperator
82. Evolved ANN 5
83. Forgetful Grudger
84. Hard Tit For 2 Tats
85. Nice Average Copier
86. Hard Go By Majority: 20
87. Tricky Defector
88. Evolved ANN 5 Noise 05
89. Forgetful Fool Me Once: 0.05
90. Nydegger
91. Fortress4
92. Hard Go By Majority: 40
93. Tullock: 11
94. Evolved FSM 4
95. Predator
96. π
97. Soft Go By Majority
98. Two Tits For Tat
99. GTFT: 0.33
100. Hard Go By Majority: 5
101. VeryBad
102. General Soft Grudger: n=1,d=4,c=2
103. Opposite Grudger
104. Hesitant QLearner
105. Once Bitten
106. Willing
107. Evolved FSM 16
108. Hard Tit For Tat
109. Prober 2
110. Soft Go By Majority: 10
111. Hopeless
112. Omega TFT: 3, 8
113. Prober
114. Inverse
115. Winner12
116. Evolved FSM 16 Noise 05
117. Inverse Punisher
118. Soft Go By Majority: 20
119. Prober 3
120. EvolvedLookerUp2_2_2
121. PSO Gambler Mem1
122. Punisher
123. EvolvedLookerUp1_1_1
124. Soft Go By Majority: 40
125. Joss: 0.9
126. Prober 4
127. Soft Go By Majority: 5
128. Pun1

129. Raider
130. Winner21
131. Level Punisher
132. ϕ
133. Limited Retaliate: 0.1, 20
134. PSO Gambler 2_2_2
135. Random Hunter
136. Gradual
137. Limited Retaliate 2: 0.08, 15
138. PSO Gambler 1_1_1
139. Random: 0.5
140. Gradual Killer: ('D', 'D', 'D', 'D', 'D', 'C', 'C')
141. Limited Retaliate 3: 0.05, 20
142. Win-Shift Lose-Stay: D
143. FSM Player: [(0, 'C', 0, 'C'), (0, 'D', 3, 'C'), (1, 'C', 5, 'D'), (1, 'D', 0, 'C'), (2, 'C', 3, 'C'), (2, 'D', 2, 'D'), (3, 'C', 4, 'D'), (3, 'D', 6, 'D'), (4, 'C', 3, 'C'), (4, 'D', 1, 'D'), (5, 'C', 6, 'C'), (5, 'D', 3, 'D'), (6, 'C', 6, 'D'), (6, 'D', 6, 'D'), (7, 'C', 7, 'D'), (7, 'D', 5, 'C')], 1, C
144. Math Constant Hunter
145. Remorseful Prober: 0.1
146. Grofman
147. Win-Stay Lose-Shift: C
148. Firm But Fair
149. Naive Prober: 0.1
150. Resurrection
151. Feld: 1.0, 0.5, 200
152. Grudger
153. GrudgerAlternator
154. Worse and Worse
155. Fool Me Forever
156. Worse and Worse 2
157. MEM2
158. Retaliate: 0.1
159. Grumpy: Nice, 10, -10
160. Worse and Worse 3
161. Evolved HMM 5
162. Hard Prober
163. Retaliate 2: 0.08
164. ZD-Extort-2: 0.1111111111111111, 0.5