Reviving, reproducing and revisiting Axelrod's second tournamentt

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1 Introduction

2 Reviving the tournament

The original source code for Axelrod's second tournament was written in Fortan (some contributors submitted code in Basic), this was subsequently published at [2]. This website maintained by the University of Michigan Center for the Study of Complex Systems was last updated (at the time of writing) in 1996. The source code available consists of a single file `TourExec1.1.f`.

For the purposes of that work this Fortran code was minimally modified so that it would run on a modern Fortran compiler. Furthermore, each strategy was extracted in to a single modular file which follows modern best practice and makes analysis more readable. This can be found at `https://github.com/Axelrod-Python/TourExec` and has been archived at [3].

Further to this, a Python library has been written that enables an interface to the Axelrod library described in the previous section. This library is referred to as the `Axelrod_fortran` library and is available at `https://github.com/Axelrod-Python/axelrod-fortran` and the specific version used for this work is [4]. This library has the fortran code of the original tournament as a dependency but otherwise offers a straightforward to install (using standard scientific python packages) and use option for the study of the strategies of [1]. This is made possible thanks to the translation of the compiled Fortran in to C and then magic happens and you type and it works.

3 Reproducing the tournament

The original tournaments from [1] have been recreated:

- Matches have length from $\{63, 77, 151, 308, 157\}$;
- Players do not know the number of rounds in a given match;
- A total of 25000 repetitions across the various match lengths have been carried out.

The scores across all repetitions are shown in Figure 1.

Whilst the results mainly agree with the original reported results, some strategies show distinct outliers:

- k61r
- k90r
- k82r

Note that there are various approaches to calculating the ranks/results, Figure 2 shows the rankings calculated in various ways as well as the mean absolute error (MAE).

The top 15 strategies in the reproduced tournament are shown in Table 1.

In [1], a linear regression model is used to identify 5 strategies, the scores against which are good predictors of the overall performance. The reported R^2 value is 0.979 (indicating 97% of variance accounted for by the model). The coefficients of this model are shown in Table 2.

Given the discrepancy in results shown in Figure 2 and Table 1 it is not surprising to see that this model no longer performs as well with $R^2 = 0.7430$.

Fitting a new model to the same 5 strategies gives the coefficients shown in Table 3 with $R^2 = 0.8771$

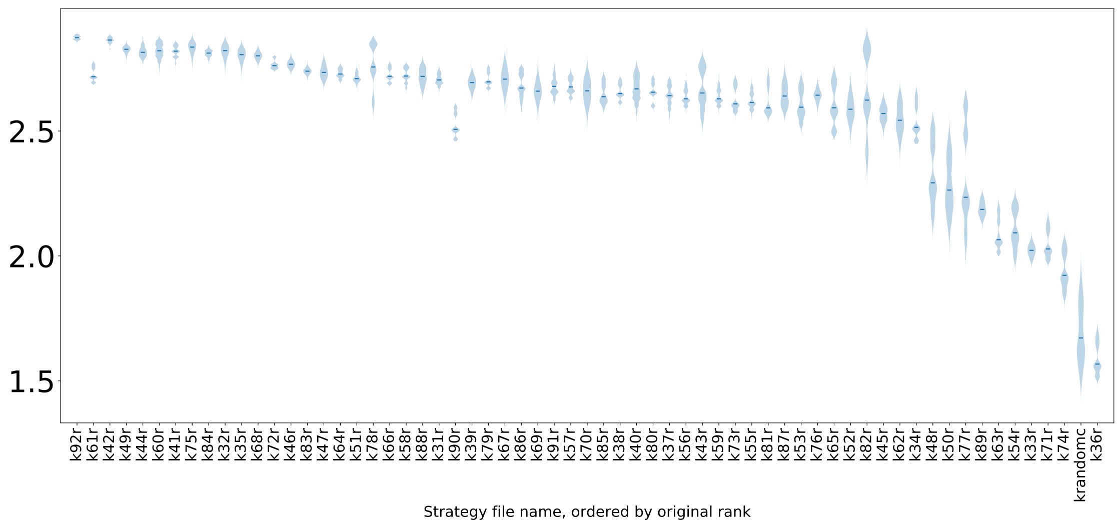

Figure 1: The scores per turn of each strategy.

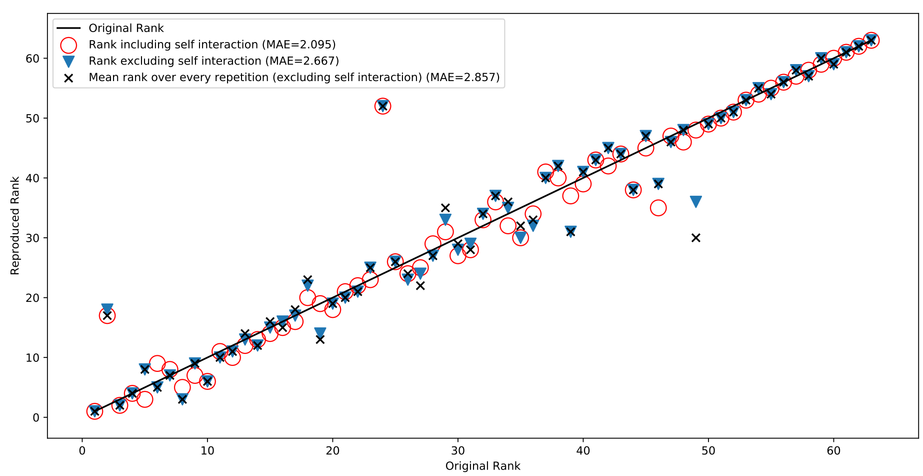

Figure 2: The rankings of each strategy.

	Author	Scores	Rank	Original Rank
k92r	Anatol Rapoport	2.8785	1	1
k61r	Danny C Champion	2.7189	17	2
k42r	Otto Borufsen	2.8342	2	3
k49r	Rob Cave	2.8261	4	4
k44r	William Adams	2.8264	3	5
k60r	Jim Graaskamp and Ken Katzen	2.8105	9	6
k41r	Herb Weiner	2.8106	8	7
k75r	Paul D Harrington	2.8241	5	8
k84r	T Nicolaus Tideman and Paula Chieruzz	2.8124	7	9
k32r	Charles Kluepfel	2.8168	6	10
k35r	Abraham Getzler	2.8021	11	11
k68r	Fransois Leyvraz	2.8089	10	12
k72r	Edward C White Jr	2.7764	12	13
k46r	Graham J Eatherley	2.7721	13	14
k83r	Paul E Black	2.7427	14	15

Table 1: Top 15 strategies in the reproduced tournament

Strategies	Coefficients
k69r	0.202
k91r	0.198
k40r	0.110
k76r	0.072
k67r	0.086
Intercept	0.795

Table 2: Linear model described in [1] with $R^2 = 0.7430$

Strategies	Coefficients	p-value	F-value
k69r	0.087	1.16446e-08	43.4383
k91r	0.203	2.52065e-10	57.1495
k40r	0.192	2.31169e-12	76.4987
k76r	0.060	0.0197037	5.73629
k67r	0.119	2.14365e-06	27.4145
Intercept	0.858	NA	NA

Table 3: Linear model fitted to the same 5 strategies described in [1] with $R^2 = 0.8771$

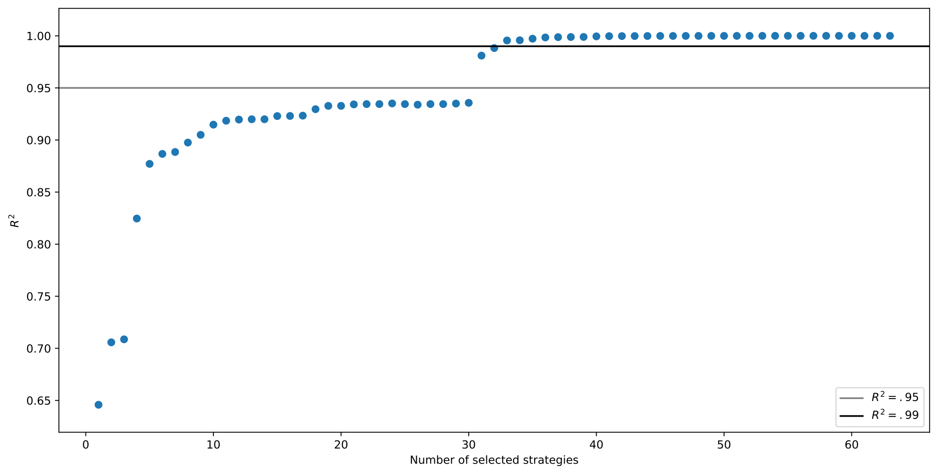

Figure 3: R^2 for models obtained using recursive feature elimination.

Using recursive feature elimination It is possible to select the features (strategies) that give the best prediction for a given number of features. The R^2 versus the number of features is shown in Figure 3.

Tables 4 and 5 show the coefficients for linear models fitted to 5 and 31 strategies with $R^2 = 0.8771$ and $R^2 = 0.9810$ respectively (31 strategies is the smallest number of strategies for which $R^2 > 95$.

Strategies	Coefficients	p-value	F-value
k31r	0.243	7.65965e-05	17.9915
k79r	0.189	1.40697e-13	89.5583
k85r	0.226	1.80033e-12	77.6167
k86r	0.196	2.24003e-15	111.226
k90r	0.312	8.66302e-15	103.82
Intercept	-0.789	NA	NA

Table 4: Linear model best fitted to 5 strategies in the reproduced tournament with $R^2 = 0.8771$

The predictions of these models are shown in Figure 4.

It is clear that the effectiveness of the predictive models with 5 strategies is low for the cluster of highly performing strategies (with a score great than 2.5). To be able to obtain a good model even for high performing strategies 31 seem to provide a good predictive model.

Strategies	Coefficients	p-value	F-value
k31r	1.448232e+11	7.65965e-05	17.9915
k32r	8.778442e+09	9.83646e-20	177.78
k35r	7.516922e+09	7.89921e-13	81.3664
k38r	-5.669016e+10	4.23448e-09	46.8878
k41r	5.246840e+10	1.295e-18	158.535
k44r	-4.131287e+10	9.9521e-10	52.0338
k46r	-5.876544e+10	0.00563622	8.236
k49r	-2.013244e+10	3.67411e-19	167.737
k51r	1.670000e-01	0.00265686	9.81693
k53r	-8.921543e+09	2.41706e-08	41.0199
k56r	-2.154542e+09	1.39838e-09	50.8026
k57r	-1.030984e+10	9.80149e-07	29.6401
k58r	2.174076e+10	1.07217e-21	215.714
k59r	-7.702084e+08	1.34573e-09	50.9409
k60r	-3.435984e+09	1.99774e-24	278.766
k61r	-3.342774e+10	1.46692e-09	50.6305
k64r	-9.135289e+10	6.13343e-14	93.6698
k70r	-4.665488e+10	3.52222e-12	74.6365
k72r	-3.422830e+10	2.06958e-09	49.4008
k73r	2.251230e+10	5.23871e-11	63.2961
k79r	5.648202e+10	1.40697e-13	89.5583
k81r	-8.213551e+09	6.7549e-13	82.0903
k82r	1.420000e-01	1.9396e-06	27.6956
k83r	1.556482e+10	3.07771e-06	26.4062
k84r	1.674378e+09	6.78837e-18	146.999
k85r	1.394646e+10	1.80033e-12	77.6167
k86r	4.702882e+10	2.24003e-15	111.226
k87r	4.556353e+10	1.46303e-09	50.6401
k88r	5.294312e+10	1.02055e-14	102.945
k90r	1.012234e+11	8.66302e-15	103.82
k92r	-5.788372e+10	3.92535e-13	84.6306
Intercept	-3.540376e+11	NA	NA

Table 5: Linear model best fitted to 31 strategies in the reproduced tournament with $R^2 = 0.9810$

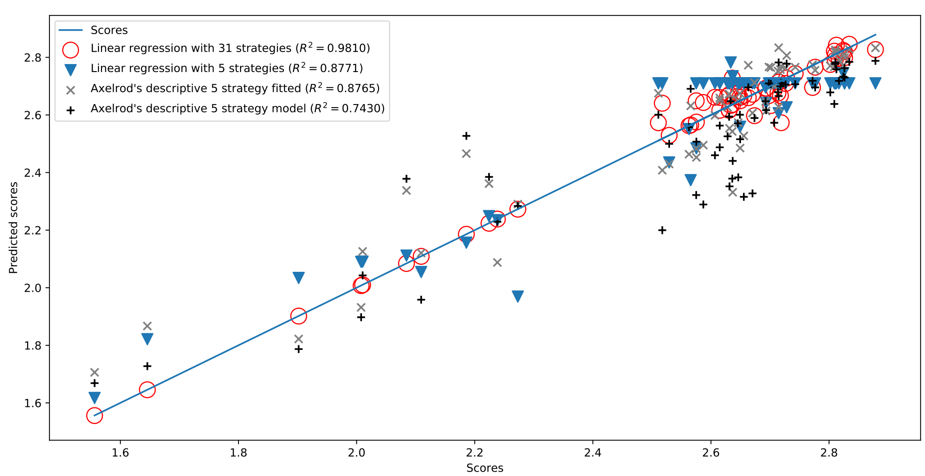

Figure 4: Predicting the performance of strategies using the 4 models discussed

4 Revisiting the tournament

4.1 Running with an extra invitation

4.1.1 Known strategies

4.1.2 Trained strategies

4.2 Further tournaments

4.2.1 Running with extortion

4.2.2 Running a large tournament

5 Conclusion

Acknowledgements

References

[1] R. Axelrod. More Effective Choice in the Prisoner's Dilemma. *Journal of Conflict Resolution*, 24(3):379–403, 1980.

[2] R. Axelrod. Complexity of cooperation web site. http://www-personal.umich.edu/ axe/research/Software/CC/CC2.html, 1996.

[3] Owen Campbell and Vince Knight. Axelrod-python/tourexec: v0.3.0. https://doi.org/10.5281/zenodo.834917, July 2017.

[4] Owen Campbell, Vince Knight, and Marc Harper. Axelrod-Python/axelrod-fortran: v0.3.1 (2017-08-04). https://doi.org/10.5281/zenodo.838980, August 2017.

A List of original players

1. k31r - Original rank: 23. Authored by Gail Grisell
2. k32r - Original rank: 10. Authored by Charles Kluepfel
3. k33r - Original rank: 59. Authored by Harold Rabbie
4. k34r - Original rank: 52. Authored by James W Friedman
5. k35r - Original rank: 11. Authored by Abraham Getzler
6. k36r - Original rank: 63. Authored by Roger Hotz
7. k37r - Original rank: 37. Authored by George Lefevre
8. k38r - Original rank: 34. Authored by Nelson Weiderman
9. k39r - Original rank: 25. Authored by Tom Almy
10. k40r - Original rank: 35. Authored by Robert Adams
11. k41r - Original rank: 7. Authored by Herb Weiner
12. k42r - Original rank: 3. Authored by Otto Borufsen
13. k43r - Original rank: 39. Authored by R D Anderson
14. k44r - Original rank: 5. Authored by William Adams
15. k45r - Original rank: 50. Authored by Michael F McGurrin
16. k46r - Original rank: 14. Authored by Graham J Eatherley
17. k47r - Original rank: 16. Authored by Richard Hufford
18. k48r - Original rank: 53. Authored by George Hufford
19. k49r - Original rank: 4. Authored by Rob Cave
20. k50r - Original rank: 54. Authored by Rik Smoody
21. k51r - Original rank: 18. Authored by John William Colbert
22. k52r - Original rank: 48. Authored by David A Smith
23. k53r - Original rank: 45. Authored by Henry Nussbacher
24. k54r - Original rank: 58. Authored by William H Robertson
25. k55r - Original rank: 42. Authored by Steve Newman

26. k56r - Original rank: 38. Authored by Stanley F Quayle
27. k57r - Original rank: 31. Authored by Rudy Nydegger
28. k58r - Original rank: 21. Authored by Glen Rowsam
29. k59r - Original rank: 40. Authored by Leslie Downing
30. k60r - Original rank: 6. Authored by Jim Graaskamp and Ken Katzen
31. k61r - Original rank: 2. Authored by Danny C Champion
32. k62r - Original rank: 51. Authored by Howard R Hollander
33. k63r - Original rank: 57. Authored by George Duisman
34. k64r - Original rank: 17. Authored by Brian Yamachi
35. k65r - Original rank: 47. Authored by Mark F Batell
36. k66r - Original rank: 20. Authored by Ray Mikkelson
37. k67r - Original rank: 27. Authored by Craig Feathers
38. k68r - Original rank: 12. Authored by Fransois Leyvraz
39. k69r - Original rank: 29. Authored by Johann Joss
40. k70r - Original rank: 32. Authored by Robert Pebly
41. k71r - Original rank: 60. Authored by James E Hall
42. k72r - Original rank: 13. Authored by Edward C White Jr
43. k73r - Original rank: 41. Authored by George Zimmerman
44. k74r - Original rank: 61. Authored by Edward Friedland
45. k75r - Original rank: 8. Authored by Paul D Harrington
46. k76r - Original rank: 46. Authored by David Gladstein
47. k77r - Original rank: 55. Authored by Scott Feld
48. k78r - Original rank: 19. Authored by Fred Mauk
49. k79r - Original rank: 26. Authored by Dennis Ambuehl and Kevin Hickey
50. k80r - Original rank: 36. Authored by Robyn M Dawes and Mark Batell
51. k81r - Original rank: 43. Authored by Martyn Jones
52. k82r - Original rank: 49. Authored by Robert A Leyland
53. k83r - Original rank: 15. Authored by Paul E Black
54. k84r - Original rank: 9. Authored by T Nicolaus Tideman and Paula Chieruzz
55. k85r - Original rank: 33. Authored by Robert B Falk and James M Langsted
56. k86r - Original rank: 28. Authored by Bernard Grofman
57. k87r - Original rank: 44. Authored by E E H Schurmann
58. k88r - Original rank: 22. Authored by Scott Appold
59. k89r - Original rank: 56. Authored by Gene Snodgrass
60. k90r - Original rank: 24. Authored by John Maynard Smith
61. k91r - Original rank: 30. Authored by Jonathan Pinkley
62. k92r - Original rank: 1. Authored by Anatol Rapoport
63. krandomc - Original rank: 62. Authored by None