Reviving, reproducing and revisiting Axelrod's second tournamentt

September 19, 2017

1 Introduction

The Prisoner's Dilemma has been the centre of the study of cooperative behaviour since Robert Axelrod's seminal work in the 1980s [1, 2, 4]. In this work, Robert Axelrod invited fellow academics to write computer code to play the Iterated Prisoner's Dilemma. This initial research has subsequently lead to a huge area of research aiming to understand the emergence of cooperative behaviour. A good overview of the variety of research areas is given in [9].

In the Prisoner's Dilemma, each player chooses simultaneously and independently between cooperation (C) or defection (D). The payoffs of the game are defined by the matrix $\begin{pmatrix} R & S \\ T & P \end{pmatrix}$, where $T > R > P > S$ and $2R > T + S$. The PD is a one round game, but is commonly studied in a manner where the prior outcomes matter. This repeated form is called the Iterated Prisoner's Dilemma (IPD).

In Robert Axelrod's tournaments the particular values of $R = 3, P = 1, S = 0, T = 5$ were used. Strategies were submitted written in either Fortran or Basic (in which case they were translated to Fortran). The very first tournament [1] involved 14 submissions (complemented by a fifteenth random strategy). Famously, the winner of this tournament was Tit For Tat: a strategy that mimics the opponent's previous action. The only sources for this work are the paper itself, all of the source code is apparently lost. In the second tournament [2] 64 strategies were submitted and again: Tit For Tat was declared the winner. From a computation archaeological point of view this tournament was far superior as all of the source code was kept and made available for use, it can be found at `http://www-personal.umich.edu/~axe/research/Software/CC/CC2/TourExec1.1.f.html`.

This work describes the process of reviving and using these strategies in a modern software framework ([12]): a Python library with over 200 strategies and a large number of analytical procedures which has been used in ongoing research [7, 8].

Importantly, reviving the strategies is not done through a manual exercise of reverse engineering the Fortran code which would be prone to mistakes. This is done by calling the original functions ensuring the **best possible reproduction and analysis of Robert Axelrod's original work**. This will be described in more detail in Section 2.

Results and further analyses pertaining to reproducing the tournament will be given in Section 3. Finally, Section 4 will extend the analysis to include contemporary research topics:

- Experimenting with adding one of over 196 other strategies from [12] to see how the results change.
- Training strategies to perform at a high level using Genetic Algorithms.
- Investigating the overall performance of Zero determinant strategies [10].
- Running a tournament with all considered strategies (more than 250).

This work contributes to the game theoretic literature by providing the first exercise in reproducing reported results that have been at the core of the study of cooperation. Furthermore it also provides a contemporary lense which, amongst other things concludes with one of the largest Iterated Prisoner's Dilemma tournaments. Finally, by reviving the original code and making it available to use in [12] it is now possible to use the original strategies of Robert Axelrod's second tournament in a modern framework which for example allows for the study of topological and evolutionary variants.

2 Reviving the tournament

As described in Section 1, the original source code for Axelrod's second tournament was written in Fortan (some contributors submitted code in Basic), this was subsequently published at [3]. This website maintained by the University of

Michigan Center for the Study of Complex Systems was last updated (at the time of writing) in 1996. The source code available consists of a single file `TourExec1.1.f`.

For the purposes of this work this Fortran code was minimally modified so that it would run on a modern Fortran compiler. Furthermore, each strategy was extracted in to a single modular file which follows modern best practice and makes analysis more readable. This can be found at `https://github.com/Axelrod-Python/TourExec` and has been archived at [5].

A Python library has been written that enables an interface to the Axelrod library described in the previous section. This library is referred to as the `Axelrod_fortran` library and is available at `https://github.com/Axelrod-Python/axelrod-fortran` and the specific version used for this work is [6]. This library has the fortran code of the original tournament as a dependency but otherwise offers a straightforward to install (using standard scientific python packages) and use option for the study of the strategies of [2]. This is made possible thanks to the translation of the compiled Fortran in to C which can be used directly by the Python software. For this to work there are a variety of minor translations that need to take place. As documented at `http://www-personal.umich.edu/~axe/research/Software/CC/CC2/TourExec1.1.f.html` the Fortran code implies that each strategy is a Fortran function which takes the following inputs.

- `J`: The opponent's previous move: 0 corresponds to a defection and 1 to a cooperation.
- `M`: The current turn number (starting at 1).
- `K`: The player's current cumulative score.
- `L`: The opponent's current cumulative score.
- `R`: A random number between 0 and 1: used by stochastic strategies.
- `JA`: The player's previous move.

For example, Figure 1 shows the source code for `k92r.f` also known as Tit For Tat.

```
      FUNCTION K92R(J,M,K,L,R, JA)
C BY ANATOL RAPOPORT
C TYPED BY AX 3/27/79 (SAME AS ROUND ONE TIT FOR TAT)
c replaced by actual code, Ax 7/27/93
c  T=0
c   K92R=ITFTR(J,M,K,L,T,R)
      k92r=0
      k92r = j
c test 7/30
c   write(6,77) j, k92r
c77   format(' test k92r. j,k92r: ', 2i3)
      RETURN
      END
```

Figure 1: Fortran Source code for original Tit For Tat strategy submitted to Axelrod's second tournament.

The `Axelrod` library takes advantage of the modern Object Oriented framework in Python. Each strategy is a class with agent based behaviour. The input to each player is simply the opponent with both holding their respective history of plays. In the newly written `Axelrod_fortran` library a class inherited from the base `Axelrod` class is written that interfaces with the Fortran strategies and the Python requirements. This includes, for example passing an initial move required by the Fortran player: using the original Fortran code as reference (see Figure 2) it is assumed that the assumed prior move is a cooperation (which is in line with Figure 1).

Figure 3 shows a diagrammatic representation of the interface between the Python strategy and the Fortran function.

The major advantage of this approach is that at no point has any subjectivity been added to the process of replicating Axelrod's second tournament. Indeed for some strategies the only description available is the Fortran code itself, thus they are being run and used as available. To the authors' knowledge this is the best possible way to replicate Axelrod's work which is the subject of the next section.

```
      Do 20 Game = 1,5
            RowGameSc = 0
            ColGameSc = 0
            JA = 0          ! Row's previous move, reported to column
            JB = 0          ! Col's previous move, reported to row
```

Figure 2: A portion of the original tournament code `https://github.com/Axelrod-Python/TourExec/blob/master/src/tournament/AxTest.f` setting the default previous move to a cooperation and score to 0.

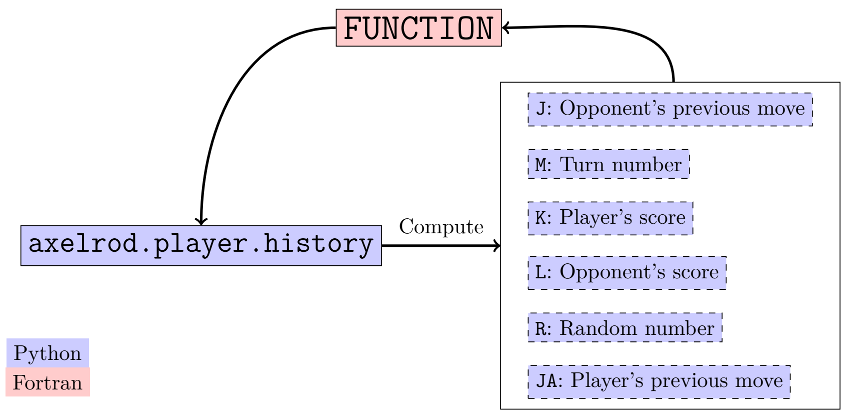


Figure 3: The interface between the Python axelrod library and the Fortran code

3 Reproducing the tournament

From [2], the following characteristics of the original tournament have been identified:

- Matches have length from $\{63, 77, 151, 308, 157\}$;
- Players do not know the number of rounds in a given match;
- A total of 25000 repetitions across the various match lengths have been carried out.

Note that there is some lack of clarity in [2] as to the length of the matches:

> *"As announced in the rules, the length of the games was determined probabilistically with a .00346 chance of ending with each given move. This parameter was chosen so that the expected median length of a game would be 200 moves. In practice, each pair of players was matched five times, and the lengths of these five games were determined once and for all by drawing a random sample. The resulting random sample from the implied distribution specified that the five games for each pair of players would be of lengths 63, 77, 151, and 308 moves. Thus the average length of a game turned out to be somewhat shorter than expected at 151 moves."*

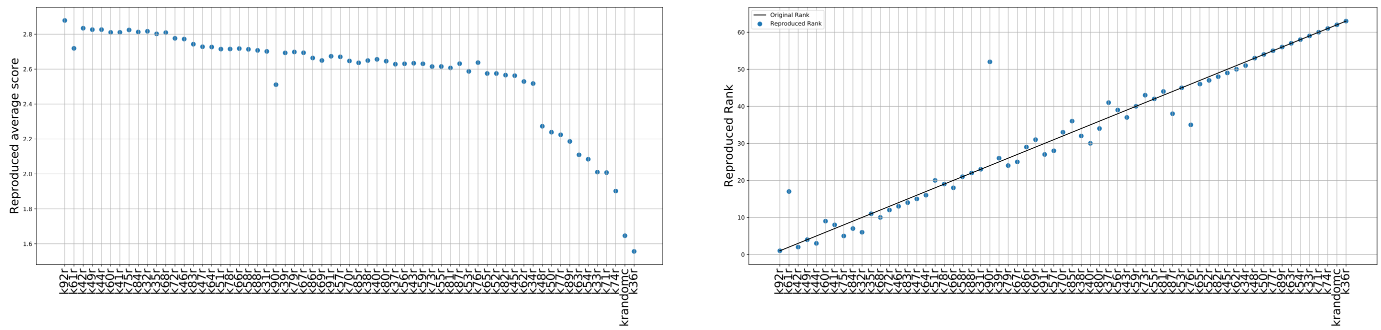
As only four match length samples were specified but the average was given, a fifth match length of 157 is assumed. It seems that the only stochastic smoothing used was these five repetitions, without a known seed it is not possible to replicate, thus the original tournament is repeated a total of 25000 for each match length.

The replicated scores and corresponding rankings of each strategy across all repetitions are shown in Figure 4.

The top 15 strategies in the reproduced tournament are shown in Table 1.

Whilst the results mainly agree with the original reported results, some strategies are distinct outliers `k61r` and `k90r`:

- `k61r`: Is referred to as Champion: cooperates for the first 10 moves, plays tit for tat for the next fifteen and then will cooperate unless: the other player defected on the previous move, the other player cooperated less than 60% and a random number between 0 and 1 is greater that the other player's cooperation rate. When run on a modern Fortran compiler this strategy does not behave as expected.



(a) Average score per turn of each strategy.

(b) Ranking of each strategy.

Figure 4: Replicated tournament with strategies ordered by original rank

	Author	Scores	Rank	Original Rank
k92r	Anatol Rapoport	2.8785	1	1
k61r	Danny C Champion	2.7189	17	2
k42r	Otto Borufsen	2.8342	2	3
k49r	Rob Cave	2.8261	4	4
k44r	William Adams	2.8262	3	5
k60r	Jim Graaskamp and Ken Katzen	2.8102	9	6
k41r	Herb Weiner	2.8106	8	7
k75r	Paul D Harrington	2.8241	5	8
k84r	T Nicolaus Tideman and Paula Chieruzz	2.8125	7	9
k32r	Charles Kluepfel	2.8166	6	10
k35r	Abraham Getzler	2.8018	11	11
k68r	Fransois Leyvraz	2.8090	10	12
k72r	Edward C White Jr	2.7763	12	13
k46r	Graham J Eatherley	2.7721	13	14
k83r	Paul E Black	2.7427	14	15

Table 1: Top 15 strategies in the reproduced tournament

- `k90r`: Is described as the well known Tit For Two Tats, however as written the Fortran code includes an error that implies that it is essentially a strategy that always cooperates.

In [2], a linear regression model is used to identify 5 strategies, the scores against which are good predictors of the overall performance. The reported R^2 value is 0.979 (indicating 97% of variance accounted for by the model). The coefficients of this model are shown in Table 2.

Strategies	Coefficients
k69r	0.202
k91r	0.198
k40r	0.110
k76r	0.072
k67r	0.086
Intercept	0.795

Table 2: Linear model described in [2] with $R^2 = 0.7427$

Given the discrepancy in results shown in Figure **??** and Table 1 it is not surprising to see that this model no longer performs as well with $R^2 = 0.7427$.

Fitting a new model to the same 5 strategies gives the coefficients shown in Table 3 with $R^2 = 0.9526$

Strategies	Coefficients	p-value	F-value
k69r	0.086	1.2156e-08	43.2943
k91r	0.204	2.47554e-10	57.2183
k40r	0.192	2.26171e-12	76.5961
k76r	0.060	0.0194647	5.75981
k67r	0.119	2.13401e-06	27.4271
Intercept	0.857	NA	NA

Table 3: Linear model fitted to the same 5 strategies described in [2] with $R^2 = 0.9526$

Using recursive feature elimination It is possible to select the features (strategies) that give the best prediction for a given number of features. The R^2 versus the number of features is shown in Figure 5.

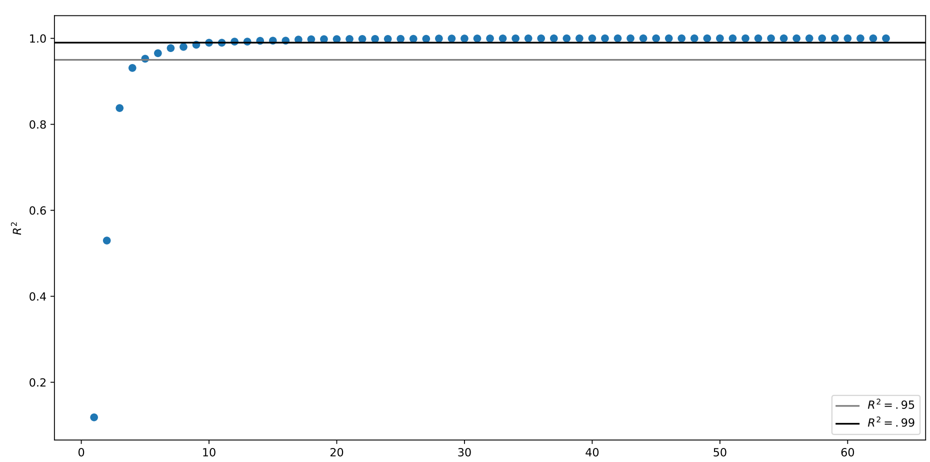


Figure 5: R^2 for models obtained using recursive feature elimination.

Tables 4 and 5 show the coefficients for linear models fitted to 5 and 12 strategies with $R^2 = 0.9526$ and $R^2 = 0.9924$ respectively (12 strategies is the smallest number of strategies for which $R^2 > 95$.

Strategies	Coefficients	p-value	F-value
k46r	0.252	0.00574885	8.19518
k53r	0.130	2.40311e-08	41.0388
k56r	0.154	1.40719e-09	50.78
k82r	0.189	1.92039e-06	27.7236
k85r	0.109	1.81982e-12	77.5683
Intercept	0.398	NA	NA

Table 4: Linear model best fitted to 5 strategies in the reproduced tournament with $R^2 = 0.9526$

Strategies	Coefficients	p-value	F-value
k39r	0.060	0.00338223	9.30339
k41r	0.077	1.30729e-18	158.467
k45r	0.063	0.207909	1.62009
k46r	0.137	0.00574885	8.19518
k53r	0.064	2.40311e-08	41.0388
k56r	0.061	1.40719e-09	50.78
k63r	0.040	0.315611	1.02382
k66r	0.116	1.44122e-16	127.302
k71r	0.051	0.16658	1.95999
k74r	0.051	0.453942	0.56803
k82r	0.044	1.92039e-06	27.7236
k85r	0.130	1.81982e-12	77.5683
Intercept	0.347	NA	NA

Table 5: Linear model best fitted to 12 strategies in the reproduced tournament with $R^2 = 0.9924$

The predictions of these models are shown in Figure 6.

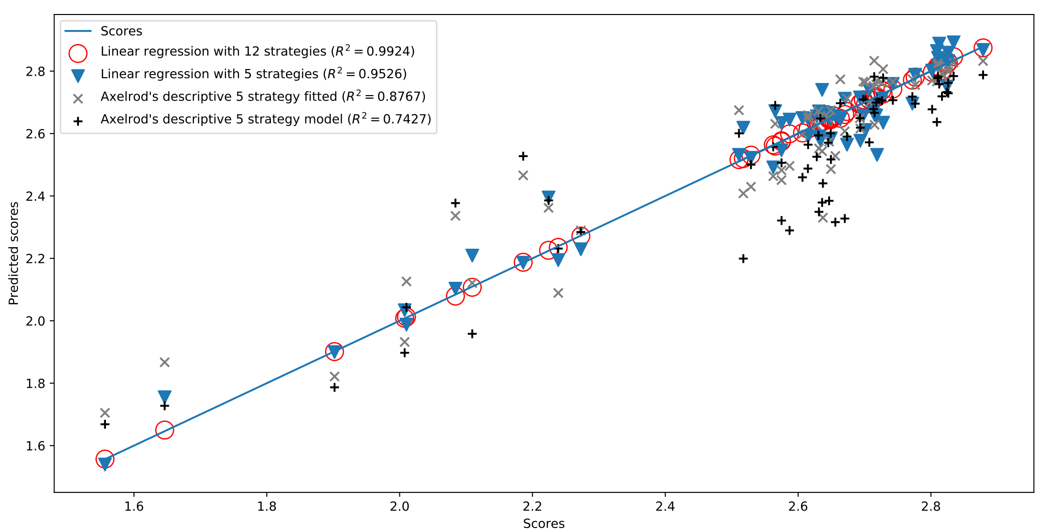


Figure 6: Predicting the performance of strategies using the 4 models discussed

It is clear that the effectiveness of the predictive models with 5 strategies is low for the cluster of highly performing strategies (with a score great than 2.5). To be able to obtain a good model even for high performing strategies 12 seem to provide a good predictive model.

The overall cooperation rate of the tournament is 0.750 . Figure 7 shows the cooperation rate versus the rank of the strategy.

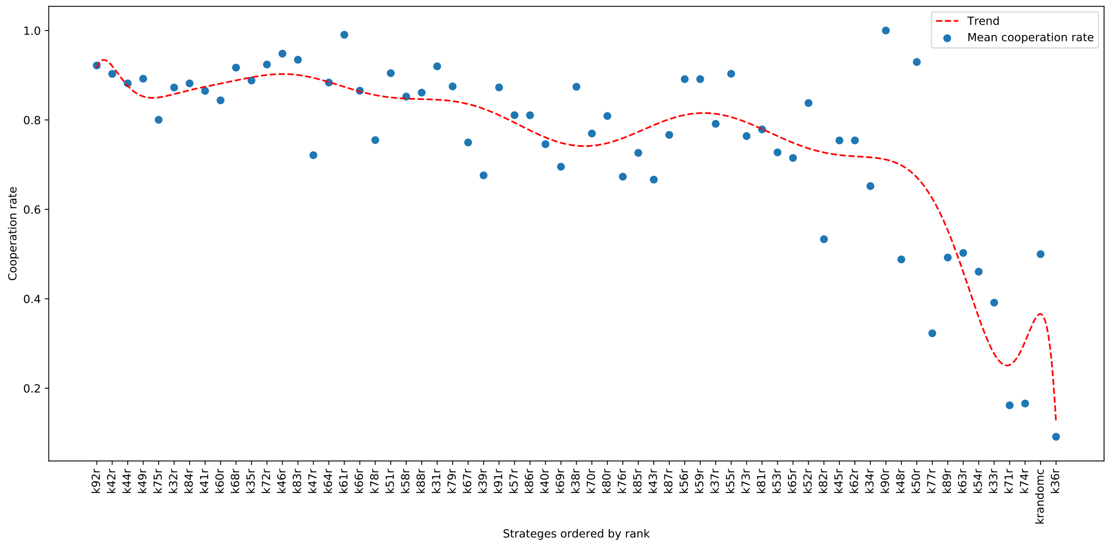


Figure 7: Cooperation rate versus rank

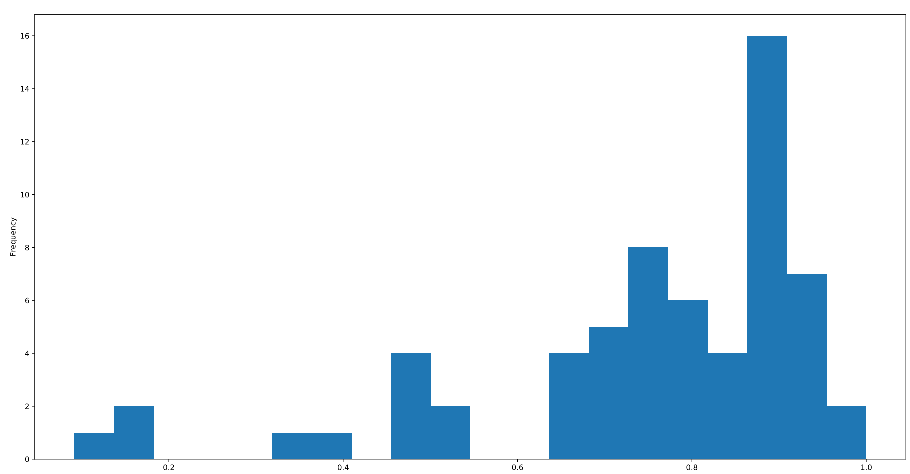


Figure 8: Distribution of cooperation rates

Figure 9 shows the pair wise cooperation rates. There is one strategy ‘k42r’ that defects against itself but cooperates against most other strategies. Also, it is clear that the low performing strategies are defecting against each other.

4 Revisiting the tournament

4.1 Running with an extra invitation

4.1.1 Known strategies

The tournament is run with every strategy of the Axelrod library. Every tournament (corresponding to each strategy) was run for 10750 repetitions. Table 6 shows the top ranking strategies.

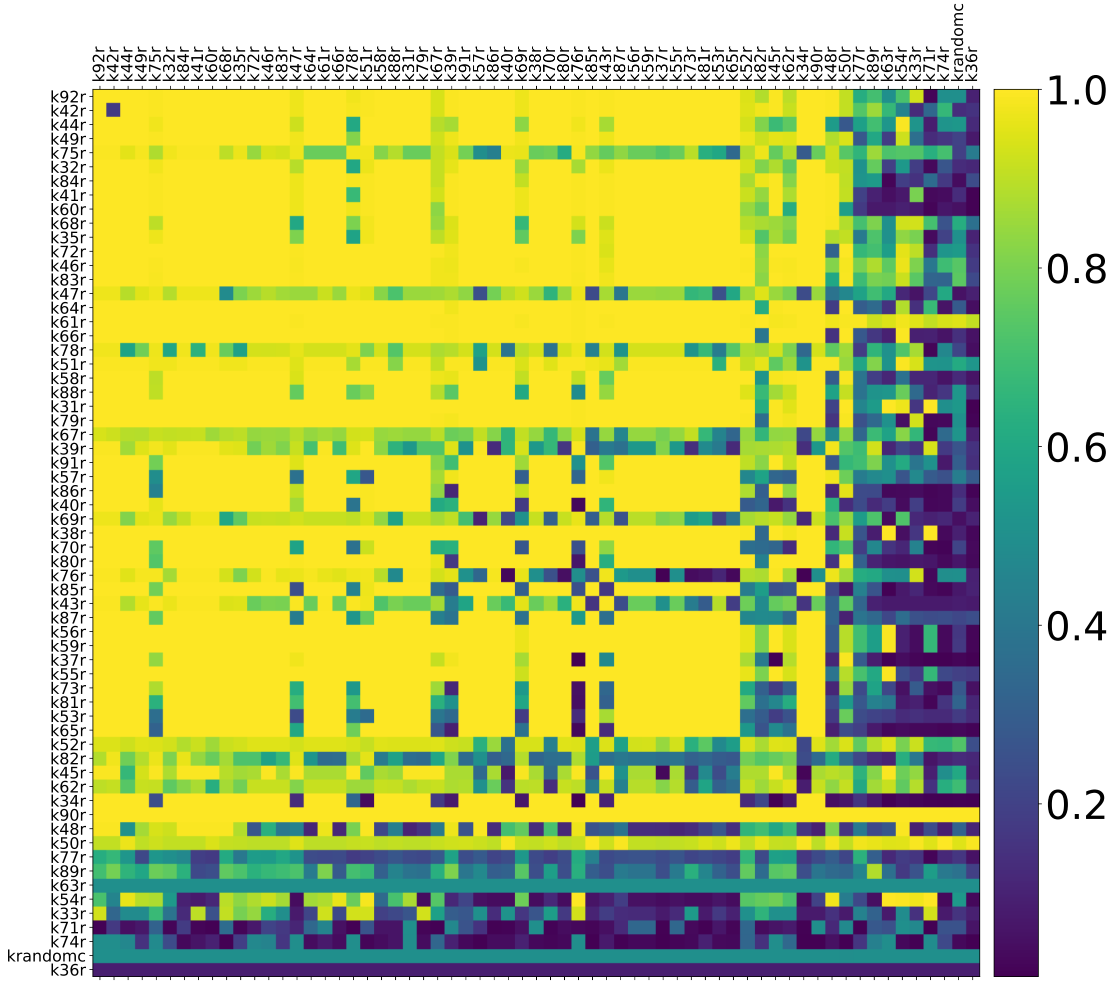


Figure 9: Cooperation rates between each pair of players (ordered by rank)

Name	Cooperation Rate against opponents	Cooperation Rate from opponents	Library Rank	Rank	Score	Winner
Meta Majority Long Memory	0.893	0.904	67	2	2.853	k92r
Omega TFT	0.888	0.904	16	2	2.854	k92r
GTFT	0.946	0.927	58	2	2.855	k92r
Forgiving Tit For Tat	0.925	0.916	70	2	2.858	k92r
Gradual	0.758	0.717	20	2	2.859	k92r
Resurrection	0.755	0.786	65	2	2.861	k92r
Meta Majority Finite Memory	0.911	0.914	97	2	2.865	k92r
Meta Majority	0.893	0.900	78	2	2.867	k92r
Firm But Fair	0.942	0.925	75	2	2.869	k92r
ZD-GTFT-2	0.941	0.929	49	2	2.876	k92r
Soft Joss	0.935	0.927	48	2	2.878	k92r
Adaptive Tit For Tat	0.923	0.923	74	2	2.880	k92r
PSO Gambler 2_2_2 Noise 05	0.866	0.880	19	3	2.833	k92r
Stein and Rapoport	0.864	0.882	44	7	2.816	k92r
Spiteful Tit For Tat	0.856	0.876	29	8	2.815	k92r
Doubler	0.922	0.892	68	12	2.784	k92r
EugineNier	0.824	0.857	31	12	2.788	k92r
Meta Majority Memory One	0.857	0.869	98	12	2.792	k92r
GrudgerAlternator	0.828	0.835	62	12	2.793	k92r
ZD-GEN-2	0.918	0.898	60	12	2.800	k92r

Table 6: Performance of extra strategy in Axelrod's original tournament

Figure 10 shows the rank of the extra strategy against it's rank in the library tournament.

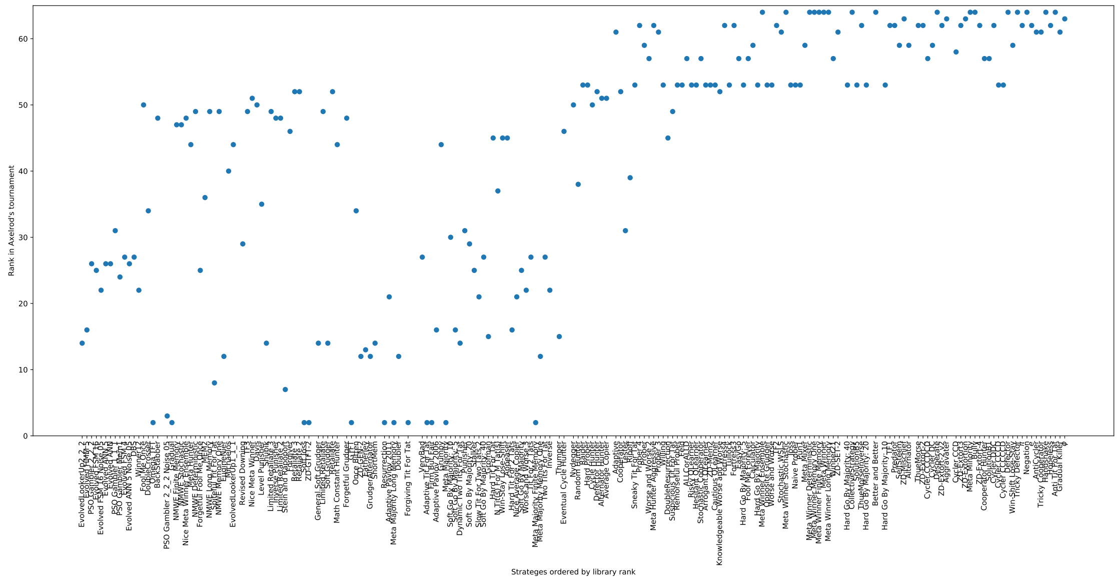


Figure 10: Ranks of extra strategy

4.1.2 Trained strategies

Using the various strategy sets and weights from the previous regression analysis it is possible to train strategies to compete in the tournament. Figure 11 show the mean utility of the strategies trained for this work. The strategies trained are 10 state Finite State machines.

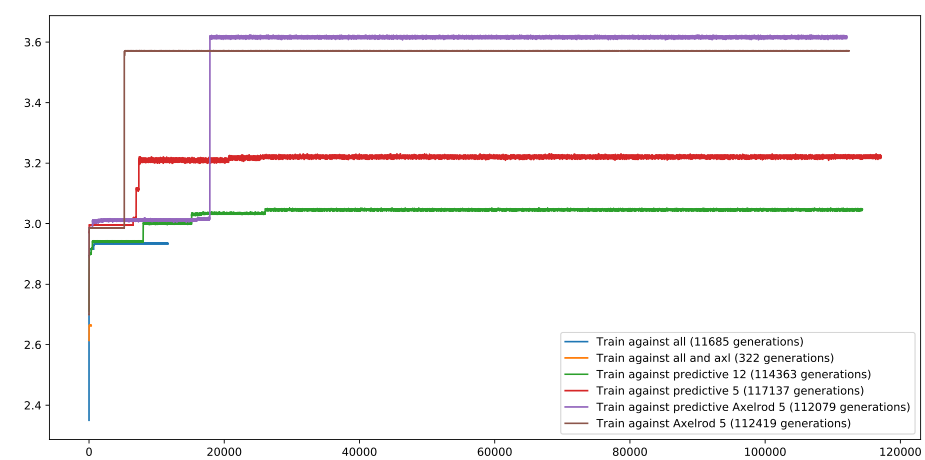


Figure 11: The maximum score per generation over the training periods

The relative score is not necessarily interpretable (the weights of the particular regression model skew this). Table 7 shows the results of the trained strategy in the tournament.

Training environment	Cooperation Rate against opponents	Cooperation Rate from opponents	Rank	Score	Winner
Full Players	0.919	0.917	1	2.933	Trained strategy
Full Players With Axl	0.946	0.919	2	2.839	k92r
Predictive 5 Strategies	0.790	0.746	41	2.621	k92r
Representative Strategies	0.599	0.652	52	2.479	k92r
Predictive 12 Strategies	0.758	0.656	53	2.423	k92r
Predictive With Axelrod 5 Strategies	0.777	0.687	53	2.384	k92r

Table 7: Results of trained strategy based on environment.

4.2 Further tournaments

4.2.1 Running with extortion

Since the work of [10] a lot of interest has been shown to Zero Determinant strategies. In [11] a small tournament is presented pitting these against each other. Table 8 shows the rankings of the top 15 strategies when including all the Zero Determinant strategies from [11] over 25000 repetitions.

	Original Author	Scores	Rank	Original Rank	Reproduced Rank
ZD-GTFT-2	NA	2.8365	1	NA	NA
GTFT	NA	2.8212	2	NA	NA
k92r	Anatol Rapoport	2.8146	3	1	1
k42r	Otto Borufsen	2.8045	4	3	2
k75r	Paul D Harrington	2.7936	5	8	5
k49r	Rob Cave	2.7893	6	4	4
k44r	William Adams	2.7840	7	5	3
k68r	Fransois Leyvraz	2.7717	8	12	10
k41r	Herb Weiner	2.7647	9	7	8
k46r	Graham J Eatherley	2.7612	10	14	13
k32r	Charles Kluepfel	2.7611	11	10	6
k72r	Edward C White Jr	2.7591	12	13	12
k35r	Abraham Getzler	2.7507	13	11	11
k84r	T Nicolaus Tideman and Paula Chieruzz	2.7490	14	9	7
k60r	Jim Graaskamp and Ken Katzen	2.7382	15	6	9

Table 8: Top 15 strategies in the tournament composed of the original strategies and the Zero Determinant strategies from [11]

The overall cooperation rate of this tournament is 0.737 and the various cooperation rates are shown in Figure 12 shows the cooperation rates of each strategy (ordered by rank).

Figure 14 shows the pair wise cooperation rates.

4.2.2 Running a large tournament

Table 9 shows the rankings of the top 20 strategies when including all strategies over 9500 repetitions. The cooperation rate for the tournament that pits all the library strategies against each other (without the Fortran ones) is 0.601.

The overall cooperation rate of this tournament is 0.632 and the various cooperation rates are shown in Figure 15 shows the cooperation rates of each strategy (ordered by rank).

Figure 17 shows the pair wise cooperation rates.

5 Conclusion

Acknowledgements

References

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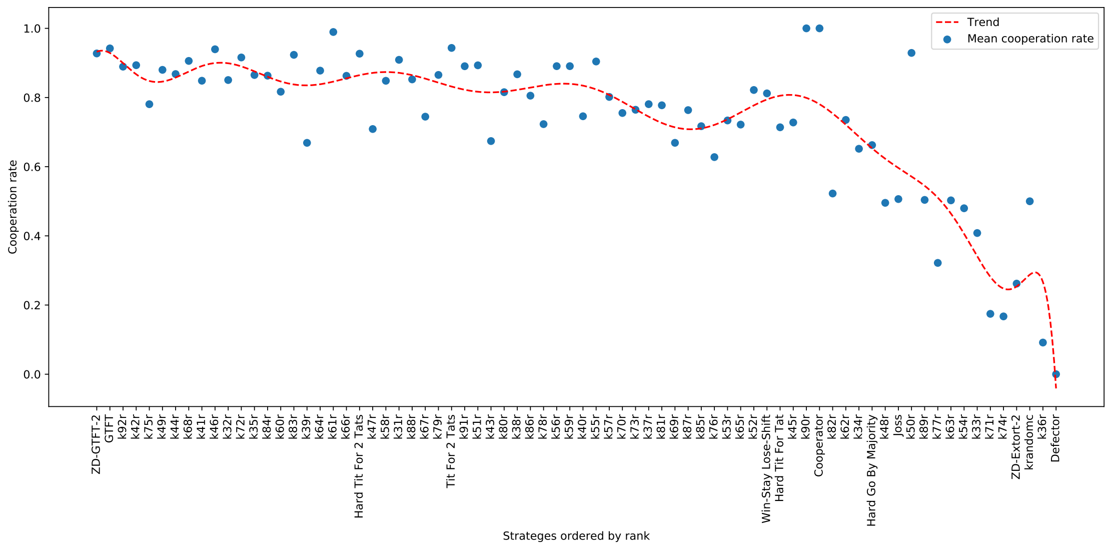

Figure 12: Cooperation rate versus rank for the Stewart and Poltkin tournament

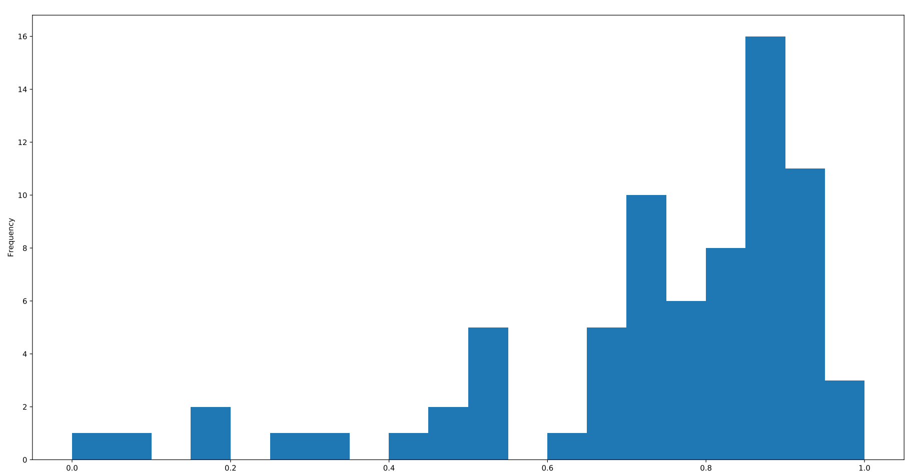

Figure 13: Distribution of cooperation rates for the Stewart and Plotkin tournament

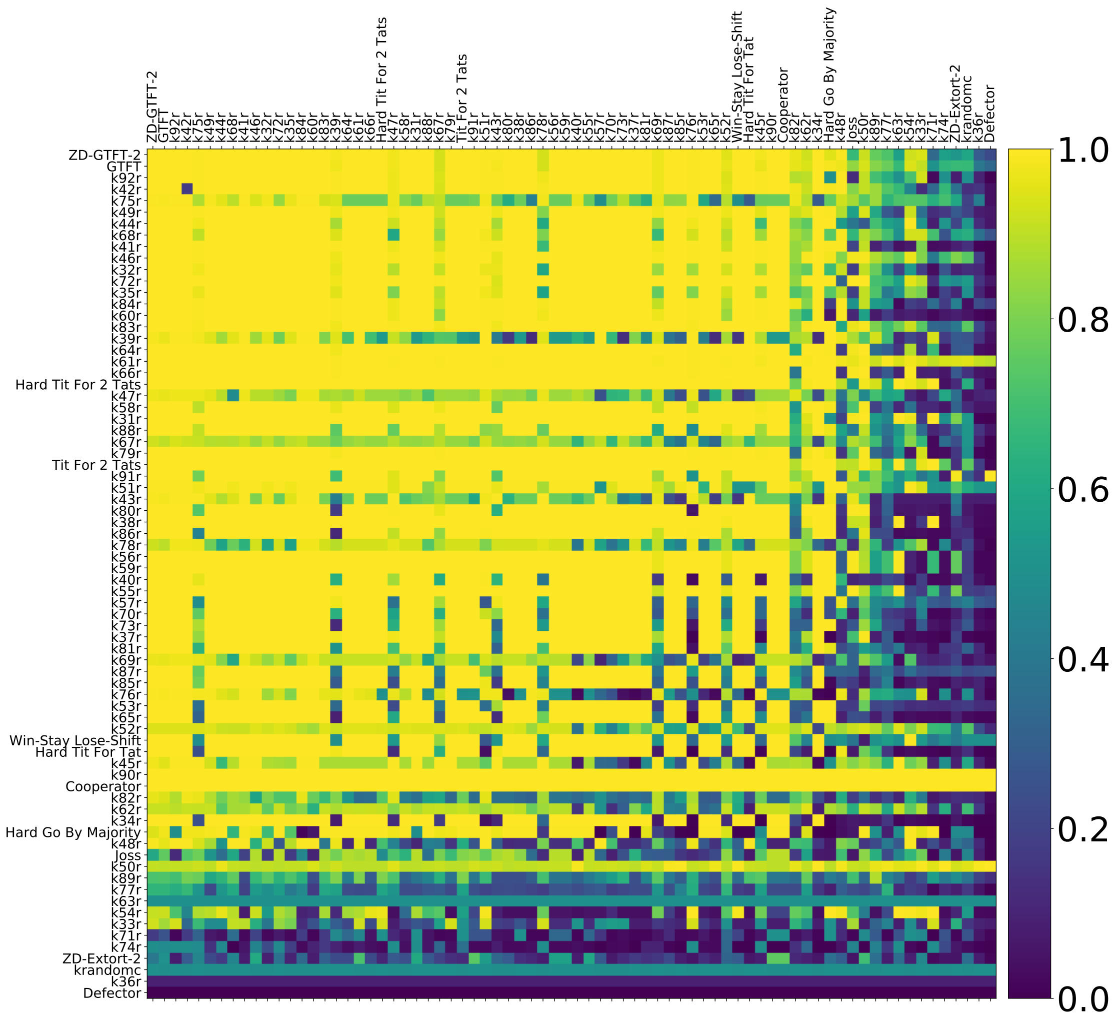

Figure 14: Cooperation rates between each pair of players (ordered by rank) for the Stewart and Plotkin tournament

	Original Author	Scores	Rank	Library Rank	Original Rank	Reproduced Rank
EvolvedLookerUp2_2_2	NA	2.8383	1	1	NA	NA
Evolved HMM 5	NA	2.8267	2	2	NA	NA
Evolved FSM 16 Noise 05	NA	2.8101	3	5	NA	NA
Evolved FSM 16	NA	2.8058	4	4	NA	NA
PSO Gambler 2_2_2	NA	2.8045	5	3	NA	NA
Evolved ANN	NA	2.7936	6	7	NA	NA
Evolved ANN 5	NA	2.7922	7	6	NA	NA
Omega TFT	NA	2.7896	8	16	NA	NA
PSO Gambler Mem1	NA	2.7863	9	9	NA	NA
Evolved FSM 4	NA	2.7826	10	10	NA	NA
PSO Gambler 1_1_1	NA	2.7819	11	8	NA	NA
PSO Gambler 2_2_2 Noise 05	NA	2.7742	12	19	NA	NA
Gradual	NA	2.7677	13	20	NA	NA
Evolved ANN 5 Noise 05	NA	2.7651	14	11	NA	NA
DBS	NA	2.7640	15	12	NA	NA
Winner12	NA	2.7608	16	13	NA	NA
k85r	Robert B Falk and James M Langsted	2.7452	17	NA	33	36
Spiteful Tit For Tat	NA	2.7357	18	29	NA	NA
k80r	Robyn M Dawes and Mark Batell	2.7349	19	NA	36	34
k87r	E E H Schurmann	2.7302	20	NA	44	38

Table 9: Top 20 strategies in the tournament when using all available strategies

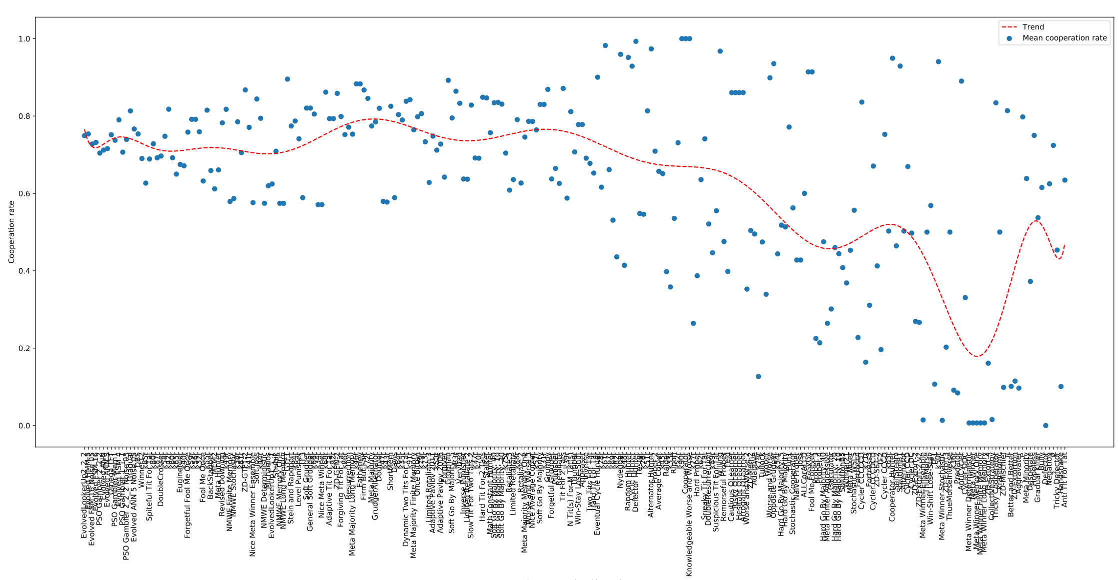

Figure 15: Cooperation rate versus rank for tournament with all available strategies

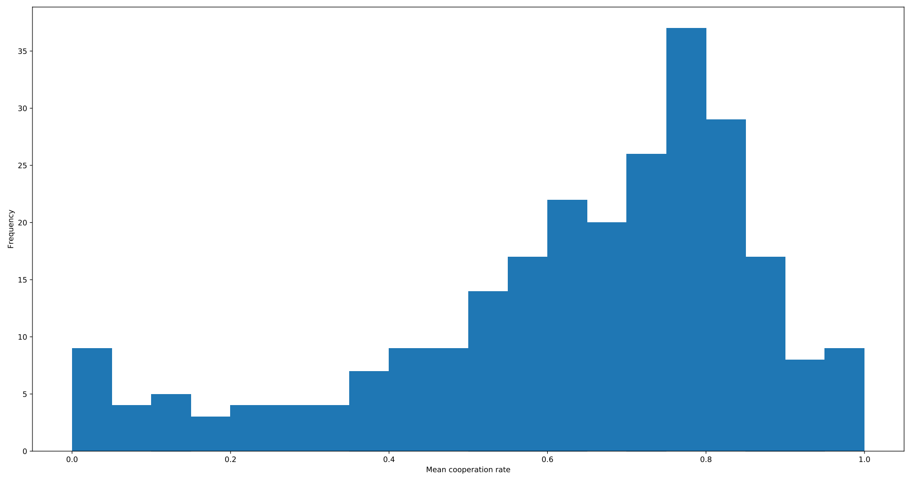

Figure 16: Distribution of cooperation rates for the full tournament.

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[12] The Axelrod project developers. Axelrod-python/axelrod: v3.3.0. https://doi.org/10.5281/zenodo.836439, July 2017.

A List of original players

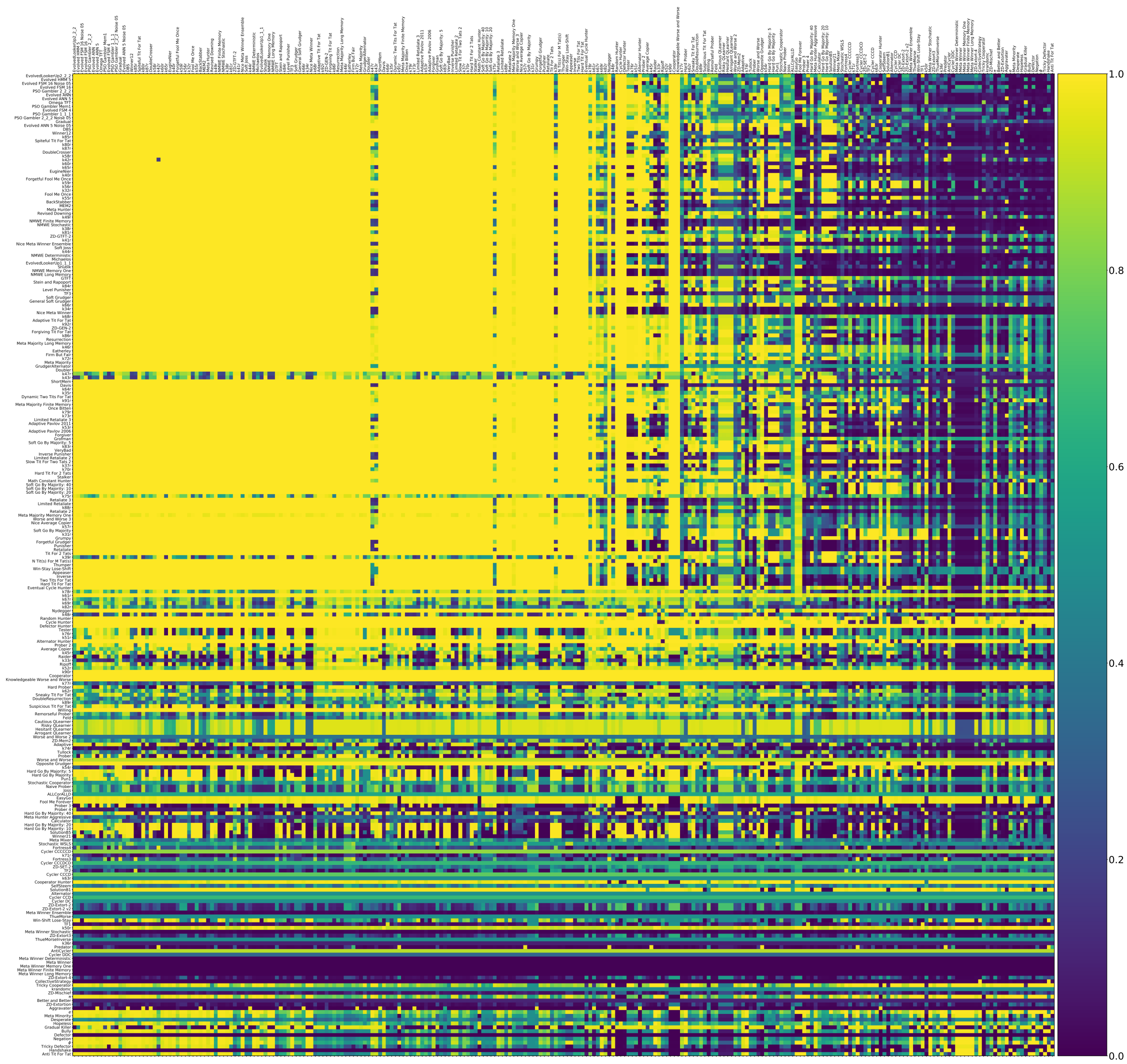

Figure 17: Cooperation rates between each pair of players (ordered by rank) for tournament with all available strategies

1. k31r - Original rank: 23. Authored by Gail Grisell
2. k32r - Original rank: 10. Authored by Charles Kluepfel
3. k33r - Original rank: 59. Authored by Harold Rabbie
4. k34r - Original rank: 52. Authored by James W Friedman
5. k35r - Original rank: 11. Authored by Abraham Getzler
6. k36r - Original rank: 63. Authored by Roger Hotz
7. k37r - Original rank: 37. Authored by George Lefevre
8. k38r - Original rank: 34. Authored by Nelson Weiderman
9. k39r - Original rank: 25. Authored by Tom Almy
10. k40r - Original rank: 35. Authored by Robert Adams
11. k41r - Original rank: 7. Authored by Herb Weiner
12. k42r - Original rank: 3. Authored by Otto Borufsen
13. k43r - Original rank: 39. Authored by R D Anderson
14. k44r - Original rank: 5. Authored by William Adams
15. k45r - Original rank: 50. Authored by Michael F McGurrin
16. k46r - Original rank: 14. Authored by Graham J Eatherley
17. k47r - Original rank: 16. Authored by Richard Hufford
18. k48r - Original rank: 53. Authored by George Hufford
19. k49r - Original rank: 4. Authored by Rob Cave
20. k50r - Original rank: 54. Authored by Rik Smoody
21. k51r - Original rank: 18. Authored by John William Colbert
22. k52r - Original rank: 48. Authored by David A Smith
23. k53r - Original rank: 45. Authored by Henry Nussbacher
24. k54r - Original rank: 58. Authored by William H Robertson
25. k55r - Original rank: 42. Authored by Steve Newman
26. k56r - Original rank: 38. Authored by Stanley F Quayle
27. k57r - Original rank: 31. Authored by Rudy Nydegger
28. k58r - Original rank: 21. Authored by Glen Rowsam
29. k59r - Original rank: 40. Authored by Leslie Downing
30. k60r - Original rank: 6. Authored by Jim Graaskamp and Ken Katzen
31. k61r - Original rank: 2. Authored by Danny C Champion
32. k62r - Original rank: 51. Authored by Howard R Hollander
33. k63r - Original rank: 57. Authored by George Duisman
34. k64r - Original rank: 17. Authored by Brian Yamachi
35. k65r - Original rank: 47. Authored by Mark F Batell
36. k66r - Original rank: 20. Authored by Ray Mikkelson
37. k67r - Original rank: 27. Authored by Craig Feathers
38. k68r - Original rank: 12. Authored by Fransois Leyvraz
39. k69r - Original rank: 29. Authored by Johann Joss
40. k70r - Original rank: 32. Authored by Robert Pebly
41. k71r - Original rank: 60. Authored by James E Hall
42. k72r - Original rank: 13. Authored by Edward C White Jr
43. k73r - Original rank: 41. Authored by George Zimmerman
44. k74r - Original rank: 61. Authored by Edward Friedland
45. k75r - Original rank: 8. Authored by Paul D Harrington
46. k76r - Original rank: 46. Authored by David Gladstein
47. k77r - Original rank: 55. Authored by Scott Feld
48. k78r - Original rank: 19. Authored by Fred Mauk
49. k79r - Original rank: 26. Authored by Dennis Ambuehl and Kevin Hickey
50. k80r - Original rank: 36. Authored by Robyn M Dawes and Mark Batell
51. k81r - Original rank: 43. Authored by Martyn Jones
52. k82r - Original rank: 49. Authored by Robert A Leyland
53. k83r - Original rank: 15. Authored by Paul E Black
54. k84r - Original rank: 9. Authored by T Nicolaus Tideman and Paula Chieruzz
55. k85r - Original rank: 33. Authored by Robert B Falk and James M Langsted

56. k86r - Original rank: 28. Authored by Bernard Grofman
57. k87r - Original rank: 44. Authored by E E H Schurmann
58. k88r - Original rank: 22. Authored by Scott Appold
59. k89r - Original rank: 56. Authored by Gene Snodgrass
60. k90r - Original rank: 24. Authored by John Maynard Smith
61. k91r - Original rank: 30. Authored by Jonathan Pinkley
62. k92r - Original rank: 1. Authored by Anatol Rapoport
63. krandomc - Original rank: 62. Authored by None