

# Reviving, reproducing and revisiting Axelrod’s second tournament

August 10, 2017

## 1 Introduction

## 2 Reviving the tournament

The original source code for Axelrod’s second tournament was written in Fortran (some contributors submitted code in Basic), this was subsequently published at [2]. This website maintained by the University of Michigan Center for the Study of Complex Systems was last updated (at the time of writing) in 1996. The source code available consists of a single file `TourExec1.1.f`.

For the purposes of that work this Fortran code was minimally modified so that it would run on a modern Fortran compiler. Furthermore, each strategy was extracted in to a single modular file which follows modern best practice and makes analysis more readable. This can be found at <https://github.com/Axelrod-Python/TourExec> and has been archived at [3].

Further to this, a Python library has been written that enables an interface to the Axelrod library described in the previous section. This library is referred to as the `Axelrod_fortran` library and is available at <https://github.com/Axelrod-Python/axelrod-fortran> and the specific version used for this work is [4]. This library has the fortran code of the original tournament as a dependency but otherwise offers a straightforward to install (using standard scientific python packages) and use option for the study of the strategies of [1]. This is made possible thanks to the translation of the compiled Fortran in to C and then magic happens and you type and it works.

## 3 Reproducing the tournament

The original tournaments from [1] have been recreated:

- Matches have length from {63, 77, 151, 308, 157};
- Players do not know the number of rounds in a given match;
- A total of 25000 repetitions across the various match lengths have been carried out.

The scores across all repetitions are shown in Figure 1.

Whilst the results mainly agree with the original reported results, some strategies show distinct outliers:

- k61r
- k90r
- k82r

Note that there are various approaches to calculating the ranks/results, Figure 2 shows the rankings calculated in various ways as well as the mean absolute error (MAE).

The top 15 strategies in the reproduced tournament are shown in Table 1.

In [1], a linear regression model is used to identify 5 strategies, the scores against which are good predictors of the overall performance. The reported  $R^2$  value is 0.979 (indicating 97% of variance accounted for by the model). The coefficients of this model are shown in Table 2.

Given the discrepancy in results shown in Figure 2 and Table 1 it is not surprising to see that this model no longer performs as well with  $R^2 = 0.7427$ .

Fitting a new model to the same 5 strategies gives the coefficients shown in Table 3 with  $R^2 = 0.9526$

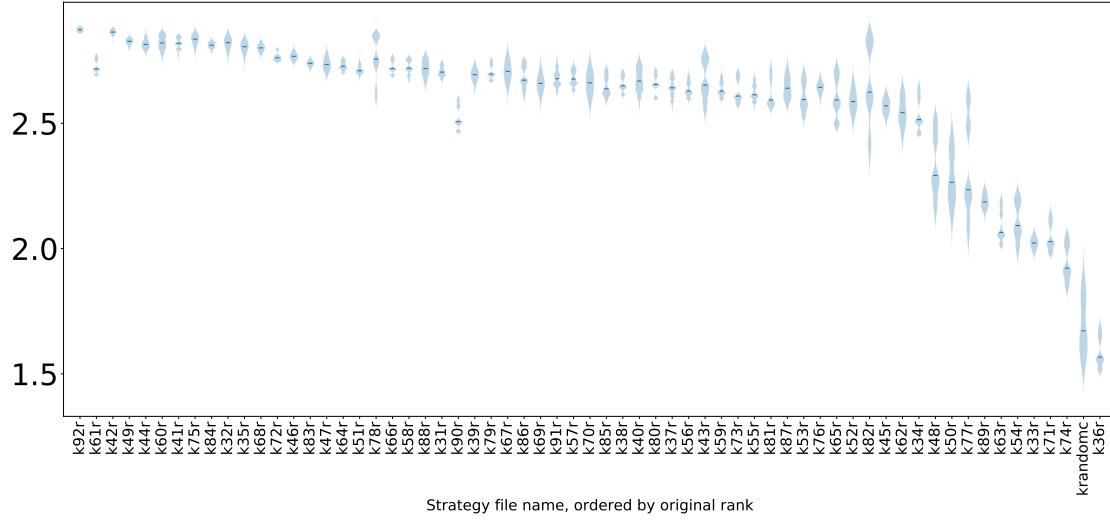


Figure 1: The scores per turn of each strategy.

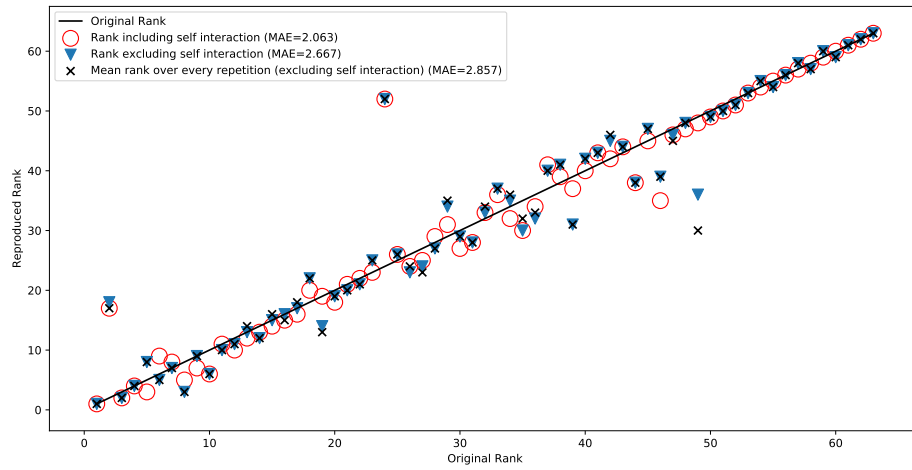


Figure 2: The rankings of each strategy.

	Author	Scores	Rank	Original Rank
k92r	Anatol Rapoport	2.8785	1	1
k61r	Danny C Champion	2.7189	17	2
k42r	Otto Borufsen	2.8342	2	3
k49r	Rob Cave	2.8261	4	4
k44r	William Adams	2.8262	3	5
k60r	Jim Graaskamp and Ken Katzen	2.8102	9	6
k41r	Herb Weiner	2.8106	8	7
k75r	Paul D Harrington	2.8241	5	8
k84r	T Nicolaus Tideman and Paula Chieruzz	2.8125	7	9
k32r	Charles Kluepfel	2.8166	6	10
k35r	Abraham Getzler	2.8018	11	11
k68r	Fransois Leyvraz	2.8090	10	12
k72r	Edward C White Jr	2.7763	12	13
k46r	Graham J Eatherley	2.7721	13	14
k83r	Paul E Black	2.7427	14	15

Table 1: Top 15 strategies in the reproduced tournament

Strategies	Coefficients
k69r	0.202
k91r	0.198
k40r	0.110
k76r	0.072
k67r	0.086
Intercept	0.795

Table 2: Linear model described in [1] with  $R^2 = 0.7427$

Strategies	Coefficients	$p$ -value	$F$ -value
k69r	0.086	1.2156e-08	43.2943
k91r	0.204	2.47554e-10	57.2183
k40r	0.192	2.26171e-12	76.5961
k76r	0.060	0.0194647	5.75981
k67r	0.119	2.13401e-06	27.4271
Intercept	0.857	NA	NA

Table 3: Linear model fitted to the same 5 strategies described in [1] with  $R^2 = 0.9526$

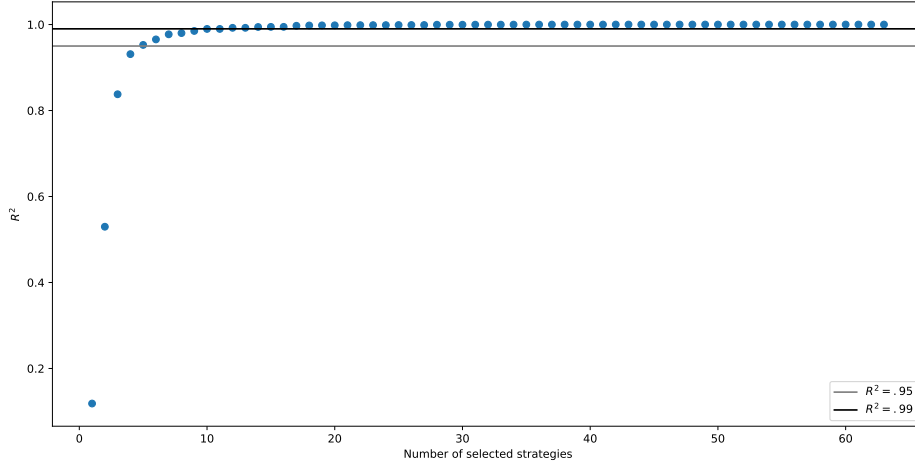


Figure 3:  $R^2$  for models obtained using recursive feature elimination.

Using recursive feature elimination It is possible to select the features (strategies) that give the best prediction for a given number of features. The  $R^2$  versus the number of features is shown in Figure 3.

Tables 4 and 5 show the coefficients for linear models fitted to 5 and 12 strategies with  $R^2 = 0.9526$  and  $R^2 = 0.9924$  respectively (12 strategies is the smallest number of strategies for which  $R^2 > 95$ ).

Strategies	Coefficients	$p$ -value	$F$ -value
k46r	0.252	0.00574885	8.19518
k53r	0.130	2.40311e-08	41.0388
k56r	0.154	1.40719e-09	50.78
k82r	0.189	1.92039e-06	27.7236
k85r	0.109	1.81982e-12	77.5683
Intercept	0.398	NA	NA

Table 4: Linear model best fitted to 5 strategies in the reproduced tournament with  $R^2 = 0.9526$

Strategies	Coefficients	$p$ -value	$F$ -value
k39r	0.060	0.00338223	9.30339
k41r	0.077	1.30729e-18	158.467
k45r	0.063	0.207909	1.62009
k46r	0.137	0.00574885	8.19518
k53r	0.064	2.40311e-08	41.0388
k56r	0.061	1.40719e-09	50.78
k63r	0.040	0.315611	1.02382
k66r	0.116	1.44122e-16	127.302
k71r	0.051	0.16658	1.95999
k74r	0.051	0.453942	0.56803
k82r	0.044	1.92039e-06	27.7236
k85r	0.130	1.81982e-12	77.5683
Intercept	0.347	NA	NA

Table 5: Linear model best fitted to 12 strategies in the reproduced tournament with  $R^2 = 0.9924$

The predictions of these models are shown in Figure 4.

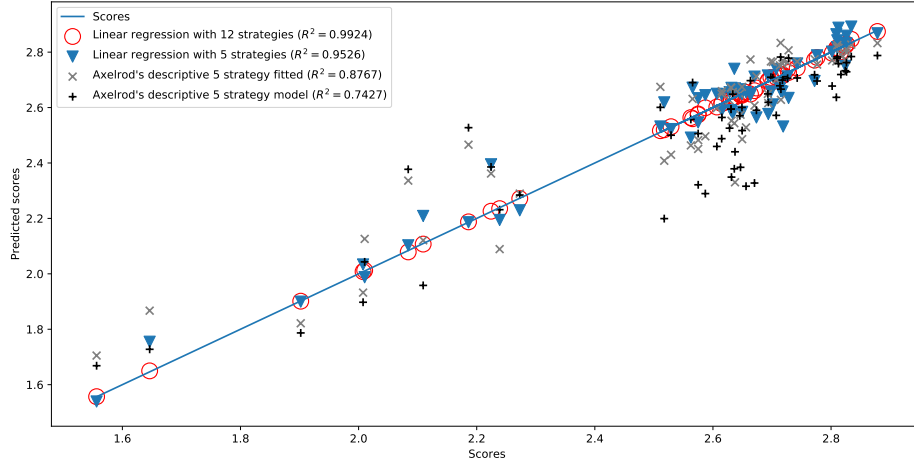


Figure 4: Predicting the performance of strategies using the 4 models discussed

It is clear that the effectiveness of the predictive models with 5 strategies is low for the cluster of highly performing strategies (with a score great than 2.5). To be able to obtain a good model even for high performing strategies 12 seem to provide a good predictive model.

The overall cooperation rate of the tournament is 0.750 . Figure 5 shows the cooperation rate versus the rank of the strategy.

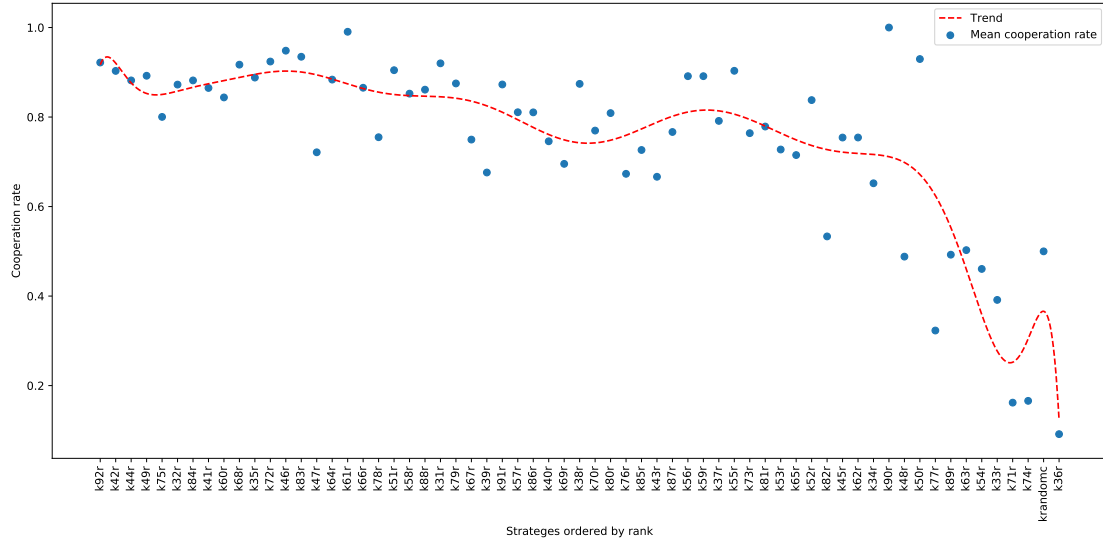


Figure 5: Cooperation rate versus rank

Figure 7 shows the pair wise cooperation rates. There is one strategy 'k42r' that defects against itself but cooperates against most other strategies. Also, it is clear that the low performing strategies are defecting against each other.

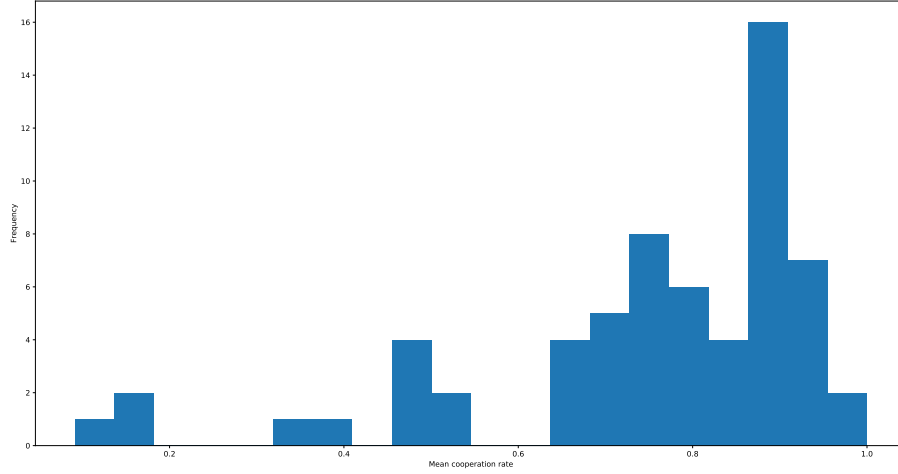


Figure 6: Distribution of cooperation rates

## 4 Revisiting the tournament

### 4.1 Running with an extra invitation

#### 4.1.1 Known strategies

The tournament is run with every strategy of the Axelrod library. Every tournament (corresponding to each strategy) was run for 2000 repetitions. Table 6 shows the top ranking strategies.

Name	Cooperation Rate against opponents	Cooperation Rate from opponents	Library Rank	Rank
Gradual	0.906	0.911	20	2
PSO Gambler 2.2.2 Noise 05	0.866	0.881	19	3
EvolvedLookerUp2.2.2	0.839	0.842	1	14
Evolved HMM 5	0.831	0.836	2	16
Evolved FSM 16 Noise 05	0.819	0.832	5	22
Winner12	0.754	0.791	13	23
PSO Gambler Mem1	0.816	0.824	9	24
Evolved FSM 16	0.812	0.817	4	25
PSO Gambler 2.2.2	0.794	0.808	3	26
Evolved ANN 5	0.828	0.828	6	26
Evolved ANN	0.820	0.825	7	26
Evolved ANN 5 Noise 05	0.849	0.838	11	26
Evolved FSM 4	0.886	0.840	10	27
PSO Gambler 1.1.1	0.769	0.791	8	31
MEM2	0.728	0.776	27	34
Fool Me Once	0.720	0.753	14	49

Table 6: Performance of extra strategy in Axelrod’s original tournament

Figure 8 shows the rank of the extra strategy against it’s rank in the library tournament.

#### 4.1.2 Trained strategies

### 4.2 Further tournaments

#### 4.2.1 Running with extortion

Since the work of [5] a lot of interest has been shown to Zero Determinant strategies. In [6] a small tournament is presented pitting these against each other. Table 7 shows the rankings of the top 15 strategies when including all the Zero

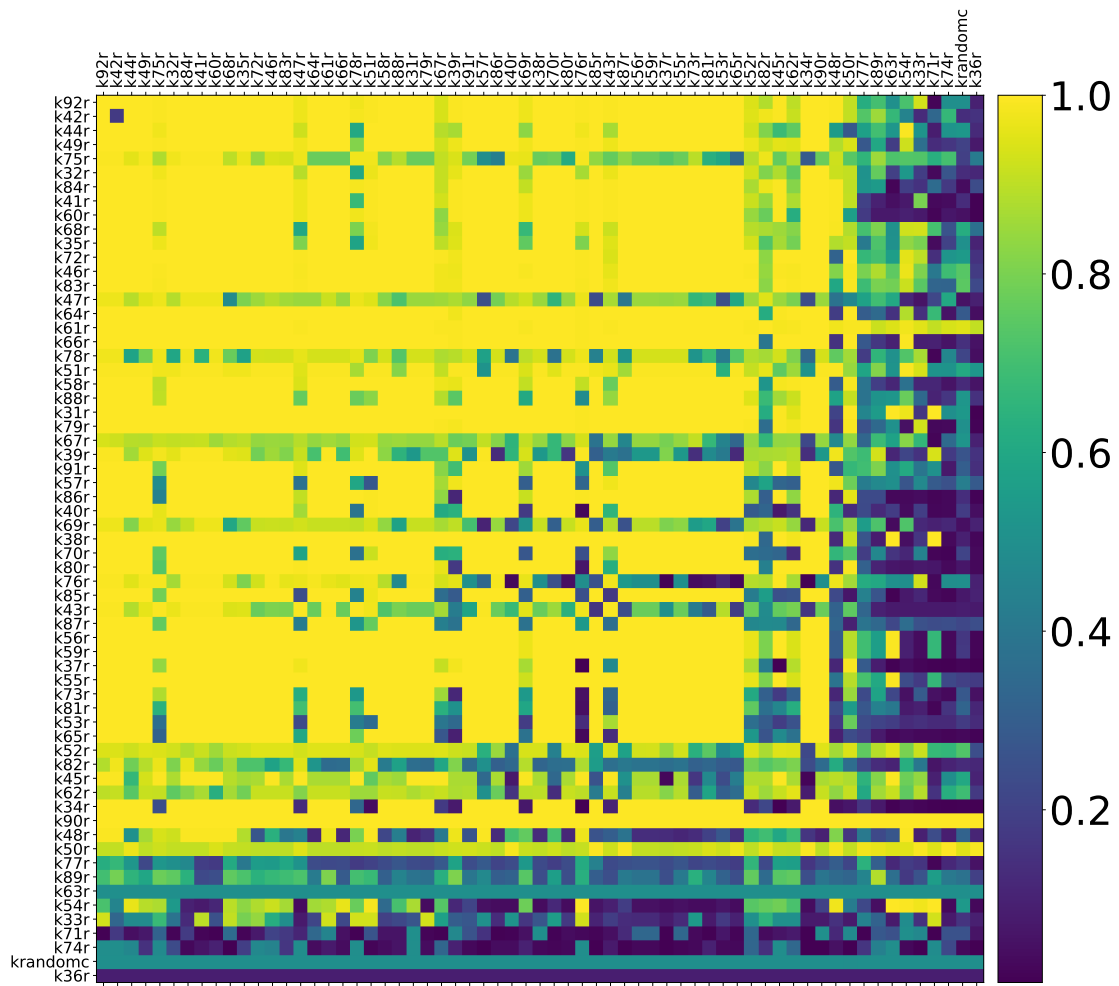


Figure 7: Cooperation rates between each pair of players (ordered by rank)

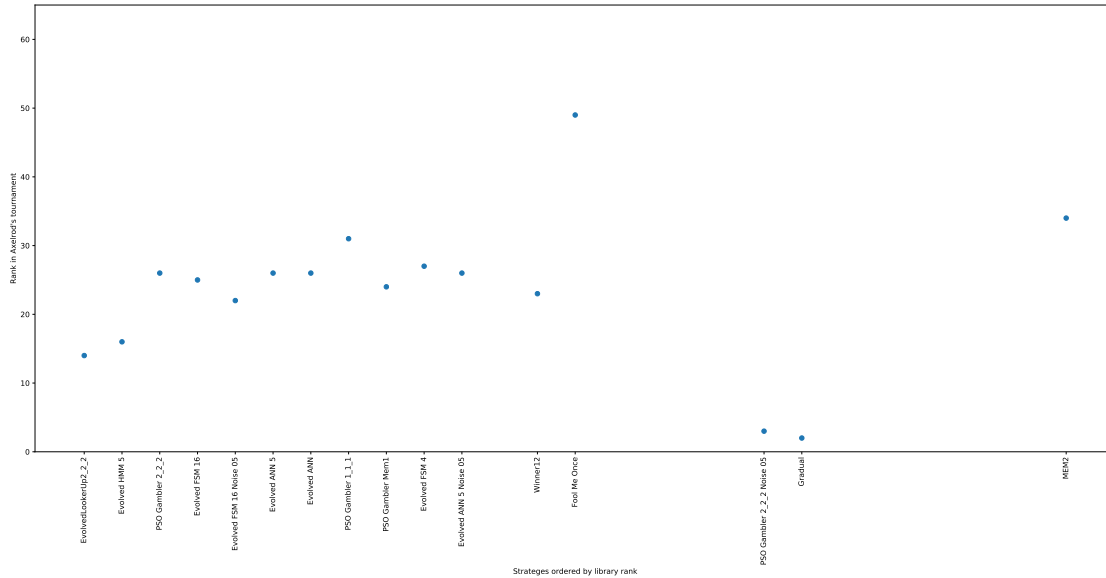


Figure 8: Ranks of extra strategy

Determinant strategies from [6] over 25000 repetitions.

	Original Author	Scores	Rank	Original Rank	Reproduced Rank
ZD-GTFT-2	NA	2.8365	1	NA	NA
GTFT	NA	2.8212	2	NA	NA
k92r	Anatol Rapoport	2.8146	3	1	1
k42r	Otto Borufsen	2.8045	4	3	2
k75r	Paul D Harrington	2.7936	5	8	5
k49r	Rob Cave	2.7893	6	4	4
k44r	William Adams	2.7840	7	5	3
k68r	Fransois Leyvraz	2.7717	8	12	10
k41r	Herb Weiner	2.7647	9	7	8
k46r	Graham J Eatherley	2.7612	10	14	13
k32r	Charles Kluepfel	2.7611	11	10	6
k72r	Edward C White Jr	2.7591	12	13	12
k35r	Abraham Getzler	2.7507	13	11	11
k84r	T Nicolaus Tideman and Paula Chieruzz	2.7490	14	9	7
k60r	Jim Graaskamp and Ken Katzen	2.7382	15	6	9

Table 7: Top 15 strategies in the tournament composed of the original strategies and the Zero Determinant strategies from [6]

The overall cooperation rate of this tournament is 0.737 and the various cooperation rates are shown in Figure 9 shows the cooperation rates of each strategy (ordered by rank).

Figure 11 shows the pair wise cooperation rates.

#### 4.2.2 Running a large tournament

Table 8 shows the rankings of the top 20 strategies when including all strategies over 4750 repetitions. The cooperation rate for the tournament that pits all the library strategies against each other (without the Fortran ones) is 0.601.

The overall cooperation rate of this tournament is 0.635 and the various cooperation rates are shown in Figure 12 shows the cooperation rates of each strategy (ordered by rank).

Figure 14 shows the pair wise cooperation rates.



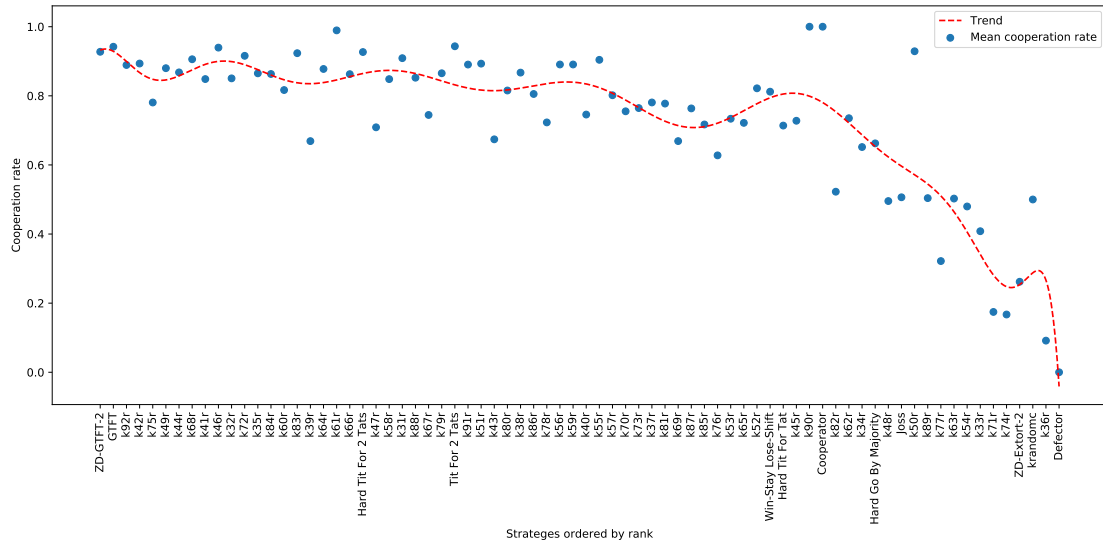


Figure 9: Cooperation rate versus rank for the Stewart and Poltkin tournament

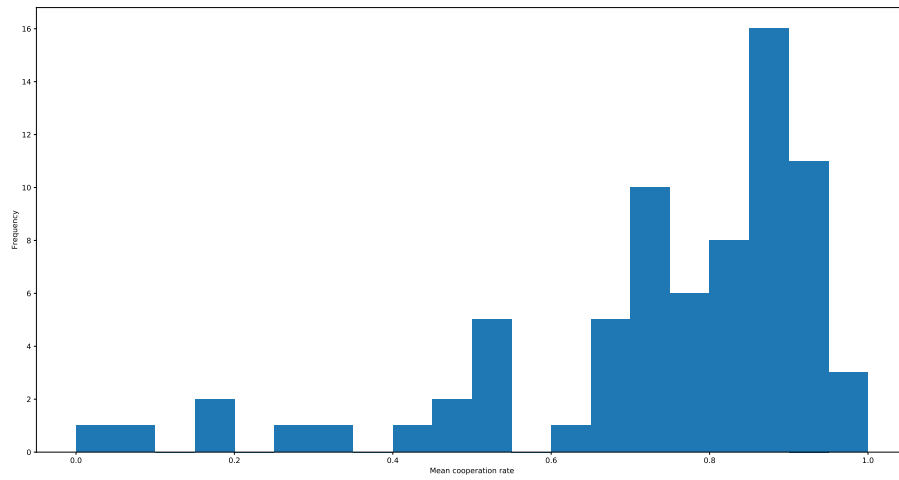


Figure 10: Distribution of cooperation rates for the Stewart and Plotkin tournament

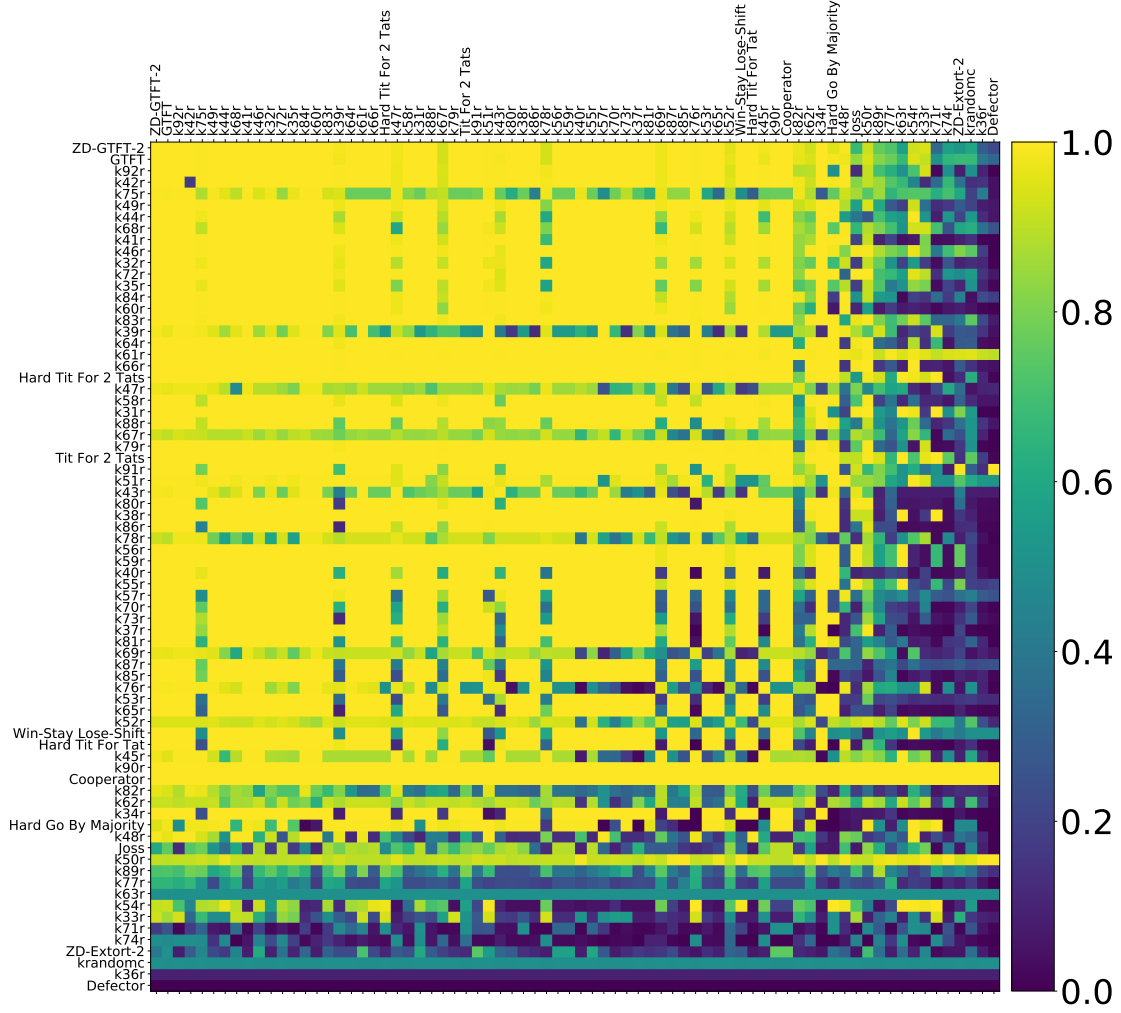


Figure 11: Cooperation rates between each pair of players (ordered by rank) for the Stewart and Plotkin tournament

	Original Author	Scores	Rank	Library Rank	Original Rank	Reproduced Rank
EvolvedLookerUp2.2.2	NA	2.8397	1	1	NA	NA
Evolved HMM 5	NA	2.8278	2	2	NA	NA
Evolved FSM 16 Noise 05	NA	2.8117	3	5	NA	NA
Evolved FSM 16	NA	2.8073	4	4	NA	NA
PSO Gambler 2.2.2	NA	2.8057	5	3	NA	NA
Evolved ANN	NA	2.7951	6	7	NA	NA
Evolved ANN 5	NA	2.7939	7	6	NA	NA
Omega TFT	NA	2.7911	8	16	NA	NA
PSO Gambler Mem1	NA	2.7877	9	9	NA	NA
Evolved FSM 4	NA	2.7841	10	10	NA	NA
PSO Gambler 1.1.1	NA	2.7833	11	8	NA	NA
PSO Gambler 2.2.2 Noise 05	NA	2.7760	12	19	NA	NA
Gradual	NA	2.7695	13	20	NA	NA
Evolved ANN 5 Noise 05	NA	2.7669	14	11	NA	NA
DBS	NA	2.7659	15	12	NA	NA
Winner12	NA	2.7628	16	13	NA	NA
k85r	Robert B Falk and James M Langsted	2.7471	17	NA	33	36
Spiteful Tit For Tat	NA	2.7375	18	29	NA	NA
k80r	Robyn M Dawes and Mark Batell	2.7370	19	NA	36	34
k87r	E E H Schurmann	2.7315	20	NA	44	38

Table 8: Top 20 strategies in the tournament when using all available strategies

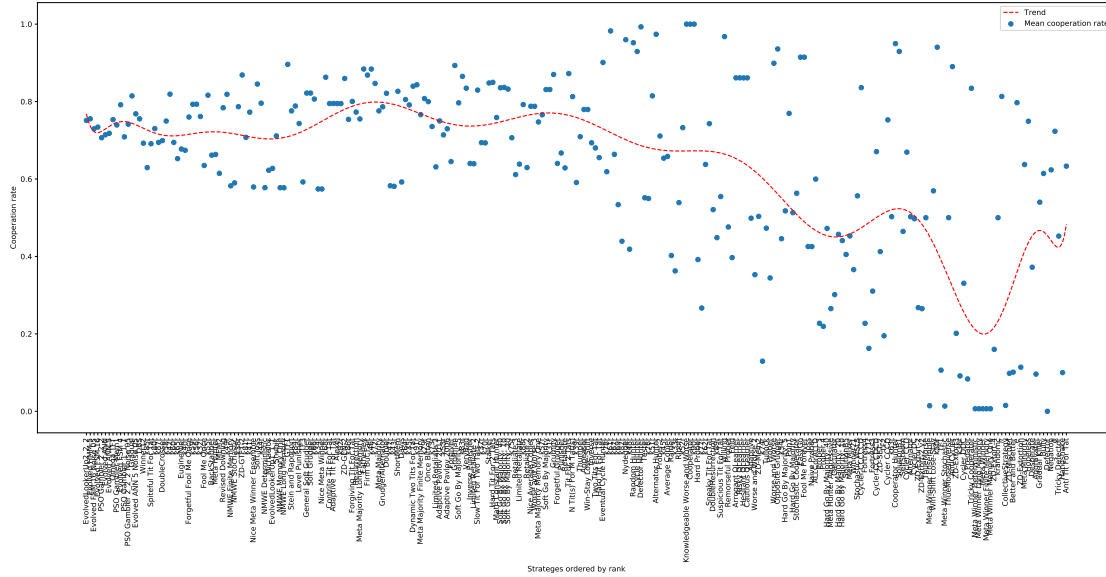


Figure 12: Cooperation rate versus rank for tournament with all available strategies

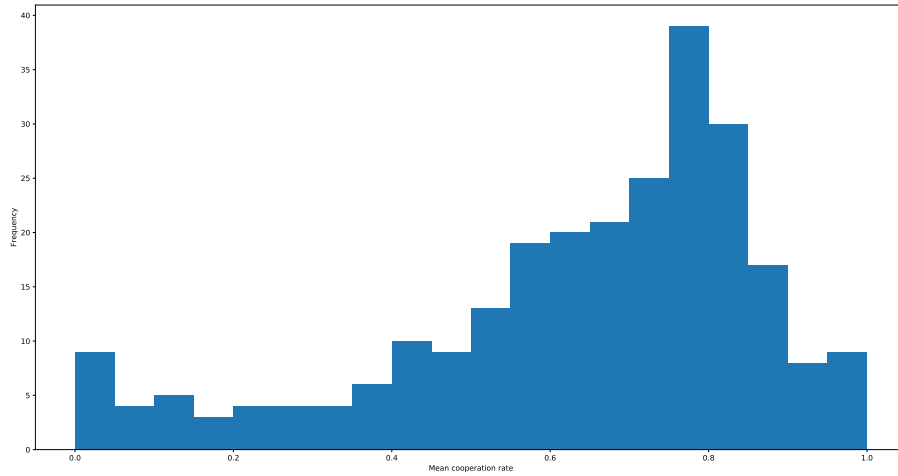


Figure 13: Distribution of cooperation rates for the full tournament.

## 5 Conclusion

## Acknowledgements

## References

- [1] R. Axelrod. More Effective Choice in the Prisoner’s Dilemma. *Journal of Conflict Resolution*, 24(3):379–403, 1980.
- [2] R. Axelrod. Complexity of cooperation web site. <http://www-personal.umich.edu/~axe/research/Software/CC/CC2.html>, 1996.
- [3] Owen Campbell and Vince Knight. Axelrod-python/tourexec: v0.3.0. <https://doi.org/10.5281/zenodo.834917>, July 2017.
- [4] Owen Campbell, Vince Knight, and Marc Harper. Axelrod-Python/axelrod-fortran: v0.3.1 (2017-08-04). <https://doi.org/10.5281/zenodo.838980>, August 2017.
- [5] William H Press and Freeman J Dyson. Iterated Prisoner’s Dilemma contains strategies that dominate any evolutionary opponent. *Proceedings of the National Academy of Sciences of the United States of America*, 109(26):10409–13, 2012.
- [6] a. J. Stewart and J. B. Plotkin. Extortion and cooperation in the Prisoner’s Dilemma. *Proceedings of the National Academy of Sciences*, 109(26):10134–10135, 2012.

## A List of original players

- |   |   |
|---|---|
| 1. k31r - Original rank: 23. Authored by Gail Grisell     | 5. k35r - Original rank: 11. Authored by Abraham Getzler  |
| 2. k32r - Original rank: 10. Authored by Charles Kluepfel | 6. k36r - Original rank: 63. Authored by Roger Hotz       |
| 3. k33r - Original rank: 59. Authored by Harold Rabbie    | 7. k37r - Original rank: 37. Authored by George Lefevre   |
| 4. k34r - Original rank: 52. Authored by James W Friedman | 8. k38r - Original rank: 34. Authored by Nelson Weiderman |
|   | 9. k39r - Original rank: 25. Authored by Tom Almy         |

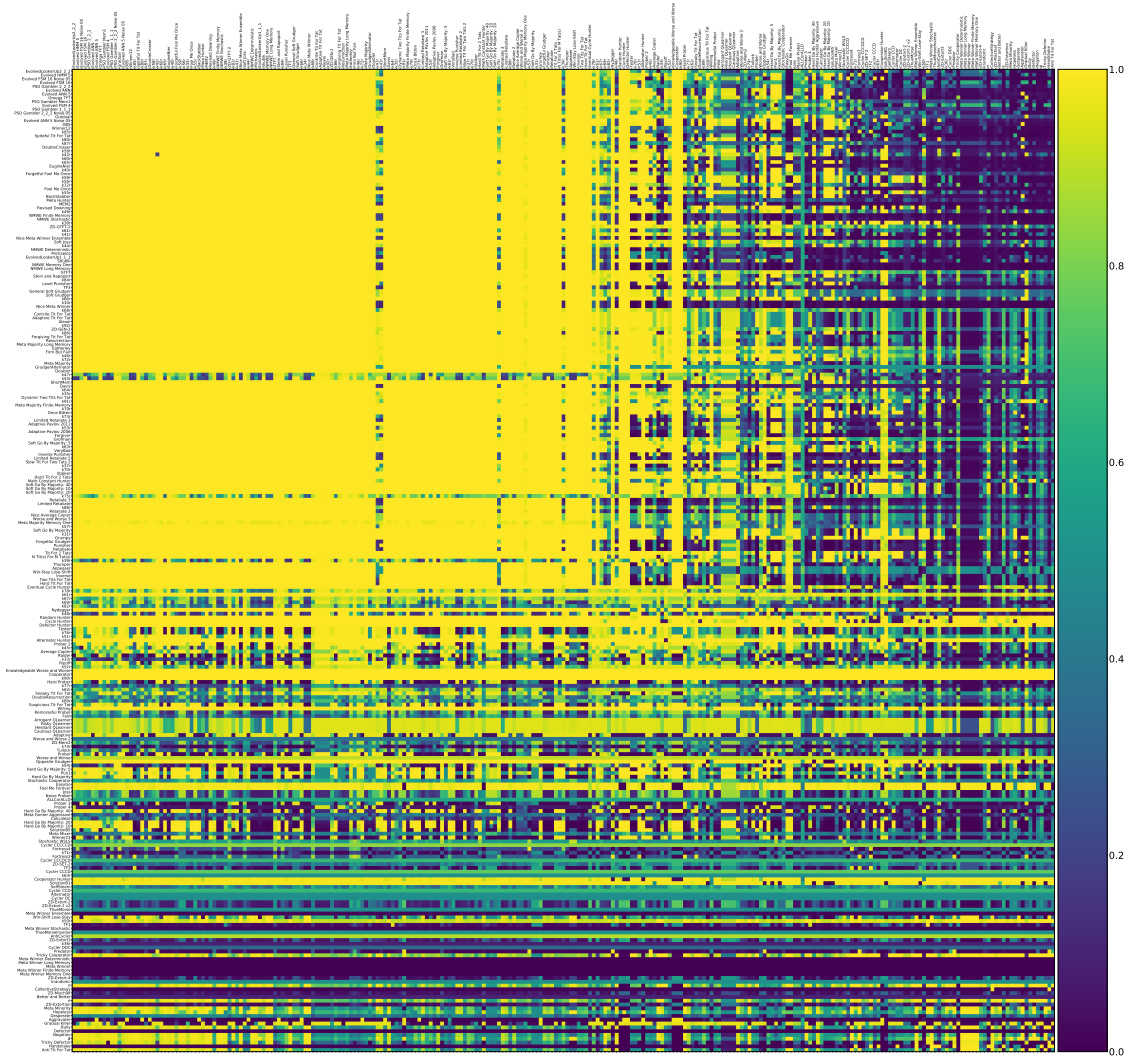


Figure 14: Cooperation rates between each pair of players (ordered by rank) for tournament with all available strategies

10. k40r - Original rank: 35. Authored by Robert Adams
11. k41r - Original rank: 7. Authored by Herb Weiner
12. k42r - Original rank: 3. Authored by Otto Borufsen
13. k43r - Original rank: 39. Authored by R D Anderson
14. k44r - Original rank: 5. Authored by William Adams
15. k45r - Original rank: 50. Authored by Michael F McGurrian
16. k46r - Original rank: 14. Authored by Graham J Eatherley
17. k47r - Original rank: 16. Authored by Richard Hufford
18. k48r - Original rank: 53. Authored by George Hufford
19. k49r - Original rank: 4. Authored by Rob Cave
20. k50r - Original rank: 54. Authored by Rik Smoody
21. k51r - Original rank: 18. Authored by John William Colbert
22. k52r - Original rank: 48. Authored by David A Smith
23. k53r - Original rank: 45. Authored by Henry Nussbacher
24. k54r - Original rank: 58. Authored by William H Robertson
25. k55r - Original rank: 42. Authored by Steve Newman
26. k56r - Original rank: 38. Authored by Stanley F Quayle
27. k57r - Original rank: 31. Authored by Rudy Nydegger
28. k58r - Original rank: 21. Authored by Glen Rowsam
29. k59r - Original rank: 40. Authored by Leslie Downing
30. k60r - Original rank: 6. Authored by Jim Graaskamp and Ken Katzen
31. k61r - Original rank: 2. Authored by Danny C Champion
32. k62r - Original rank: 51. Authored by Howard R Hollander
33. k63r - Original rank: 57. Authored by George Duisman
34. k64r - Original rank: 17. Authored by Brian Yamachi
35. k65r - Original rank: 47. Authored by Mark F Batell
36. k66r - Original rank: 20. Authored by Ray Mikkelsen
37. k67r - Original rank: 27. Authored by Craig Feathers
38. k68r - Original rank: 12. Authored by Francois Leyvraz
39. k69r - Original rank: 29. Authored by Johann Joss
40. k70r - Original rank: 32. Authored by Robert Pebly
41. k71r - Original rank: 60. Authored by James E Hall
42. k72r - Original rank: 13. Authored by Edward C White Jr
43. k73r - Original rank: 41. Authored by George Zimmerman
44. k74r - Original rank: 61. Authored by Edward Friedland
45. k75r - Original rank: 8. Authored by Paul D Harrington
46. k76r - Original rank: 46. Authored by David Gladstein
47. k77r - Original rank: 55. Authored by Scott Feld
48. k78r - Original rank: 19. Authored by Fred Mauk
49. k79r - Original rank: 26. Authored by Dennis Ambuehl and Kevin Hickey
50. k80r - Original rank: 36. Authored by Robyn M Dawes and Mark Batell
51. k81r - Original rank: 43. Authored by Martyn Jones
52. k82r - Original rank: 49. Authored by Robert A Leyland
53. k83r - Original rank: 15. Authored by Paul E Black
54. k84r - Original rank: 9. Authored by T Nicolaus Tide-man and Paula Chieruzz
55. k85r - Original rank: 33. Authored by Robert B Falk and James M Langsted
56. k86r - Original rank: 28. Authored by Bernard Grofman
57. k87r - Original rank: 44. Authored by E E H Schurmann
58. k88r - Original rank: 22. Authored by Scott Appold
59. k89r - Original rank: 56. Authored by Gene Snodgrass
60. k90r - Original rank: 24. Authored by John Maynard Smith
61. k91r - Original rank: 30. Authored by Jonathan Pinkley
62. k92r - Original rank: 1. Authored by Anatol Rapoport
63. krandmc - Original rank: 62. Authored by None