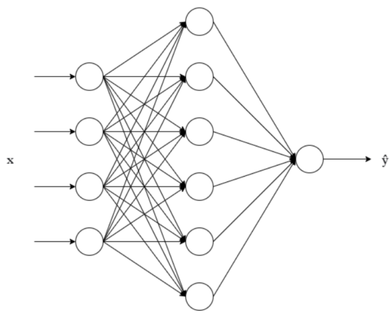
1 Logistic Regression

This is the first post in a series that will lead to an implementation of a deep artificial neural network from scratch. This post will focus on the mathematical foundations behind the logistic regression. Throughout we will work with a training set $\mathbf{X}_{train} \in \mathbb{R}^{n \times m}$ with n training samples with m dimensions. We will be trying to solve a binary decision problem $\mathbf{y}_i \in \{0, 1\}$.



1.1 Logistic Regression as a Single-Layer Neural Network

Something tt immediately clear to me when I started machine learning was how a logistic regression could be seen as a single-layer- or a single-neuron neural network. In this context, a single-layer neural network has an input layer for the training samples, one set of weights $\theta \in \mathbb{R}^m$, an activation function (the sigmoid in this case) and an output (layer).

The image below shows a typical representation of a fully-connected neural network with one hidden layer. The next chapter will go into more detail about these, for now you can just accept this representation as a given.

$$\underset{\theta}{\operatorname{argmax}} \prod_{i=1}^{N} p(y_i | x_i, \theta) \tag{1}$$

$$= \prod_{i=1}^{N} g(\theta^\top \mathbf{x}_i)^{y^i} (1 - g(\theta^\top \mathbf{x}_i))^{1-y^i} \tag{2}$$